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FAKULTÄT II
FINANCE AND BANKING

Introduction to Corporate Finance: Discounted Cash Flow Valuation

Readings:

Hillier et al., Chapter 4

Overview of Lecture

- Valuation: The One-Period Case
- Valuation: The Multi-Period Case
- Compounding Periods
- Simplifications
- What is a Firm Worth?

Valuation

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Key question underlying this part of the course:

How do we know whether a particular business decision
(investing in an R&D project, launching a marketing campaign, selling property…)
“adds value”?

Valuation: The One-Period Case
An Example

- You are trying to sell a piece of undeveloped land in Wales. There are two potential buyers who offer the following:
  - A offers to pay £10,000 for the property immediately.
  - B offers to buy the property for £11,424 in exactly one year from now.

- Both buyers are honest and financially solvent, so you have no fear that the offer you select will fall through.

- Which offer should you choose?

Valuation: The One-Period Case
The Time Value of Money

- Basically, the underlying question is: How do we evaluate cash flows occurring at different points in time?

- Cash flows (i.e., money) can always be exchanged for goods and services. Thus, they represent consumption opportunities.

- Most people are impatient: They prefer consuming a good or a service today to consuming it tomorrow.

- This marginal preference for present over future consumption is called "time preference".

⇨ Cf. Irving Fisher, The Theory of Interest, 1930 (reprinted 1997), pp. 61-62

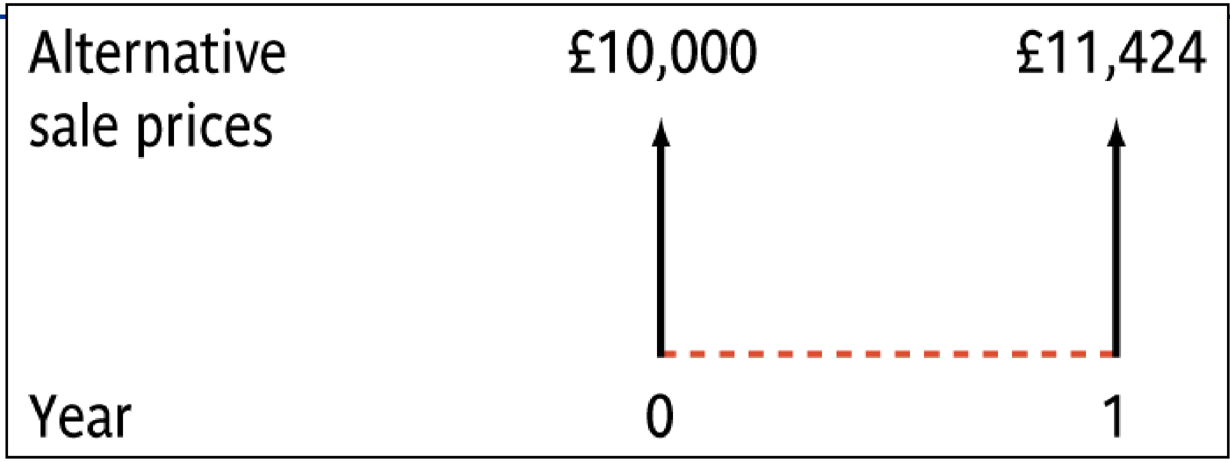
Valuation: The One-Period Case
The Time Value of Money

- Time preference implies that if we decide not to consume something today, we want to consume more of it in the future.
- Regarding money, our preference for present cash flows over future cash flows is called the "time value of money".
- The price at which we exchange present money for future money is called the interest rate.
- Thus, when comparing cash flows occurring at different points in time, we have to take into account the time value of money, which is expressed by the interest rate.

⇨ Cf. Irving Fisher, The Theory of Interest, 1930 (reprinted 1997), pp. 61-62

Valuation: The One-Period Case
Solving the Example

- The offers:

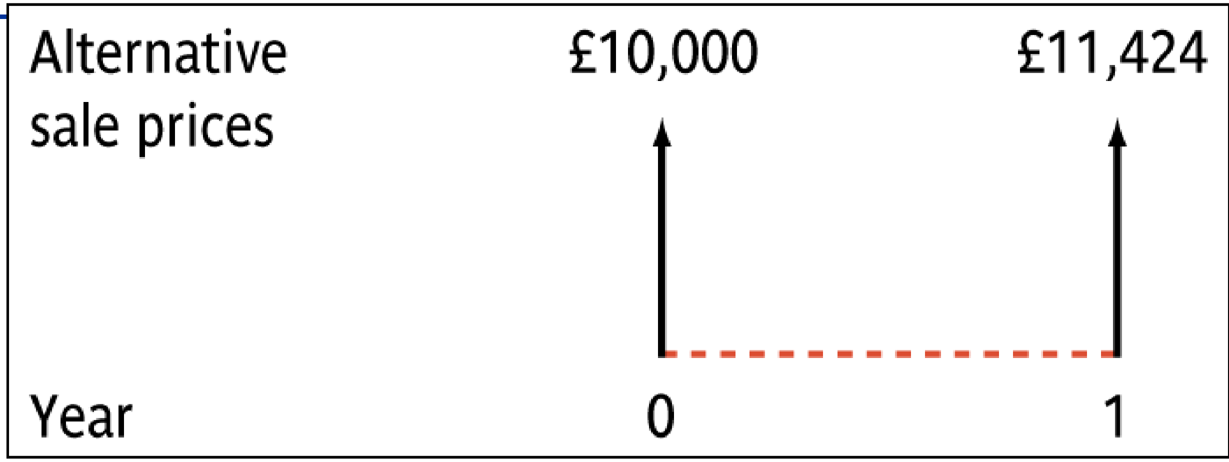


- Assume that your bank's interest rate on deposits is 12%
- In this case, you could sell the land to A today and invest £10,000 at 12%. At the end of the year, you would have: £10,000 + (0.12 x £10,000) = £10,000 x 1.12 = £11,200
- £11,200 is the future value (FV), or compound value, of this investment strategy

↳ Compared with £11,424, selling to investor B is better

Valuation: The One-Period Case
Solving the Example

- The offers:



- Alternatively, you could ask how much money you would have to invest today at 12% to receive £11,424 in one year. This yields the present value (PV) of B's offer:
  PV x 1.12 = £11,424 ⇔ PV = £11,424 / 1.12 = £10,200

↳ Compared with £10,000 now from buyer A, selling to investor B is better (again)

⇨ Both approaches give the same decision

General Formulae for Single Period Valuation

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$$FV = C_0(1+r)$$

$$PV = \frac{C_1}{1+r}$$

where *FV* Future value of a cash flow

PV Present value of a cash flow

C_t Cash flow at date *t*

r Rate of return that you require on your investment. It is also referred to as the discount rate.

