Gravitational Waves from Cosmological Phase Transitions

Cyril Lagger

2019 Sydney Particle Physics and Cosmology meeting

Introduction

What are Gravitational Waves?

- A prediction of General Relativity and Einstein Field Equations:

$$8\pi G\, T_{\mu\nu} = G_{\mu\nu}$$

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- A prediction of General Relativity and Einstein Field Equations:

$$8\pi G \quad \text{[Energy-matter density]} \quad = \quad \text{[Space-time curvature]}$$

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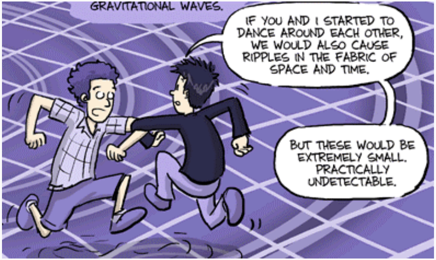
- Very weak coupling



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What are Gravitational Waves?

- A prediction of General Relativity and Einstein Field Equations:

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- Very weak coupling
- Gravitational wave source

Astrophysical?
Cosmological?
...?

What are Gravitational Waves?

- A prediction of General Relativity and Einstein Field Equations:

$$8\pi G \text{ [Energy-matter density]} = \text{[Space-time curvature]}$$

- Very weak coupling
- Gravitational wave source
- Gravitational wave properties

Frequency - Amplitude?
Transient or stochastic?

Detector design?

GWs as a probe of the Universe

Gravitational Wave (GW) detections by LIGO/Virgo are promising for theoretical physics:

- confirm a prediction of General Relativity
- allow us to test GR (and its modifications) in a strong and dynamical regime
- suggest to look for other sources of GWs in relation to particle physics: phase transitions, inflation cosmic strings,...

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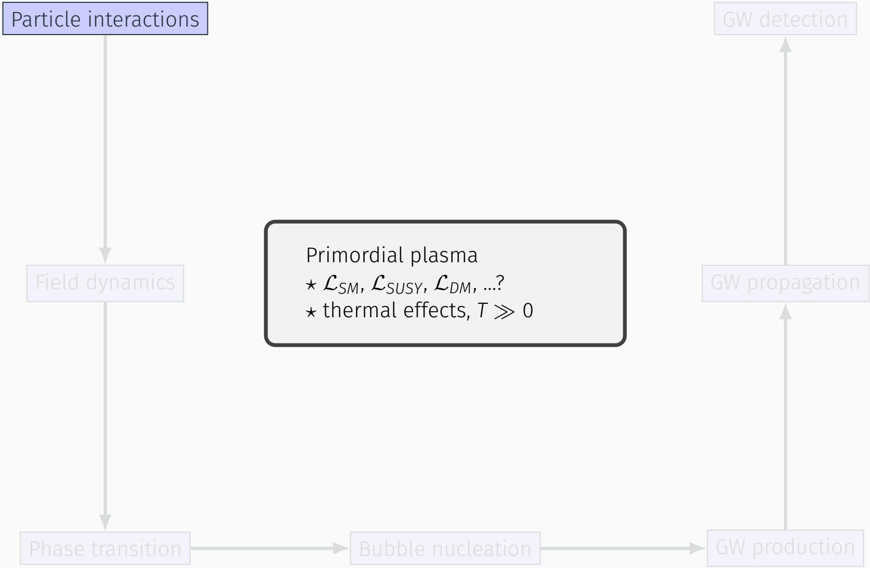
Main topic of this talk:

- exploring beyond the Standard Model physics with GWs from phase transitions

From quantum fields to Gravitational Waves!


