Dynamical Clockwork Axions

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USyd Webinar, 7th September 2017

Based on arXiv: 1706.04529, accepted by JHEP

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Outline

What is Clockwork?

- A mechanism for generating **exponentially suppressed** couplings from a theory with only $\mathcal{O}(1)$ parameters

- Equivalently, generate an interaction scale much larger than the dynamical scale of the theory

Key features

1. $U(1)^{N+1}$ symmetry, either global or local
2. Breaking $U(1)^{N+1} \to U(1)_0$, either explicitly or spontaneously, by **asymmetric nearest neighbour** couplings
3. SM couples only to the last Clockwork site
4. Dynamics of remnant $U(1)_0$ symmetry gives exponential suppression

Scalar Clockwork[1]

- $N+1$ complex scalars, ϕ_j, **global $\mathbf{U(1)^{N+1}}$ symmetry**
- Spontaneous symmetry breaking at high scale, f
- Asymmetric **explicit breaking** to $U(1)_0$ by nearest neighbour coupling at much lower scale,

$$V = \sum_{j=0}^{N} \left(-m^2 |\phi_j|^2 + \frac{\lambda}{4} |\phi_j|^4 \right) + \sum_{j=0}^{N-1} \frac{\epsilon}{f^{q-3}} \phi_j^\dagger \phi_{j+1}^q + h.c. \quad (1)$$

[1] Choi & Im 1511.00132; Kaplan & Rattazzi 1511.01827; Giudice & McCullough 1610.07962

Scalar Clockwork

- High-energy Lagrangian,

$$V = \sum_{j=0}^{N} \left(-m^2 |\phi_j|^2 + \frac{\lambda}{4} |\phi_j|^4 \right) + \sum_{j=0}^{N-1} \frac{\epsilon}{f^{q-3}} \phi_j^\dagger \phi_{j+1}^q + h.c. \quad (2)$$

- Parametrise in terms of NGBs, $\phi_j \to U_j \equiv f e^{i\pi_j/\sqrt{2}f}$, then

$$V = -2\epsilon f^4 \sum_{j=0}^{N-1} \cos \left(\frac{\pi_j - q\pi_{j+1}}{\sqrt{2}f} \right)^2 \quad (3)$$

- Orthogonal rotation to mass basis $\pi_j = O_{jk} a_k$, with $m_{a_0} = 0$,

$$\begin{aligned} O_{j0} - qO_{j+1,0} &= 0 \\ \Rightarrow O_{j0} &= \frac{\mathcal{N}}{q^j}, \ \mathcal{N} \approx 1 \end{aligned} \quad (4)$$

- a_0 component of π_j decreases **exponentially** with j

Scalar Clockwork

- Final step: couple Clockwork to the SM at the **final site**, e.g. to a dimension-4 operator of SM fields:

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{SM} + \mathcal{L}_{CW} + \frac{\pi_N \mathcal{O}_{SM}^{d=4}}{32\pi^2 f} + h.c. \\ &\approx \mathcal{L}_{SM} + \mathcal{L}_{CW} + \frac{a_0 \mathcal{O}_{SM}^{d=4}}{32\pi^2 q^N f} + \sum_{j=1}^{N} C_j \frac{a_j \mathcal{O}_{SM}^{d=4}}{32\pi^2 f} + h.c., \end{aligned} \tag{5}$$

  where $C_j \sim \mathcal{O}(1)$.

- Coupling of NGB to SM suppressed by q^N

- New effective scale is $f_{eff} \approx q^N f \gg f$

The Strong CP problem

- The SM Lagrangian includes the CP-violating term,

$$\mathcal{L}_\theta = \frac{\theta_{QCD}}{32\pi^2} G\tilde{G} \tag{6}$$

- Measured coefficient of this term is

$$\bar{\theta} = \theta_{QCD} - arg(\det[Y_d Y_u]), \tag{7}$$

and experimentally (neutron EDM) we know $|\bar{\theta}| < 10^{-10}$

- QCD term and EW term should be unrelated!
- Severe fine-tuning required: this is the **Strong CP problem**

