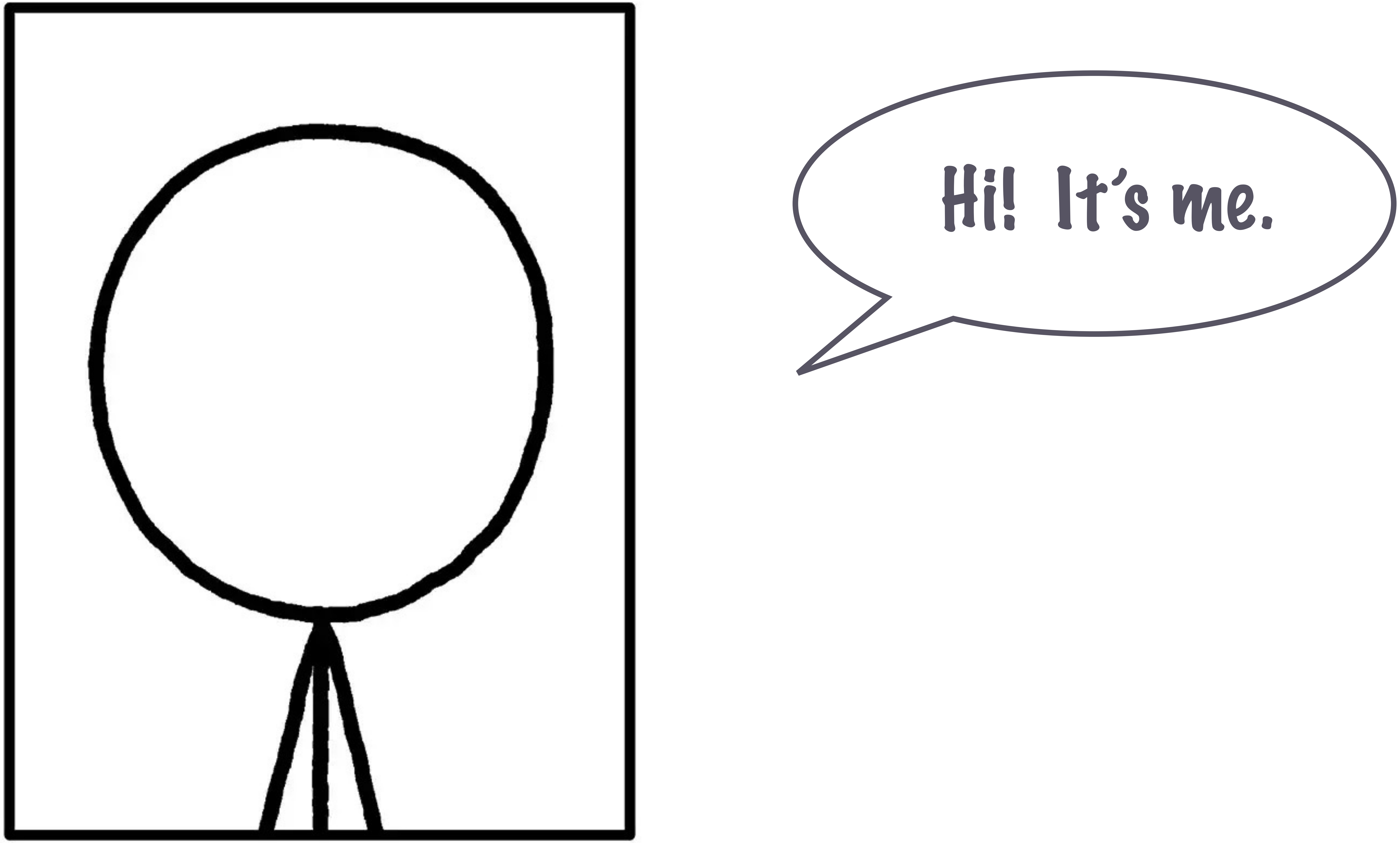
Bayesian regression modeling: Theory & practice

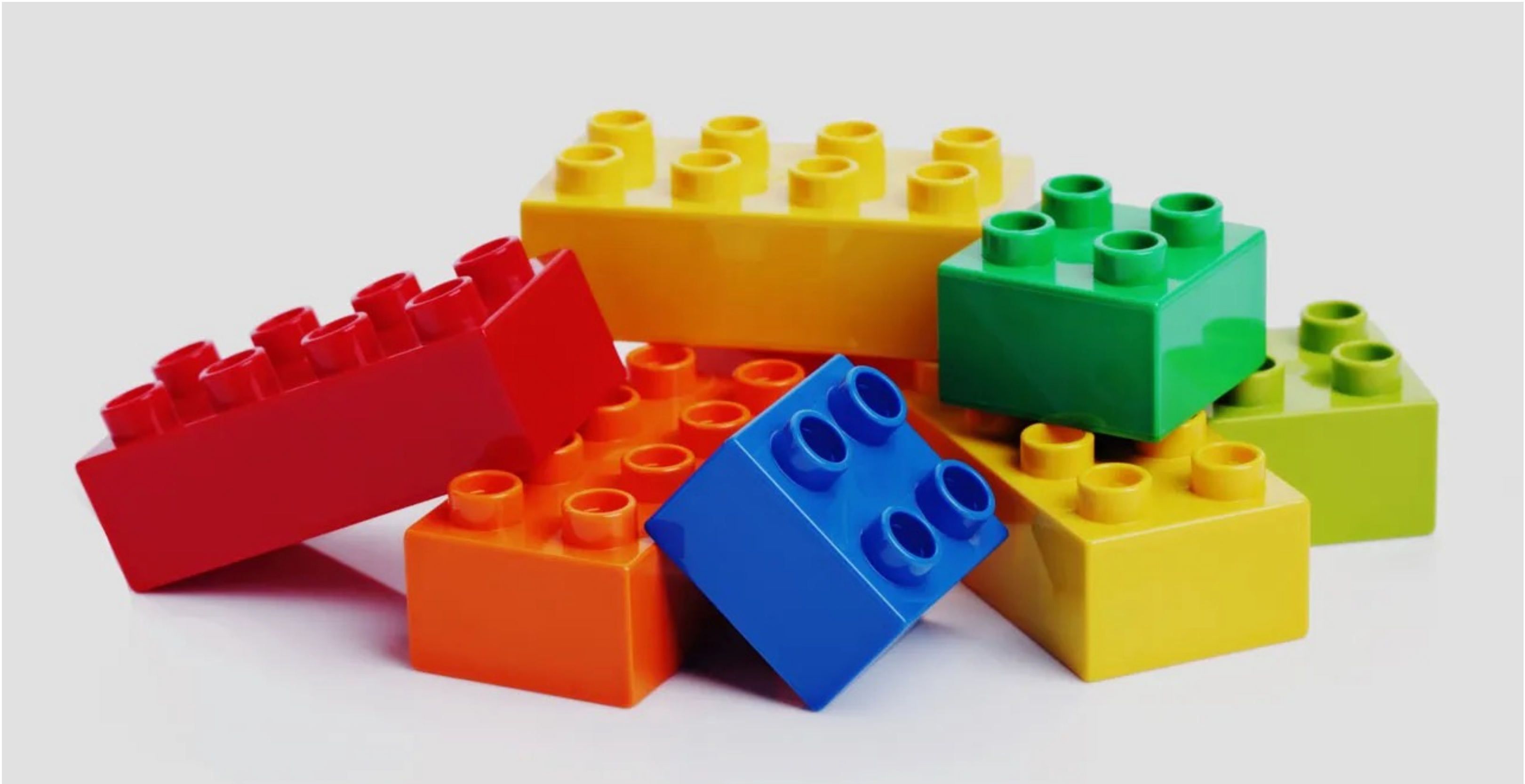
Part 1: Bayesian basics & simple linear regression

Michael Franke


Hi! It's me.

Pros
of BDA

- well-founded & totally general
- easily extensible / customizable
- more informative / insightful
- stimulates view: “models as tools”

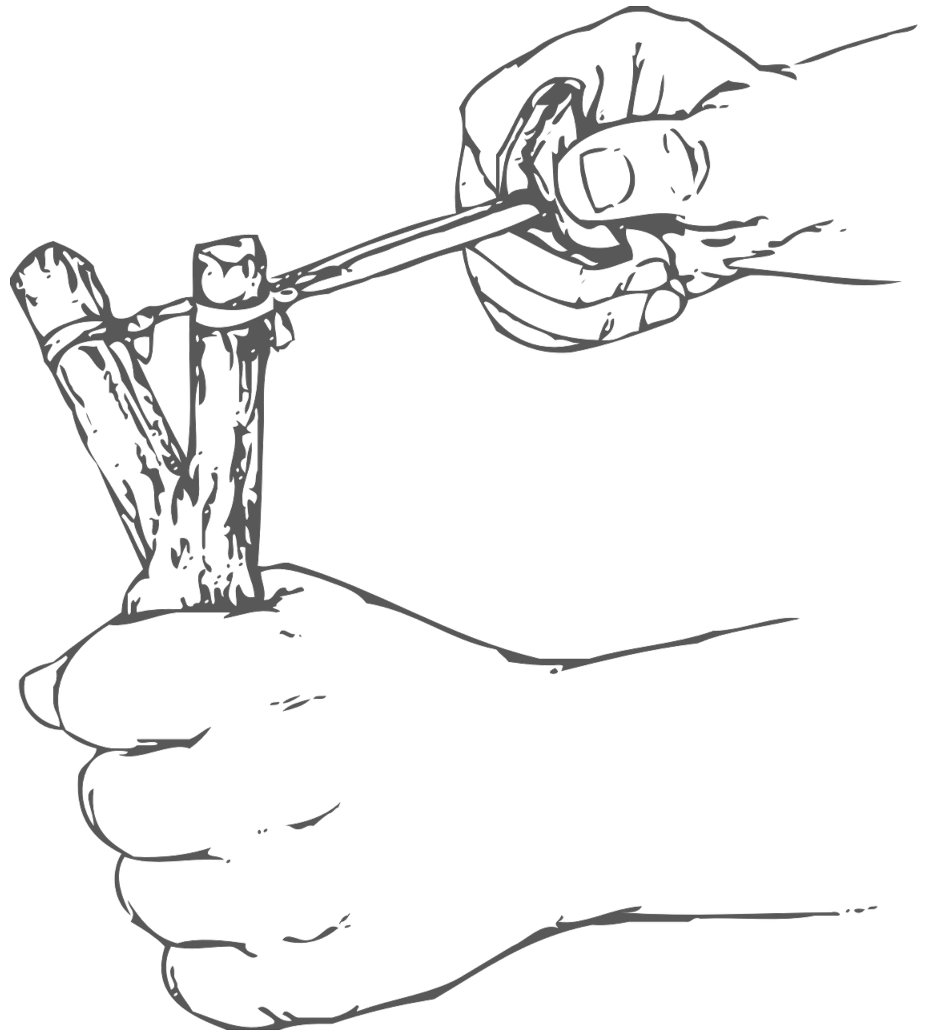


Cons
of BDA

- not yet fully digested by community
- possibly computationally complex
- less ready-made, more hands-on
- requires thinking (wait, that’s a pro!)
  - last two points less valid than 10 years ago

this course

Main learning goals



1. become comfortable in applying multi-level GLMS
    a. determine the appropriate (kind of) model for a given problem
    b. implement, run and interpret the Bayesian model
    c. draw conclusions regarding evidence for/against research questions
2. become oriented in the landscape “beyond standard LMs”
    a. know enough to determine what is relevant for you
    b. know enough to quickly find and cover what you need
3. understand key concepts of intermediate BDA
    a. multi-level modeling
    b. non-standard link functions, non-linear models …
    c. model comparison w/ Bayes factors and cross-validation
    d. Bayesian computation (MCMC)
    e. causal inference

no coins, please!

