Why $N_{\text{eff}} \neq 3.046$

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Outline

- Motivations
- Theory;
    - Active Transport
    - Finite Temperature QED
- Our work:
    - Solve ODE
    - Add next order

Motivations

- For years (2005), SM has predicted $N_{\text{eff}} = 3.046$ [1]
- Recently, [2] gave a different answer $N_{\text{eff}} = 3.052$ using a "non-perturbative" method.
- Who is right? Can we improve upon correct result? What is N_{eff}?

Effective number of neutrinos

- Simplistic view: How is the total energy density is divided up?

- Set a time: after electron annihilation epoch; before neutrinos become non-relativistic

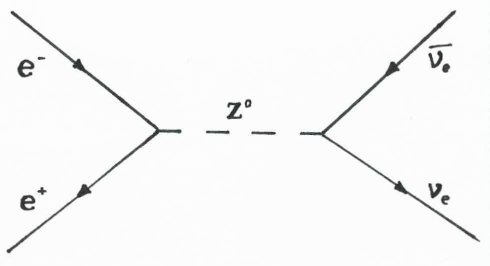
$$\rho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right]\rho_\gamma$$

- Where have these terms come from?

Deviation from $N_{\text{eff}} = 3$

$$\rho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right]\rho_\gamma$$

- Exchange energy with photons?
  - Not coupled directly.
  - Indirect – four diagrams at tree level



http://www.mushtukov.com/

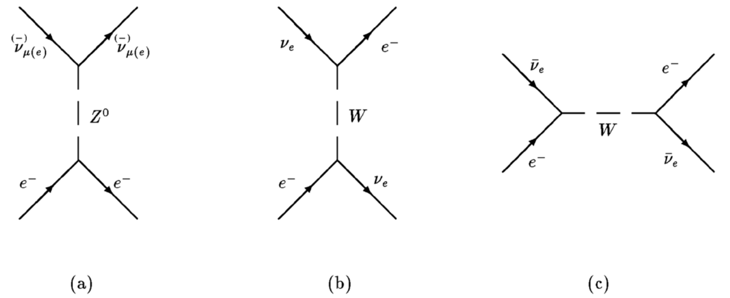


Figure 1: *Feynman diagrams for the processes of neutral current (NC) νe-scattering (a), and charged current (CC) $\nu_e e$-scattering via the exchange of a W-Boson (b,c).*

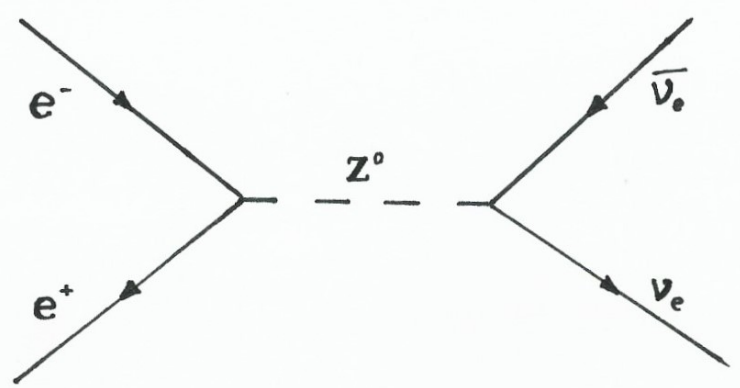
https://cds.cern.ch/record/248487/

  - T=MeV Neutrinos decouple
  - T=0.5 MeV e+- decouple.

Active transport

Neutrino decouple ~ MeV
$e^{\pm}$ annihilation ~0.5MeV

- Do all electrons annihilate instantaneously?
  - Fermi-Dirac distributions
- Are all neutrinos decoupled when electrons annihilate?
- Some energy transferred to neutrino sector
- Use continuity equation



