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Information Distortion in a Supply Chain: The Bullwhip Effect

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Consider a series of companies in a supply chain, each of whom orders from its immediate upstream member. In this setting, inbound orders from a downstream member serve as a valuable informational input to upstream production and inventory decisions. This paper claims that the information transferred in the form of "orders" tends to be distorted and can misguide upstream members in their inventory and production decisions. In particular, the variance of *orders* may be larger than that of *sales*, and distortion tends to increase as one moves upstream—a phenomenon termed "bullwhip effect." This paper analyzes four sources of the bullwhip effect: demand signal processing, rationing game, order batching, and price variations. Actions that can be taken to mitigate the detrimental impact of this distortion are also discussed.

Key words: supply chain management; information distortion; information integration; production and inventory management
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1. Introduction

Recent interest in supply chain management centers around coordination among various members of a supply chain comprising manufacturers, distributors, wholesalers and retailers. There are many examples of the benefits of coordination activities to the individual members in the supply chain (see, for example, Byrnes and Shapiro 1992, and Kurt Salmon Associates 1993).

One important mechanism for coordination in a supply chain is the information flows among members of the supply chain. These information flows have a direct impact on the production scheduling, inventory control and delivery plans of individual members in the supply chain. This paper studies the demand information flow in a supply chain and reports the systematic distortion in demand information as it is passed along the supply chain in the form of orders. The illustration in Figure 1 (based on real data but manipulated to maintain confidentiality) shows a retail store's *sales* of a product, alongside the retailer's *orders* issued to the manufacturer. The figure clearly highlights the distortion in demand information. The retailer's orders do not coincide with the actual retail sales. In this paper the *bullwhip effect or whiplash effect* refers to the phenomenon where orders to the supplier tend to have larger variance than sales to the buyer (i.e., demand distortion), and the distortion propagates upstream in an amplified form (i.e., variance amplification).

The bullwhip phenomenon has been recognized in many diverse markets. Procter & Gamble found that the diaper orders issued by the distributors have a degree of variability that cannot be explained by consumer demand fluctuations alone. At Hewlett-Packard, the orders placed to the printer division by resellers have much bigger swings and variations than customer demands, and the orders to the company's integrated circuit division have even worse swings. Also, it is often said that the DRAM market faces a much higher volatility than the computer market. (See Lee et al. 1997 for more evidence of the bullwhip effect.)

The distortion of demand information implies that the manufacturer who only observes its immediate order data will be misled by the amplified demand patterns, and this has serious cost implications. For instance, the manufacturer incurs excess raw materials cost due to unplanned purchases of supplies, additional manufacturing expenses created by excess capacity, inefficient utilization and overtime, excess warehousing expenses and additional transportation costs due to inefficient scheduling and premium shipping rates. Trade estimates suggest that these activities can result in excess costs in the range between 12.5% to 25% (Kurt Salmon Associates 1993).

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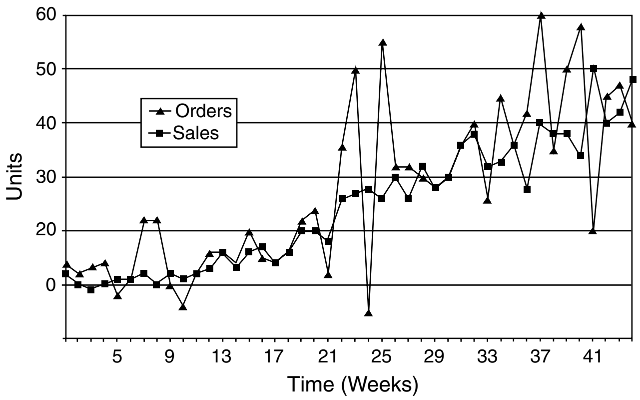
Figure 1 Orders vs. Sales



By another measure, the inefficiencies bear part responsibility for the $75 billion to $100 billion worth of inventory caught between various members of the $300 billion (annual) grocery industry (Fuller et al. 1993).

Overall, these numbers allude to the potential efficiency gains achievable through improvement in information flow design. To this end, this paper attempts to explore what drives the bullwhip effect. We propose four sources of the bullwhip effect—demand signal processing, rationing game, order batching and price variations. Our choice of these effects is dictated by the fact that they are common effects in distribution channels. Retailers routinely use demand realizations as signals/predictors of future demand. Order batching is a routine part of retail buyers' decision process because the buyers are constantly trying to gain economies in pricing (i.e., volume discounts) and transportation. Note that the demand signal processing and order batching effects are related in the sense that they are outputs of traditional inventory management models at the retail level. Rationing is common in product markets during the growth phase of the product life-cycle when demand outstrips supply. And, price promotions are endemic in mature product categories reflecting moves by manufacturers in a market share war. Note also that the rationing and price variation effects are related in as much as they represent channel members' reactions to market forces. We show that each of these effects is capable of generating rational behaviors that result in the bullwhip effect. Since in reality some combination of these effects characterize the marketplace, it is important for all players in the supply chain to realize the impact of these forces and take measures to improve the coordination among members in the supply chain.

Identification of the causes for the bullwhip effect leads to prescriptions for alleviating the detrimental impact of this phenomenon. The paper argues that sales information available in the form of orders received from the downstream member should be used with great caution. Sell-through data and information on inventory status at downstream nodes are keys to improving channel coordination and dampening the bullwhip effect. The growing popularity of sharing sell-through and inventory data among members of the supply chain supports this argument.

The rest of the paper is organized as follows. The next section provides a brief survey on the related literature. Section 3 studies the four causes of the bullwhip effect. In §4 we discuss the managerial implications for channel coordination activities and provide concluding remarks.

2. Related Literature

The basic phenomenon is not new and has been known to management scientists for some time. Forrester (1961) illustrates the effect in a series of case studies, and points out that it is a consequence of industrial dynamics or time varying behaviors of industrial organizations. In other words, the basic form and policies used by an organization can give rise to characteristic and undesirable behaviors in the supply chain.