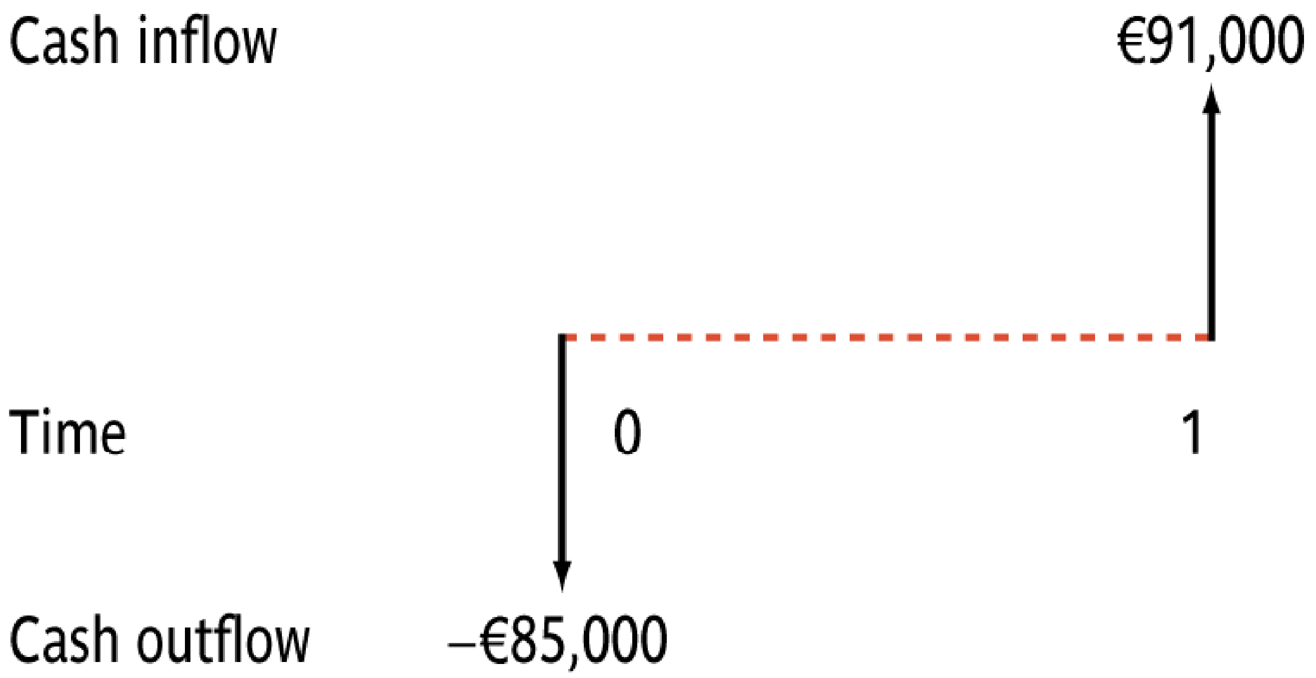
Another Example

- You are a financial analyst at a leading German real estate firm. You are thinking about recommending to invest in a piece of land that costs €85,000.

- You are certain that next year the land will be worth €91,000, a sure €6,000 gain.

- Given that the guaranteed interest rate in the bank is 10 percent, should your company undertake the investment in land?

Solution



$$\text{Present value} = \frac{€91{,}000}{1.10} = €82{,}727.27$$

⇨Do not purchase the property

Net Present Value

- Frequently, you want to calculate the incremental cost or benefit from adopting an investment.

- For previous example:

$$-€85{,}000 + \frac{€91{,}000}{1.10} = -€2{,}273$$

Cost of land today

Present value of next year's sales price

Net present value

Net Present Value

Net present value formula:

$$NPV = -Cost + PV$$

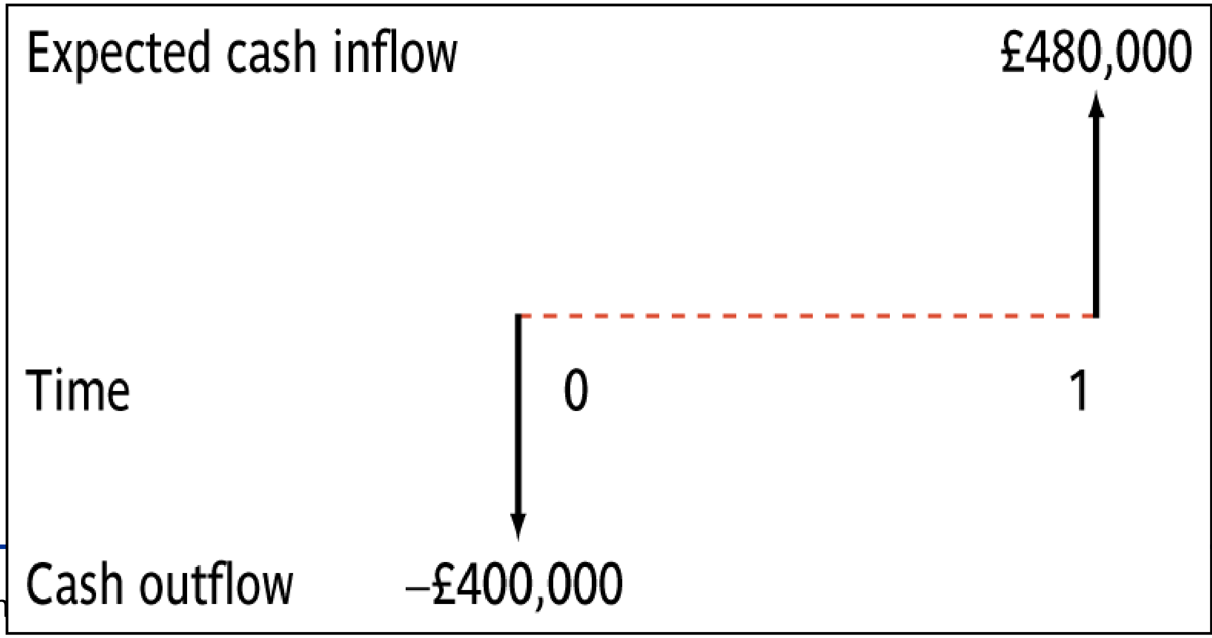
If an investment's net present value is positive, the investment should be made.

If an investment's net present value is negative, it should be rejected.

Example: Uncertainty and Valuation

- Professional Artworks plc is a firm that speculates in modern paintings. The manager is thinking of buying an original Picasso for £400,000 with the intention of selling it at the end of one year. The bank interest rate is 10 percent.

- The manager expects that the painting will be worth £480,000 in one year. The relevant expected cash flows are depicted below:



Example: Uncertainty and Valuation

a) Discount Cash Flows at 10 percent:

$$\frac{£480,000}{1.10} = £436,364$$

⇨ Looks good, but the 10 percent interest is for riskless cash flows. We need a higher discount rate to reflect the riskiness of the investment.

b) Discount Cash Flows at 25 percent:

$$\frac{£480,000}{1.25} = £384,000$$

⇨ Do not buy the painting!