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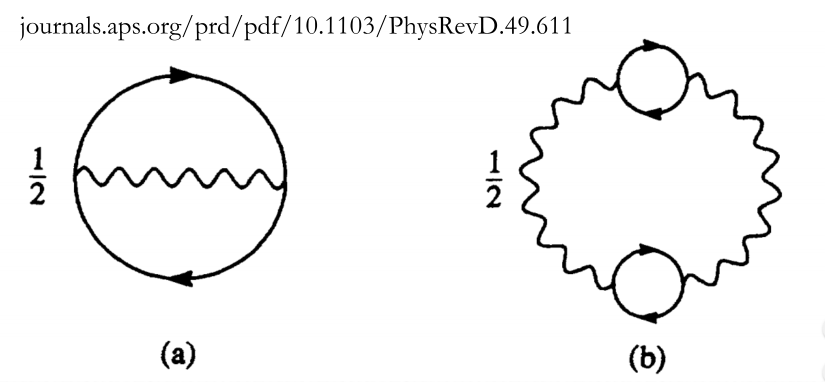
Finite-temperature QED

$$\rho_R = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right]\rho_\gamma$$

- Electrons and photons gain self energy through QED.

- Thermal masses instructive mentally



- $m_e^2 \to m_e^2 + \delta m_e^2(T), m_\gamma^2 \to \delta m_\gamma^2(T)$.

- Not fully relativistic – lowers ρ_γ but ρ_ν same $\therefore N_{\text{eff}}$ increases

New Variables

$$\boxed{\begin{gathered} x = m_e R(T) \\ y = pR(T) \\ z = T_\gamma R(T) \end{gathered}}$$

- Go to co-moving variables

$$x = m_e R(T)$$
$$y = pR(T)$$
$$z = T_\gamma R(T)$$

- $R(T)$ is some normalized scale factor $\propto a$
  - $R(T_d) = \frac{1}{T_\nu}$
- Relativistic temperatures evolve $\propto \frac{1}{a}$
- $p \propto \frac{1}{a}$
- Need P and ρ. Related.

$$\rho = -P + T\frac{\partial P}{\partial T}$$

Continuity Equation

$$x = m_e/T_\nu$$
$$y = p/T_\nu$$
$$z = T_\gamma/T_\nu$$

- Normally $\frac{d}{dt}\rho = -3H(\rho + P)$.

$$\bar{P} = P\left(\frac{x}{m_e}\right)^4$$

- Need to track H. Tough.

$$\bar{\rho} = \rho\left(\frac{x}{m_e}\right)^4$$

- Co-moving continuity equation $\frac{d}{dx}\bar{\rho} = \frac{1}{x}(\bar{\rho} - 3\bar{P})$
- When does this approach work?

$$\boxed{\frac{d}{dx}\bar{\rho} = \frac{1}{x}(\bar{\rho} - 3\bar{P})}$$

ODE

$$\boxed{\begin{array}{c} x = m_e/T_\nu \\ y = p/T_\nu \\ z = T_\gamma/T_\nu \end{array}}$$

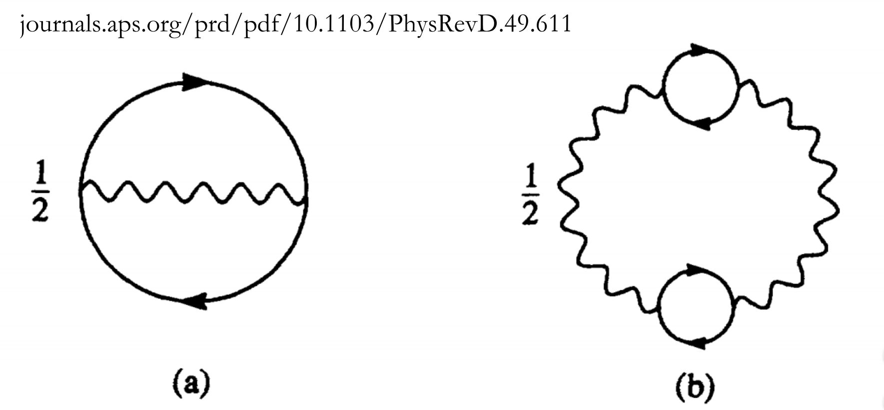
- $\frac{dz}{dx} = \frac{\frac{1}{x}\left(\bar{\rho}^0 - 3\bar{P}^0\right) - \frac{\partial\bar{\rho}^0}{\partial x} + G_1(x,z)}{\frac{\partial\bar{\rho}^0}{\partial z} + G_2(x,z)}$ eg $P^0 = \frac{T}{\pi^2}\int_0^\infty \mathrm{d}p\, p^2 \ln\left[\frac{\left(1+\mathrm{e}^{-E_e/T}\right)^2}{\left(1-\mathrm{e}^{-E_\gamma/T}\right)}\right]$
- G's encode QED corrections. Interplay.
- Want $\frac{dz}{dx}$, tells us how z (T_γ/T_ν) evolves. $\Delta N_{\mathrm{eff}} = -12\frac{\delta z}{z_0}$, $\delta z = z_{\mathrm{fin}} - z_0$, $z_0 = \left(\frac{11}{4}\right)^{1/3}$.