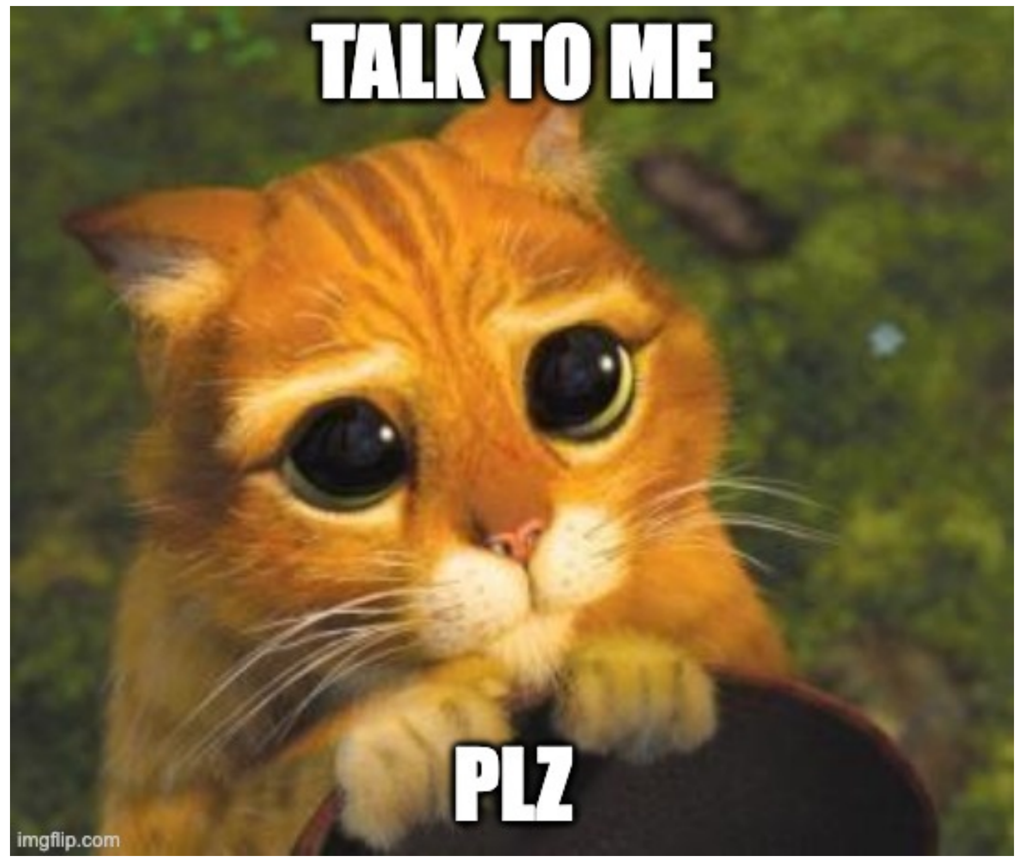
How this works

- class from 2:00 — 6:00 pm
  - w/ some shorter and one longer break
  - some live demos
- web-book w/ tutorials and exercises
  - live version: here
  - GitHub code: here
- discussion of exercises
  - on Slack (or other means)
  - first part of class
- contribute (if you want)
  - e.g., send pull-requests to **populate the cheat sheet**

Here is ~~the~~ a plan

Day 1	Day 2	Day 3	Day 4	Day 5
course structure BDA basics	priors predictions	GLMs & beyond	model comparison	non-linear models
Bayesian regression categorical predictors	multi-level models	Bayesian computation	causal inference	🛠️

exercises exercises exercises exercises exercises exercises exercises

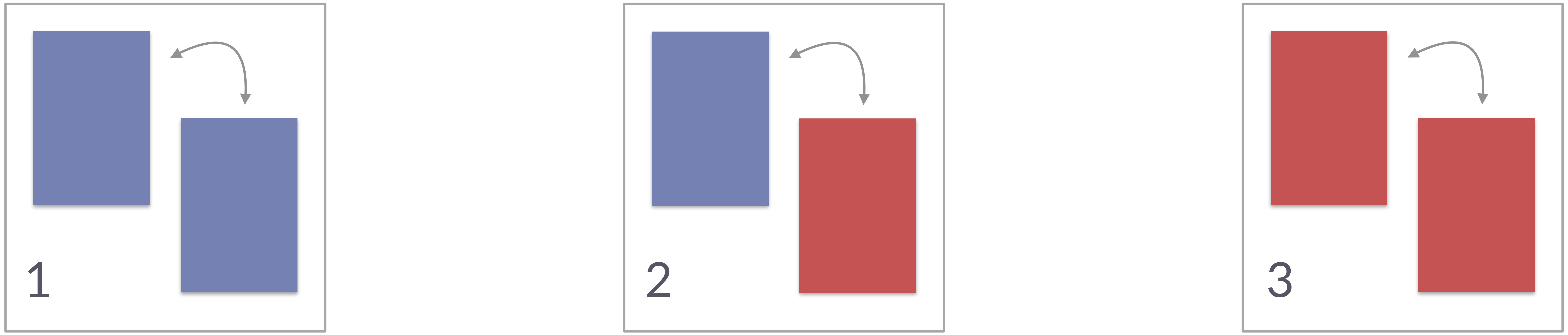


Bayesian modeling

Three-card problem

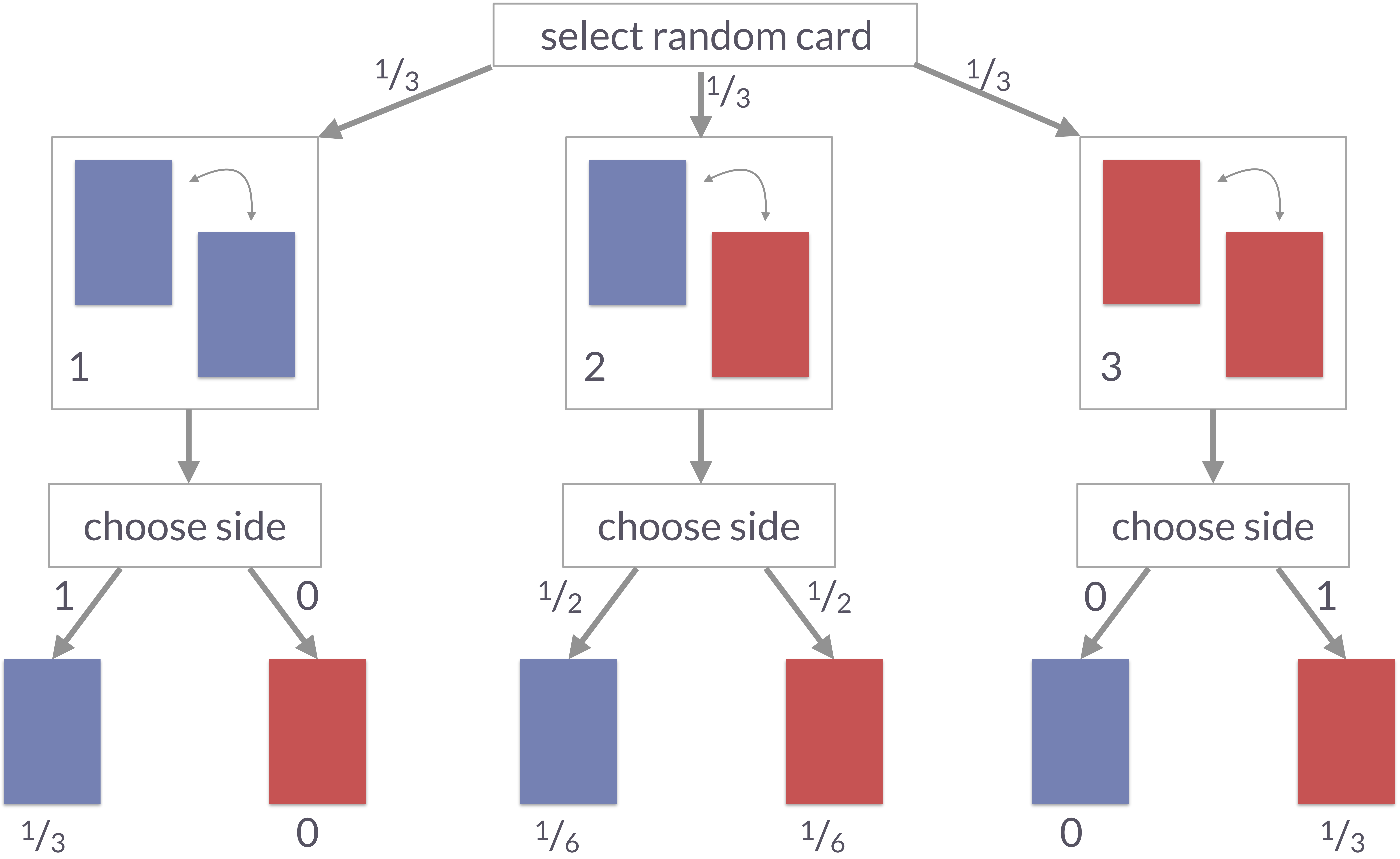
problem statement

- Sample a card (uniformly at random).
- Choose a side of that card to reveal (uniformly at random).
- What's the probability that the side you do not see is **BLUE**, given that the side you see is **BLUE**?



Three-card problem

data-generating process



Conditional probability and Bayes rule

for the three-card problem

- conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

- Bayes rule

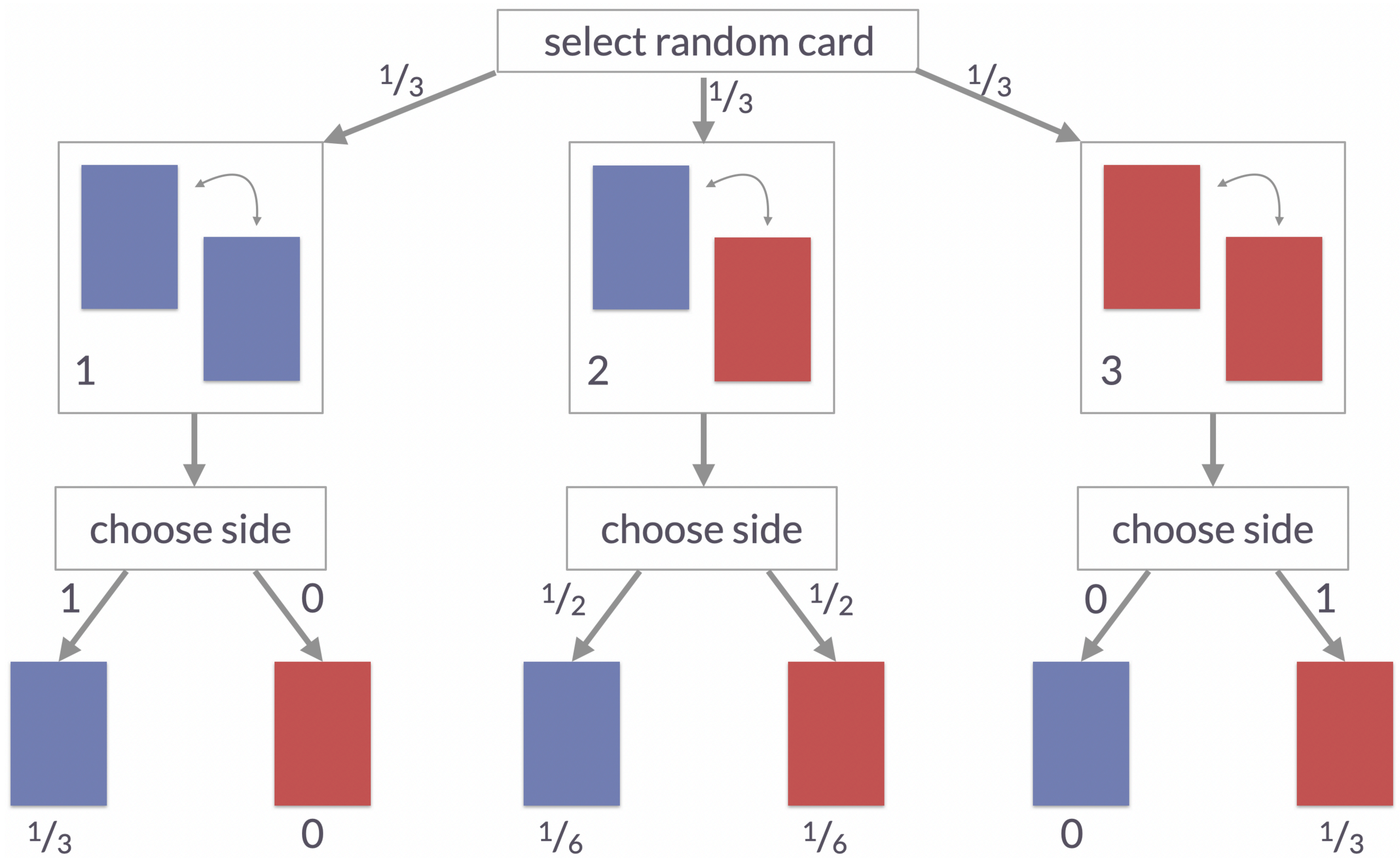
$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

