Magnetic Monopoles in the Standard Model and Implications for Electroweak Baryogenesis

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Overview

- Magnetic monopoles review
  - Dirac monopoles
  - 't Hooft-Polyakov monopoles
- Topological stability of monopoles in the Standard Model
- Mass predictions: constraining the Born-Infeld parameters
- Implications for electroweak baryogenesis
- Conclusions

Magnetic Monopoles

- No magnetic monopoles $\nabla \cdot \boldsymbol{B} = 0$
- Strong theoretical reasons to believe they exist

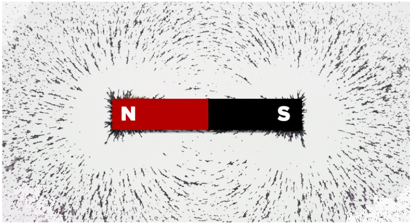

Figure 1: A standard bar magnet.

Dirac Monopoles

Dirac (1931)

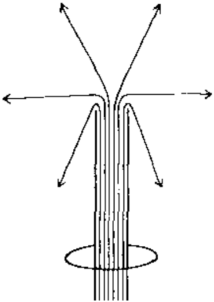

Figure 2: A Dirac string.

$$A_\phi = m\frac{(1-\cos\theta)}{r\sin\theta} \qquad \Longrightarrow \qquad \mathbf{B} = m\frac{\hat{r}}{r^2}$$

$$\boxed{em = \frac{n}{2}, \quad n \in \mathbb{Z}}$$

't Hooft-Polyakov Monopoles

- Georgi-Glashow (1972) model - SO(3) + triplet scalar field Φ
- Potential $V(\Phi) = \frac{\lambda}{4}(\Phi^a\Phi^a - v^2)^2$ with minimum

$$\Phi^a\Phi^a = (\Phi^1)^2 + (\Phi^2)^2 + (\Phi^3)^2 = v^2$$

  an S^2 **vacuum manifold** $\mathcal{V}$

- Can choose $\Phi = (0, 0, v) \Rightarrow$ SSB

't Hooft-Polyakov Monopoles

't Hooft (1974) & Polyakov (1974)

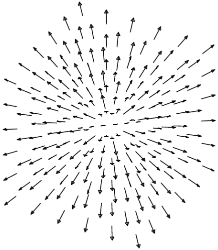

Figure 3: Another solution for Φ: a "hedgehog" configuration.

- Φ: S^2 (spatial ∞) $\to S^2(\mathcal{V})$, falls under $\pi_2(S^2) = \mathbb{Z}$
- Finite energy solution, effect on gauge fields at large distances $\Rightarrow$ solution consistent with monopole with $m = \frac{1}{e}$

The Model - Concepts

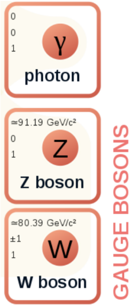

Figure 4: The electroweak gauge bosons of the Standard Model.

- SU(2) × U(1) electroweak part with complex scalar doublet $\Phi = (\phi_1 + i\phi_2, \phi_3 + i\phi_4)^T$

$$\boxed{\mathcal{V} : \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = v^2}$$

The Model - Concepts

- $\pi_2(S^3) = 1$ no topological charge, decays to constant vacuum solution $\Rightarrow$ no monopoles
- **But** may have a hybrid solution between Dirac and 't Hooft-Polyakov monopoles - Cho & Maison (1997)
- The mass is formally infinite - need to regularise

Monopole Ansatz

- Static solutions representing magnetic monopoles

$$\mathbf{\Phi} = \frac{1}{\sqrt{2}}\rho(r)\begin{pmatrix} i\sin(\theta/2)e^{-i\phi} \\ -i\cos(\theta/2) \end{pmatrix}$$

$$\mathbf{A}_\mu = \frac{1}{g_2}(f(r)-1)\hat{r}\times\partial_\mu\hat{r}$$

$$B_\mu = -\frac{1}{g_1}(1-\cos\theta)\partial_\mu\phi$$

Born-Infeld Electrodynamics

- For a point charge $U = \frac{1}{2}\int_V |\mathbf{E}|^2 d^3r = \frac{q^2}{8\pi}\int_0^\infty \frac{1}{r^2} dr$