Axion solution

- Make $\overline{\theta}$ a **dynamical field**
- Impose a $U(1)_{PQ}$ symmetry, which must be
  a. Axial
  b. Spontaneously broken at scale $f_a \gg \Lambda_{QCD}$
  c. Explicitly broken by the **QCD anomaly**, i.e. $aG\tilde{G}$ term
- The $U(1)_{PQ}$ pNGB is the axion, defined by $\overline{\theta} = \frac{a}{f_a}$
- Axion potential from the QCD anomaly is

$$V(a) \sim -\Lambda_{QCD}^4 \cos\left(\frac{a}{f_a}\right), \tag{8}$$

and this takes $\overline{\theta} \to \mathbf{0}$ dynamically

Axion phenomenology

- Axions are a CDM candidate when produced via the **misalignment mechanism**, with relic abundance given by

$$\Omega_a h^2 \approx 0.07 \alpha_i^2 \left(\frac{f_a}{10^{12}\ \mathrm{GeV}} \right)^{7/6}, \tag{9}$$

  where $\alpha_i \sim \mathcal{O}(1)$ is the initial misalignment angle

- Upper bound on f_a from overproduction of DM, lower bound from ν burst duration in SN 1987A:

$$4 \times 10^8\ \mathrm{GeV} \lesssim f_a \lesssim 10^{12}\ \mathrm{GeV} \tag{10}$$

- $m_a \sim 10 \left(\frac{10^{12}\ \mathrm{GeV}}{f_a} \right) \mu\mathrm{eV}$ from QCD anomaly

KSVZ axion[2]

- Simple construction of axion model: $U(1)_{PQ}$ must be
  a. axial $\Rightarrow$ introduce vector-like fermion, ψ
  b. anomalous w.r.t. to QCD $\Rightarrow$ give ψ QCD charge
  c. spontaneously broken $\Rightarrow$ add a complex scalar, σ, with a VEV
- Giving ψ_L, ψ_R, σ appropriate charges, we can write

$$\mathcal{L} \supset y\overline{\psi}_L \sigma \psi_R + h.c., \tag{11}$$

and the VEV of $\sigma \sim \frac{f_a}{\sqrt{2}} e^{ia/f_a}$ breaks $U(1)_{PQ}$ spontaneously

- Below the scale f_a, axion couples to QCD,

$$\mathcal{L} \supset \frac{aG\tilde{G}}{32\pi^2 f_a}, \tag{12}$$

which generates the cosine potential that takes $\overline{\theta} \to 0$

[2] Kim, J.E., Phys. Rev. Lett. 43 (1979) 103; Shifman, M.A., Vainstein, V.I., and Zakharov, V.I., Nucl. Phys. B 166 (1980) 4933

The Clockwork axion

- Recall Scalar Clockwork model
- If **only** π_N is coupled to QCD anomaly, we have

$$\begin{aligned} \mathcal{L} &= \frac{\pi_N}{32\pi^2 f} G\tilde{G} \\ &\approx \frac{a_0}{32\pi^2 q^N f} G\tilde{G} + \ldots, \end{aligned} \tag{13}$$

  i.e. an effective axion decay constant $f_a = q^N f \gg f$.

- So it's possible to set f at $\mathcal{O}(\text{TeV})$ and connect EW/LHC-scale physics to axion physics
- Shift in $\overline{\theta}$ from $U(1)_{PQ}$-breaking gravity terms depends on the size of $\frac{f}{M_{Pl}}$, so lower dynamical scale **better protects** $\overline{\theta}$

But . . .

- This is a nice example of the Clockwork in action, but we may wonder:
    - Where do all these scalars come from?
    - What is the origin of the scale f? Is it stable against quantum corrections?
    - What is the effect of gravity on the $U(1)^{N+1}$ symmetry?
    - What is the size of q? (not predicted)
    - How can we distinguish this axion model?
- In short, many **open theory and phenomenology questions**

Composite axion[3]

- Motivation: avoid elementary scalars, generate f_a dynamically
- Introduce strongly-coupled gauge $SU(N_c)_a$, condenses at Λ_a
- Vector-like fermions $Q_{L,R} \sim (\mathbf{N_c}, \mathbf{3})$ and $\psi_{L,R} \sim (\mathbf{N_c}, \mathbf{1})$ under $SU(N_c)_a \times SU(3)_{QCD}$
- There is a $U(1)_A^{(Q)} \times U(1)_A^{(\psi)}$ symmetry, which is broken:
  - spontaneously by fermion bilinears, $\langle \overline{Q}_{Li} Q_{Ri} \rangle = \langle \overline{\psi}_L \psi_R \rangle \sim \Lambda_a^3$
  - explicitly by $SU(N_c)_a$ and $SU(3)_{QCD}$ **anomalies**

[3] Kim, J.E., Phys. Rev. D31 (1985) 1733.