Particle interactions
GW detection
Field dynamics
Primordial plasma
$\star$ $\mathcal{L}_{SM}$, $\mathcal{L}_{SUSY}$, $\mathcal{L}_{DM}$, ...?
$\star$ thermal effects, $T \gg 0$
GW propagation
Phase transition
Bubble nucleation
GW production

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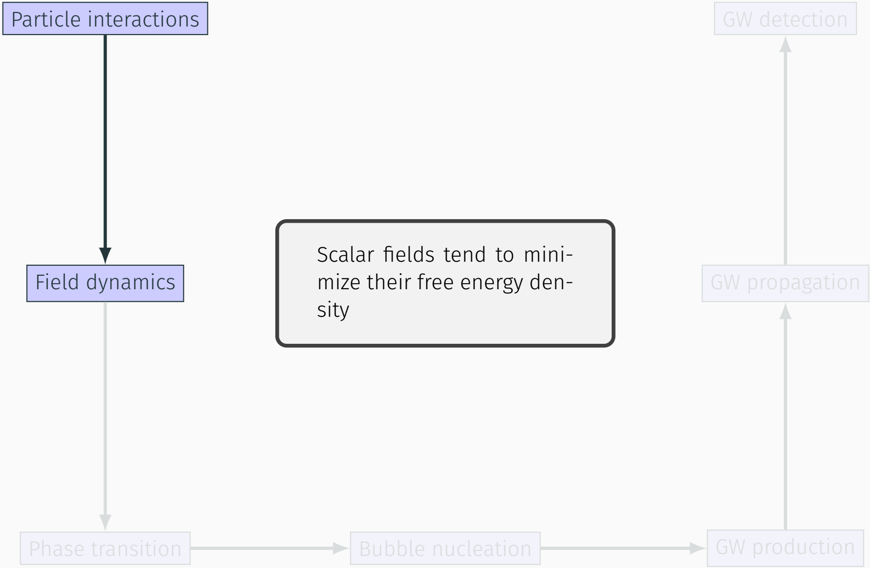
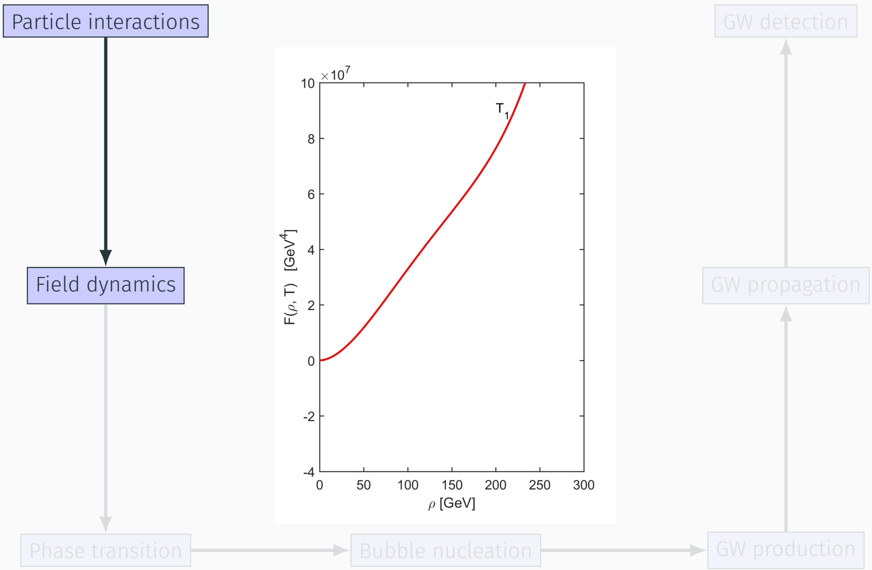


Scalar fields tend to minimize their free energy density


Particle interactions
Field dynamics
Phase transition
Bubble nucleation
GW production
GW propagation
GW detection
$\times 10^7$
10
8
6
4
2
0
-2
-4
0
50
100
150
200
250
300
T_1
F(ρ, T) [GeV⁴]
ρ [GeV]

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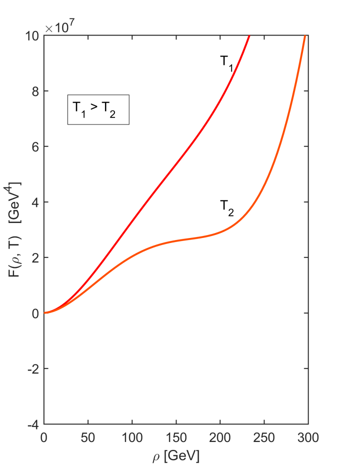