- Theory to avoid divergence - Born & Infeld (1934)

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \rightarrow \beta^2\left[1 - \sqrt{1 + \frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16\beta^4}(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2}\right]$$

- $[\beta] = (\text{energy})^2$, $\beta \rightarrow \infty$ gives standard Maxwell theory

Monopole Mass

- For static configurations the energy is given by $E = -L =$ monopole mass

$$E = 4\pi \int_0^\infty dr\ \beta_1^2 \left[\sqrt{r^4 + \frac{1}{g_1^2 \beta_1^2}} - r^2 \right] (\approx 77.1\sqrt{\beta_1})$$
$$+ \beta_2^2 \left[\sqrt{r^4 + \frac{(f^2-1)^2}{g_2^2 \beta_2^2} + \frac{2f'^2 r^2}{g_2^2 \beta_2^2}} - r^2 \right]$$
$$+ \frac{1}{2}(r\rho')^2 + \frac{\lambda r^2}{8}(\rho^2 - \rho_0^2)^2 + \frac{1}{4} f^2 \rho^2$$

Constraining the Born-Infeld Parameters

- Expand the Born-Infeld term

$$\mathcal{L}_{BI} = \beta^2\left[1 - \sqrt{1 + \frac{1}{2\beta^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{16\beta^4}(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2}\right]$$
$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{32\beta^2}((F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + (F_{\mu\nu}F^{\mu\nu})^2) - \cdots$$

- Products of 4 terms $\partial_\mu A$ - contributes to $\gamma\gamma$ scattering amplitude

- Can expand non-Abelian BI factor - contributes to heavy boson WW scattering amplitude

- Constraints on β_1, β_2 give a **lower bound** $\mathrm{M}_m \sim$ **9 − 11 TeV**

Monopoles and the Electroweak Phase Transition

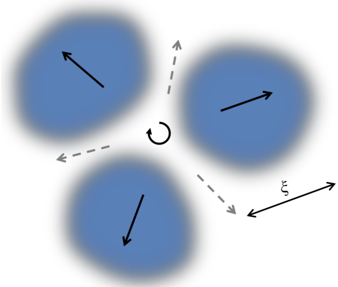

Figure 5: The production of monopoles during the electroweak phase transition via the Kibble (1976) mechanism.

- Production of monopoles drives surrounding plasma **out of equilibrium**

Monopoles and the Electroweak Phase Transition

- Interactions between monopoles/antimonopoles/SM particles provide a new source of **B + L number violation** (Rubakov (1982), Callan (1982))

- Δ B $\neq 0$ at the conclusion of the EW phase transition requires $M_m \gtrsim 0.9 \times 10^4$ TeV (Arunasalam & Kobakhidze (2017))

- Abundance of monopoles and nucleosynthesis constraints gives $M_m \lesssim 2.3 \times 10^4$ TeV (Arunasalam & Kobakhidze (2017))

Sakharov (1967) Conditions and CP Violation

- **CP violation** accommodated by monopoles through the physical 'θ-term'

$$\mathcal{L}_\theta = \theta_{ew} F^a_{\mu\nu} \widetilde{F}^{a\mu\nu}$$

- θ_{ew} can be constrained by precision measurements of the electric dipole moment of known particles

- The **baryon asymmetry parameter** can be expressed

$$\eta_B = \kappa \theta_{ew} \frac{n_0}{s}$$

Conclusions

- **Magnetic monopoles in SM** - regularise mass using Born-Infeld theory
- Topologically **stable solution**
- **Lower bound** on monopole **mass** $\sim \mathbf{9-11}$ **TeV**
- Monopoles can play an **important role** in **electroweak baryogenesis**