Composite axion

- $U(1)_{PQ}$ is linear combination broken **only** by QCD anomaly
- Q, ψ have PQ charges $+1, -3$ respectively
  - *recall* $Q_{L,R} \sim (\mathbf{N_c}, \mathbf{3})$, $\psi_{L,R} \sim (\mathbf{N_c}, \mathbf{1})$
- Light pNGB is the composite axion, $a \sim \frac{\overline{Q}\gamma^5 Q - 3\overline{\psi}\gamma^5\psi}{\sqrt{10}}$
- Same coupling to QCD as before, with $f_a \sim \Lambda_a$, and $\overline{\theta} \to 0$

Extending the composite axion model with Clockwork

- For the composite axion, we introduced an additional, asymptotically-free $SU(N_c)$ and 2 vector-like fermions
- Let's **Clockwork** this, and introduce
  - N copies of asymptotically-free $SU(N_c)$
  - $N + 1$ vector-like fermions

Clockwork realisation in strongly-coupled gauge theories

- In Composite axion model, $\psi \sim (\mathbf{N_c}, \mathbf{1})$, $Q \sim (\mathbf{N_c}, \mathbf{3})$

Our Clockwork extension is:

	$SU(N_c)_1$	$SU(N_c)_2$	$SU(N_c)_3$	$\dots$	$SU(N_c)_N$	$SU(3)_{QCD}$
ψ_0	$\mathbf{N_c}$	$\mathbf{1}$	$\mathbf{1}$	$\dots$	$\mathbf{1}$	$\mathbf{1}$
ψ_1	$\mathbf{R}$	$\mathbf{N_c}$	$\mathbf{1}$	$\dots$	$\mathbf{1}$	$\mathbf{1}$
ψ_2	$\mathbf{1}$	$\mathbf{R}$	$\mathbf{N_c}$	$\dots$	$\mathbf{1}$	$\mathbf{1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
ψ_{N-1}	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\dots$	$\mathbf{N_c}$	$\mathbf{1}$
ψ_N	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\dots$	$\mathbf{R}$	$\mathbf{3}$

- Notably, this arrangement mimics the Clockwork's **asymmetric nearest-neighbour** couplings

The Clockwork realisation

- For each ψ_j there is a $U(1)_A^{(\psi_j)}$ symmetry, overall there is a $U(1)_A^{N+1}$ symmetry
- Since the $SU(N_c)_j$ confine, the fermion bilinear condensates,

$$\langle \overline{\psi}_i \psi_i \rangle \approx \Lambda^3, \tag{14}$$

spontaneously break the $U(1)_A^{N+1}$

- Assume Λ are all the same, in fact RG running drives them together
- The NGBs, π_j, associated with SSB of $U(1)_A^{(\psi_j)}$ are given by

$$\overline{\psi}_j \psi_j \sim \Lambda^3 e^{i\pi_j} \tag{15}$$

Preserved $U(1)_{PQ}$ symmetry

- $U(1)_A^{(\psi_j)}$ **explicitly broken** by $SU(N_c)_j$ and $SU(N_c)_{j+1}$ anomalies
- However, there is a **preserved**, anomaly-free $U(1)_A$, with current

$$j_A^\mu = \sum_{j=0}^{N} q_j \times j_j^\mu; \qquad j_j^\mu = \frac{1}{2}\overline{\psi}_j \gamma_5 \gamma^\mu \psi_j. \tag{16}$$

- As in scalar example, there is one NGB and N massive pNGBs
- Identify exact NGB as the axion and exact $U(1)_A$ as $U(1)_{PQ}$

Anomaly calculations

- The $U(1)_{PQ}$ charges are $q_j \approx q^{-j}$, with the Clockwork factor

$$q = -\frac{2T(\mathbf{R})N_c}{d(\mathbf{R})}, \tag{17}$$

where $T(\mathbf{R})$, $d(\mathbf{R})$ are the Dynkin index and dimension of $\mathbf{R}$

- The axion component in the pNGBs is

$$\pi_j = q_j \frac{a}{f}, \tag{18}$$

where $f \sim \Lambda/4\pi$

Group Theory and Clockwork factor

- Choices of N_c, $\mathbf{R}$ are restricted by: a) $SU(N_c)$ must be asymptotically-free; b) need $|q| > 1$

- To ensure asymptotic freedom, require

$$11N_c - 4N_c T(\mathbf{R}) - 2d(\mathbf{R}) > 0 \tag{19}$$

- Should choose $\mathbf{R} \equiv \mathbf{A_2}$ and $N_c = 4, 5$, giving

$$q = -\frac{4}{3}\ (N_c = 4); \qquad q = -\frac{3}{2}\ (N_c = 5) \tag{20}$$

Realising the QCD axion

	$SU(N_c)_1$	$SU(N_c)_2$	$SU(N_c)_3$	$\dots$	$SU(N_c)_N$	$SU(3)_{QCD}$
ψ_0	$\mathbf{N_c}$	$\mathbf{1}$	$\mathbf{1}$	$\dots$	$\mathbf{1}$	$\mathbf{1}$
ψ_1	$\mathbf{R}$	$\mathbf{N_c}$	$\mathbf{1}$	$\dots$	$\mathbf{1}$	$\mathbf{1}$
ψ_2	$\mathbf{1}$	$\mathbf{R}$	$\mathbf{N_c}$	$\dots$	$\mathbf{1}$	$\mathbf{1}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
ψ_N	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\dots$	$\mathbf{R}$	$\mathbf{3}$