In an inventory management experimental context, Sterman (1989) reports evidence of the bullwhip effect in the "Beer Distribution Game." The experiment involves a supply chain with four players who make independent inventory decisions without consultation with other chain members, relying only on orders from the neighboring player as the sole source of communications. Under the linear cost structure, the experiment shows that the variances of orders amplify as one moves up in the supply chain, confirming the bullwhip effect. Sterman (1989) interprets the phenomenon as a consequence of players' systematic irrational behavior, or "misperceptions of feedback."

Like Forrester (1961) and Sterman (1989), our interest lies in understanding the causes and managerial implications of the bullwhip effect. However, our work differs from previous research in several respects. Unlike Forrester (1961) or Sterman (1989), we develop simple mathematical models of supply chains that capture essential aspects of the institutional structure and optimizing behaviors of members. We demonstrate through the models that the bullwhip effect is an outcome of the *strategic interactions* among *rational* supply chain members. Hence, the key difference between the previous work and ours is in the behaviors of members. Forrester *assumes* certain behaviors of the members, and Sterman's members lack full rationality and are prone to misperceptions, while the members in our model are rational and optimizing. We employ mathematical models to explain the outcome of rational decision making, as opposed to deriving as optimal decision rule for managers.

These differences in approaches lead to insights that have different implications for optimal practice and/or policy. Previous findings suggest that progress can be made in mitigating the bullwhip effect through modifications in behavioral practice (Forrester) and/or individual education (Sterman). Our results suggest that companies attempting to gain control of the bullwhip effect are better served by attacking the institutional and inter-organizational infrastructure and related processes.

Economists (e.g., Holt et al. 1960, Blinder 1982, and Blanchard 1983) also made a major contribution to our understanding of the bullwhip phenomenon. The role of inventory is to act as a buffer to smooth production in response to demand fluctuations. It enables the manufacturer to exploit economies in production and minimize total costs. This argument then suggests that the variance in the production time series should be smaller than the variance in the demand time series. Empirical studies based on macroeconomics data (e.g., Blanchard 1983), however, show the opposite result—the variance of production is greater than that of demand. To explain the discrepancies, Caplin (1985) and Blinder (1982, 1986) show that the use of (s, S) type inventory policies by retailers results in the variance of replenishment orders exceeding the variance of demand. Kahn (1987) shows that the presence of positive serial correlation in demand and backlogging also results in the bullwhip effect. Our work complements the above works, but it differs from them in that it approaches the issue from a managerial, as opposed to macroeconomics, perspective. It describes multiple sources of the bullwhip effect within the framework of classical inventory theory developed in the production and operations management literature.

3. The Causes of the Bullwhip Effect

This section describes four causes of the bullwhip effect—demand signal processing, the rationing game, order batching, and price variations. To see how these causes contribute to the bullwhip effect, we start with an idealized situation. Consider a multiperiod inventory system that is operated under a periodic review policy. Suppose that the following conditions hold

(i) past demands are not used for forecasting (e.g., when demands are stationary),

(ii) resupply is infinite with a fixed lead time,

(iii) there is no fixed order cost, and

(iv) purchase cost of the product is stationary over time.

It is clear that the above conditions give rise to the familiar result that the order-up-to-S policy is optimal, where S is a constant. With such a result, the order quantity in each period is exactly equal to the demand of the previous period, and hence orders and demand have the same variance. It turns out that the relaxation of these four conditions, one at a time, corresponds to the four causes of the bullwhip effect. Demand signal processing refers to the situation where demand is nonstationary and one uses past demand information to update forecasts. The rationing game refers to the strategic ordering behavior of buyers when supply shortage is anticipated. When the fixed order cost is nonzero, ordering in every period would be uneconomical, and batching of orders would occur. Finally, price variations refer to nonconstant purchase prices of the product.

Demand signal processing and order batching are inter-related since they are driven by each member trying to optimize internal operations of inventory management. The rationing game and price variations are also related to each other since they both reflect the member's reaction to the market dynamics.

3.1. Demand Signal Processing

We here focus on the retailer-supplier relationship, although the analysis is applicable to the wholesaler-distributor or the distributor-manufacturer relationships as well. Consider a multiperiod inventory model where demand is nonstationary over time, and demand forecasts are updated based on observed demand. With nonstationary demand, the order-up-to point for period t for the inventory system would also be nonstationary. Suppose, for example, that the retailer experiences a surge of demand in one period. She will interpret it as a signal of high future demand, adjust the demand forecast, and place a large order. Examples of such demand signaling models include the Bayesian updating model by Azoury (1985), and serially correlated demand models of Kahn (1987) and Miller (1986).

Let S_t be the order-up-to point for period t. If the demand surge happened in period $t-1$, then in period t the retailer will order a quantity to bring the inventory back to the original level S_{t-1}, plus an additional quantity $\Delta_t > 0$ to reflect the update of her future demands, which leads to a different order-up-to point S_t $(=S_{t-1} + \Delta_t)$. A symmetric argument yields that a low demand observed by the retailer will translate to an order that is lower than the original low demand. Either way the demand observed at the retailer is transmitted to the supplier in an exaggerated form. As the retailer processes demand signals, the original sales information is distorted and its variance amplifies when passed upstream to the supplier. Additionally, a long lead time in replenishing orders from upstream tends to aggravate the distortion even further.

To pursue the argument more rigorously, we consider a retailer's single-item multiperiod inventory problem (as in Heyman and Sobel 1984, p. 75). The retailer orders a single item from a supplier each period. There is a delay of ν periods between ordering and receiving the goods representing the order lead time and the transit time from the supplier to the retailer's site. To simplify the analysis, we also assume that excess inventory can be returned without cost. (Kahn 1987 also implicitly makes this assumption.)

The timing of the events is as follows. At the beginning of period t, a decision to order a quantity z_t is made. We call this time point the "decision point" for period t. Next, the goods ordered ν periods ago arrive. Lastly, demand is realized, and the available inventory is used to meet the demand. Excess demand is backlogged. Let h, π, and c denote the unit holding cost, the unit shortage penalty cost and the unit ordering cost, respectively. We represent the decision variable as S_t, which is the amount is stock plus on order (including those in transit) after the decision z_t has been made in period t. Let β be the cost discount factor per period. We will use the notation x' to denote $\max(0, x)$.

