Quadratic Bezier

Setup: Linear interpolation (Lerp)

Let A and B be the start and end points and t be $\mathbb{R} \in [0, 1]$.

$$L(A, B, t) = A + (B - A)t$$

Quadratic Bezier (in terms of Lerp)

Let A and C be the start and end points, B be the control point, and t be $\mathbb{R} \in [0, 1]$.

$$\begin{aligned} L_0 &= L(A, B, t) \\ L_1 &= L(B, C, t) \\ Q(A, B, C, t) &= L(L_0, L_1, t) \end{aligned}$$

Quadratic Bezier (explicit form)

$$\begin{aligned} L_0 &= L(A, B, t) = A + (B - A)t \\ L_1 &= L(B, C, t) = B + (C - B)t \end{aligned}$$

$$\begin{aligned} Q(A, B, C, t) &= L(L_0, L_1, t) \\ &= L_0 + (L_1 - L_0)t \\ &= [A + (B - A)t] + ([B + (C - B)t] - [A + (B - A)t])t \\ &= [A + Bt - At] + ([B + Ct - Bt] - [A + Bt - At])t \\ &= A + Bt - At + Bt + Ct^2 - Bt^2 - At - Bt^2 + At^2 \\ \boldsymbol{Q(A, B, C, t)} &= \boldsymbol{(A - 2B + C)t^2 + (2B - 2A)t + A} \end{aligned}$$

Find roots using quadratic formula

$$\begin{aligned} Q(A, B, C, t) &= (A - 2B + C)t^2 + (2B - 2A)t + A \\ a &= A - 2B + C \\ b &= 2B - 2A \\ c &= A \\ \text{Quadratic formula} : t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

Quadratic Bezier Derivative

$$\begin{aligned} Q(A, B, C, t) &= (A - 2B + C)t^2 + (2B - 2A)t + A \\ \boldsymbol{Q'(A, B, C, t)} &= \boldsymbol{2(A - 2B + C)t + (2B - 2A)} \end{aligned}$$