Phase transition

Bubble nucleation

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GW propagation

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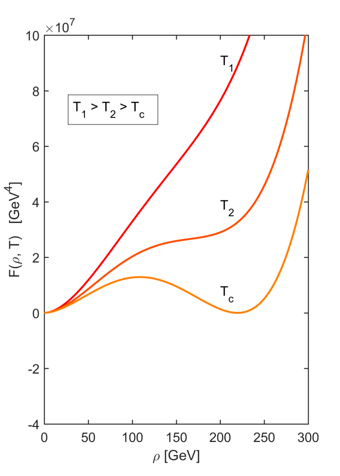

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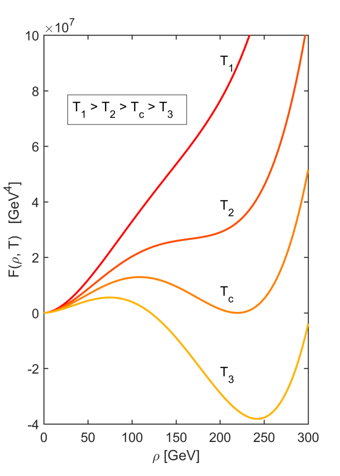

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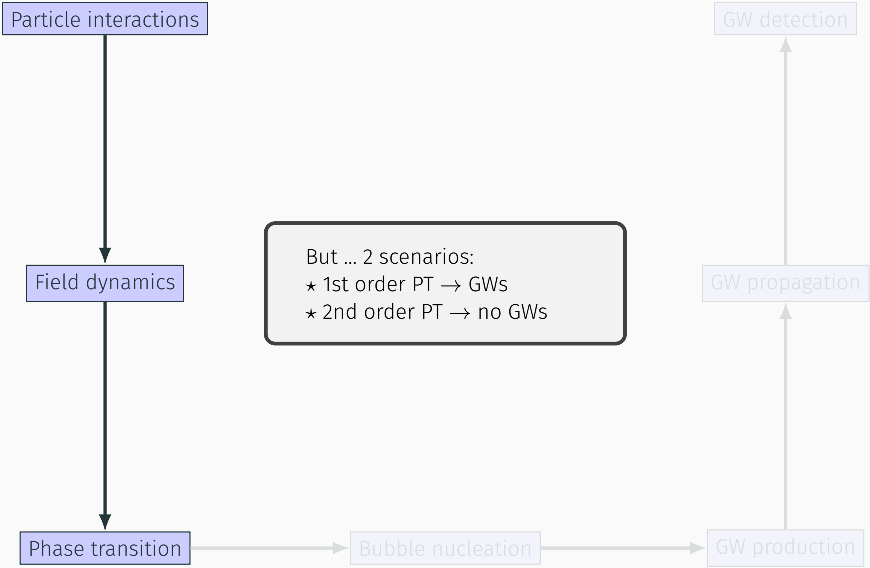

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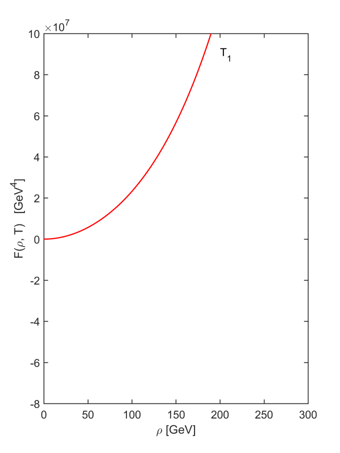

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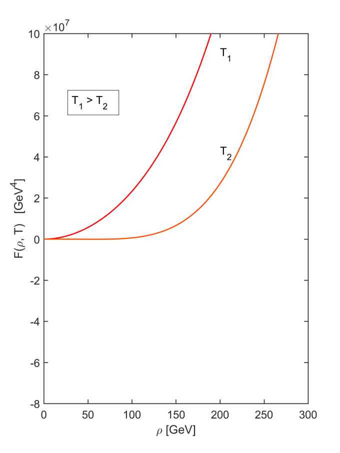