Also, borrowing Kahn's (1987) demand model, we assume that the retailer faces serially correlated demands which follow the process

$$D_t = d + \rho D_{t-1} + u_t,$$

Where D_t is the demand in period t, ρ is a constant satisfying $-1 < \rho < 1$, and u_t is independent and identically normally distributed with zero mean and variance σ^2. We assume that σ is significantly smaller than d, so that the probability of a negative demand is negligible.

The cost-minimization problem in an arbitrary period (normalized at 1) is formulated as

$$\min_{(S_t)} \left[\sum_{t=1}^{\infty} \beta^{t-1} E_1 \left[c z_t + \beta^\nu g\left(S_t, \sum_{i=t}^{t+\nu} D_i \right) \right] \right]$$

where

$$g\left(S_t, \sum_{i=t}^{t+\nu} D_i\right) = h \cdot \left(S_t - \sum_{i=t}^{t+\nu} D_i\right)^+ + \pi \cdot \left(\sum_{i=t}^{t+\nu} D_i - S_t\right)^+;$$

i.e., the discounted holding and shortage cost ν periods later (see Heyman and Sobel 1984, pp. 75–78, for its derivation). The expectation operator E_1 denotes the expectation taken at the decision point of period 1, conditional on the demand realizations D_i, $i = 0, -1, -2$.

Theorem 1. *In the above setting, we have*

(a) *If $0 < \rho < 1$, the variance of retail orders is strictly larger than that of retail sales; i.e.,* $\mathrm{Var}(Z_1) > \mathrm{Var}(D_0)$.

(b) *If $0 < \rho < 1$, the larger the replenishment lead time, the larger the variance of orders, i.e.,* $\mathrm{Var}(z_1)$ *strictly increases in* ν.

Proof. Heyman and Sobel (1984, pp. 75–78) show that the program can be solved by solving

$$\min_{(S_t)} \sum_{t=1}^{\infty} \beta^{t-1} E_1[G(S_t)],$$

where $G(S) = c(1-\beta)S + \beta^\nu E_1[g(S, \sum_{i=1}^{\nu+1} D_i)]$. Also, they show that

$$S_1^* = Q_{\nu+1}^{-1}\left[\frac{\pi - c(1-\beta)/\beta^\nu}{h+\pi}\right], \tag{3.1}$$

where $Q_{\nu+1}$ denotes the distribution function of $\sum_{i=1}^{\nu+1} D_i$ (conditional on the demand record up to period 0, in our case).

Now note that

$$\begin{aligned} D_k &= d + \rho D_{k-1} + u_k = d(1+\rho) + \rho^2 D_0 + (\rho u_{k-1} + u_k) \\ &= \cdots = d\frac{1-\rho^k}{1-\rho} + \rho^k D_0 + \sum_{i=1}^{k} \rho^{k-i} u_i \quad \text{for } k \geq 1. \end{aligned}$$

Thus, $\sum_{i=1}^{\nu+1} D_i$ at the decision point in period 1 is a random variable distributed according to $N(M, \Sigma)$, where

$$M := d \sum_{k=1}^{\nu+1} \frac{1-\rho^k}{1-\rho} + \frac{\rho(1-\rho^{\nu+1})}{1-\rho} D_0,$$

and $\Sigma := \sum_{k=1}^{\nu+1} \sum_{i=1}^{k} \rho^{2(k-i)} \sigma^2$. Thus, from (3.1), we have

$$\begin{aligned} S_1^* &= d \sum_{k=1}^{\nu+1} \frac{1-\rho^k}{1-\rho} + \frac{\rho(1-\rho^{\nu+1})}{1-\rho} D_0 \\ &\quad + K^* \sigma \sqrt{\sum_{k=1}^{\nu+1} \sum_{i=1}^{k} \rho^{2(k-i)}}, \end{aligned} \tag{3.2}$$

where

$$K^* = \Phi^{-1}\left(\frac{\pi - c(1-\beta)/\beta^\nu}{h+\pi}\right)$$

for the standard normal distribution function Φ. From (3.2), the optimal order amount z_1^* is given by

$$z_1^* = S_1^* - S_0^* + D_0 = \frac{\rho(1-\rho^{\nu+1})}{1-\rho}(D_0 - D_{-1}) + D_0. \tag{3.3}$$

On the RHS of the last equation, the first term denotes the adjustment of expected demand based on D_0, while the second term is the one-for-one replenishment of the inventory demanded in the last period.

Thus, we have

$$\begin{aligned} \mathrm{Var}(z_1) &= \mathrm{Var}(D_0) + \left(\frac{\rho(1-\rho^{\nu+1})}{1-\rho}\right)^2 \mathrm{Var}(D_0 - D_{-1}) \\ &\quad + 2\frac{\rho(1-\rho^{\nu+1})}{1-\rho} \mathrm{cov}(D_0 - D_{-1}, D_0). \end{aligned} \tag{3.4}$$

Noting the independence between D_{-1} and u_0, it can be shown that $\text{Var}(D_0) = \text{Var}(D_{-1}) = \sigma^2/(1-\rho^2)$, $\text{Var}(D_0 - D_{-1}) = 2\sigma^2/(1+\rho)$, and $\text{cov}(D_0 - D_{-1}, D_0) = \sigma^2/(1+\rho)$.

Hence,

$$\text{Var}(z_1) = \text{Var}(D_0) + \frac{2\rho(1-\rho^{\nu+1})(1-\rho^{\nu+2})}{(1+\rho)(1-\rho)^2}\sigma^2 > \text{Var}(D_0). \quad (3.5)$$

This proves part (a). Part (b) is also straightforward from (3.5). □

The theorem says that the variance amplification takes place when the retailer adjusts the order-up-to level based on the demand signals. Also, the degree of amplification increases in the replenishment lead time. Note that, when $0 < \rho < 1$ and $\nu = 0$, we have $\text{Var}(z_1) = \text{Var}(D_0) + 2\rho$, showing that demand variability amplification exists, even when lead time is zero.