- Applied to three-card problem:

$$P(\text{card 1} \mid \text{blue}) = \frac{P(\text{blue} \mid \text{card 1})\ P(\text{card 1})}{P(\text{blue})}$$

$$= \frac{1 \times \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

"reasoning from observed effect to latent cause via a model of the data-generating process"



Bayesian data analysis

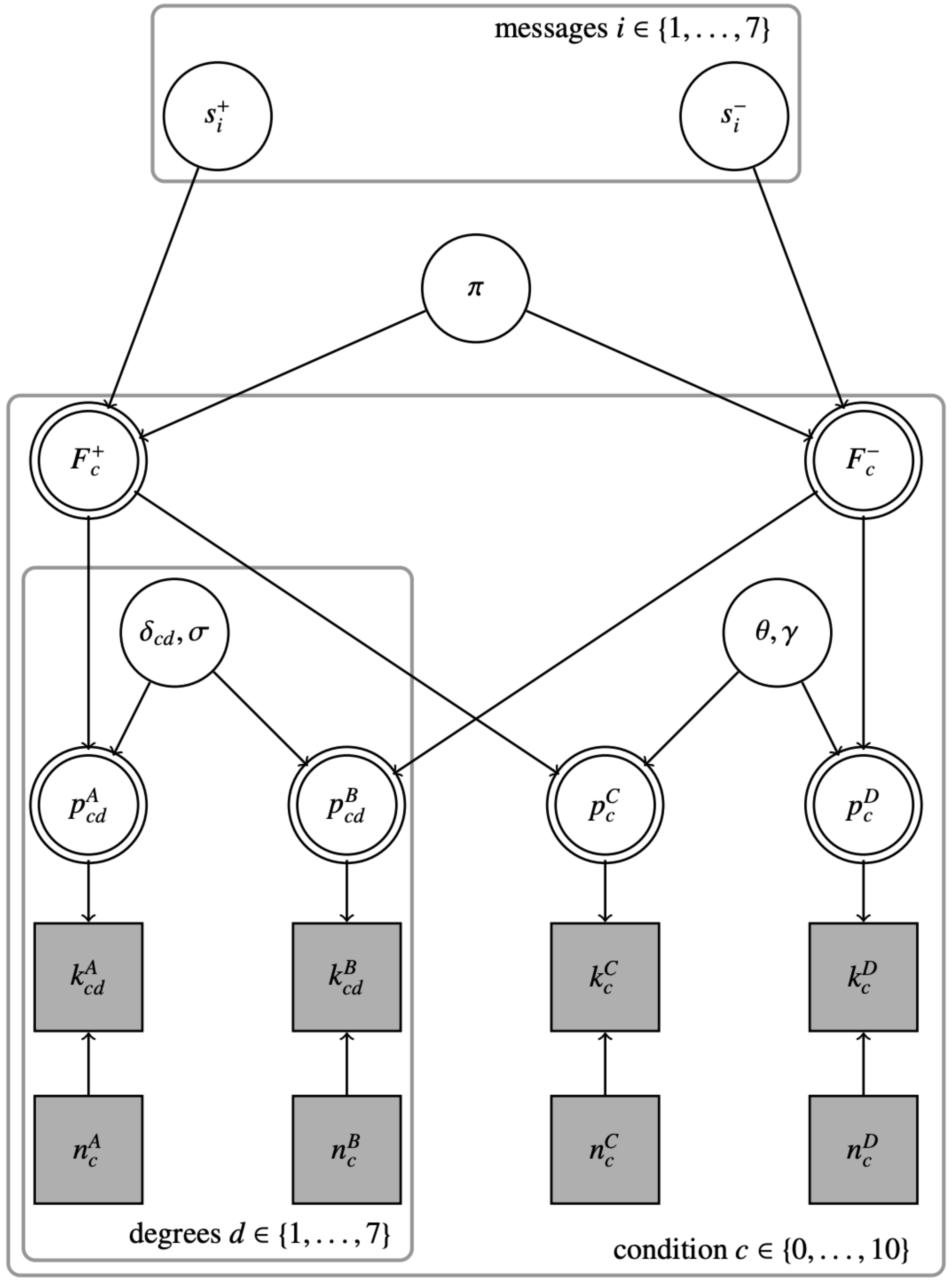
in a nutshel

- BDA is about what we *should* believe given:
  - some observable data, and
  - our model of how this data was generated (a.k.a. **the data-generating process**)
- our best friend will be **Bayes rule**
  - e.g., for **parameter inference**:

$$\underbrace{P(\theta \mid D)}_{\text{posterior}} \propto \underbrace{P(\theta)}_{\text{prior}} \times \underbrace{P(D \mid \theta)}_{\text{likelihood}}$$

  - or, for **model comparison**:

$$\underbrace{\frac{P(M_1 \mid D)}{P(M_2 \mid D)}}_{\text{posterior odds}} = \underbrace{\frac{P(D \mid M_1)}{P(D \mid M_2)}}_{\text{Bayes factor}} \underbrace{\frac{P(M_1)}{P(M_2)}}_{\text{prior odds}}$$



$$s_i^{+/-} \sim \text{Beta}(1,1)$$

$$\tfrac{1}{\pi} \sim \text{Gamma}(0.5, 0.5)$$

$$F_c^{+/-} = \text{F}(c\,;\,\vec{s}^{\,+/-}, \pi)$$

$$\sigma \sim \text{Uniform}(0, 0.4)$$

$$\delta_{d \in \{1,\dots,6\}} \sim \text{Normal}(d/7, 14)$$
$$\delta_0 = -\infty; \quad \delta_7 = \infty$$

$$\theta \sim \text{Normal}(0.5, 0.2)$$

$$\tfrac{1}{\gamma} \sim \text{Gamma}(1, 1)$$

$$p_{cd}^{A/B} = \int_{\delta_{cd-1}}^{\delta_{cd}} \text{Normal}(x, F_c^{+/-}, \sigma)\, \mathrm{d}x$$

$$p_c^{C/D} = (1 + \exp(-\gamma(F_c^{+/-} - \theta)))^{-1}$$

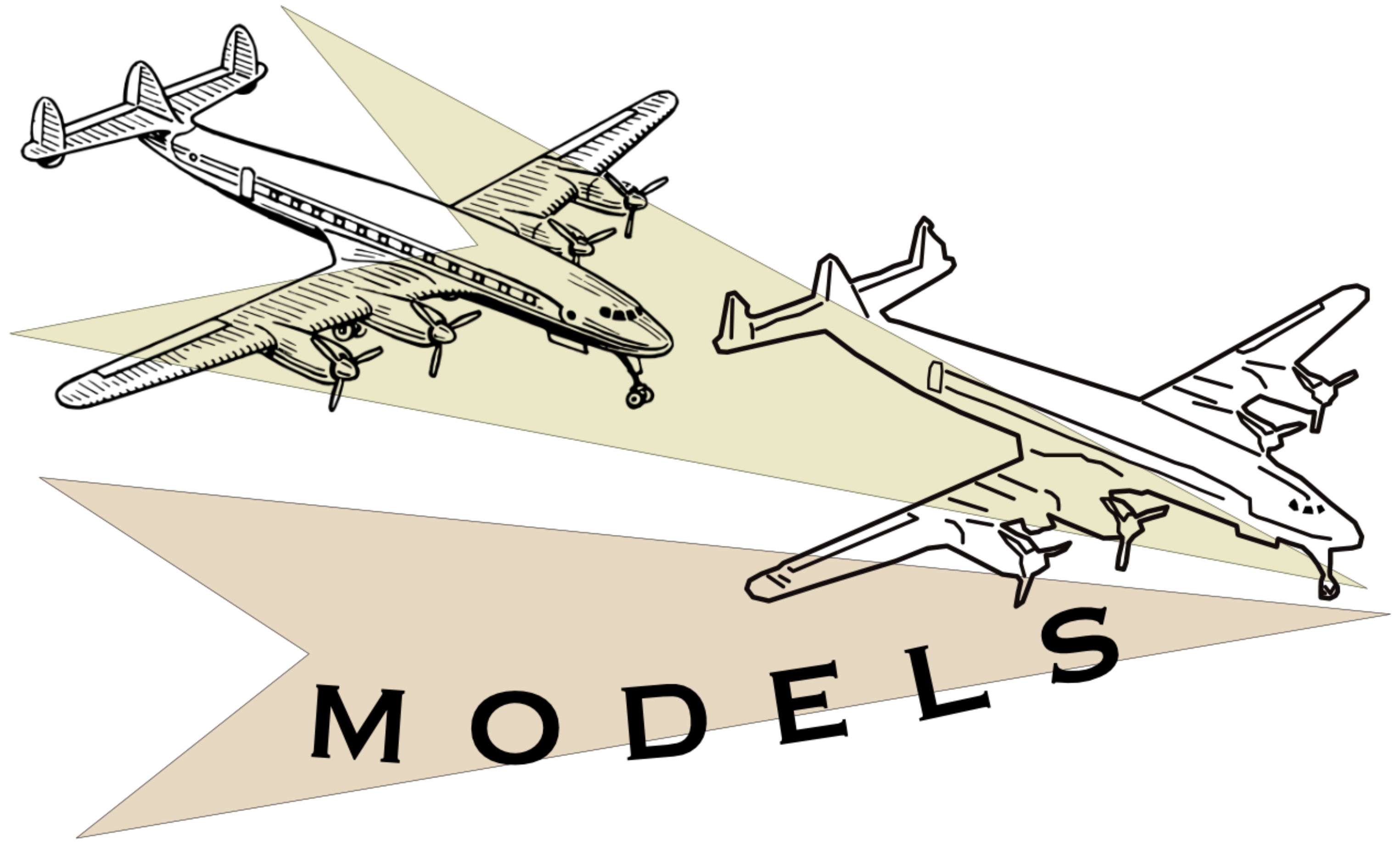
$$k_{cd}^{A/B} \sim \text{Multinomial}(p_{cd}^{A/B}, n_c^{A/B})$$

