CS 4780/5780 Homework 9

November 30, 2018

Problem 1: Neural Nets and kernel machines.

Assume you are given a 1-hidden layer neural network, with M hidden nodes. Let $\vec{x}$ denote the input, the hidden nodes $h_m(\vec{x})$ (for $m = 1, \ldots, M$) and the output $H(\vec{x})$ We could view a one-hidden-layer neural network as kernelized linear regression with a learned kernel. What is the kernel function?

Problem 2: Convolutional Process

1. Suppose you have a filter of size $k \times k$ and stride s. When you apply this filter to a $n \times n$ input with padding $pdeng$, what is the dimension of the output feature map?

2. Suppose you are given an 3×3 matrix from one patch of an image I and a 3×3 convolutional kernel K. In the image matrix, each entry represents the gray scale color of a corresponding pixel.

$$I = \begin{bmatrix} 3 & 1 & 1 \\ 3 & 0 & 2 \\ 4 & 4 & 0 \end{bmatrix}$$

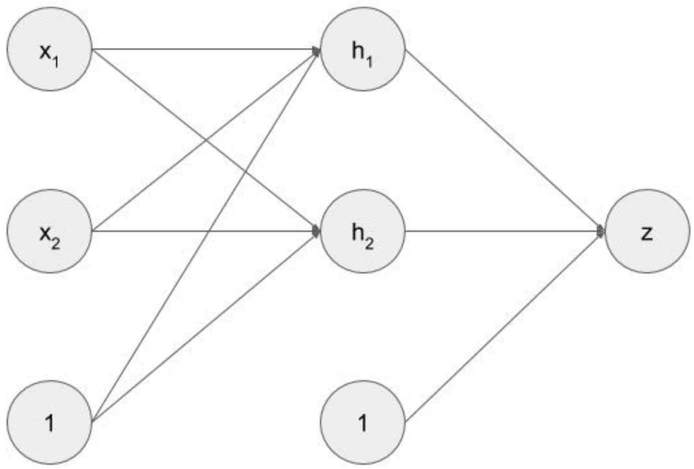
and

$$K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

What is the output of applying the filter to the input with stride 1 without padding?

Problem 3: RELU-network

In this question, you are going to explore a 2-layer fully connected network with RELU activation function.



Suppose you have the architecture above, namely,

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$z = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} f(h_1) \\ f(h_2) \\ 1 \end{bmatrix}$$

$$t = \sigma(z)$$

where $f(h_i) = max(0, h_i)$, $\sigma(z) = \frac{1}{1+e^{-z}}$ and t is the output of the network.

(a) Suppose $\begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & -1 & 0 \end{bmatrix}$ and $\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}$. Draw the decision boundary of the network, namely, $\sigma(z) = 0.5$ within the range $[-10, 10] \times [-10, 10]$. Please indicate the positive and negative side of the boundary.

(b) Assume the weights in (a), what is the prediction for $[x_1, x_2]^T = [1, 1]^T$?

(c) Usually, the weights $W = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{11} & w_{12} & w_{13} \end{bmatrix}$ and $V = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$ have to be learned using Stochastic Gradient Descent. For neural network binary classification, a common loss function is the cross entropy loss

$$l(y, t) = -(y \log(t) + (1 - y) \log(1 - t))$$

where y is the true label for a sample and t is the output of the neural network. In order to use SGD, we have to derive the gradient with respect to W and V. Show that for a single training example, $x = [x_1, x_2]$

$$
\begin{aligned}
\frac{\partial l}{\partial v_i} &= (t-y)f(h_i) \text{ for } i \neq 3 \\
\frac{\partial l}{\partial v_3} &= (t-y) \\
\frac{\partial l}{\partial w_{ij}} &= (t-y)v_i\mathbb{I}(h_i > 0)x_j \text{ for } j \neq 3 \\
\frac{\partial l}{\partial w_{i3}} &= (t-y)v_i\mathbb{I}(h_i > 0)
\end{aligned}
$$

where $\mathbb{I}(\cdot)$ is the indicator function.