Valuation: The Multi-Period Case
Future Value and Compounding

- Suppose you deposit €1 for one year at a rate of 9%. How much will it amount to in one year?

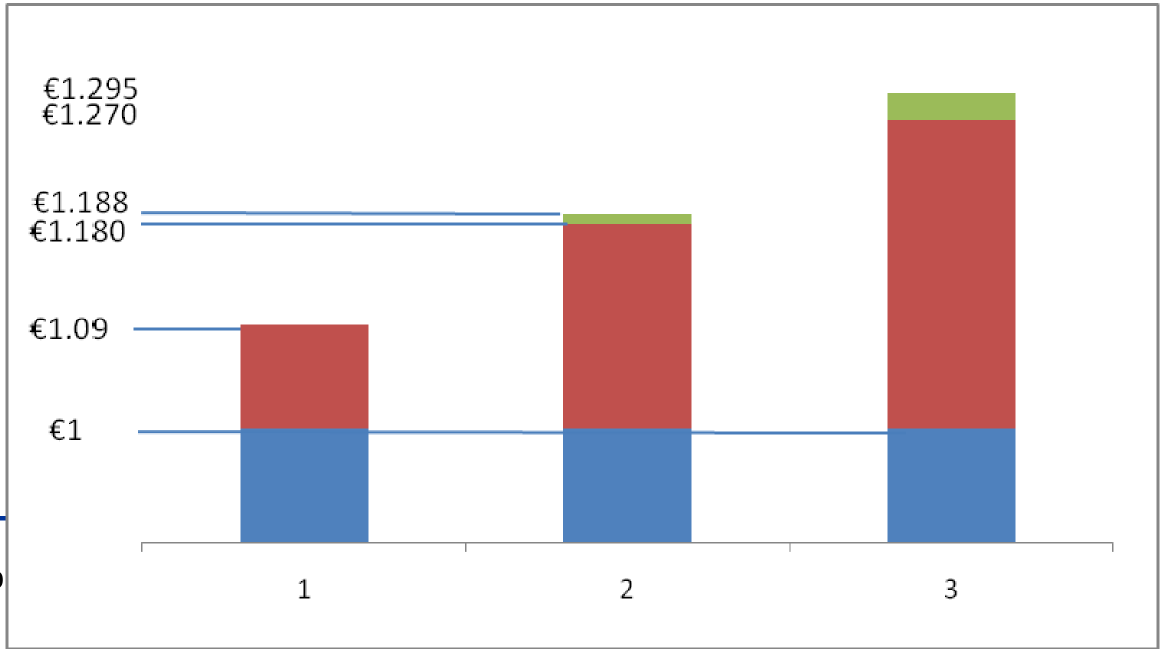
  $€1 \times (1 + r) = €1 \times 1.09 = €1.09$

- What happens if you leave it in the account for another year?

  $€1 \times (1 + r) \times (1 + r) = €1 \times (1 + r)^2 = 1 + 2r + r^2$

  $€1 \times (1.09) \times (1.09) = €1 \times (1.09)^2 = €1 + €0.18 + €0.0081$

  $= €1.1881$



Multi-Period Formula

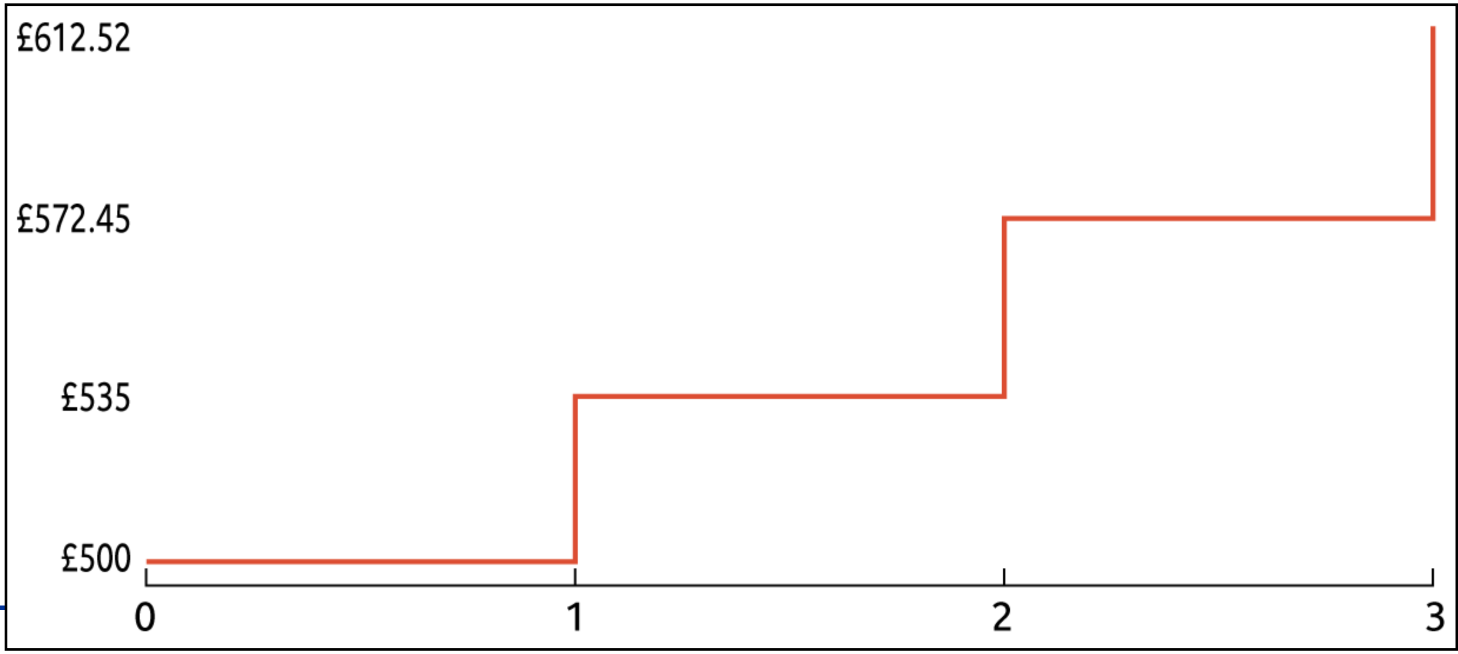
Future Value of an Investment:

$$FV = C_0(1+r)^T$$

Example 4.3: Interest on Interest

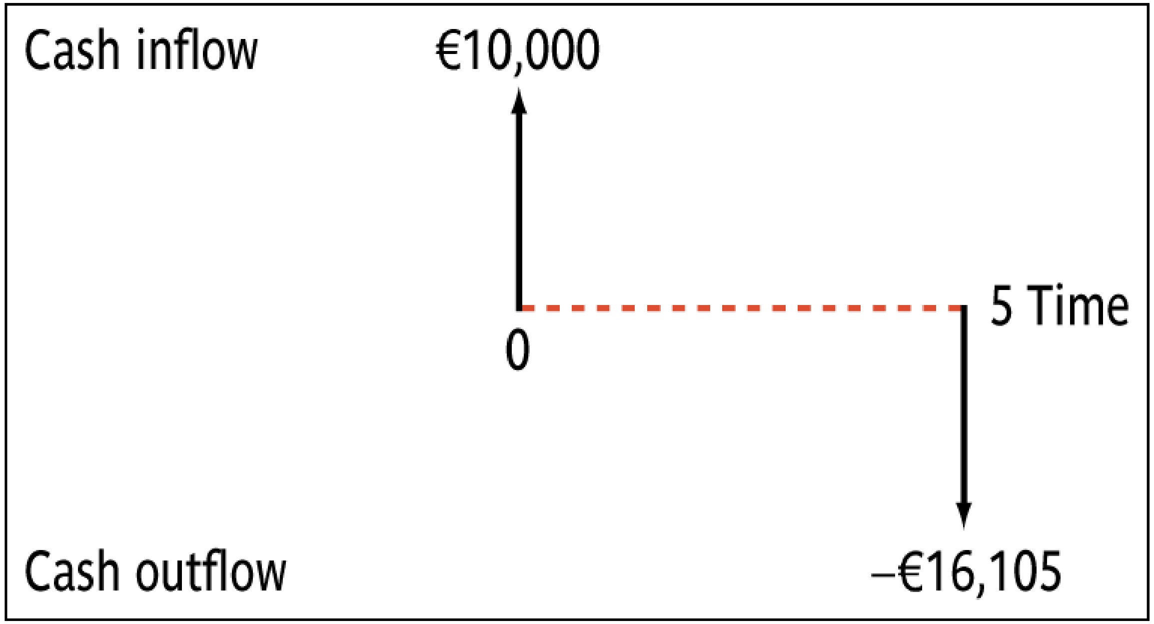
You have put €500 in a savings account at Deutsche Bank. The account earns 7 percent, compounded annually. How much will you have at the end of three years?

Answer: €500 $\times$ 1.07 $\times$ 1.07 $\times$ 1.07 = €500 $\times(1.07)^3$ = €612.52



Example 4.5: Finding the Rate

- You have recently won €10,000 in the lottery. You don't need the money right now, but you want to buy a car in five years. You estimate that the car will cost €16,105 at that time. Thus, the cash flows are as follows:



- At which interest rate do you have to invest the money to be able to afford the car?

Solution

- The ratio of purchase price to initial cash is:

$$\frac{€16.105}{€10.000} = 1.6105$$

- Thus, you must earn an interest rate that allows €1 to become €1.6105 in five years: €10,000 $\times$ (1 + r)^5 = €16,105, where r is the interest rate needed to purchase the car.

- Because €16,105/€10,000 = 1.6105, we have:

$$(1+r)^5 = 1.6105$$
$$r = 10\%$$

The Power of Compounding

Simple Interest

r = 8.47 percent

Invest £1 for 208 Years

Interest = £0.0847

Value of investment at end of 208 years:

£1 + (208 x £0.0847)

= £1 + £17.62

= £18.62

Compound Interest

r = 8.47 percent

Invest £1 for 208 Years

Value of investment at end of 208 years

FV = £1(1.0847)^{208}

= £22,100,655

⇨ Quite a difference!

Example 4.6: Manhattan Island

- New Netherlands, in 1626, allegedly bought Manhattan Island for 60 guilders' worth of trinkets from native Americans. It is reported that 60 guilders was worth about \$24 at the prevailing exchange rate. Was this a good deal for the Native Americans?

- If the Native Americans had sold the trinkets at a fair market value and invested the \$24 at 5 percent (tax free), it would now, about 385 years later, be worth more than \$2.5 billion.

- Today, Manhattan is undoubtedly worth more than \$2.5 billion, so at a 5 percent rate of return the native Americans got the worst of the deal.

- What if the interest rate was 10 percent?

Example 4.6: Manhattan Island

C = \$24

T = 385 years

r = 10%

⇨ $FV = \$24(1+r)^T = 24 \times 1.1^{385} = 207{,}202{,}517{,}436{,}662{,}000$

$\cong$ \$207 quadrillion

⇨ \$207 quadrillion is more than all the real estate in the world is worth today.

Present Value and Discounting

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Present Value of Investment:

$$\boxed{PV = C_T \cdot \frac{1}{(1+r)^T} = \frac{C_T}{(1+r)^T}}$$

where C_T is cash flow at date T and
r is the appropriate discount rate.

The term $\frac{1}{(1+r)^T}$ is called present value factor.

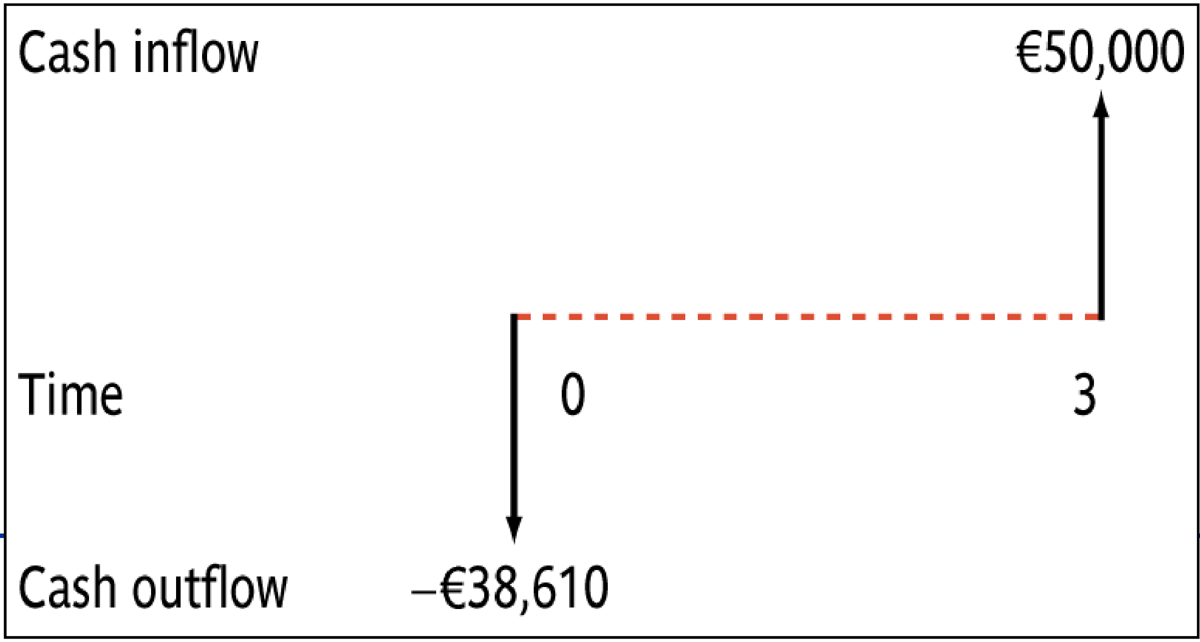
Example 4.7: Multi-Period Discounting

- You will receive €10,000 three years from now. You can earn 8 percent on your investments, so the appropriate discount rate is 8 percent. What is the present value of your future cash flow?

$$
\begin{aligned}
\text{PV} &= €10,000 \times \left(\frac{1}{1.08}\right)^3 \\
&= €10,000 \times .7938 \\
&= €7,938
\end{aligned}
$$

Example 4.8: Finding the Rate

- A customer of Chaffkin GmbH wants to buy a ship today. Rather than paying immediately, he will pay €50,000 in three years. It will cost Chaffkin GmbH €38,610 to build the ship immediately. The relevant cash flows to Chaffkin GmbH are displayed below. At what interest rate would Chaffkin GmbH neither gain nor lose on the sale?