- $U(1)_{PQ}$ is broken by the QCD anomaly, and the Clockwork axion arises as desired:

$$\mathcal{L} \supset \frac{d(\mathbf{R})\pi_N}{32\pi^2} G\tilde{G} \approx \frac{d(\mathbf{R})a}{32\pi^2 q^N f} G\tilde{G} + \dots, \tag{21}$$

i.e. effective axion decay constant is $f_a \approx \frac{q^N}{d(\mathbf{R})} f \gg f$

Alternative model: contact connection

Can we have a larger Clockwork factor? Yes!

	$SU(N_c)_1$
Q_1	$\mathbf{R}_Q$
ψ_1	$\mathbf{R}_\psi$

	$SU(N_c)_2$
Q_2	$\mathbf{R}_Q$
ψ_2	$\mathbf{R}_\psi$

	$SU(N_c)_3$
Q_3	$\mathbf{R}_Q$
ψ_3	$\mathbf{R}_\psi$

$\cdots$

	$SU(N_c)_N$
Q_N	$\mathbf{R}_Q$
ψ_N	$\mathbf{R}_\psi$

- Contact interactions (enforced by a $\mathbb{Z}_m^{N-1}$ symmetry)

$$\mathcal{L}_{contact} = \frac{1}{M_*^2} \sum_{j=1}^{N-1} \left(\overline{\psi}_{Lj}\psi_{Rj}\right)^\dagger \left(\overline{Q}_{Lj+1} Q_{Rj+1}\right) + h.c. \qquad (22)$$

Contact-connection model

- $U(1)_A^{2N}$ broken spontaneously by fermion condensates,

$$\overline{Q}_j Q_j \sim \Lambda^3 e^{i\pi_j}; \qquad \overline{\psi}_j \psi_j \sim \Lambda^3 e^{i\xi_j} \tag{23}$$

- Contact interactions and anomalies explicitly breaks $U(1)_A^{2N} \to U(1)_A \equiv U(1)_{PQ}$, so there is one exact NGB
- Preserved $U(1)_A$ current given by

$$j_A^\mu = \sum_{j=1}^{N} \left(q_{Qj} j_{Qj}^\mu + q_{\psi j} j_{Q\psi}^\mu \right), \qquad j_{fj}^\mu = \frac{1}{2} \overline{f} \gamma_5 \gamma^\mu f \tag{24}$$

- Axion component in pNGB modes is in geometric progression

$$\begin{aligned} \pi_j &\approx q_{Qj} \frac{a}{f} + \ldots, & \xi_j &\approx q_{\psi j} \frac{a}{f} + \ldots, \text{ with} \\ q_{Qj} &= q^{1-j}, & q_{\psi j} &= q^{-j}. \end{aligned} \tag{25}$$

- This model admits a larger Clockwork factor,

$$q = -\frac{T(\mathbf{R}_\psi)}{T(\mathbf{R_Q})} \sim \mathcal{O}(N_c) \gg 1. \tag{26}$$

Realising the QCD axion

. . .

	$SU(N_c)_N$
Q_N	$\mathbf{R}_Q$
ψ_N	$\mathbf{R}_\psi$

	$SU(3)_{\mathrm{QCD}}$
Q_{N+1}	$\mathbf{3}$

- Can introduce N_c flavours of Q_{N+1} with $q_{Q_{N+1}} = q_{\psi_N}$
- $U(1)_A$ is broken by the QCD anomaly, and

$$\mathcal{L} \supset \frac{N_c q_{Q_{N+1}} \pi_{N+1}}{32\pi^2} G\tilde{G} \approx \frac{N_c a}{32\pi^2 q^N f} G\tilde{G} + \ldots, \qquad (27)$$

i.e. effective axion decay constant is $f_a \approx \frac{q^N}{N_c} f \gg f$

Why these models are nice

- We can connect EW/LHC-scale physics to axion physics
- Better protection of $\bar{\theta}$ against cut-off (depends on $\frac{f}{M_{Pl}}$)
- Specific to these realisations:
  - Provides a symmetry explanation for the convenient nearest neighour couplings
  - Initial scale, Λ, is stable against quantum corrections
  - $U(1)_{PQ}$ arises as accidental symmetry from the gauge symmetries and fermion content
  - Predicts Clockwork factor
  - Consequently, interesting phenomenology ...

