

Quadratic Bezier

Setup: Linear interpolation (Lerp)

Let A and B be the start and end points and t be $\mathbb{R} \in [0, 1]$.

$$L(A, B, t) = A + (B - A)t$$

Quadratic Bezier (in terms of Lerp)

Let A and C be the start and end points, B be the control point, and t be $\mathbb{R} \in [0, 1]$.

$$L_0 = L(A, B, t)$$

$$L_1 = L(B, C, t)$$

$$Q(A, B, C, t) = L(L_0, L_1, t)$$

Quadratic Bezier (explicit form)

$$L_0 = L(A, B, t) = A + (B - A)t$$

$$L_1 = L(B, C, t) = B + (C - B)t$$

$$\begin{aligned} Q(A, B, C, t) &= L(L_0, L_1, t) \\ &= L_0 + (L_1 - L_0)t \\ &= [A + (B - A)t] + ([B + (C - B)t] - [A + (B - A)t])t \\ &= [A + Bt - At] + ([B + Ct - Bt] - [A + Bt - At])t \\ &= A + Bt - At + Bt + Ct^2 - Bt^2 - At - Bt^2 + At^2 \\ Q(A, B, C, t) &= (A - 2B + C)t^2 + (2B - 2A)t + A \end{aligned}$$

Find roots using quadratic formula

$$Q(A, B, C, t) = (A - 2B + C)t^2 + (2B - 2A)t + A$$

$$a = A - 2B + C$$

$$b = 2B - 2A$$

$$c = A$$

$$\text{Quadratic formula : } t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$