Example 4.8: Finding the Rate

- The ratio of construction cost (present value) to sale price (future value) is: $€38{,}610 / €50{,}000 = 0.7722$

- The interest rate that allows €1 to be received in three years to have a present value of €0.7722 is 9 percent.

- Why? Because: $0.7722 \cdot (1+r)^3 = 1 \Leftrightarrow r = \sqrt[3]{\frac{1}{0.7722}} - 1 = 0.09$

- Which is equivalent to:

$$38{,}610 \cdot (1+r)^3 = 50{,}000 \Leftrightarrow r = \sqrt[3]{\frac{50{,}000}{38{,}610}} - 1 = 0.09$$

Calculating Net Present Value

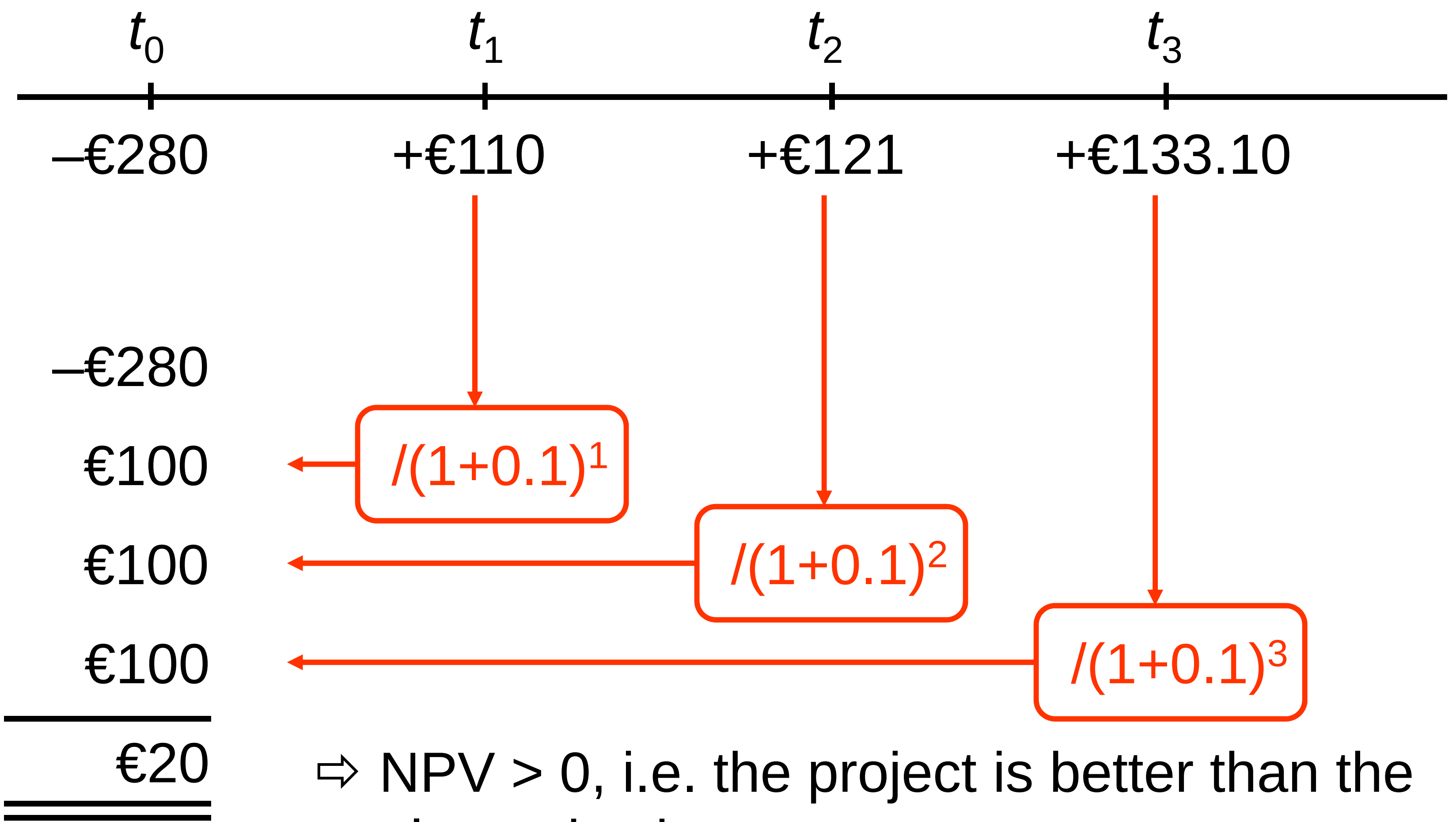
- Consider a project with the following cash flows:

t_0	t_1	t_2	t_3
–€280.00	+€110.00	+€121.00	+€133.10

- The interest rate on an alternative investment (e.g., bank a deposit) is 10%.

- Should you invest in the project?

Calculating Net Present Value



⇨ NPV > 0, i.e. the project is better than the alternative investment

Calculating Net Present Value: The Algebraic Formula

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$$NPV = -C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_T}{(1+r)^T}$$

$$= -C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

where C_t is cash flow at date t,
T is the date of the last cash flow, and
r is the appropriate discount rate.

Note that we assume that each cash flow occurs at the end of a period, and that the distance between two subsequent periods is always the same.

Calculating Net Present Value

"If I invest in the project, the cash flows will occur during the next three years. But I want to spend the additional 20 bucks from the project right now. What do I have to do?"

⇨ Borrow €300 from a bank at an interest rate of 10% for three years, and use the cash flows from the project to repay this loan:

Year	Loan	Interest payment	Loan re-payment	Loan cash flow	Project cash flow	Net cash flow
0	300	0	0	300	-280	20
1	220	-30	-80	-110	110	0
2	121	-22	-99	-121	121	0
3	0	-12.10	-121	-133.10	133.10	0

Digression: Assumptions Underlying Net Present Value

- The basic NPV formula is based on the assumption of perfect capital markets. Although textbook definitions of “capital market perfection” differ, they usually contain the following elements:
  - There are no taxes or transaction costs.
  - Access to the capital markets is not restricted.
  - All investors have the same information about borrowing and lending opportunities.
  - There are so many investors that the actions of a single investor do not influence the market price of an investment opportunity.

