Mathematics Journal

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Quantum Computation - Intro

This chapter will consist of quantum bits and gates.

1.1 Quantum Bits

Classical computation is built upon *bits.* In the world of Quantum computation and quantum information, we have an analogous concept known as a *quantum bit*, or *qubit.*

Definition 1 (Classical Bit)**.** A *classical bit* is a unit of information that takes on one of two states, commonly denoted 0 or 1. The term bit is a portmanteau of the term binary digit.

Definition 2 (Quantum Bit)**.** A quantum bit, also known as a qubit has a state. Two possible states for a qubit are $|0\rangle$ and $|1\rangle$. The notation "$|\ \rangle$" is known as *Dirac notation* named after Paul Dirac.

The main difference between bits and qubits is that a qubit can take on a state other than $|0\rangle$ or $|1\rangle$. A qubit can take the form that is a linear combination of these two states, this is referred to as a superposition.

An example of a superposition is the following

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{1.1}$$

where $\alpha, \beta \in \mathbb{C}$.

We can consider the state of a qubit as a vector in $\mathbb{C}^2$. The states $|0\rangle$ and $|1\rangle$ are known as computation basis states and form an orthogonal basis for this vector space.

While a bit's state can be determined by examination, the same is not true for a qubit. That is, one cannot determine the values α and β. When we measure a qubit we either get the result 0, with probability $|\alpha|^2$, or the result 1, with probability $|\beta|^2$. Since the total probability must sum to 1, we have that $|\alpha|^2 + |\beta|^2 = 1$. Returning to the vector space interpretation, we have that the qubit's state is a unit vector in $\mathbb{C}^2$.

Example 1. A qubit can have the following state,

$$\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle ,$$

when measured the state gives the result 0 fifty percent of the time, as well as the result 1 fifty percent of the time. This state is commonly denoted $|+\rangle$.

We can think about qubits using a different geometric interpretation by rewriting Equation (1.1) as

$$|\psi\rangle = e^{i\gamma} \left[\cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right) |1\rangle \right], \tag{1.2}$$

where $\theta, \phi, \gamma \in \mathbb{R}$. We will later see that we can ignore the factor of $e^{i\gamma}$, because it has no observable effects. We can now write

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right) |0\rangle + e^{i\phi} \sin\left(\frac{\theta}{2}\right) |1\rangle , \tag{1.3}$$

where the numbers θ and ϕ define a point on the three-dimensional unit sphere. We will refer to this as the *Bloch sphere.*

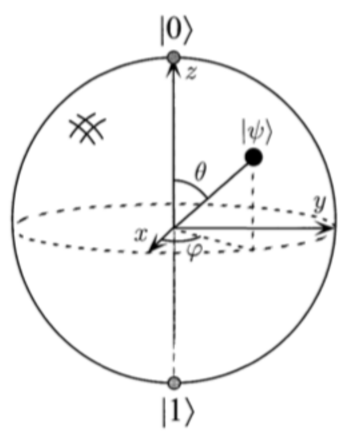

Figure 1.1: Bloch Sphere REFERENCE

1.1.1 Multiple Qubits

With two classical bits, there are four possible states: 00, 01, 10, and 11. Similarly, qubits have the four corresponding computation basis states: $|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$.

A pair of qubits can also exist in superposition. This can be represented as the following

$$|\phi\rangle = \alpha_{00}\,|00\rangle + \alpha_{01}\,|01\rangle + \alpha_{10}\,|10\rangle + \alpha_{11}\,|11\rangle\,.$$

Where each α_{ij} is a complex coefficient, known as the *amplitude*. The measurement result for each of the four states occurs with probability $|\alpha_{ij}|^2$