$$k_c^{C/D} \sim \text{Binomial}(p_c^{C/D}, n_c^{C/D})$$

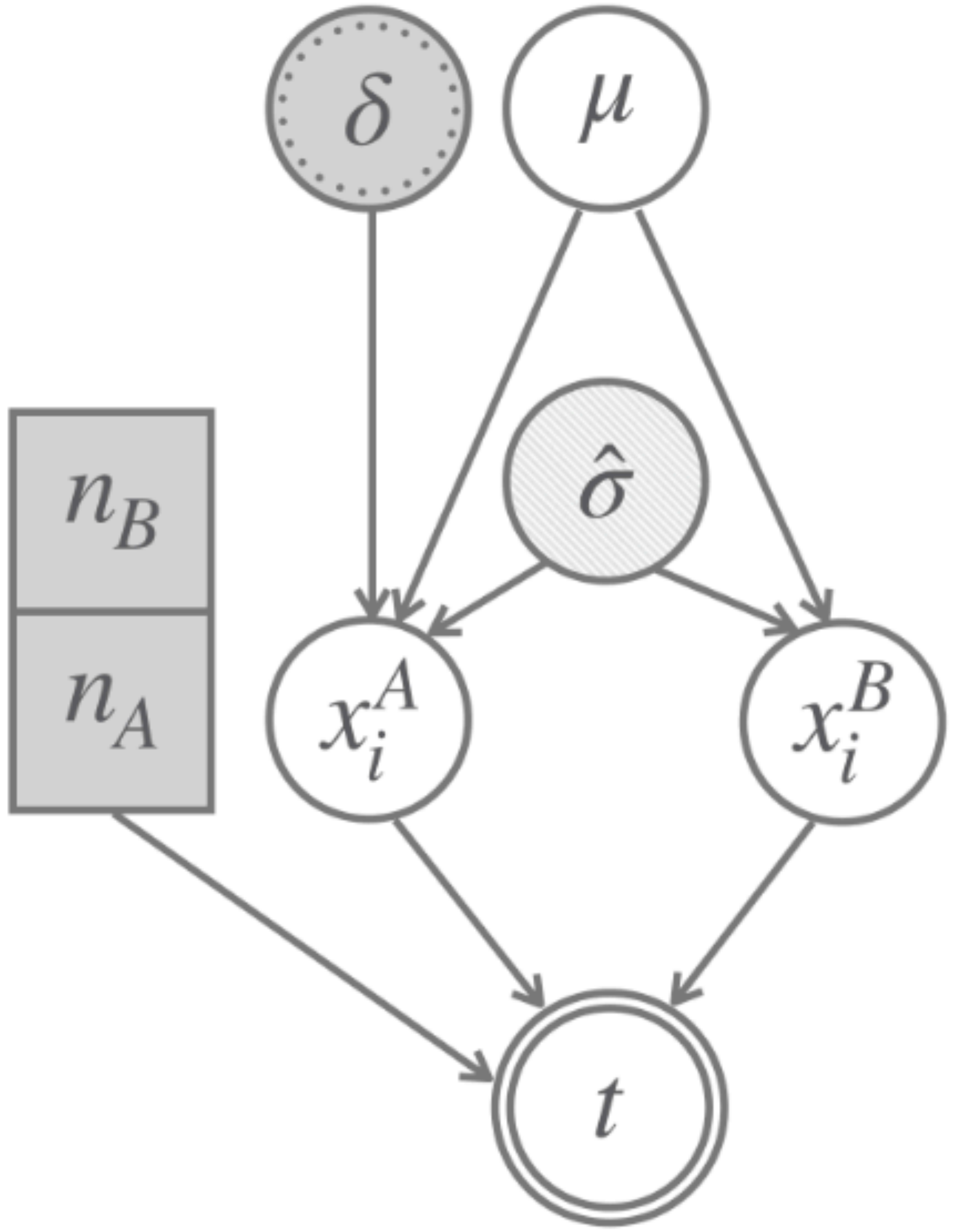
Statistical models

likelihoods from a data-generating process

- A **statistical model** is a condensed formal representation, following common conventional practices of formalization, of the assumptions we make about what the data is and how it might have been generated by some (usually: stochastic) process.
- "All models are wrong, but some are useful." (Box 1979)
- a **Bayesian statistical model** of stochastic process generating data D consists of:
  - a vector of **parameters** θ
  - a **likelihood function**: $P(D \mid \theta)$
  - a **prior** distribution: $P(\theta)$
- among other things, we can use a model for **inference**:
  - **posterior** distribution: $P(\theta \mid D) \propto P(D \mid \theta)\ P(\theta)$



MODELS are everywhere



$$x_i^A \sim \text{Normal}(\mu + \delta, \sigma)$$

$$x_i^B \sim \text{Normal}(\mu, \sigma)$$

$$\hat{\sigma} = \sqrt{\frac{(n_A - 1)\hat{\sigma}_A^2 + (n_B - 1)\hat{\sigma}_B^2}{n_A + n_B - 2}\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}$$

$$t = \left((\bar{x}_A - \bar{x}_B) - \delta\right) \cdot \frac{1}{\hat{\sigma}}$$

Sampling distribution:
$t \sim \text{Student-t}(\nu = n_A + n_B - 2)$

model of the data-generating process buried inside a two-sample t-test

read more here

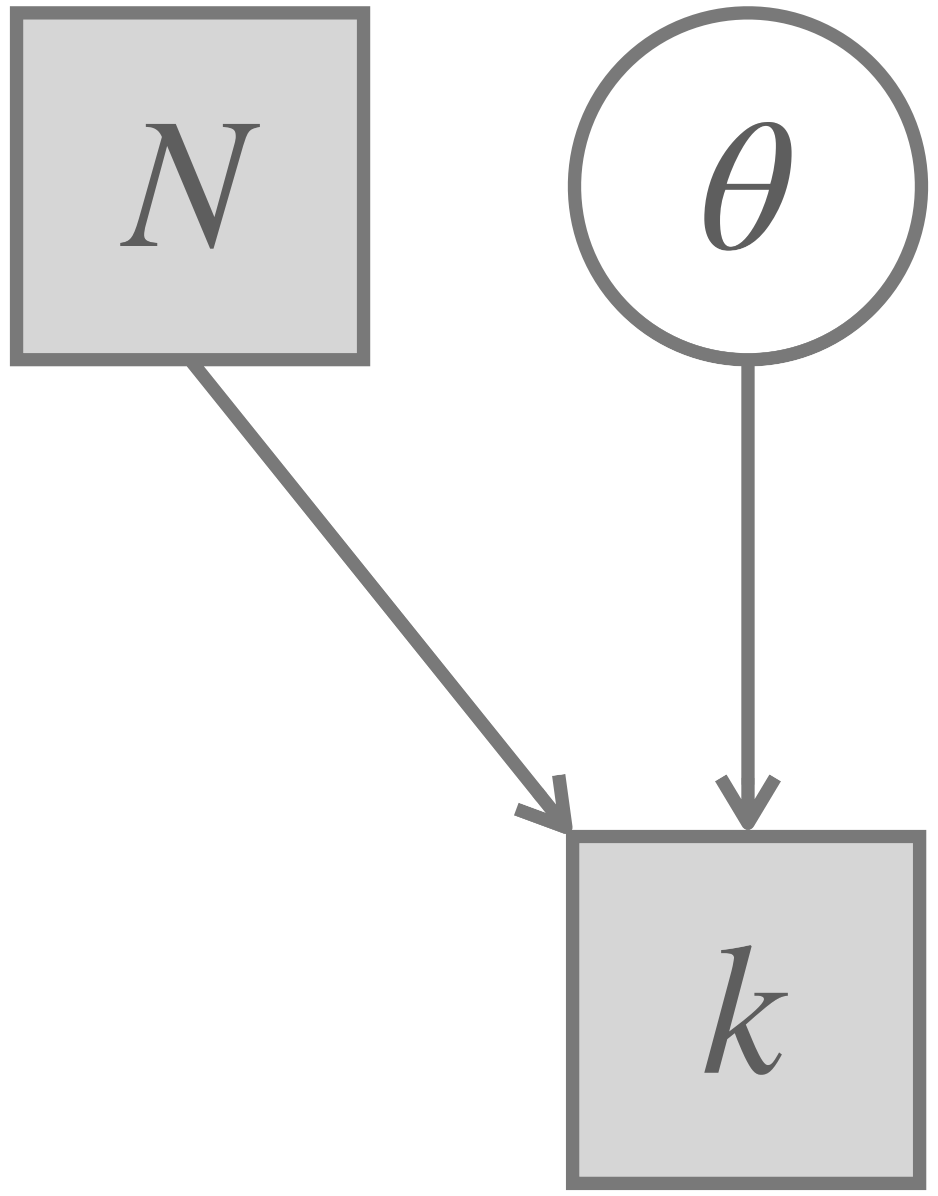
Binomial model

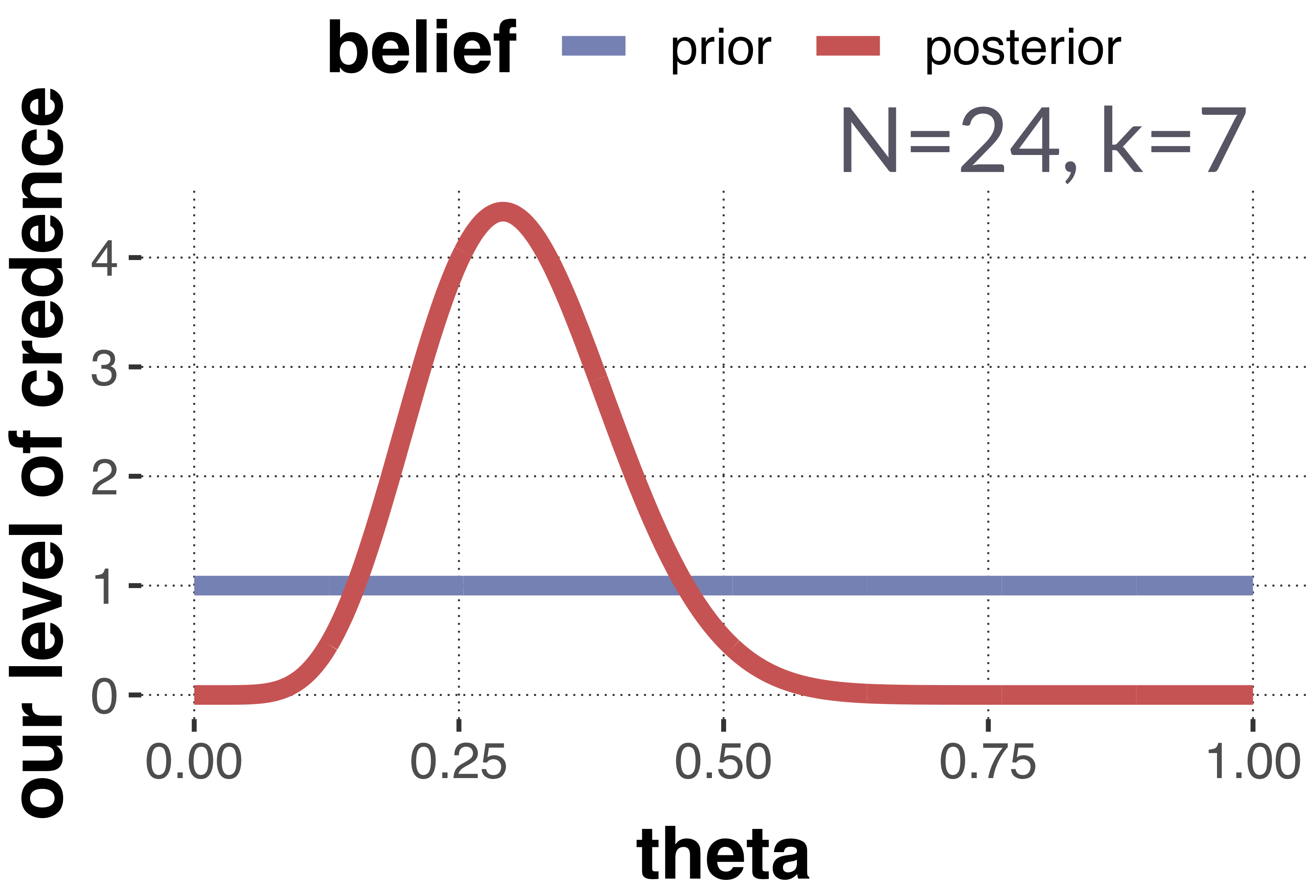
the 'coin-flip' model

- data: pair of numbers $D = \{k, N\}$
  - N is the number of tosses
  - k is the number of heads (successes)
- variable:
  - θ is the number of heads (successes)
- uninformed prior:

  $\theta \sim \text{Beta}(1,1)$
- likelihood function:

  $k \sim \text{Binomial}(\theta, N)$
- conventions for model graphs:
  - circles / squares: continuous / discrete variables
  - white / gray nodes: latent / observed variables