The managerial implications of the theorem become apparent in the light of the institutional settings of a traditional distribution channel. Typically, an upstream supplier relies only on the order data from the downstream retailer. Our result shows that such an arrangement will cause the supplier to lose track of the true demand pattern at the retail end. Moreover, the supplier's inventory control based on this distorted information will inevitably suffer. This highlights the needs for information sharing among supply chain members. If sales and inventory data are shared among chain members, the supply chain as a whole can implement echelon-based inventory control which can yield superior performance to installation-based inventory control.

The theorem also implies that the bullwhip effect will propagate in an amplified form upwards the supply chain if each member in the supply chain processes order signals coming from below. Thus, "double forecasting" can be a key driver of the bullwhip effect.

This result sheds some light on the interpretation of the bullwhip phenomenon in the beer game experimental setting. Sterman (1989) attributes it to the "misperceptions of feedback" by the players. More specifically, the players tend to disregard the inventory in pipeline they ordered earlier and keep on ordering more. While we (from our personal experience with the game) agree that this is a part of what is happening in the experiment, the theorem proposes an alternative hypothesis that the bullwhip phenomenon of the beer game may be driven by rational decision making. Given insufficient information about the demand pattern, the players justifiably process demand signals. This could create the bullwhip phenomenon, and three periods of replenishment lead time may aggravate it.

It should be admitted that the "costless return" assumption is not a traditional one in the inventory literature. One may ask whether and when this technical assumption plays a critical role in driving the result. To answer this, note that the assumption is violated when $z_1^* < 0$. But straightforward algebra shows that

$$z_1^* = \frac{\rho(1-\rho^{\nu+1})}{1-\rho}(D_0 - D_{-1}) + D_0$$
$$= E_1[D_{\nu+1}] + \frac{1-\rho^{\nu+1}}{1-\rho}u_0.$$

Thus, the assumption is violated if the error term u_0 falls sufficiently below zero to offset the positive value $E_1[D_{\nu+1}]$. But recall that the normality assumption of u_t subsumes that the mean of the demand is significantly larger than the standard deviation to avoid a negative demand. In particular, if $\nu = 0$, $z_1^* = E_1[D_1] + u_0$. Hence, the costless return assumption may not be critical to the theorem, if the replenishment lead time ν, the variance σ^2 and the correlation factor ρ are small, and/or d is large.

3.2. The Rationing Game

Consider a product whose demand potentially exceeds supply due to limitation in production capacity or uncertainty of production yield. Under the shortage situation, the manufacturer would ration the supply of the product to satisfy the retailers' orders. To try to secure more units, each retailer will issue an order which exceeds in quantity what the retailer would order if the supply of the product is unlimited. Below we develop a simple one-period model (an extended newsvendor model) with multiple retailers to illustrate that indeed this is what the retailers would do rationally under the situation.

A manufacturer supplies a single product to N identical retailers indexed by $n = 1, 2, \ldots, N$. Retailer n first observes her demand distribution $\Phi(\cdot)$ and places an order z_n at time 0. Then, the manufacturer delivers the product at time 1. The manufacturer's output μ is a random variable, distributed according to $F(\cdot)$.

In case the total amount of orders $\sum_{n=1}^{N} z_n$ exceeds the realized output μ, the manufacturer allocates the output to retailers in proportion to their orders. The order quantity from retailer i is denoted by z_i, and the total retail orders are denoted by Q (i.e., $Q := \sum_j z_j$). If the realized capacity μ is smaller than Q, retailer i will receive $z_i\mu/Q$ due to the allocation.

Let $C_i(z_1, \ldots, z_i, \ldots, z_N)$ denote the expected cost to retailer i when retailer i chooses the order quantity z_i for $i = 1, 2, \ldots, N$. A Nash equilibrium is defined as the order quantities $(z_1^*, z_2^*, \ldots, z_N^*)$ chosen by retailers who each takes others' decisions as given and chooses z_i to minimize the expected cost. That is, $(z_1^*, z_2^*, \ldots, z_N^*)$ is a Nash equilibrium if, for each $i = 1, 2, \ldots, N$,

$$z_i^* = \arg\min_{z_i} C_i(z_1^*, \ldots, z_{i-1}^*, z_i, z_{i+1}^*, \ldots, z_N^*). \quad (3.6)$$

Thus, if the first-order condition approach is valid, the equilibrium must satisfy for each i:

$$\frac{dC_i}{dz_i}(z_1^*, \ldots, z_{i-1}^*, z_i, z_{i+1}^*, \ldots, z_N^*)\Big|_{z_i = z_i^*} = 0. \quad (3.7)$$

Since every retailer is symmetric, we will focus on the symmetric Nash equilibrium where $z_i^* = z^*$ for each $i = 1, 2, \ldots, N$. That is, z^* satisfies

$$\frac{dC_i}{dz_i}(z^*, \ldots, z^*, z_1, z^*, \ldots, z^*)\Big|_{z_i = z^*} = 0. \quad (3.7')$$

To derive the equilibrium, consider retailer i who takes the other retailers' ordering strategies z_j^* $(j \neq i)$ as given and chooses z_i to minimize the expected cost

$$\begin{aligned} C_i = \int_{\mu=0}^{Q} \bigg[p \int_{\mu z_i/Q}^{\infty} \left(\xi - \frac{\mu z_i}{Q} \right) d\Phi(\xi) \\ + h \int_0^{\mu z_i/Q} \left(\frac{\mu z_i}{Q} - \xi \right) d\Phi(\xi) \bigg] dF(\mu) \\ + (1 - F(Q)) \bigg[p \int_{z_i}^{\infty} (\xi - z_i)\, d\Phi(\xi) \\ + h \int_0^{z_i} (z_i - \xi)\, d\Phi(\xi) \bigg], \end{aligned} \quad (3.8)$$

where $Q = \sum_{j \neq i} z_j^* + z_i$. Note that the decision z_i must be taken before the capacity μ is realized. Hence, there are two possible scenarios depending on the supply condition. One is when the supply μ falls short of the total demand Q, and retailer i is allocated the amount $z_i \mu / Q$. The first term on the RHS of (3.8) is the expected cost times the probability under this shortage scenario. The other scenario is when the realized supply is sufficient to meet the total demand. The second term on the RHS of (3.8) is the expected cost times the probability of this scenario.