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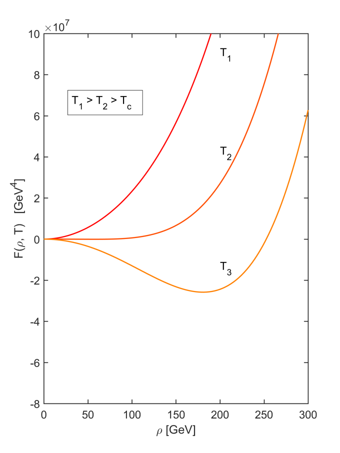

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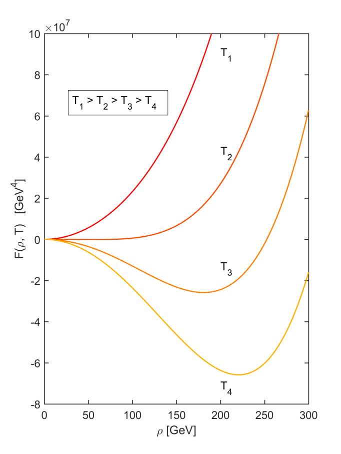

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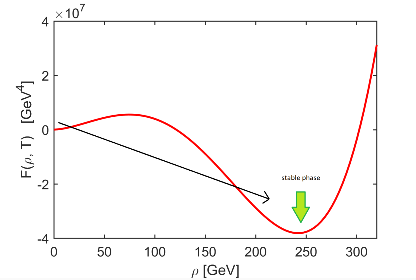

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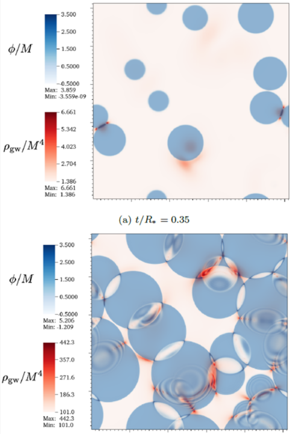

(a) $t/R_* = 0.35$

(b) $t/R_* = 0.66$

[D. Cutting, M. Hindmarsh, D. Weir, arXiv:1802.05712]

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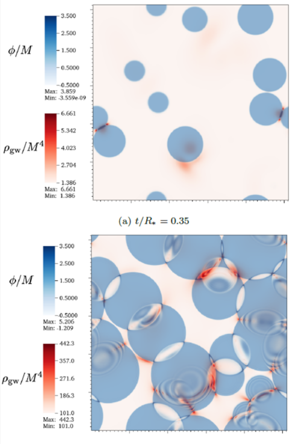

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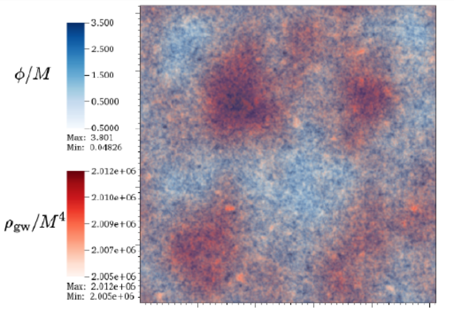

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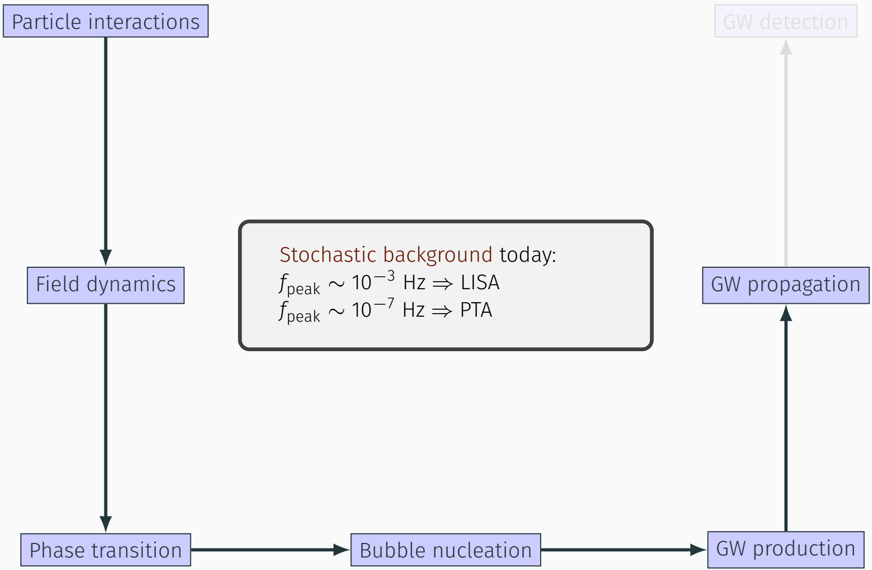