  ➯ Are these assumptions realistic?

- In particular, the assumption of perfect capital markets implies that the interest rates for borrowing and lending money are identical.

  ➯ Is this condition realistic? Is it necessary?

Compounding Periods

Sometimes interest is charged
more frequently than once per year:

- Semi-Annually (twice a year)
- Quarterly (4 times a year)
- Monthly (12 times a year)
- Weekly (52 times a year)
- Daily (365 times a year)
- Continuous

Formula for Compounding More Than Once a Year

- Compounding a one-year investment *m* times a year provides end-of-year wealth of:

$$FV = C_0\left(1+\frac{r}{m}\right)^m$$

where C_0 is the initial investment and r is the stated annual interest rate.

- The stated annual interest rate is the annual interest rate without consideration of compounding.

Example 4.11: Effective Annual Rate

- What is the end-of-year wealth if you receive a stated annual interest rate of 24 percent compounded monthly on a €1 investment?
- Solution: $$€1\left(1+\frac{0.24}{12}\right)^{12} = €1(1.02)^{12} = €1.2682$$
- The annual rate of return is 26.82 percent. This annual rate of return is called either the effective annual rate (EAR) or the effective annual yield (EAY).
- The general formula for EAR is: $$\boxed{EAR = \left(1+\frac{r}{m}\right)^{m} - 1}$$

Compounding over Many Years

- To calculate the future value of an investment over one or more years (*T*), the following formula applies:

$$FV = C_0\left(1+\frac{r}{m}\right)^{mT}$$

Example 4.13: Multi-Year Compounding

- You are investing €5,000 at a stated annual interest rate of 12 percent per year, compounded quarterly, for five years. What is your wealth at the end of five years?

$$€5,000 \times \left(1 + \frac{.12}{4}\right)^{4 \times 5} = €5,000 \times (1.03)^{20}$$

$$= €5,000 \times 1.8061 = €9,030.50$$

The Annual Percentage Rate

Many loans have front or back end fees relating to management costs, administration, etc

In the EU, all loans must state the effective interest rate that includes all costs, not just the interest payments

This is known as the Annual Percentage Rate (APR). It is calculated by solving the standard present value formula for APR.

Thus: $$PV = C_0 + \frac{C_1}{1+\boxed{APR}} + \frac{C_2}{(1+\boxed{APR})^2} + \ldots + \frac{C_T}{(1+\boxed{APR})^T} = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+\boxed{APR})^t}$$

Continuous Compounding

- To calculate the future value of an investment that is compounded every infinitesimal instant over one or more years (*T*), the following formula applies:

$$FV = C_0 e^{rT}$$

where e^x is the exponential function,
$e \approx 2.7182818$

Examples 4.15 & 4.16: Continuous Compounding

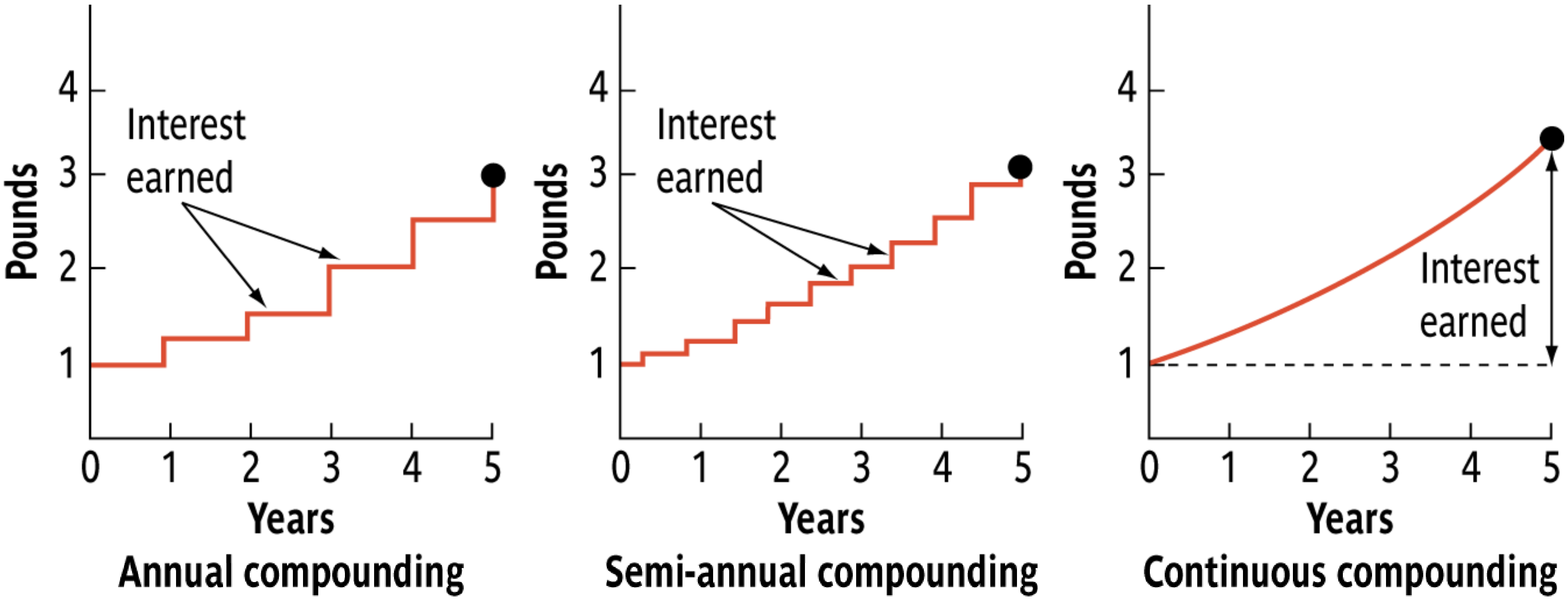
- You invested £1,000 at a continuously compounded rate of 10 percent for one year. What is the value of your wealth at the end of one year?

$$£1{,}000 \times e^{0.10} = £1{,}000 \times 1.1052 = £1{,}105.2$$

- You invested £1,000 at a continuously compounded rate of 10 percent for two years. How much did this amount to at the end of two years?

$$£1{,}000 \times e^{0.10 \times 2} = £1{,}000 \times e^{0.20} = £1{,}221.40$$

Annual, Semi-Annual and Continuous Compounding



Example 4.17: Present Value with Continuous Compounding

- A crossword competition is going to pay you €1,000 at the end of four years. If the annual continuously compounded rate of interest is 8 percent, what is the present value of this payment?

$$€1{,}000 \times \frac{1}{e^{.08 \times 4}} = €1{,}000 \times \frac{1}{1.3771} = €726.16$$