Clockwork axion phenomenology

Several interesting avenues:

- Collider signatures
- Cosmology
- Coupling to photons

LHC phenomenology

- Spectrum of new coloured particles depends on model
- Dynamical model and one realisation of contact-connection model predicts colour octet scalar meson with mass $m_8 \sim \frac{g_s \Lambda}{4\pi}$, which can be pair produced or singly produced via

$$\mathcal{L} \supset \frac{g_s^2}{4\pi\Lambda} S_8 G \tilde{G} \tag{28}$$

- Dijet search in Run II can constrain production cross section to $\sigma \lesssim \mathcal{O}(0.1)$ pb, testing $m_8 \sim$ TeV [4]
- $SU(N_c)$ baryons and other hadrons generally too heavy to be found at LHC

[4] See e.g. Belyaev, A. et al., JHEP 1701 (2017) 094

LHC phenomenology

- Contact-connection model may include vector-like quarks, Q_{N+1}, with mass $m_{Q_{N+1}} \sim \mathcal{O}(\Lambda^3/M_*^2)$, that interact with the SM, e.g. via

$$\mathcal{L} \supset \epsilon \overline{q}_{Li} \tilde{H} Q_{N+1R} \tag{29}$$

- ATLAS and CMS set $m_{T,B} \gtrsim 800$ GeV at 95% CL

- CMS expects to set $m_{T,B} \gtrsim 1.85$ TeV at 95% CL with 3000 fb^{-1} of $\sqrt{s} = 14$ TeV data

Cosmology

- Coherent oscillation of axion field provides a relic abundance,

$$\Omega_a h^2 \approx 0.07\alpha_i^2 \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^{7/6}, \tag{30}$$

  for initial misalignment angle, $\alpha_i \in [-\pi, \pi]$
- If $f = 1$ TeV (recall $f_a \approx q^N f$), correct relic density
  - For $N \sim \mathcal{O}(50)$ in $N_c = 5$ dynamical model
  - For $N \sim \mathcal{O}(15)$ in $N_c = 4$ contact-connection model
- Large domain wall number due to fractional clockwork factor, $N_{DW} \sim s^N$, where $q = r/s$
- To avoid domain wall problem, require $U(1)_{PQ}$ be broken during inflation and not restored afterwards

Baryonic DM

- Each fermion, ψ_j, Q_j has associated $U(1)_V$ symmetry in contact-connection model
- Lightest baryon under each $U(1)_V$ is a **stable bound state** with mass $m_{\mathcal{B}} \sim \mathcal{O}(N_c \Lambda)$
- Large relic density if reheating temperature lies in the window

$$T_F \lesssim T_R \lesssim \Lambda, \tag{31}$$

where the baryonic freeze-out temperature is given by

$$x_F \sim \frac{1}{2} \log(x_F) + 35 - \log\left(\frac{m_{\mathcal{B}}}{\text{TeV}}\right); \quad x_F \equiv \frac{m_{\mathcal{B}}}{T_F} \tag{32}$$

- Then relic density of baryonic dark matter is

$$\Omega_{\mathcal{B}} h^2 \sim 0.1 \frac{N}{15} \left(\frac{m_{\mathcal{B}}}{20 \text{ TeV}}\right)^2 \tag{33}$$

Coupling to photons

- Suppose ψ_j or Q_j has $\mathcal{O}(1)$ charge under $U(1)_Y$
- Then axion-photon coupling goes as

$$\mathcal{L} \sim \frac{a}{32\pi^2 q^j f} F\tilde{F} \approx \frac{q^{N-j} a}{32\pi^2 f_a} F\tilde{F} + \dots \qquad (34)$$

- Axion can have much larger coupling to photons than conventional models![5]
- Makes axion more 'visible' in haloscopes, e.g. ADMX

[5] For more details, see Farina, M. et al., JHEP 1701 (2017) 095

Summary

- Clockwork is an interesting mechanism to generate exponentially small couplings
- We have constructed models where Clockwork emerges in a sequence of strongly-coupled gauge theories, and applied them to the composite axion
- Can link LHC-scale physics to axion physics with stable f_a, protection of $\overline{\theta}$, prediction of Clockwork factor
- Range of phenomenology, from the collider to the cosmos, and the possibility that the axion has a large coupling to photons
- Still early days for Clockwork: many unexplored realisations/applications!

Questions or comments?