Stochastic background today:
$f_{\text{peak}} \sim 10^{-3}$ Hz $\Rightarrow$ LISA
$f_{\text{peak}} \sim 10^{-7}$ Hz $\Rightarrow$ PTA

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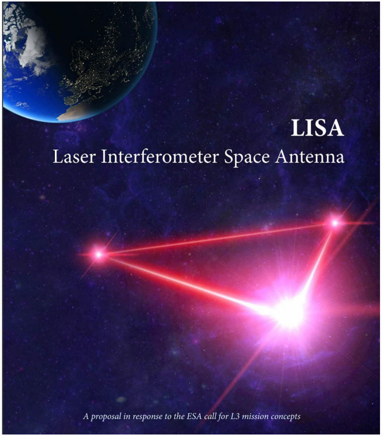
Particle interactions → Field dynamics → Phase transition → Bubble nucleation → GW production → GW propagation → GW detection



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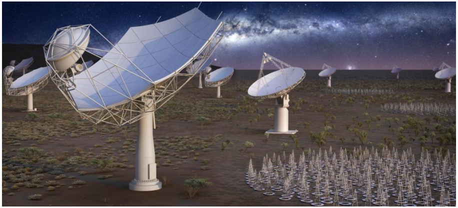

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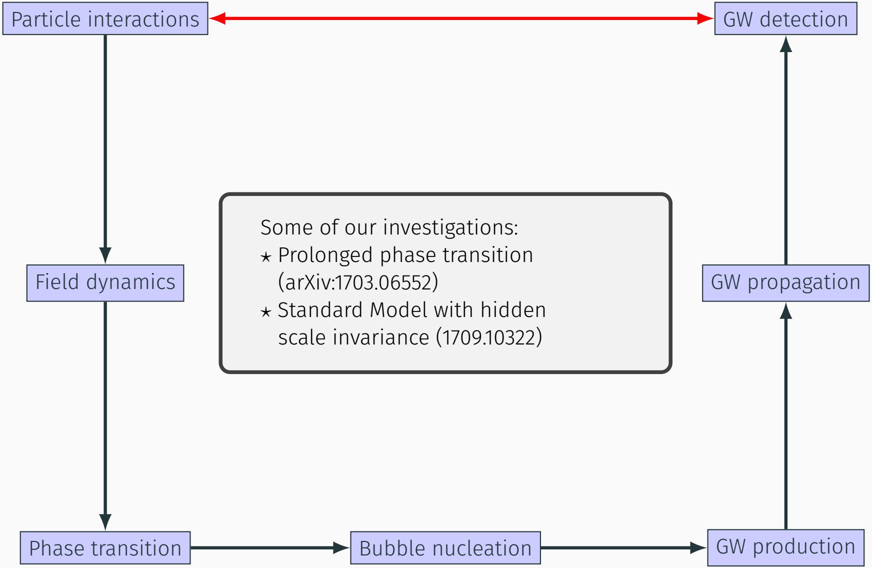

Some of our investigations:

⋆ Prolonged phase transition (arXiv:1703.06552)

⋆ Standard Model with hidden scale invariance (1709.10322)

GW background from bubble collisions

Mechanism and dimensional analysis

Stochastic background from three sources [C. Caprini et al., JCAP 1604 (2016) no.04 001]:

$$h^2\Omega_{\mathrm{GW}}(f) \simeq h^2\Omega_{col} + h^2\Omega_{sw} + h^2\Omega_{\mathrm{MHD}}$$