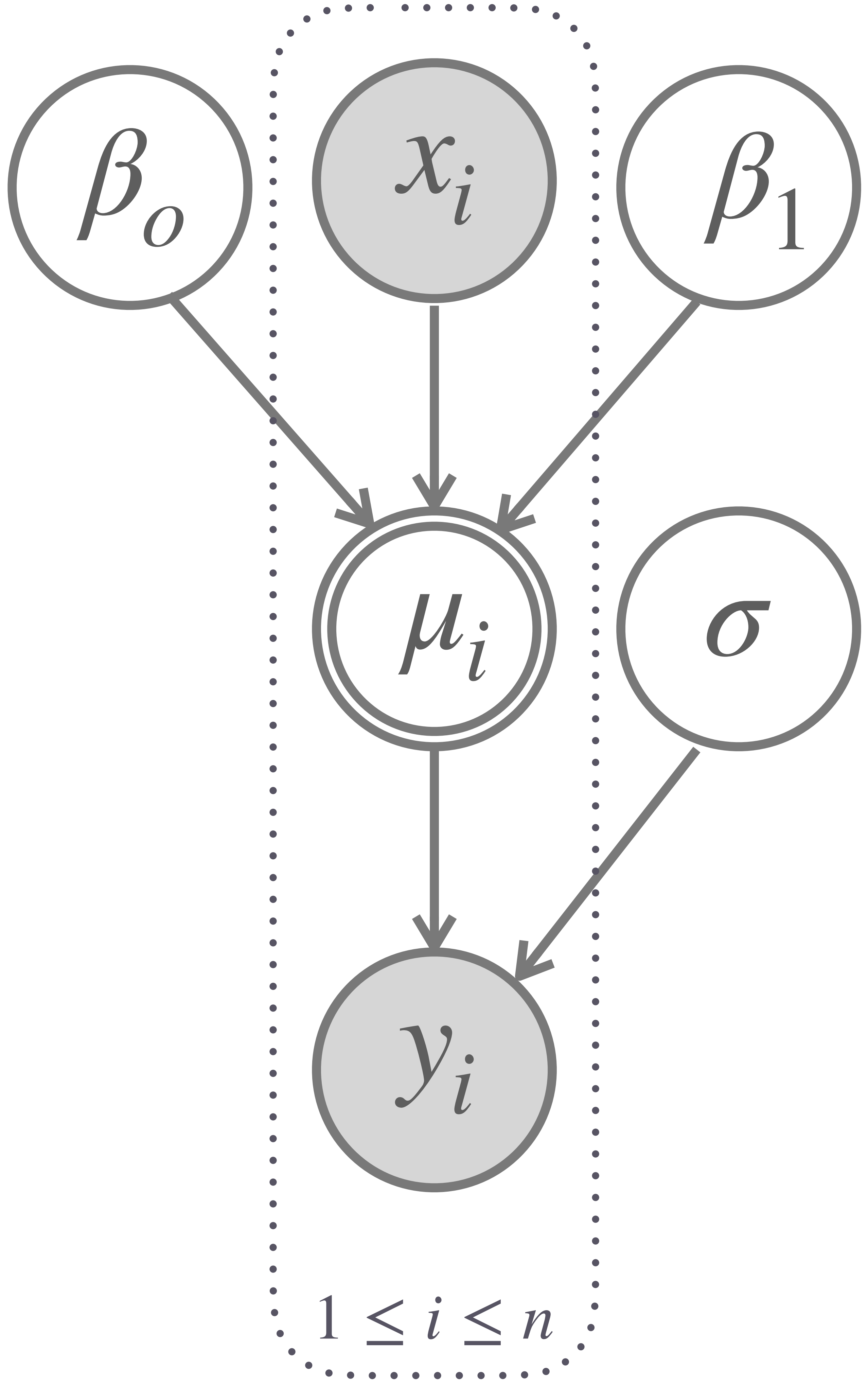
Simple linear regression model

for a single predictor variable

- data: n pairs of numbers $D = \{\langle x_1, y_1 \rangle, \ldots \langle x_n, y_n \rangle\}$
  - x_i is the i-th observation of the **independent / predictor variable**
  - y_i is the i-th observation of the **dependent / response variable**
- parameters:
  - β_0 is the **intercept** parameter
  - β_1 is the **slope** parameter
  - σ is the standard deviation of a normal distribution
- derived variable: [shown in node w/ double lines]
  - μ_i is the linear predictor for observation i
- priors (uninformed):

  $\beta_0, \beta_1 \sim \text{Uniform}(-\infty, \infty) \qquad \log(\sigma^2) \sim \text{Uniform}(-\infty, \infty)$
- likelihood:

  $y_i \sim \text{Normal}(\mu_i, \sigma) \qquad \mu_i = \beta_0 + x_1 \cdot \beta_1$



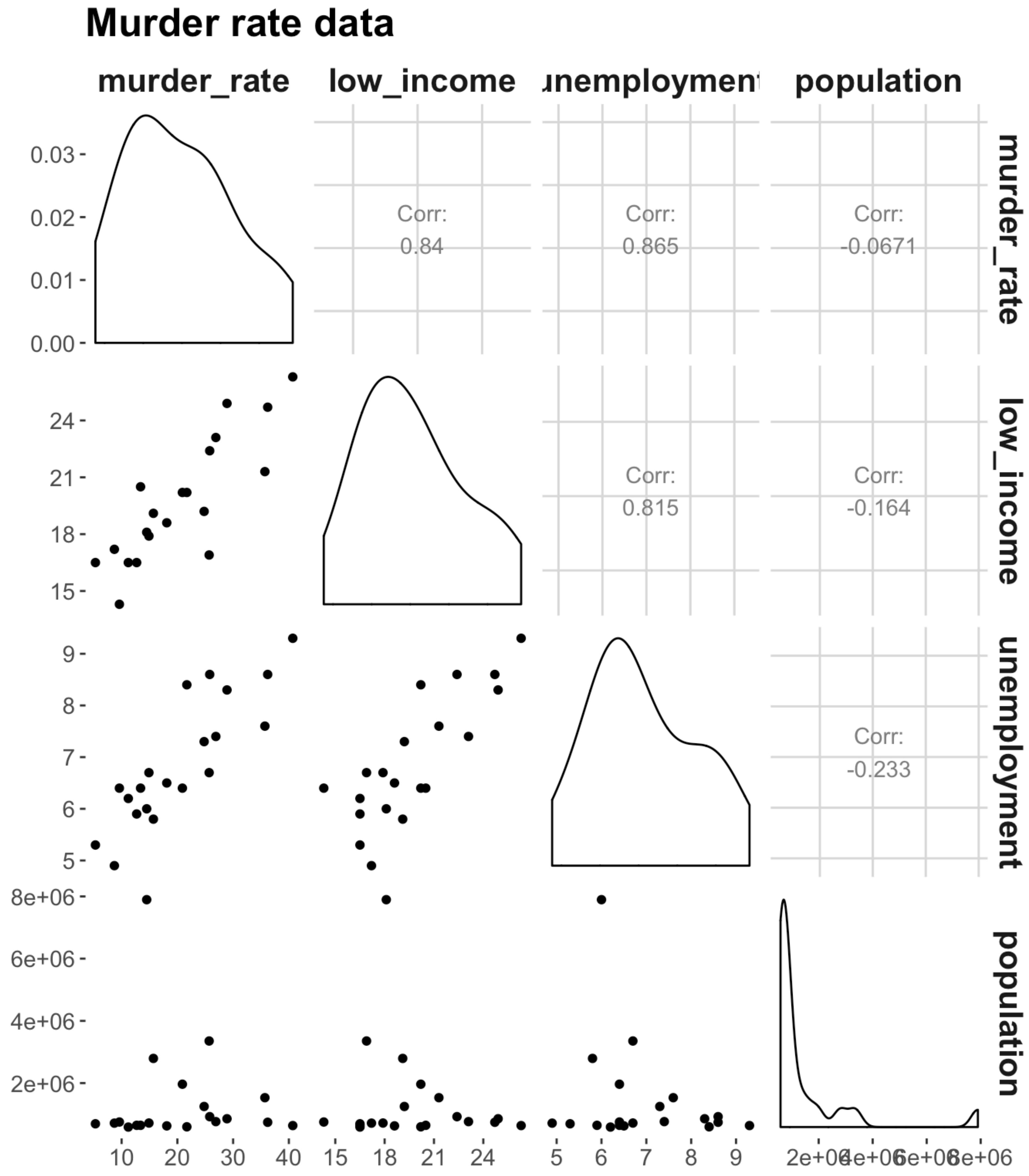
Simple linear regression

likelihood & Bayesian posterior

Murder data

annual murder rate, average income, unemployment rates and population

```
## # A tibble: 20 x 4
##    murder_rate low_income unemployment population
##          <dbl>      <dbl>        <dbl>      <dbl>
##  1        11.2       16.5          6.2     587000
##  2        13.4       20.5          6.4     643000
##  3        40.7       26.3          9.3     635000
##  4         5.3       16.5          5.3     692000
##  5        24.8       19.2          7.3    1248000
##  6        12.7       16.5          5.9     643000
##  7        20.9       20.2          6.4    1964000
##  8        35.7       21.3          7.6    1531000
##  9         8.7       17.2          4.9     713000
## 10         9.6       14.3          6.4     749000
## 11        14.5       18.1          6      7895000
## 12        26.9       23.1          7.4     762000
## 13        15.7       19.1          5.8    2793000
## 14        36.2       24.7          8.6     741000
## 15        18.1       18.6          6.5     625000
## 16        28.9       24.9          8.3     854000
## 17        14.9       17.9          6.7     716000
## 18        25.8       22.4          8.6     921000
## 19        21.7       20.2          8.4     595000
## 20        25.7       16.9          6.7    3353000
```