Its first order condition is given by

$$\begin{aligned} \frac{dC_i}{dz_i} = \int_{\mu=0}^{Q} \left[-p + (p+h)\Phi\left(\frac{\mu z_i}{Q} \right) \right] \mu \left(\frac{1}{Q} - \frac{z_i}{Q^2} \right) dF(\mu) \\ + [1 - F(Q)][-p + (p+h)\Phi(z_i)] = 0, \end{aligned} \quad (3.9)$$

where we used Leibnitz's rule and $dQ/dz_i = 1$. To establish the pseudo-convexity of C_i, consider C_i at z_i^0 satisfying Equation (3.9). It must be that $-p + (p+h)\Phi(z_i^0) \geq 0$, for otherwise, we would have $-p + (p+h)\Phi(\mu z_i^0/Q) < 0$ for all $\mu \leq Q$, resulting in $dC_i/dz_i < 0$. Hence, in order to have $dC_i/dz_i = 0$ at z_i^0, we must have $-p + (p+h)\Phi(z_i^0) \geq 0$. Now, consider the second derivative of C_i w.r.t. z_i at z_i^0:

$$\begin{aligned} \frac{d^2 C_i}{dz_i^2} = [-p + (p+h)\Phi(z_i^0)] f(Q) \\ + [1 - F(Q)](p+h)\phi(z_i^0) \geq 0. \end{aligned}$$

This establishes the pseudo-convexity of C_i. It follows that the solution z_i^0 to Equation (3.9) is the optimal order quantity z_i^*.

From (3.7)′ and (3.8), therefore, the symmetric equilibrium z^* must satisfy

$$\begin{aligned} \int_0^{N \cdot z^*} \left[-p + (p+h)\Phi\left(\frac{\mu}{N} \right) \right] \mu \left(\frac{1}{N \cdot z^*} - \frac{1}{N^2 \cdot z^{*2}} \right) dF(\mu) \\ + [1 - F(z^* \cdot N)][-p + (p+h)\Phi(z^*)] = 0. \end{aligned} \quad (3.10)$$

Theorem 2. *In the above setting, $z' \leq z^*$, where z' is the solution of the newsvendor problem.*[3] *Further, if $F(\cdot)$ and $\Phi(\cdot)$ are strictly increasing, the inequality strictly holds.*

The theorem states that the optimal order quantity for the retailer in the rationing game exceeds the order quantity in the traditional newsvendor problem. This in turn implies the bullwhip effect when the mean demand changes over time. Retailers' equilibrium order quantity may be identical or close to the newsvendor solution for low-demand periods, while it will be larger than the newsvendor solution for high-demand periods. Hence, the variance is amplified at the retailer.

This is prominent when rationing is combined with demand signal processing. Consider the same setting as §3.1, where each retailer processes demand signals. Suppose a retailer observes a significant increase in sales. The retailer processes demand signal and orders more than observed sales. In addition, she anticipates that other retailers may have observed a similar signal and have issued large orders to the manufacturer. She assesses a positive probability of supply shortage and thus also issues a larger order. Hence, a large retail sale leads to an even larger order, due to the combination of demand signal processing and shortage anticipation. Note that the rationing game is triggered only at an upswing of demand. Overall, the rationing game amplifies the bullwhip effect even more than in the presence of demand signal processing only.

To see the impact of the rationing game on variance amplification, consider a supply chain consisting of three layers—a manufacturer, multiple wholesalers, and multiple retailers. If the manufacturer appears in short supply, wholesalers will play the rationing game to get a large share of the supply. Assessing

[3] The newsvendor solution z satisfies $-p + (p+h)\Phi(z) = 0$.

a possibility of the wholesaler not getting enough from the manufacturer, retailers also play the same game. The demand and its variance are amplified as one moves up the supply chain. From the members' rational behaviors, the supply chain experiences variance amplification.

3.3. Order Batching

Consider a periodic review stationary demand system with full backlogging at a retailer. The retailer would thus use an order up to level to monitor its inventory. This implies that he would order an amount equal to the previous review cycle's demand in every review cycle.

Suppose there exist N retailers each using a periodic review system with the review cycle equal to R periods. Suppose that the demands for retailer j in period k is ξ_{jk}, which is i.i.d. with mean m and variance σ^2 for each retailer. Depending on whether and how retailers' order cycles are independent or correlated, consider three cases—(a) random ordering, (b) (positively) correlated ordering, and (c) balanced ordering.

Case 1. *Random Ordering.* Demands from retailers are independent. Let n be a random variable denoting the number of retailers who order in a randomly chosen period. Thus, n is a binomial variable with parameters N and $1/R$. Hence, $E(n) = N/R$ and $\text{Var}(n) = N(1/R)(1-1/R)$. Also, let Z_t^r denote the total orders from n retailers in period t, i.e.,

$$Z_t^r := \sum_{j=1}^{n} \sum_{k=t-R}^{t-1} \xi_{jk}.$$

Then we have

$$\begin{aligned} E(Z_t^r) &= E[E(Z_t^r \mid n)] = E[nRm] = Nm, \quad \text{and} \\ \text{Var}(Z_t^r) &= E[\text{Var}(Z_t^r \mid n)] + \text{Var}\,[E(Z_t^r \mid n)] \\ &= E[nR\sigma^2] + \text{Var}\,[nRm] \\ &= N\sigma^2 + R^2 m^2 \frac{N}{R}\left(1 - \frac{1}{R}\right) \\ &= N\sigma^2 + m^2 N(R-1) \ge N\sigma^2 \end{aligned} \tag{3.11}$$

If $R = 1$, then the variance of demand as seen by the supplier (orders placed by the retailer) would be the same as the retailer's demand. As R increases, the variance of demand as seen by the supplier increases. Also $\text{Var}(Z_t^r)$ increases in N, while the coefficient of variation decreases in N.

Case 2. *Positively Correlated Ordering.* For positively correlated ordering, we consider the extreme case in which all retailers order in the same period (e.g., when R is a week, all retailers order on Monday with probability $1/R$, and not on other days of the week).