Simplifications

Perpetuity

Growing Perpetuity

Annuity

Growing Annuity

Perpetuity Formulae

- Perpetuity: A constant stream of cash flows that never ends
- PV of a perpetuity: $PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots = \frac{C}{r}$
- PV of a growing perpetuity: $PV = \frac{C}{r-g}$

where g is the perpetual growth rate

Example 4.18: Perpetuities

- Consider a perpetuity paying £100 a year. If the relevant interest rate is 8 percent, what is the value of the cash flow stream?

$$PV = \frac{£100}{.08} = £1,250$$

- Now suppose that interest rates fall to 6 percent. What is the value now?

$$PV = \frac{£100}{.06} = £1,666.67$$

Example: Growing Perpetuities

- Imagine an apartment building where cash flows to the landlord after expenses will be €100,000 next year. These cash flows are expected to rise at 5 percent per year. The relevant interest rate is 11 percent. What is the present value of the cash flows?

$$\frac{€100,000}{.11-.05} = €1,666,667$$

Important Points about the Perpetuity Formulae

Numerator

- First cash flow occurs in t_1, not in t_0

Discount Rate and Growth Rate

- $r > g$

Timing Assumption

- Each cash flow occurs at the end of a period, and the distance between two subsequent periods is always the same

Example 4.19: Paying Dividends

- A company is *just about* to pay a dividend of €3.00 per share. Investors anticipate that the annual dividend will rise by 6 percent a year forever. The applicable discount rate is 11 percent. What is the share price today?

$$€66.60 = €3.00 + \frac{€3.18}{.11 - .06}$$

Annuity Formulae

- Annuity: A level stream of cash flows that last for a fixed number of periods

- PV of an annuity: $$PV = C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right] = C\underbrace{\left[\frac{1-\frac{1}{(1+r)^T}}{r}\right]}$$

↳ The term in brackets is called the annuity factor, A_r^T

- PV of a growing annuity: $$PV = C\left[\frac{1-\left(\frac{1+g}{1+r}\right)^T}{r-g}\right]$$

Example 4.20: Lottery Valuation

- You have just won a competition paying £50,000 a year for 20 years. You are to receive the first payment a year from now. The competition organizers advertises this as the Million Pound Competition because £1,000,000 = £50,000 × 20. If the interest rate is 8 percent, what is the true value of the prize?

PV of Million Pound Competition:

$$\pounds 50{,}000\left[\frac{1-\dfrac{1}{1.08^{20}}}{0.08}\right]=\pounds 490{,}905$$

The Future Value of an Annuity

- FV of an annuity: $FV = C\left[\frac{(1+r)^T}{r} - \frac{1}{r}\right] = C\left[\frac{(1+r)^T - 1}{r}\right]$

- Suppose you put €3,000 per year into a Cash Investment Savings Account. The account pays 6 percent interest per year, tax-free. How much will you have when you retire in 30 years?

$$FV = €3{,}000\left[\frac{1.06^{30} - 1}{0.06}\right]$$

$$= €3{,}000 \cdot 79.0582 = €237{,}174.56$$

Annuity Formulae

Tricky Issues

- Delayed annuity
- Annuity due
- Infrequent annuity
- Equating the PV of two annuities

Example: A Delayed Annuity

- You will receive a four-year annuity of €500 per year, beginning at date 6. If the interest rate is 10 percent, what is the present value of your annuity?

1. Discount annuity back to year 5
2. Discount year 5 value of annuity back to year 0

Example: A Delayed Annuity

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- Step 1: Discount annuity to year 5

$$€500\left[\frac{1-\frac{1}{(1.04)^4}}{.10}\right] = €500 \times A^4_{.10}$$

$$= €500 \times 3.1699$$

$$= €1,584.95$$

- Step 2: Discount year 5 value back to year 0

$$\frac{€1,584.95}{(1.10)^5} = €984.13$$

Example 4.23: Annuity Due

- You receive £50,000 a year for 20 years from a competition. Assume that the first payment occurs immediately and that the discount rate is 8 percent. What is the value of the prize?

£50,000 + £50,000 × $A_{.08}^{19}$

Payment at date 0 | 19-year annuity

=£50,000 + (£50,000 × 9.6036)

=£530,180

Example 4.24: Infrequent Annuities

- You receive an annuity of £450, payable once every two years. The annuity stretches out over 20 years. The first payment occurs at date 2 – that is, two years from today. The annual interest rate is 6 percent. What is the value of this annuity?

- Determine the interest rate over a two-year period.

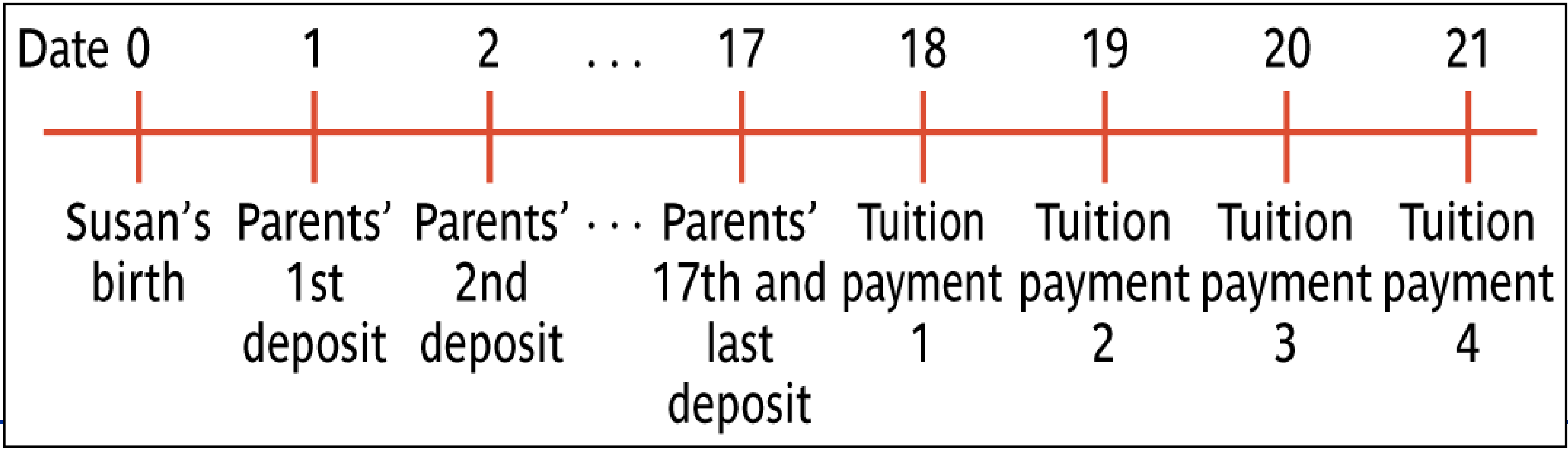
$$(1.06 \times 1.06) - 1 = 12.36\%$$

- Now calculate the present value of a £450 annuity over 10 periods, with an interest rate of 12.36 percent per period:

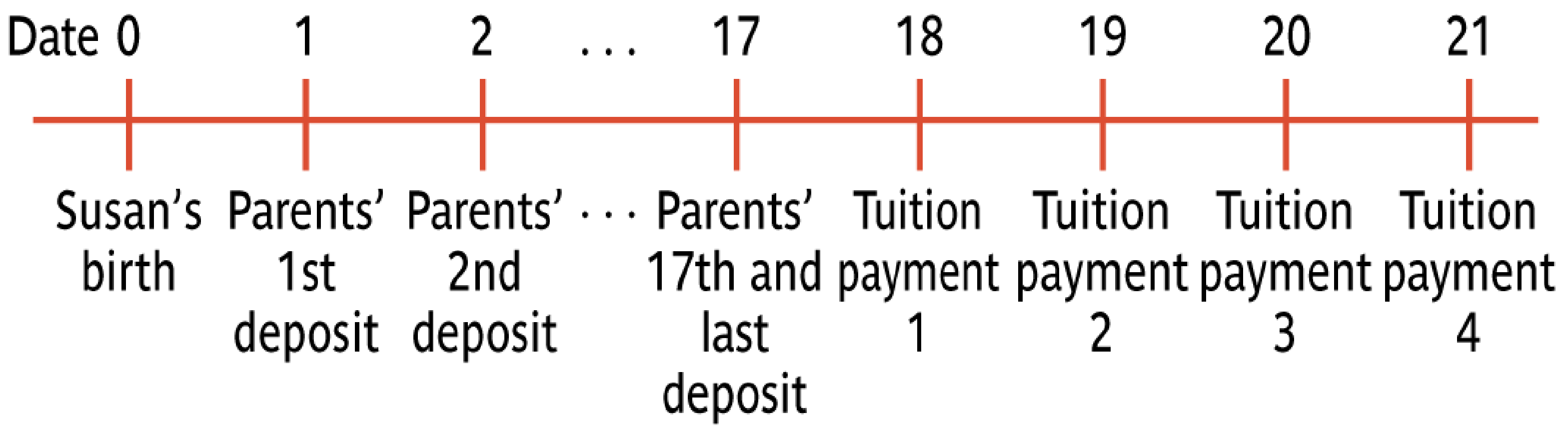
$$£450 \left[\frac{1 - \frac{1}{(1+.1236)^{10}}}{.1236} \right] = £450 \times A_{.1236}^{10} = £2,505.57$$

Example 4.25: Equating the Present Value of Two Annuities

- Harold and Helen Nash are saving for the university education of their newborn daughter, Susan.
- The Nashes estimate that university expenses will be €30,000 per year when their daughter reaches university in 18 years.
- The annual interest rate over the next few decades will be 14 percent.
- How much money must they deposit in the bank each year so that their daughter will be completely supported through four years of university?



Example 4.25: Equating the Present Value of Two Annuities



Three Steps:

1. Calculate the Year 17 Value of the university payments
2. Calculate the Year 0 value of the university payments
3. Calculate the cash flow that equates the year 1 – 17 payments to the year 0 value of the university payments

Example 4.25: Equating the Present Value of Two Annuities

1. $$€30,000 \times \left[\frac{1 - \frac{1}{(1.14)^4}}{.14} \right] = €30.000 \times A^4_{.14}$$

$$= €30,000 \times 2,9137 = €87,411$$

2. $$\frac{€87,411}{(1.14)^{17}} = €9,422.91$$

3. $$C \times A^{17}_{.14} = €9,422.91 \qquad C = \frac{€9,422.91}{6.3729} = €1,478.59$$

Example 4.26: Growing Annuities

- Stuart Gabriel, a second-year MBA student, has just been offered a job at £80,000 a year. He anticipates his salary increasing by 9 percent a year until his retirement in 40 years. Given an interest rate of 20 percent, what is the present value of his lifetime salary?

$$£80{,}000 \times \left[\frac{1 - \left(\frac{1.09}{1.20}\right)^{40}}{.20 - .09} \right] = £711{,}730.71$$

Example 4.27: More Growing Annuities

- In a previous example, Helen and Harold Nash planned to make 17 identical payments to fund the university education of their daughter, Susan. Alternatively, imagine that they planned to increase their payments at 4 percent per year. What would their first payment be?

- The first two steps of the previous Nash family example showed that the present value of the university costs was €9,422.91. These two steps would be the same here.

- However, the third step must be altered. Now we must ask, How much should their first payment be so that, if payments increase by 4 percent per year, the present value of all payments will be €9,422.91?

Example 4.27: More Growing Annuities

- Set the growing annuity formula to €9,422.91 and solve for C:

$$C\left[\frac{1-\left(\frac{1+g}{1+r}\right)^{T}}{r-g}\right]=C\left[\frac{1-\left(\frac{1.04}{1.14}\right)^{17}}{.14-.04}\right]=€9,422.91$$

- Here, C = €1,192.78.

What is a Firm Worth?

- A firm is expected to generate net cash flows (cash inflows minus cash outflows) of £5,000 in the first year and £2,000 for each of the next five years.
- The firm can be sold for £10,000 seven years from now.
- The owners of the firm would like to be able to make 10 percent on their investment in the firm. What is the value of the firm today?

What is a Firm Worth?

The Present Value of the Firm

End of Year	Net Cash Flow of the Firm	Present Value Factor (10%)	Present Value of Net Cash Flows
1	£ 5,000	.90909	£ 4,545.45
2	2,000	.82645	1,652.90
3	2,000	.75131	1,502.62
4	2,000	.68301	1,366.02
5	2,000	.62092	1,241.84
6	2,000	.56447	1,128.94
7	10,000	.51316	5,131.58
		PV	£16,569.35

Example 4.28: Firm Valuation

- Daniele's Pizza Company is contemplating investing €1 million in four new outlets in Italy.
- Massimiliano Barbi, the firm's chief financial officer (CFO), has estimated that the investments will pay out cash flows of €200,000 per year for nine years and nothing thereafter. (The cash flows will occur at the end of each year and there will be no cash flow after year 9.)
- Mr. Barbi has determined that the relevant discount rate for this investment is 15 percent. This is the rate of return that the firm can earn at comparable projects.
- Should Daniele make the investments in the new outlets?

Example 4.28: Firm Valuation

$$\begin{aligned}\text{NPV} &= -€1,000,000 + \frac{€200,000}{1.15} + \frac{€200,000}{(1.15)^2} + \cdots + \frac{€200,000}{(1.15)^9} \\ &= -€1,000,000 + €200,000 \times A^9_{.15} \\ &= -€1,000,000 + €954,316.78 \\ &= -€45,683.22\end{aligned}$$

Overview of Lecture

- Valuation: The One-Period Case
- Valuation: The Multi-Period Case
- Compounding Periods
- Simplifications
- What is a Firm Worth?