annual murders per million inhabitants

percentage inhabitants with low income

percentage inhabitants who are unemployed

total population

Predicting murder rate

no information at all

```
## # A tibble: 20 x 4
##    murder_rate low_income unemployment population
##          <dbl>      <dbl>        <dbl>      <dbl>
##  1        11.2       16.5          6.2     587000
##  2        13.4       20.5          6.4     643000
##  3        40.7       26.3          9.3     635000
##  4         5.3       16.5          5.3     692000
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##  6        12.7       16.5          5.9     643000
##  7        20.9       20.2          6.4    1964000
##  8        35.7       21.3          7.6    1531000
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## 14        36.2       24.7          8.6     741000
## 15        18.1       18.6          6.5     625000
## 16        28.9       24.9          8.3     854000
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## 18        25.8       22.4          8.6     921000
## 19        21.7       20.2          8.4     595000
## 20        25.7       16.9          6.7    3353000
```
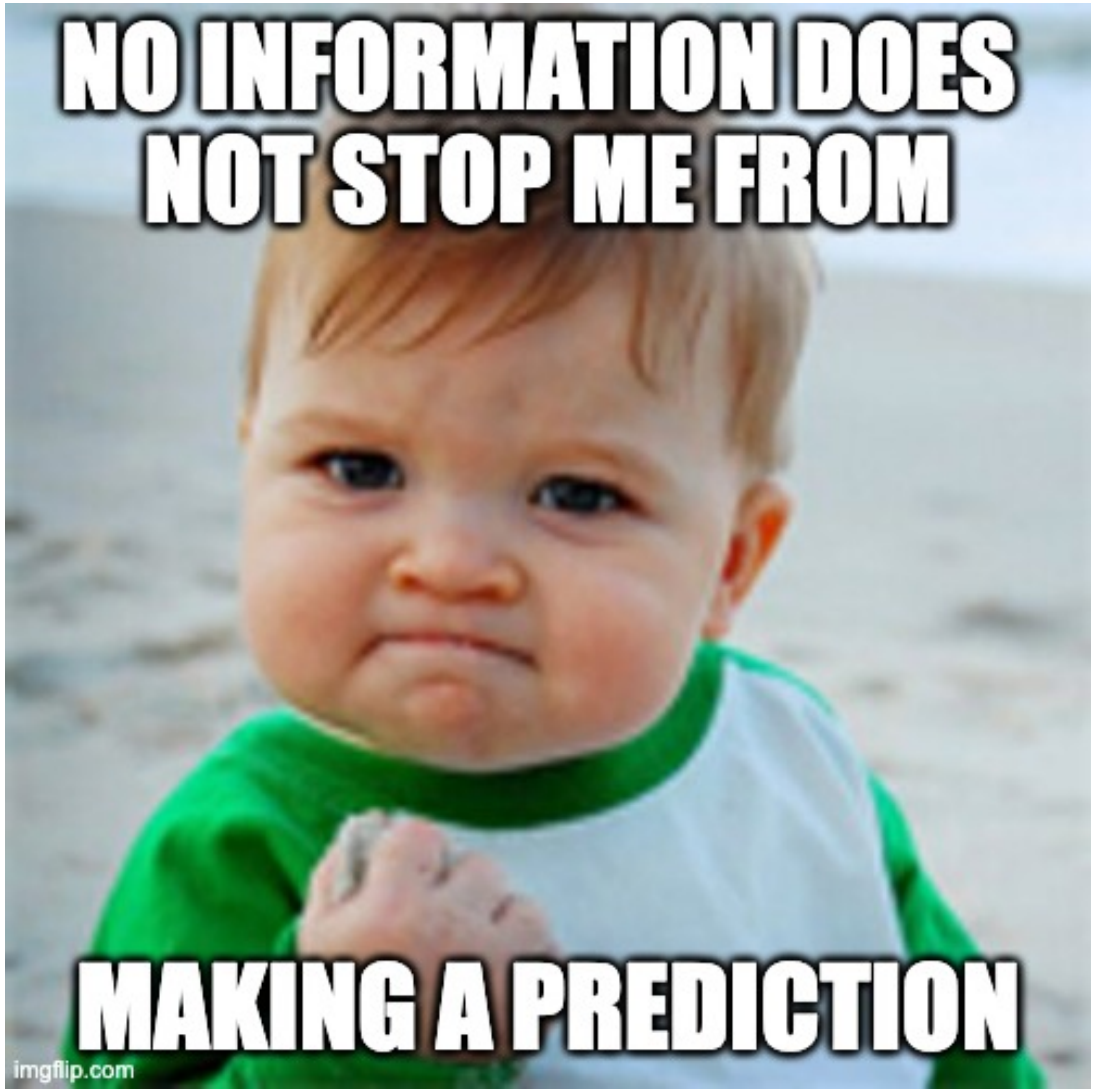
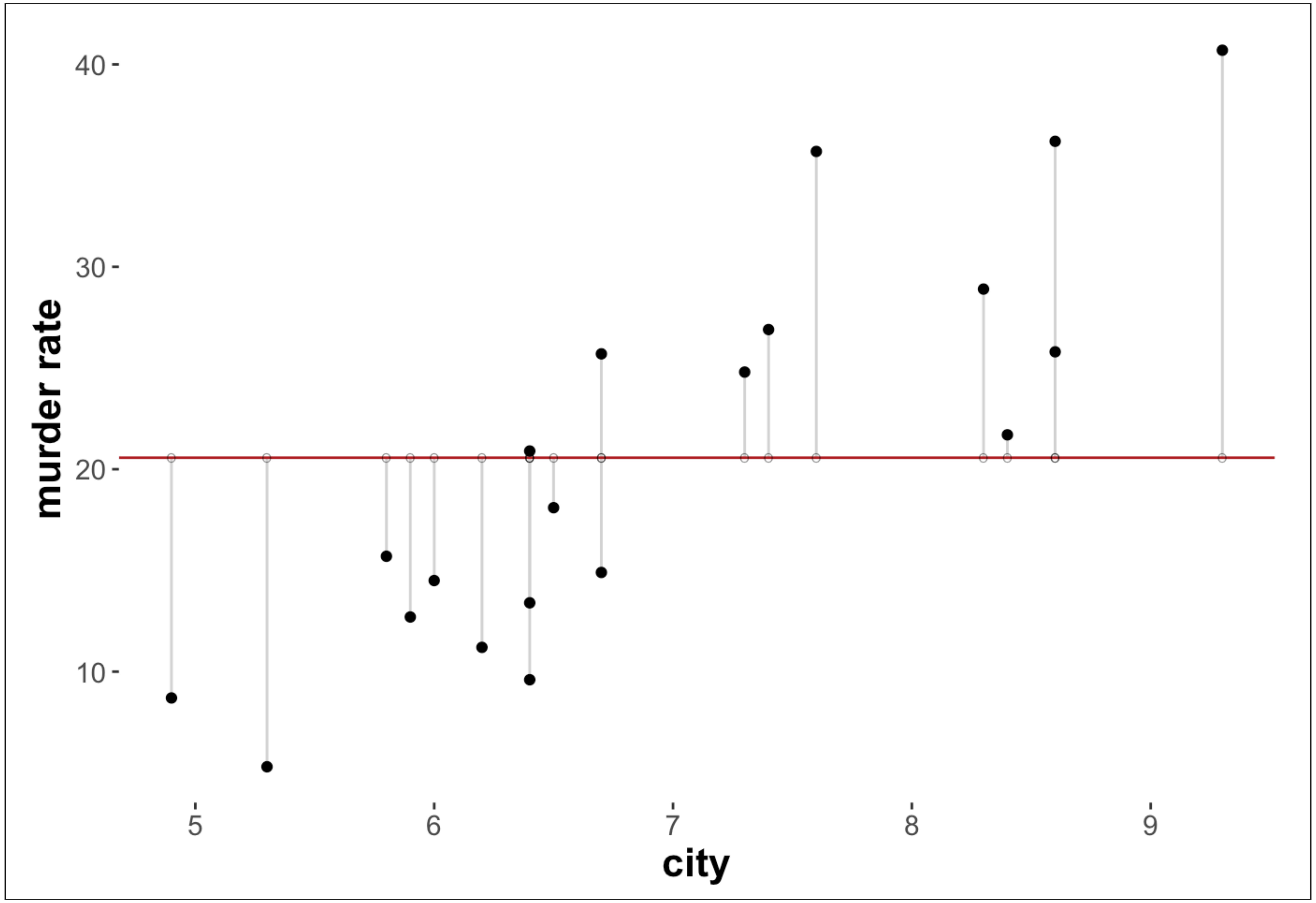



