# Mathematics Journal

Michael C.R. Byrd Jr.

August 25, 2018

# Contents

# Part I

# Calculus

# Chapter 1

# Derivatives

This is a chapter on Derivatives.

## 1.1 Rates of Change

This is the section on Rates of Change.

# Part II

# Abstract Algebra

# Chapter 1

# Groups

This is the chapter on Groups.

## 1.1 Introduction - Groups

# Part III

# Combinatorics

# Chapter 1

# Graphs

This is the chapter on Graphs.

## 1.1 Introduction - Graphs

# Part IV

# Quantum Computation

# Chapter 1

# Quantum Computation - Intro

This chapter will consist of quantum bits and gates.

## 1.1 Quantum Bits

Classical computation is built upon *bits*. In the world of Quantum computation and quantum information, we have an analogous concept known as a *quantum bit*, or *qubit*.

**Definition 1** (Classical Bit)**.** A *classical bit* is a unit of information that takes on one of two states, commonly denoted 0 or 1. The term bit is a portmanteau of the term binary digit.

**Definition 2** (Quantum Bit)**.** A quantum bit, also known as a qubit has a state. Two possible states for a qubit are $|0\rangle$ and $|1\rangle$. The notation "$|\ \rangle$" is known as *Dirac notation* named after Paul Dirac.

The main difference between bits and qubits is that a qubit can take on a state other than $|0\rangle$ or $|1\rangle$. A qubit can take the form that is a linear combination of these two states, this is referred to as a superposition.

An example of a superposition is the following

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{1.1}$$

where $\alpha, \beta \in \mathbb{C}$.

We can consider the state of a qubit as a vector in $\mathbb{C}^2$. The states $|0\rangle$ and $|1\rangle$ are known as computation basis states and form an orthogonal basis for this vector space.

While a bit's state can be determined by examination, the same is not true for a qubit. That is, one cannot determine the values $\alpha$ and $\beta$. When we measure a qubit we either get the result 0, with probability $|\alpha|^2$, or the result 1, with probability $|\beta|^2$. Since the total probability must sum to 1, we have that $|\alpha|^2 + |\beta|^2 = 1$. Returning to the vector space interpretation, we have that the qubit's state is a unit vector in $\mathbb{C}^2$.

**Example 1.** A qubit can have the following state,

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle,$$

when measured the state gives the result 0 fifty percent of the time, as well as the result 1 fifty percent of the time. This state is commonly denoted $|+\rangle$.

We can think about qubits using a different geometric interpretation by rewriting Equation (1.1) as

$$|\psi\rangle = e^{i\gamma}\left[\cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle\right], \tag{1.2}$$

where $\theta, \phi, \gamma \in \mathbb{R}$. We will later see that we can ignore the factor of $e^{i\gamma}$, because it has no observable effects. We can now write

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle, \tag{1.3}$$

where the numbers $\theta$ and $\phi$ define a point on the three-dimensional unit sphere. We will refer to this as the *Bloch sphere.*

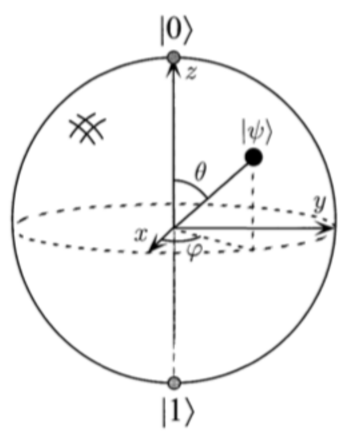



Figure 1.1: Bloch Sphere REFERENCE

### 1.1.1 Multiple Qubits

With two classical bits, there are four possible states: 00, 01, 10, and 11. Similarly, qubits have the four corresponding computation basis states: $|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$.

A pair of qubits can also exist in superposition. This can be represented as the following

$$|\phi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle.$$

Where each $\alpha_{ij}$ is a complex coefficient, known as the *amplitude*. The measurement result for each of the four states occurs with probability $|\alpha_{ij}|^2$

## 1.2 Quantum Computation

## 1.3 Quantum Algorithms

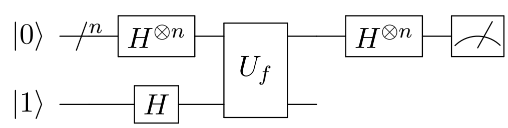
### 1.3.1 Classical Computer on Quantum

### 1.3.2 Quantum Parallelism

### 1.3.3 Deutsch's Algorithm

### 1.3.4 The Deutsch-Jozsa Algorithm

The quantum circuit for Deutsch-Jozsa algorithm.




First start with $n$ $|0\rangle$ states and one $|1\rangle$ state which can be written as

$$|\psi_0\rangle = |0\rangle^{\otimes n}|1\rangle. \tag{1.4}$$

We then pass each qubit through a Hadamard gate which leaves us with the following,

$$|\psi_1\rangle = \sum_{x\in\{0,1\}^n} \frac{|x\rangle}{\sqrt{2^n}}\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]. \tag{1.5}$$

We next evaluate the function $f$ using the $U_f : |x, y\rangle \to |x, y \oplus f(x)\rangle$ which yields

$$|\psi_2\rangle = \sum_{x} \frac{(-1)^{f(x)}|x\rangle}{\sqrt{2^n}}\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]. \tag{1.6}$$

# Part V

# Real Analysis

# Chapter 1

# Real Number System

Real Number System Chapter.

## 1.1 Real Numbers

This is the section of real numbers.

# Part VI

# Differential Geometry

# Chapter 1

# Vectors and Curves

This chapter will consist of material relevant to Vectors and Curves in Differential Geometry

## 1.1 Tangent Vectors

We will define a calculus vector as the following, $\vec{v} = \langle v_1, v_2, v_3 \rangle$. We then define a point in $\mathbb{R}^3$ as $P(p_1, p_2, p_3)$ where $\mathbb{R}^3$ is a three-dimensional vector space.

**Definition 3** (Tangent Vector)**.** A *tangent vector* $X_p$ in $\mathbb{R}^n$ is an ordered pair $X_p = \{\vec{x}, \vec{p}\}$ where $\vec{x}$ is a regular calculus vector and $\vec{p}$ is a position vector that points to the foot of $\vec{x}$. $X_p \in \mathbb{R}^n \times \mathbb{R}^n$.