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Peak frequency and amplitude of the background mainly depend on the mean bubble size $\bar{R}$ at collision and kinetic energy ρ_{kin} stored in the bubbles:

- $f_{\mathrm{peak}} \sim (\bar{R})^{-1}$
- $\Omega_{col} \sim (\bar{R}H_p)^2 \dfrac{\rho_{\mathrm{kin}}^2}{(\rho_{\mathrm{kin}} + \rho_{\mathrm{rad}})^2}$

Bubble collision simulations

Going beyond dimensional analysis with simulations (and redshift)

[S. Huber and T. Konstandin, JCAP 0809 (2008) 022]

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Amplitude:

$$h^2\Omega_{col}(f) = 1.67 \times 10^{-5} \left(\frac{100}{g_*}\right)^{1/3} \left(\frac{\beta}{H_p}\right)^{-2} \kappa_v^2 \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{0.11 v^3}{0.42 + v^2}\right) S(f)$$

$$S(f) = \frac{3.8(f/f_0)^{2.8}}{1 + 2.8(f/f_0)^{3.8}}$$

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Peak frequency:

$$f_0 = 1.65 \times 10^{-7} \left(\frac{T_p}{1\text{ GeV}}\right) \left(\frac{g_*}{100}\right)^{1/6} H_p^{-1}\beta \left(\frac{0.62}{1.8-0.1v+v^2}\right) \text{Hz}$$

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The set of parameters $(\bar{R},\ \rho_{\rm kin},\ v,\ \kappa_\nu)$ is determined by the underlying particle physics model.

Phase transitions beyond the Standard Model

Various scenarios of phase transition

In the Standard Model, both electroweak and QCD PTs are crossover

[K. Kajantie et al., Phys. Rev. Lett. 77 (1996) 2887] [Y. Aoki et al, Nature 443 (2006) 675]

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Typical BSM electroweak PT:

- $\mathcal{O}(1)$ bubbles produced per Hubble volume at $T_n \lesssim T_{EW}$
- they rapidly collide $\Rightarrow$ percolation temperature $T_p \sim T_n$
- time scale of the process much shorter than Hubble time
- $f_{\mathrm{peak}} \sim$ milliHertz $\Rightarrow$ range of LISA [C. Caprini et al., JCAP 1604 (2016) no.04 001]

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Prolonged and supercooled PT [A. Kobakhidze, CL, A. Manning, J. Yue, arXiv:1703.06552]:

- weaker nucleation probability
- less bubbles produced $\Rightarrow$ more time needed for them to collide
- $\Rightarrow T_p \ll T_n \lesssim T_{EW}$
- $f_{\rm peak} \sim 10^{-8}$ Hertz $\Rightarrow$ range of Pulsar Timing Arrays

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QCD-induced PT [S. Arunasalam, A. Kobakhidze, CL, S. Liang, A. Zhou, arXiv:1709.10322] :

- no EWPT before $T \sim T_{\rm QCD}$
- at $T_{\rm QCD}$: 1st order chiral phase transition with 6 massless quarks
- quark condensates trigger EWPT
- $f_{\rm peak} \sim 10^{-8}$ Hertz $\Rightarrow$ range of Pulsar Timing Arrays

See also: [E. Witten Nucl.Pys.B177 (1981) 477] [W. Buchmuller, D. Wyler, PLB 249 (1990) 281] [S. Iso et al., PRL 119 (2017) 141301] [B. von Harling, G. Servant, JHEP 1801 (2018) 159]