$$\boxed{\frac{dz}{dx} = \frac{\frac{1}{x}(\bar{\rho}^0 - 3\bar{P}^0) - \frac{\partial \bar{\rho}^0}{\partial x} + G_1}{\frac{\partial \bar{\rho}^0}{\partial z} + G_2}}$$

Higher order

$$\boxed{\begin{gathered} x = m_e/T_\nu \\ y = p/T_\nu \\ z = T_\gamma/T_\nu \end{gathered}}$$

- $G_1^{(i)} = \frac{1}{x}\left(\bar{\rho}^{(i)} - 3\bar{P}^{(i)}\right) - \frac{\partial \bar{\rho}^{(i)}}{\partial x}$
- $G_2^{(i)} = \frac{\partial \bar{\rho}^{(i)}}{\partial z}$
- (i) indicates $\mathcal{O}(e^i)$
- e.g $P^{(2)} = -\int_0^\infty \frac{\mathrm{d}p}{2\pi^2}\left[\frac{p^2}{E_e}\frac{\delta m_e^2}{1+\mathrm{e}^{E_e/T}} + \frac{p}{2}\frac{\delta m_\gamma^2}{\mathrm{e}^{p/T}-1}\right]$



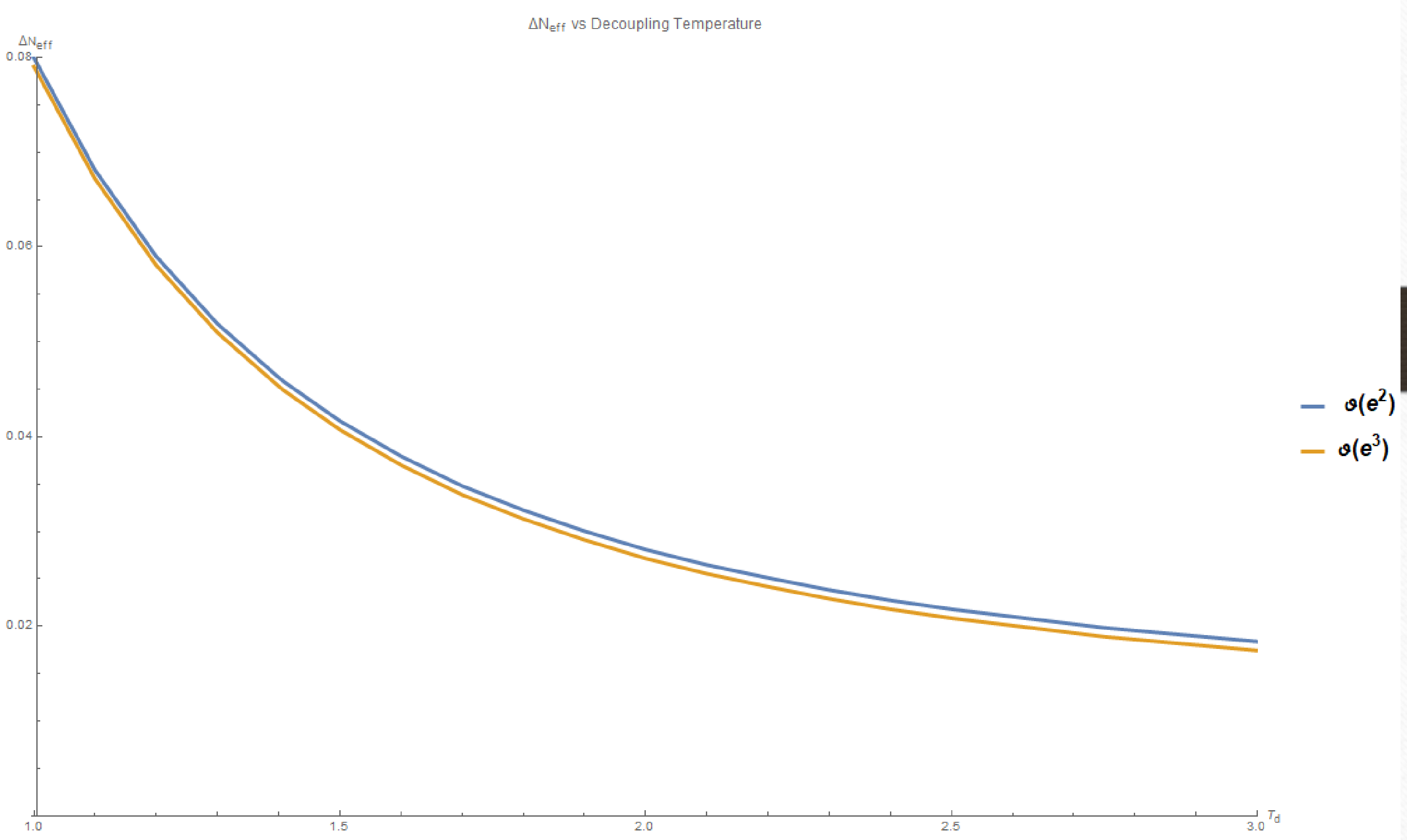
$\mathcal{O}(e^2)$ $\mathcal{O}(e^3)$

Solutions

- Wrote C++ code

to solve

- Sensitive to T_D.
- $T_D = 1.41$ MeV [3],
- $N_{\text{eff}}^{(2)} = 3.046$;
