Chapter 1

Quantum Computation - Intro

This chapter will consist of quantum bits and gates.

1.1 Quantum Bits

In classical computation, the foundation is built upon what is known as a *bit*. In the world of Quantum computation and quantum information, we have an analogous concept known as a *quantum bit*, or *qubit*.

A classical bit takes on one of two states, either 0 or 1. The qubit also has a state, the two possible states for a qubit are $|0\rangle$ and $|1\rangle$. The notation "$|\ \rangle$" is known as *Dirac notation* named after Paul Dirac.

The main difference between bits and qubits is that a qubit can take on a state other than $|0\rangle$ or $|1\rangle$. A qubit can take the form that is a linear combination of these two states, this is referred to as a superposition.

An example of a superposition is the following

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{1.1}$$

where $\alpha, \beta \in \mathbb{C}$. We can also consider the state of a qubit as a vector in $\mathbb{C}^2$. The states $|0\rangle$ and $|1\rangle$ are known as computation basis states and form an orthogonal basis for this vector space. While a bit's state can be determined by examination, the same is not true for a qubit. That is, one cannot determine the values α and β. When we measure a qubit we either get the result 0, with probability $|\alpha|^2$, or the result 1, with probability $|\beta|^2$. Since the total probability must sum to 1, we have that $|\alpha|^2 + |\beta|^2 = 1$. Returning to the vector space interpretation, we have that the qubit's state is a unit vector in $\mathbb{C}^2$.

For example, a qubit can have the following state

$$\frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle,$$

when measured the state gives the result 0 fifty percent of the time, as well as the result 1 fifty percent of the time. This state is commonly denoted $\left|+\right\rangle$.

We can think about qubits using a different geometric interpretation by rewriting Equation (1.1) as

$$\left|\psi\right\rangle = e^{i\gamma}\left[\cos\left(\frac{\theta}{2}\right)\left|0\right\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)\left|1\right\rangle\right], \tag{1.2}$$

where $\theta, \phi, \gamma \in \mathbb{R}$. We will later see that we can ignore the factor of $e^{i\gamma}$, because it has no observable effects. We can now write

$$\left|\psi\right\rangle = \cos\left(\frac{\theta}{2}\right)\left|0\right\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)\left|1\right\rangle, \tag{1.3}$$

where the numbers θ and ϕ define a point on the three-dimensional unit sphere. We will refer to this as the *Bloch sphere.*