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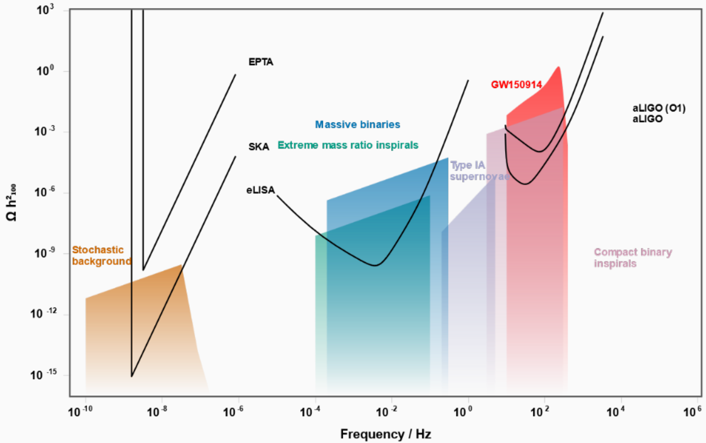


Prolonged electroweak phase transition

The model

Non-linear realization of the electroweak gauge group

[A. Kobakhidze, A. Manning, J. Yue, Int.J.Mod.Phys. D26 (2017) no.10, 1750114]

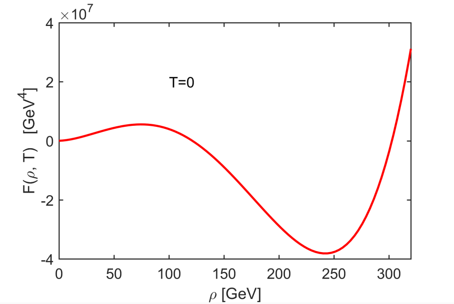
- $\mathcal{G}_{\text{coset}} = SU(2)_L \times U(1)_Y / U(1)_Q$ is gauged
- broken generators $T^i = \sigma^i - \delta^{i3}\mathbb{I}$ and Goldstone bosons $\pi^i(x)$
- physical Higgs as a singlet $\rho(x) \sim (\mathbf{1}, \mathbf{1})_0$
- SM Higgs doublet identified as $H(x) = \frac{\rho(x)}{\sqrt{2}} e^{\frac{i}{2}\pi^i(x)T^i} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad i \in \{1, 2, 3\}$

SM particle content but BSM interactions

Minimal setup (usual SM configurations except Higgs potential):

$$V^{(0)}(\rho) = -\frac{\mu^2}{2}\rho^2 + \frac{\kappa}{3}\rho^3 + \frac{\lambda}{4}\rho^4.$$

Model specified by one parameter: $\kappa = \bar{\kappa} \cdot \frac{m_h^2}{v} \sim 63.5 \cdot \bar{\kappa}$ GeV.



Phase transition dynamics

Decay probability per unit volume per unit time: $\Gamma(T) \approx A(T)e^{-S(T)}$

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Probability for a point of space-time to remain in the false-vacuum:

$$p(t) = \exp\left[-\frac{4\pi}{3}\int_{t_\star}^{t} dt'\,\Gamma(t')a^3(t')r^3(t,t')\right] \qquad r(t,t') = \int_{t'}^{t} dt''\,\frac{v(t'')}{a(t'')}$$

Percolation temperature ($\sim$ collision) [L. Leitao et al., JCAP 1210 (2012) 024]: $p(t_p) \approx 0.7$

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Number density of produced bubbles:

$$\frac{dN}{dR}(t,t_R) = \Gamma(t_R)\left(\frac{a(t_R)}{a(t)}\right)^4 \frac{p(t_R)}{v(t_R)}$$

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Nucleation temperature T_n: maximum of $\dfrac{dN}{dR}(t_p, t_R)$

Bubbles properties at collision

By definition:

- most bubbles collide at t_p
- majority of them produced at t_n

$$\Rightarrow \text{bubble physical radius: } \bar{R} = a(t_p)r(t_p, t_n)$$

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Kinetic energy stored in bubble-walls:

$$E_{\text{kin}} = \kappa_\nu \cdot 4\pi \int_{t_n}^{t_p} dt \frac{dR}{dt}(t, t_n) R^2(t, t_n) \epsilon(t)$$

- $\epsilon(t)$: latent heat ($\sim$ vacuum energy)
- κ_ν: fraction of energy going into the wall motion (vs. heating the plasma)