Here,

$$\Pr\{n = i\} = \begin{cases} 1 - 1/R & \text{for } i = 0; \\ 1/R & \text{for } i = N, \\ 0 & \text{for otherwise,} \end{cases}$$

with $E(n) = N/R$ and $\text{Var}(n) = N^2/R(1-1/R)$.

Letting $Z_t^c := \sum_{j=1}^{n} \sum_{k=t-R}^{t-1} \xi_{jk}$,

$$\begin{aligned} E(Z_t^c) &= E[E(Z_t^c \mid n)] = E[nRm] = Nm, \quad \text{and} \\ \text{Var}(Z_t^c) &= N\sigma^2 + R^2 m^2 \frac{N^2}{R}\left(1 - \frac{1}{R}\right) \\ &= N\sigma^2 + m^2 N^2 (R-1). \end{aligned} \tag{3.12}$$

Case 3. *Balanced Ordering.* The other extreme case of correlated orders is when orders from different retailers are evenly distributed in time. For ease of exposition, suppose $N = MR + k$, where M and k are integers and $0 \le k \le R$. Consider a coordination scheme such that all N retailers are divided into R groups: k groups each of size $(M+1)$, and $(R-k)$ groups each of size M. Then, all retailers in the same group place orders in a designated period within the review cycle, and no two groups order in the same period. For example, if a review cycle is a week (five days) and there are 23 retailers, then $(N, M, R, k) = (23, 4, 5, 3)$ since $23 = 4 \times 5 + 3$. Three groups each consisting of five retailers respectively order every Monday, Tuesday and Wednesday, while two groups of size four respectively order every Thursday and Friday.

Here,

$$\Pr\{n = i\} = \begin{cases} 1 - k/R & \text{for } i = M; \\ k/R & \text{for } i = M+1; \\ 0 & \text{otherwise.} \end{cases}$$

Note that

$$\begin{aligned} E(n) &= M(1-k/R) + (M+1)k/R = N/R \quad \text{and} \\ \text{Var}(n) &= (1-k/R)M^2 + (M+1)^2 k/R - (N/R)^2 \\ &= (k/R)(1-k/R). \end{aligned}$$

Letting $Z_t^b := \sum_{j=1}^{n} \sum_{k=t-R}^{t-1} \xi_{jk}$,

$$\begin{aligned} E(Z_t^b) &= E[E(Z_t^b \mid n)] = E[nRm] = Nm, \quad \text{and} \\ \text{Var}(Z_t^b) &= N\sigma^2 + R^2 m^2 \frac{k}{R}\left(1 - \frac{k}{R}\right) \\ &= N\sigma^2 + m^2 k(R-k). \end{aligned} \tag{3.13}$$

Since $k(R-k) \le N(R-1)$ for each $k = 1, 2, \ldots, R$,

$$\begin{aligned} N\sigma^2 + m^2 k(R-k) &\le N\sigma^2 + m^2 N(R-1) \\ &\le N\sigma^2 + m^2 N^2 (R-1), \end{aligned}$$

which gives the following result.

Theorem 3. (a) $E[Z_t^c] = E[Z_t^r] = E[Z_t^b] = Nm$,
(b) $\mathrm{Var}[Z_t^c] \geq \mathrm{Var}[Z_t^r] \geq \mathrm{Var}[Z_t^b] \geq N\sigma^2$, *where* Z_t^c, Z_t^r, *and* Z_t^b *are the random variables denoting the orders from N retailers, respectively, under correlated ordering, random ordering and balanced ordering.*

The theorem states that different ordering patterns generate the same expected amount of orders with different variances. Correlated ordering with all orders falling in the same period has the highest variability, while balanced ordering has the lowest variability and random ordering comes in between. In all cases, the variability of demand experienced by the supplier is higher than that experienced by the retailers (i.e., the bullwhip effect exists).

When $N = mR$, or $k = 0$, then "perfectly balanced" or "completely synchronized" retailer ordering can be achieved. Under that scenario (and only then), the variability experienced by the supplier and the retailers are identical, and the bullwhip effect disappears.

Unfortunately, the first and worst type of correlated ordering pattern is quite common in practice as a result of institutional habits. A majority of assembly plants operating on a monthly production planning cycle run MRP or DRP systems and issue orders almost at the same time. This generates the "MRP jitters" or the "hockey stick" phenomenon to manufacturers, who, for example, see more than 70% of their monthly orders arrive in the last week. In addition, salespeople tend to rush and close deals towards the end of the quarter to meet their quarterly sales target. This can also lead orders from different salespeople to be positively correlated.

The batching effect can be replicated and amplified at upstream nodes. That is, as batching at the retail level increases the variance of the demand to wholesalers, batching at the wholesale level may also increase the variance of the demand to the distributor or the manufacturer. Again, for rational reasons, the variance of orders is amplified upstream in the supply chain.

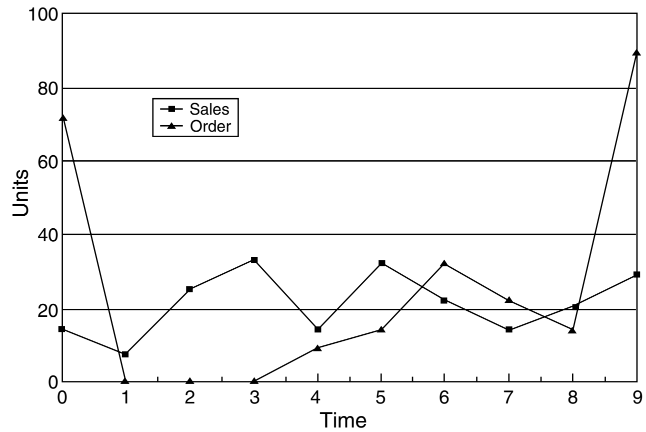
3.4. Price Variations

A retailer faces independent and identically distributed demand with density function $\phi(\cdot)$ each period. Her sole manufacturing source alternates between two prices c^L and c^H over time, where $c^L < c^H$. With probability q (or $1-q$, respectively) the price in a period will be c^L (or c^H, respectively). Facing the price variations, the retailer's inventory problem can be formulated as

$$V^i(x) = \min_{y \geq x}\Big[c^i(y-x) + L(y) + \beta\int_0^x [qV^L(y-\xi) + (1-q)V^H(y-\xi)]\phi(\xi)\,d\xi\Big], \qquad (3.14)$$

where V^i $(i = H, L)$ denotes the minimal expected discounted cost incurred throughout an infinite horizon

Figure 2 **Sales vs. Order When Price Changes**



when the current price is c^i. $L(\cdot)$ is the sum of one-period inventory and shortage costs at a given level of inventory, and is given by[4]

$$L(y) = p\int_y^\infty (\xi - y)\phi(\xi)\,d\xi + h\int_0^y (y-\xi)\phi(\xi)\,d\xi.$$

The following result reports the optimal inventory policy.