- $N_{\text{eff}}^{(3)} = 3.045$
- Similar to inc. ν osc.

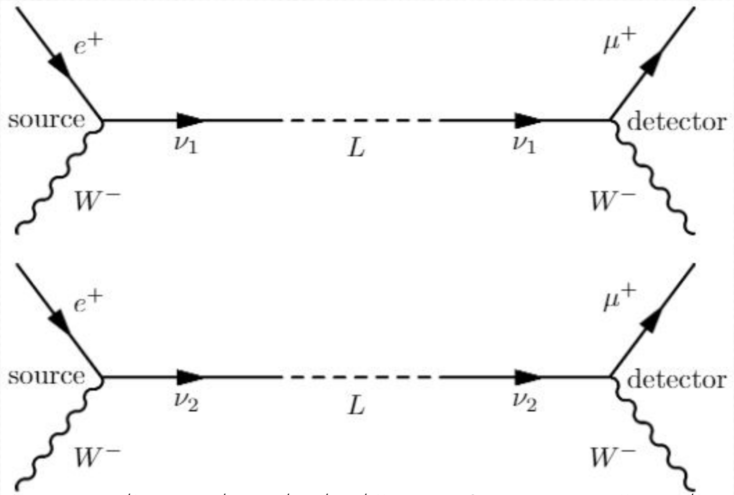


$$\frac{dz}{dx} = \frac{\frac{1}{x}\left(\bar{\rho}^0 - 3\bar{P}^0\right) - \frac{\partial\bar{\rho}^0}{\partial x} + G_1}{\frac{\partial\bar{\rho}^0}{\partial z} + G_2}$$

Future

$$x = m_e/T_\nu$$
$$y = p/T_\nu$$
$$z = T_\gamma/T_\nu$$

- Even higher order pointless?
- Include non-instantaneous neutrino decoupling
- Neutrino oscillations



physino.xyz/learning/2015/05/15/illusion-of-neutrino-oscillation/

$$\frac{dz}{dx} = \frac{\frac{1}{x}\left(\bar{\rho}^0 - 3\bar{P}^0\right) - \frac{\partial \bar{\rho}^0}{\partial x} + G_1}{\frac{\partial \bar{\rho}^0}{\partial z} + G_2}$$

Conclusion

$$x = m_e/T_\nu$$
$$y = p/T_\nu$$
$$z = T_\gamma/T_\nu$$

- What is ρ_ν/ρ_γ?
- Find P and ρ.
- Annihilations. Thermal effects
- Use continuity equation, solve for z. Get N_{eff}