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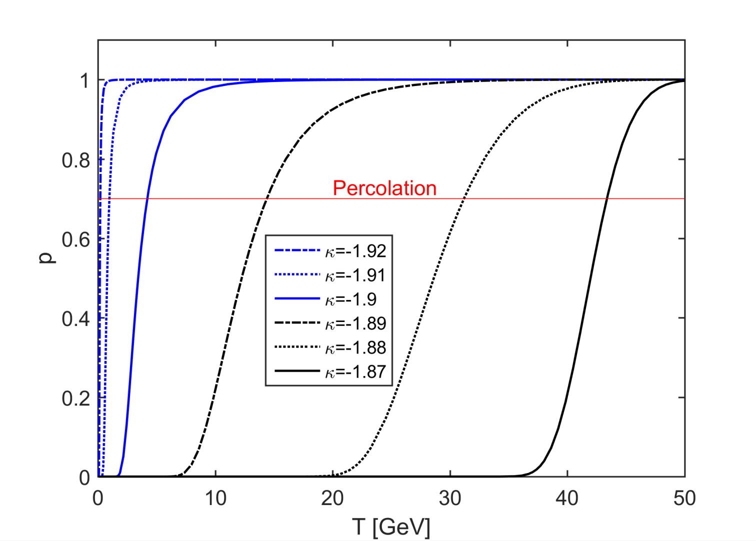
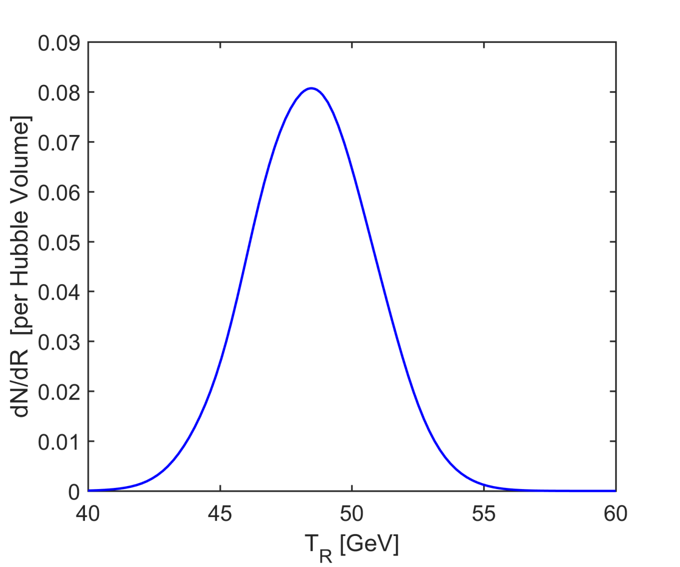
$\bar{R}$ and E_{kin}: key parameters to deduce the GW spectrum

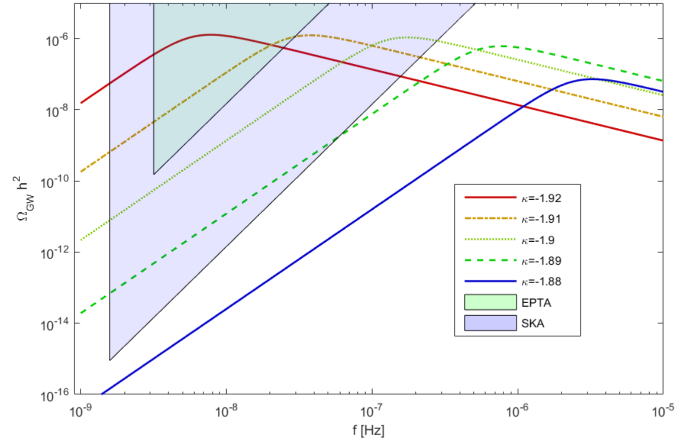
Probability $p(T)$:



Number density distribution for $|\bar{\kappa}| = 1.9$: $\Rightarrow T_n \sim 49$ GeV



Numerical results



- Current constraints: EPTA, PPTA, NANOGrav
- Possible detection: Square Kilometre Array

Conclusion

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- Gravitational Waves are allowing us to explore the fundamental laws of the Universe!

- Future detectors and theoretical investigations are likely to bring new discoveries!