Theorem 4. *The following inventory policy is optimal to the problem* (3.14): *At price* c^L, *get as close as possible to the stock level* S^L, *and at price* c^H *bring the stock level to* S^H, *where* $S^H < S^L$.

We now show that price fluctuations generate the bullwhip effect. We group periods by regenerative cycles. A regenerative cycle is defined as the interval of periods between two consecutive low-price periods, with the first low-price period excluded. See Figure 2 showing the initial inventory level, orders and sales over time. Here $S^H = 30$ and $S^L = 100$. In period 0 the manufacturer's price is low, and the retailer raises her inventory level to 100. Thus, the first period (period 1) in a regenerative cycle starts with a high inventory level $100 - \xi_0$. For the following periods, the manufacturer's price remains high, so the retailer does not order any until the inventory level hits below 30. In period 4 the inventory falls below 30, and the retailer orders a positive amount for the first time within the regenerative cycle. She orders only up to 30 and even afterwards maintains the same order-up-to level until the price goes down to c^L. In period 9 the price is reduced, and the regenerative cycle ends.

From this we can derive the following result.

Theorem 5. *In the above setting*, $\mathrm{Var}[z_t] > \mathrm{Var}[\xi]$.

[4] Here we keep the unit inventory cost h the same, regardless of the purchase price. In actuality, the unit inventory cost increases in the purchase price. We keep the present assumption to simplify the analysis. But the basic result of the section will remain the same or become even sharper if we adopt the more realistic assumption.

4. Managerial Implications and Concluding Remarks

The preceding section highlighted independent forces that lead to systematic distortions of sales information in the order-replenishment transactions of a standard supply chain. Identification of these forces aids the development of strategies to alleviate the detrimental impact of the bullwhip effect. The essential thrust of the following discussion is that a combination of activities are necessary to counter the bullwhip effect. These strategies require among other things: information sharing of sell-through and inventory status data, coordination of orders across retailers and simplification of the pricing/promotional activities of the manufacturer. Table 1 summarizes the contributing factors to the causes of the bullwhip effect, as well as the counter-measures and the state of practice in various industries.

4.1. Demand Signal Processing

Distortion of demand information arises when the retailer issues orders based on her updated demand forecast. As a result, the manufacturer loses sight of the true demand in the marketplace. The production schedule based on the distorted signals is inevitably inefficient. The distortion effect gets amplified as the number of intermediaries in the channel increase. One clear solution is to grant the manufacturer access to the demand data at the retail outlet. The grocery industry is a good illustration of demand information sharing. Electronic data interchange systems between retailers and manufacturers are becoming fairly common. These systems facilitate quick and easy transmission of demand data to upstream members of the channel. The computer industry is also making some progress in this regard. Manufacturers such as IBM, Hewlett-Packard and Apple are increasingly requesting sell-through data from their resellers.

Access to a common data set for forecasting purposes is not the total solution. Differences in forecasting methodologies will still lead to higher fluctuations in ordering and demand distortion. To eliminate the bullwhip effect, we can envision having a single member of the supply chain perform forecasting and ordering for other members. This way the supply chain can implement centralized multi-echelon inventory control system—known to be superior to independently operating site-based inventory control (Clark and Scarf 1960). The growing popularity of Vendor-Managed-Inventory (VMI) systems or Continuous Replenishment Programs (CRP) in the packaged goods industry attests to the efficiencies attainable through consolidated information processing. For instance, VMI is at the heart of the disposable diaper supply chain at P&G (stretching from their adhesive tape supplier 3M and ending with their customer Wal-Mart). Also, VMI is listed as one of the recommended practices in the Efficient Consumer Response (ECR) movement in the grocery industry (Crawford 1994).

As an aside, the direct marketing channel by a manufacturer represents a channel design that is not subject to the bullwhip effect created by demand signal processing by the distribution channel. The manufacturer in this instance has complete information on the demand pattern. Dell Computer's "Dell Direct" program and the "Consumer Direct" program of Apple Computers are good illustrations. The potential benefits of eliminating channel intermediaries has also been alluded to by Forrester earlier.

Since long replenishment leadtimes contribute to the bullwhip effect, shortening the leadtime is a direct and effective counter-measure. Lead time reduction has long been part of manufacturing strategy in various companies (Fisher 1994) and industries

Table 1 The Causes and Counter-Measures of the Bullwhip Effect

Causes	Contributing factors	Counter-measures	State of practice
Demand signaling	• No visibility of end demand	• Access sell-thru or POS data	• Sell-thru data in contracts (e.g., HP, Apple, IBM)
	• Multiple forecasts	• Single control of replenishment	• VMI (P&G and WalMart)
	• Long lead-time	• Lead-time reduction	• Quick Response mfg strategy
Order batching	• High order cost	• EDI & CAO	• McKesson, Nabisco
	• FTL economics	• Discount on assorted truckload, consolidation by 3rd party logistics	• 3rd party logistics in Europe, emerging in the US
	• Random or correlated ordering	• Regular delivery appointment	• P&G
Fluctuating prices	• High-low pricing	• EDLP	• P&G (resisted by some retailers)
	• Delivery & purchase asynchronized	• Special purchase contract	• Under study
Shortage game	• Proportional rationing scheme	• Allocate based on past sales	• Saturn, HP
	• Ignorance of supply conditions	• Shared capacity & supply information	• Scheduling sharing (HP, Motorola)
	• Unrestricted orders & free return policy	• Flexibility limited over time, capacity reservation	• HP, Sun, Seagate

(e.g., Quick Response in the apparel industry, Hammond 1990).

4.2. Rationing Game

Information distortion can arise as a consequence of strategic decisions by the retailer who assesses the possibility of being placed on allocation by the manufacturer. The order data has little or even negative informational value to the manufacturer, and he needs to exercise great care in interpreting the order signals for inventory/capacity planning.

To avoid the nonproductive gaming, one can design a different rule of allocating supply across retailers in a shortage situation. An alternative decision rule would be to allocate the supply in proportion to the retailer's market share in the previous period (i.e., retailer's share of total sales of the product). General Motors, for example, allocates products in shortage to dealers based on historic sales records. Companies such as Texas Instruments and Hewlett-Packard in the computer industry also use this method when allocation of fast moving products is necessary.

Note also that gaming can also arise due to retailers' self-protection against imaginary shortage, as opposed to real shortage. This can be prevented to a degree if the manufacturer shares production and inventory information with downstream members of the supply chain. Access to accurate information as opposed to conjecture can go a long way in diluting the motivation for gaming. Note that this alone will not resolve the resource contention problem to everyone's satisfaction in real shortage situations.

A more efficient resolution comes in the form of a contract that restricts the buyer's flexibility, since an unrestricted choice of order quantities, free return and generous order cancellation policies all contribute to gaming. For example, a common form of contract operates as follows: A buyer starts transmitting its demand forecast 18 weeks ahead of the delivery. The buyer updates the initial forecast in four weeks, but is only allowed to change up to 30% of the initial forecast. Four weeks later, the buyer updates the forecast one more time, but up to 15% of the second forecast. The third forecast becomes the binding order to the supplier. This method of sharing forecast information, risk and flexibility is observed in computer and retail companies like Hewlett-Packard, SUN, and Lands' End with their suppliers.[5] Also some companies like Seagate reserve the supplier's capacity or intermediate goods well ahead of time, so that the buyer and the supplier share risk and demand information.

[5] For mathematical analysis of such a system, see Bassok and Srinivasan (1994) and Tsay and Lovejoy (1995).

4.3. Order Batching

Batching of orders is a consequence of two factors: the periodic review process and the processing cost of a purchase transaction. Demand distortion due to the periodic review process can be alleviated by providing the manufacturer with access to sell-through data and/or inventory data at the retail level. The manufacturer uses this information to create a production schedule that is determined by sales as opposed to orders.

Another approach to mitigate the batching effect is to reduce the need for order batching by lowering the transactions cost (from the logic of EOQ model), a big portion of which is due to the paperwork and processing requirement in generating an order. EDI-based order transmission systems are a big help in reducing the ordering costs and batch size. For instance, Nabisco uses such a system to perform computer assisted ordering (CAO) which is paperless. McKesson's "Economost" ordering system is another example of reducing ordering cost for druggists (Row and Clemons 1988). A result of the system is more frequent replenishment in small batches, which in turn leads to less distortion of demand information and more efficient delivery/production schedules.

Manufacturers can influence buyers' batching decisions in other ways. One is to allow retailers to order an assortment of products to fill a truckload and offer the same volume discount. For instance, P&G allows the shipment of mixed truckloads of virtually all of its products from its plants. Another approach is coordination of delivery schedules. This moves the channel away from random or correlated ordering to balanced ordering. For instance, manufacturers like P&G reserve certain time slots (e.g., the first and third Tuesday of each month) at the buyer's gate for deliveries. The manufacturer does not only avoid queuing delays, but also level deliveries to retailers across time, thereby reducing the batching effect as experienced by the manufacturer.

A more interesting institutional development is the growth of third party logistics providers. These companies are in a position to exploit scale economies that were not accessible in traditional single buyer-seller relationships. For instance, it is now possible to create a full truckload by consolidating orders from retailers who are in geographical proximity to one another. This way while each individual buyer could find it uneconomical to generate a full truckload order by herself, consolidation allows the third party logistics provider the opportunity to do so inexpensively.

4.4. Price Variations

One way to control the bullwhip effect due to price fluctuation is to reduce the frequency as well as depth of manufacturer's trade promotions (i.e., wholesale price discounts). This prescription is at the core of the recent shift by major manufacturers (e.g., P&G, Kraft, Pillsbury, etc.) to the Every Day Low Price (EDLP) strategy in spite of the resistance by several grocery chains.

The use of practices such as VMI and CAO, in addition to a rationalized wholesale pricing policy, further reduces the benefits from forward-buying and diversion. It is also true that retailers are not always well served by the use of strategic buying. Some industry experts argue that the inventory carrying cost is not properly assessed in the overall profit analysis, so the benefit of such buying activities is often bloated. As a remedy, an activity based costing (ABC) practice can help explicitly recognize the excessive costs associated with forward-buying and diversion. Thus, the spread of ABC can help spread the EDLP cause.

The reason for strategic buying is for the buyers to capitalize on the discount offered during a short period of time, while the pain to the manufacturer is the uneven production schedule, unnecessary inventory costs and distorted demand information. In fact, the buyer's gain and the manufacturer's pain do not necessarily have to go together. The manufacturer may keep the high-low pricing practice, but instead synchronize purchase and delivery schedules. That is, the manufacturer and the buyer can sign a purchase contract, according to which the buyer agrees to buy a large quantity of goods at a discount, but the goods are delivered in multiple future time points evenly separated. This way, the manufacturer can plan production more efficiently, the buyer can enjoy her strategic buying practice, and both parties can save inventory carrying costs.

4.5. Summary

We claim that demand distortion may arise as a result of optimizing behaviors by players in the supply chain. On the normative side, the combination of sell through data, exchange of inventory status information, order coordination, and simplified pricing schemes can help mitigate the bullwhip effect. Traditionally, sales data and inventory status data have been considered to be proprietary to retailers with no obligation or reason to share it with others. But the prescription for overcoming the bullwhip effect demands that the manufacturer be given access to these data. This paper suggests this shift in attitude is desirable to the *manufacturer*, but it is silent about the interesting and challenging question of why the *retailer* should provide the manufacturer with the data. In theory, the net benefit from efficient supply chain management can be redistributed among members. The subject of how to split the gain and cost appears to deserve attention of its own.

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