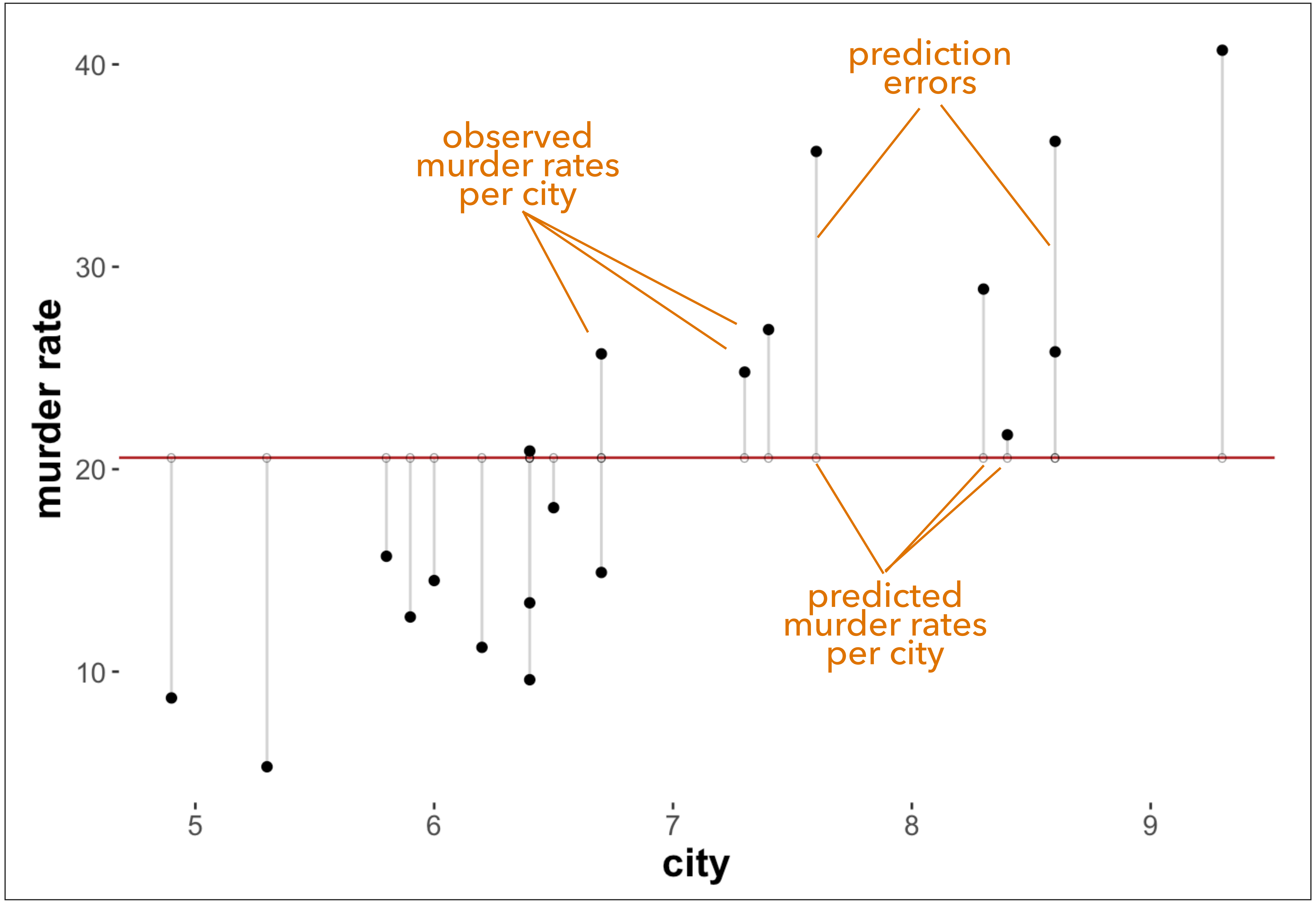
Predicting murder rate

by empirical mean



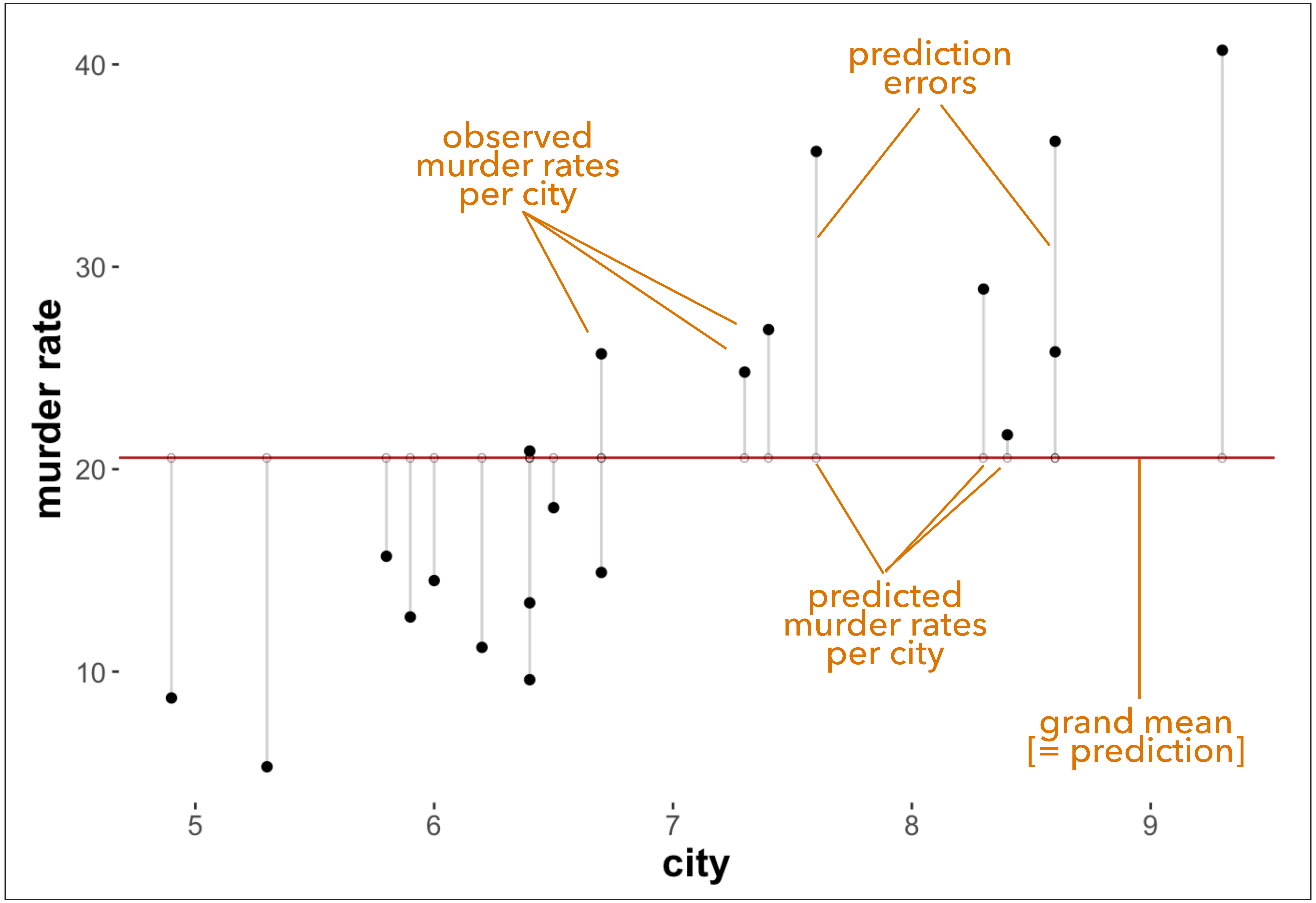
Predicting murder rate

by grand mean



Predicting murder rate

by grand mean



$$\text{TSS} = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

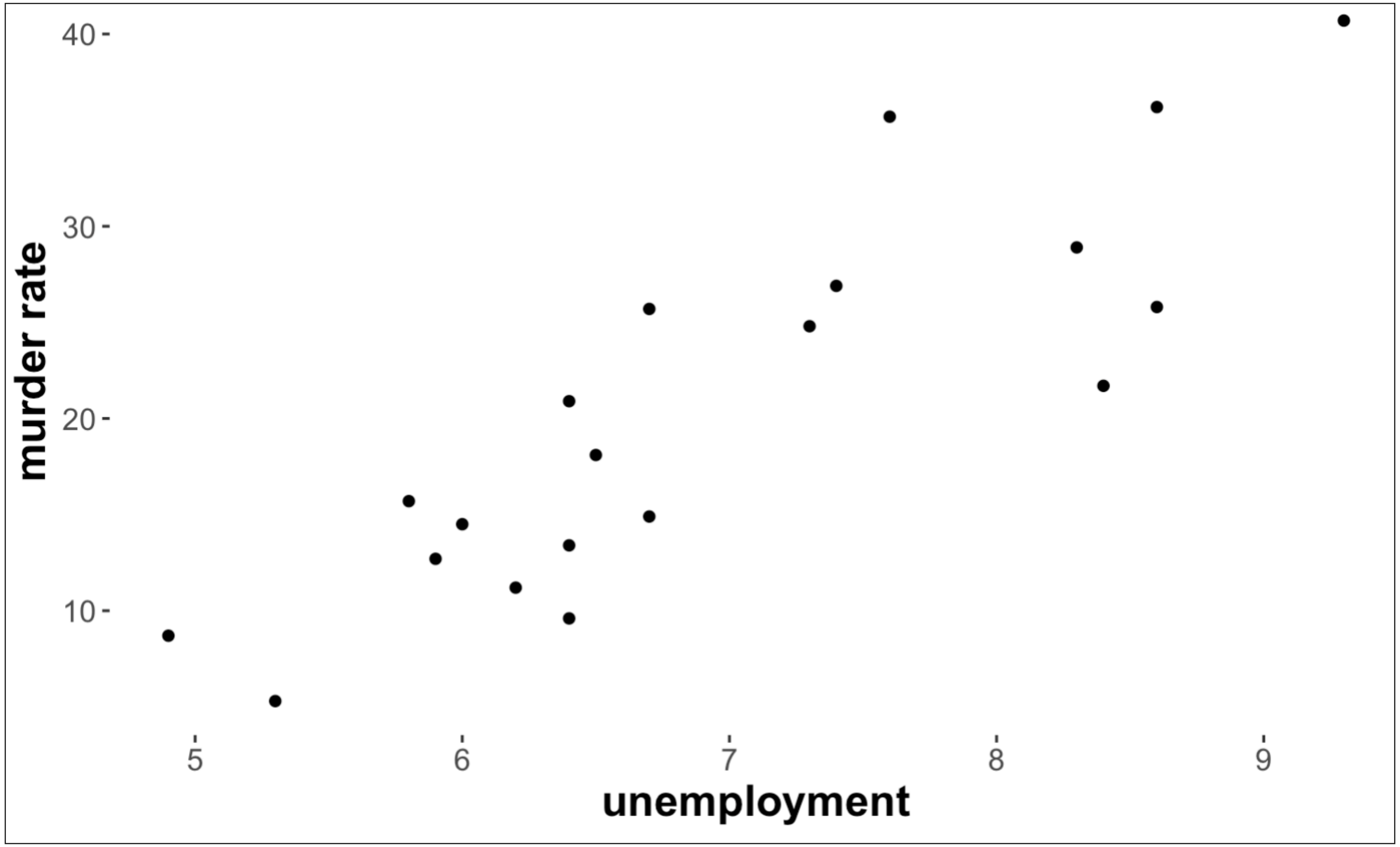
[total sum of squares]

```
y <- murder_data %>% pull(murder_rate)
n <- length(y)
tss_simple <- sum((y - mean(y))^2)
tss_simple
```

```
## [1] 1855.202
```

Predicting murder rate based on unemployment rate

some wild linear guessing



We are to predict the murder rate y_i of a randomly drawn city i. **We know that city's unemployment rate, x_i, but nothing more.**

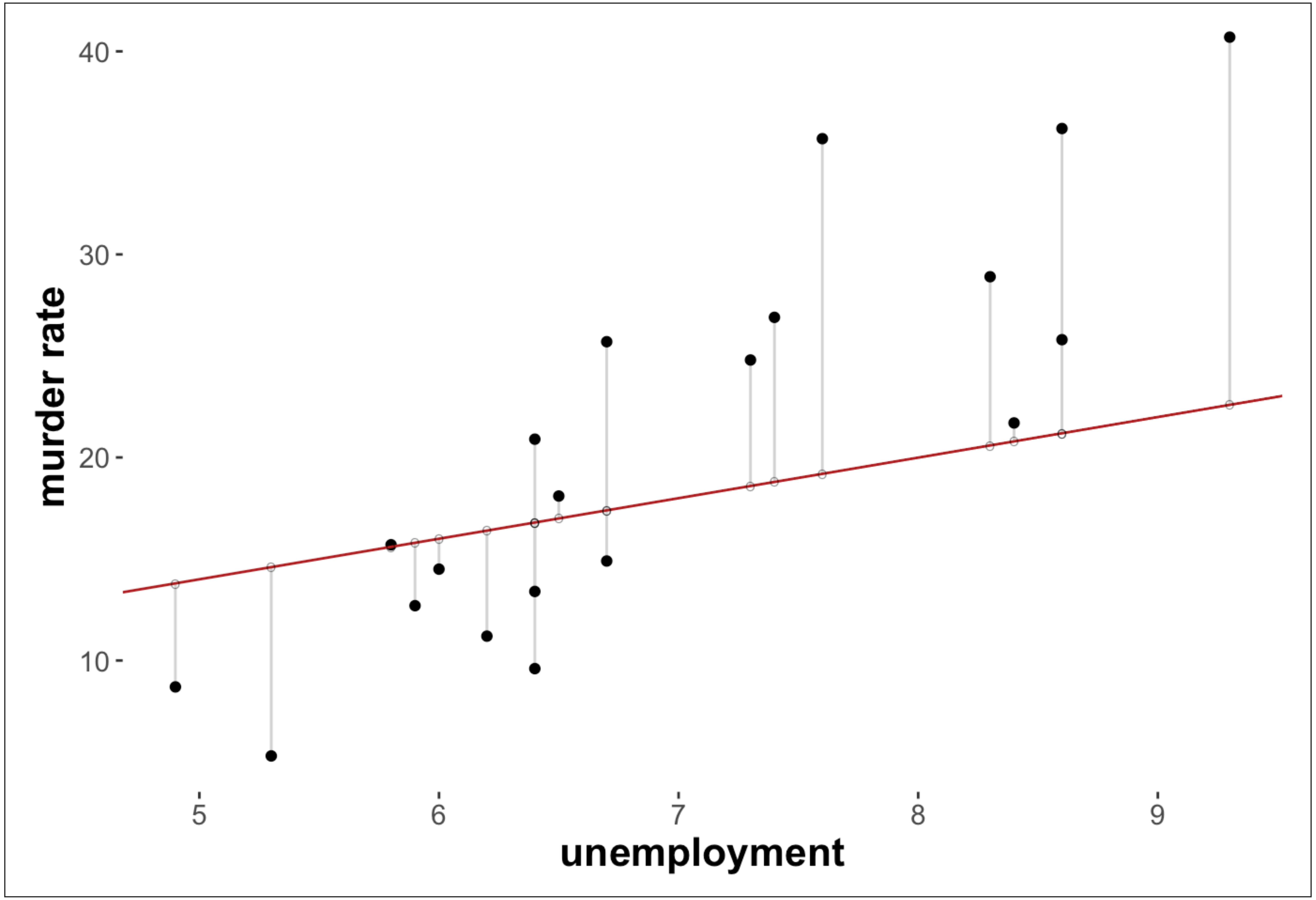
Let's just assume the following **linear relationship** to make a prediction b/c why not?!?

$$\hat{y}_i = 4 + 2x_i$$

How good is this prediction?

How good is any given prediction?

quantifying distance or likelihood



Distance-based approach

Residual Sum-of-Squares:

$$\text{RSS} = \sum_{i=1}^{n} \left(\hat{y}_i - y_i\right)^2$$

- no predictions about spread around linear predictor

Likelihood-based approach:

Normal likelihood:

$$\text{LH} = \prod_{i=1}^{n} \mathcal{N}\left(y_i \mid \mu = \hat{y}_i, \sigma\right)$$

- fully predictive

Likelihood-based simple linear regression

- likelihood:

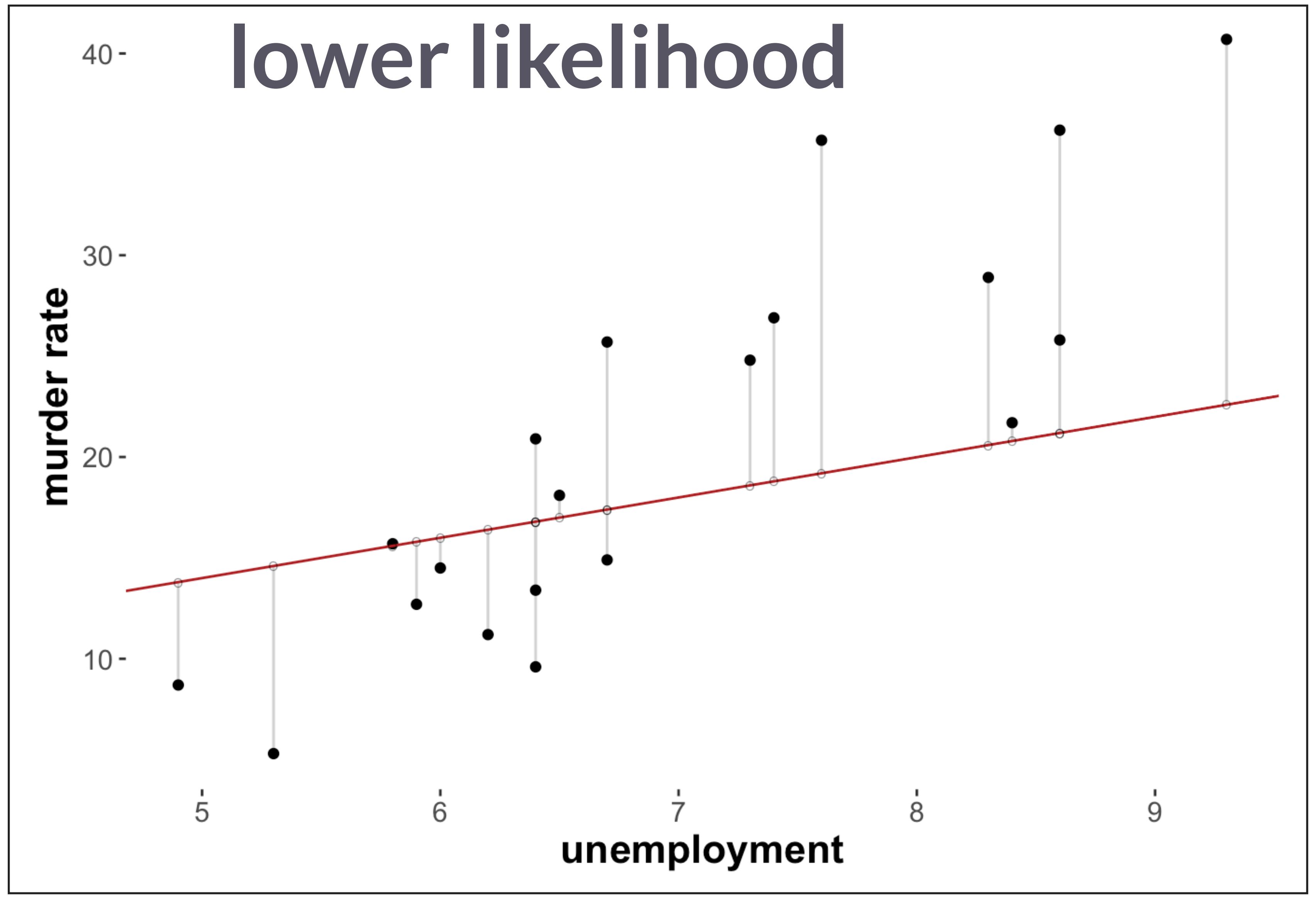
  $$y_i \sim \text{Normal}(\mu_i, \sigma)$$
  $$\mu_i = \beta_0 + x_1 \cdot \beta_1$$

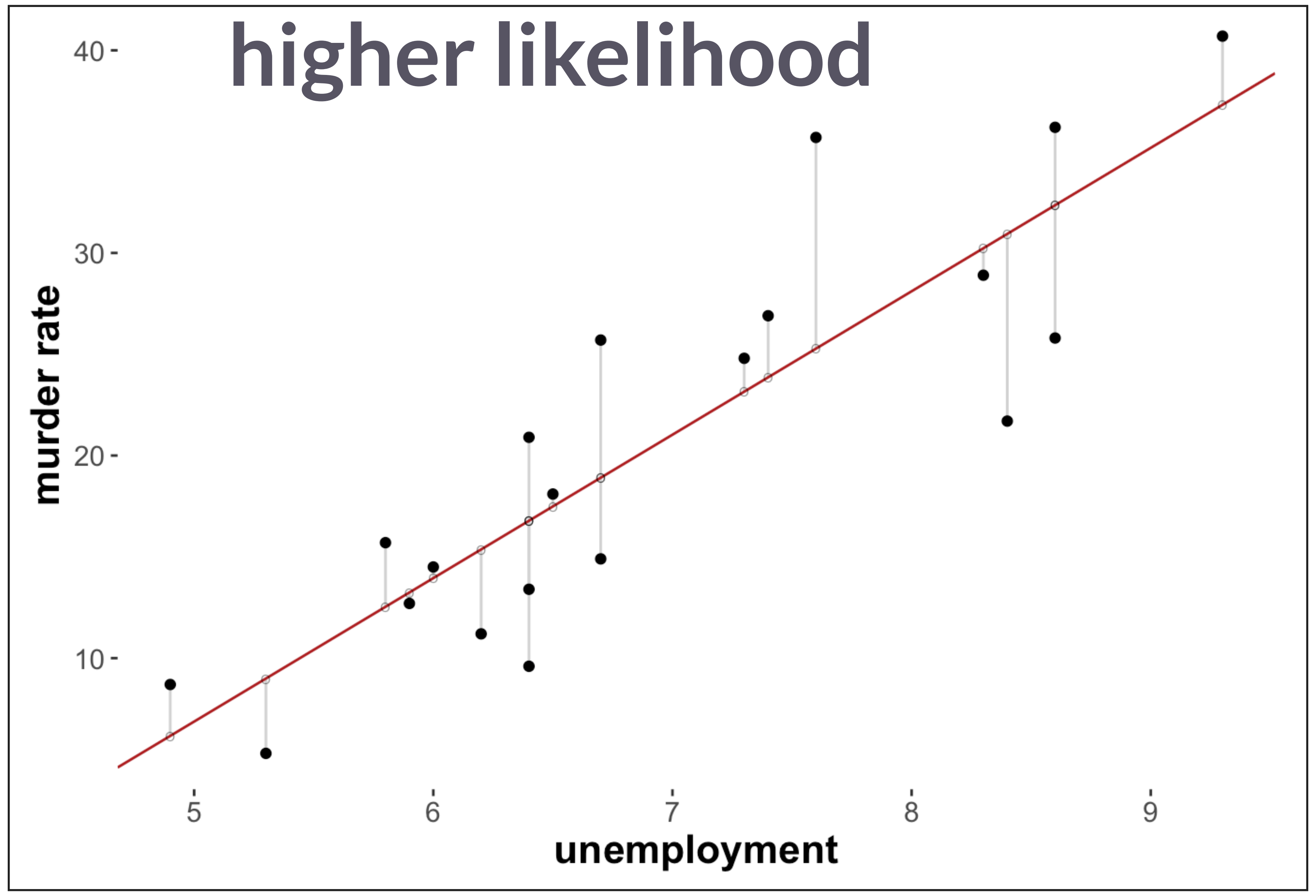
- differential likelihood:
  - parameter triples $\langle \beta_0, \beta_1, \sigma \rangle$ can be better or worse
  - higher vs. lower likelihood $P(D \mid \beta_0, \beta_1, \sigma)$
- maximum-likelihood solution:

  $$\arg\max_{\beta_0, \beta_1, \sigma} P(D \mid \beta_0, \beta_1, \sigma)$$

  - standard (frequentist) solution
  - MLE corresponds to MAP for "flat" priors
- Bayesian approach: full posterior distribution

  $$P(\beta_0, \beta_1, \sigma \mid D) \propto P(D \mid \beta_0, \beta_1, \sigma)\ P(\beta_0, \beta_1, \sigma)$$





Bayesian linear regression in R

using BRMS and Stan

- R package BRMS provides high-level interface for Bayesian linear regression
- models are specified with R's formula syntax
- returns samples from the posterior distribution
  - alternatives: MAPs, variational inference
- builds on probabilistic programming language Stan
  - powerful, cutting-edge tool for Bayesian computation
  - strong, non-commercial development team
  - many interfaces: stand-alone, R, Python, Julia, …

```
fit_brms_murder <- brm(
  # specify what to explain in terms of what
  #  using the formula syntax
  formula = murder_rate ~ unemployment,
  # which data to use
  data = murder_data
)
```

Stan

Navigating BRMS output

```r
summary(fit_brms_murder)
```

model & data we used

```
 Family: gaussian
  Links: mu = identity; sigma = identity
Formula: murder_rate ~ unemployment
   Data: murder_data (Number of observations: 20)
```

information about sampling (more on this later)

```
Samples: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
         total post-warmup samples = 4000
```

main model parameters

```
Population-Level Effects:
             Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept      -28.48      7.32   -42.05   -13.79 1.00     3014     2362
unemployment     7.07      1.04     4.97     9.04 1.00     2978     2451
```

additional model parameters

```
Family Specific Parameters:
      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sigma     5.42      0.96     3.88     7.63 1.00     2664     2196
```

information about sampling (more on this later)

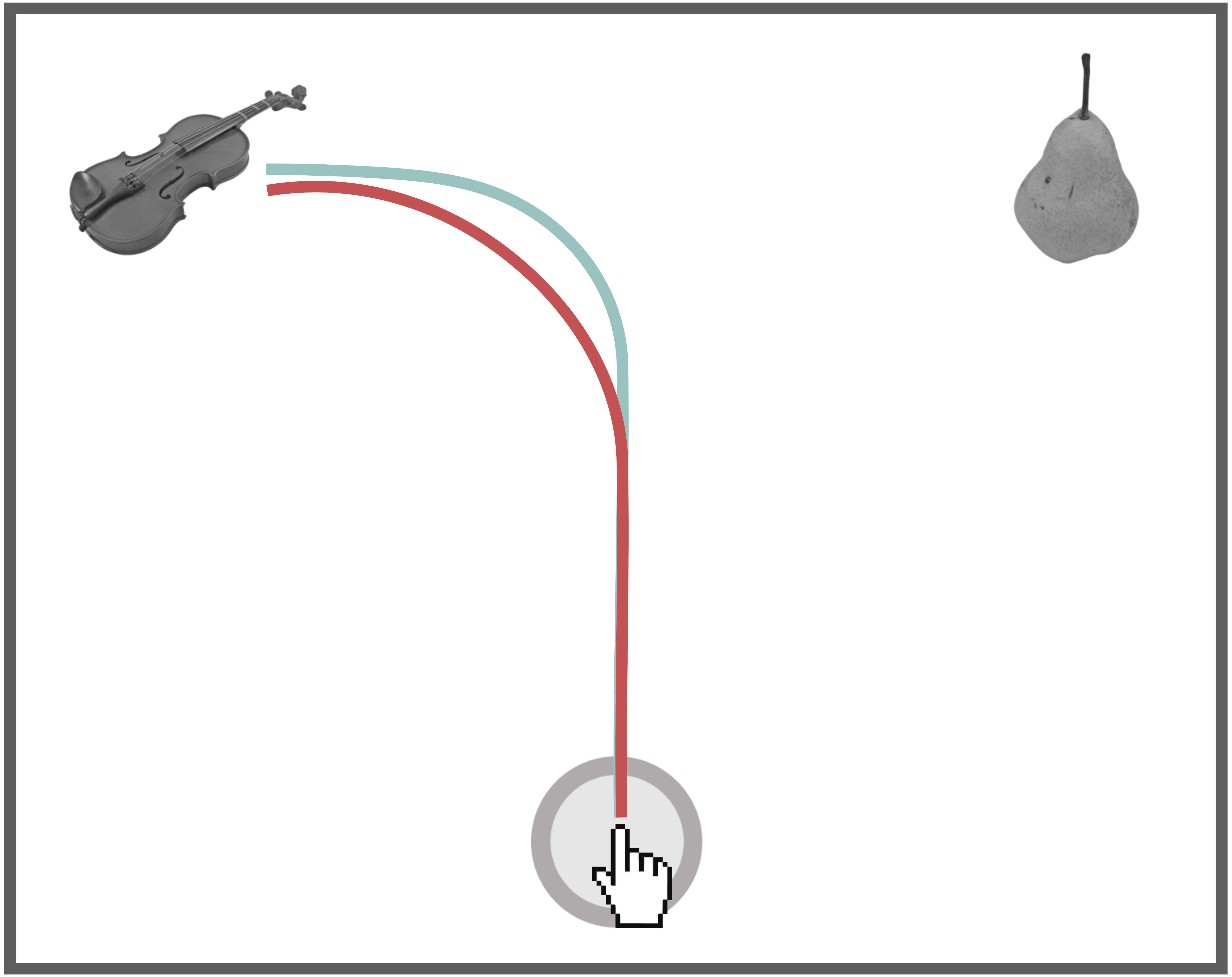
```
Samples were drawn using sampling(NUTS). For each parameter, Eff.Sample
is a crude measure of effective sample size, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
```

Mouse-tracking data on typicality in category decisions

Mouse-tracking

Hand-movement during decision making

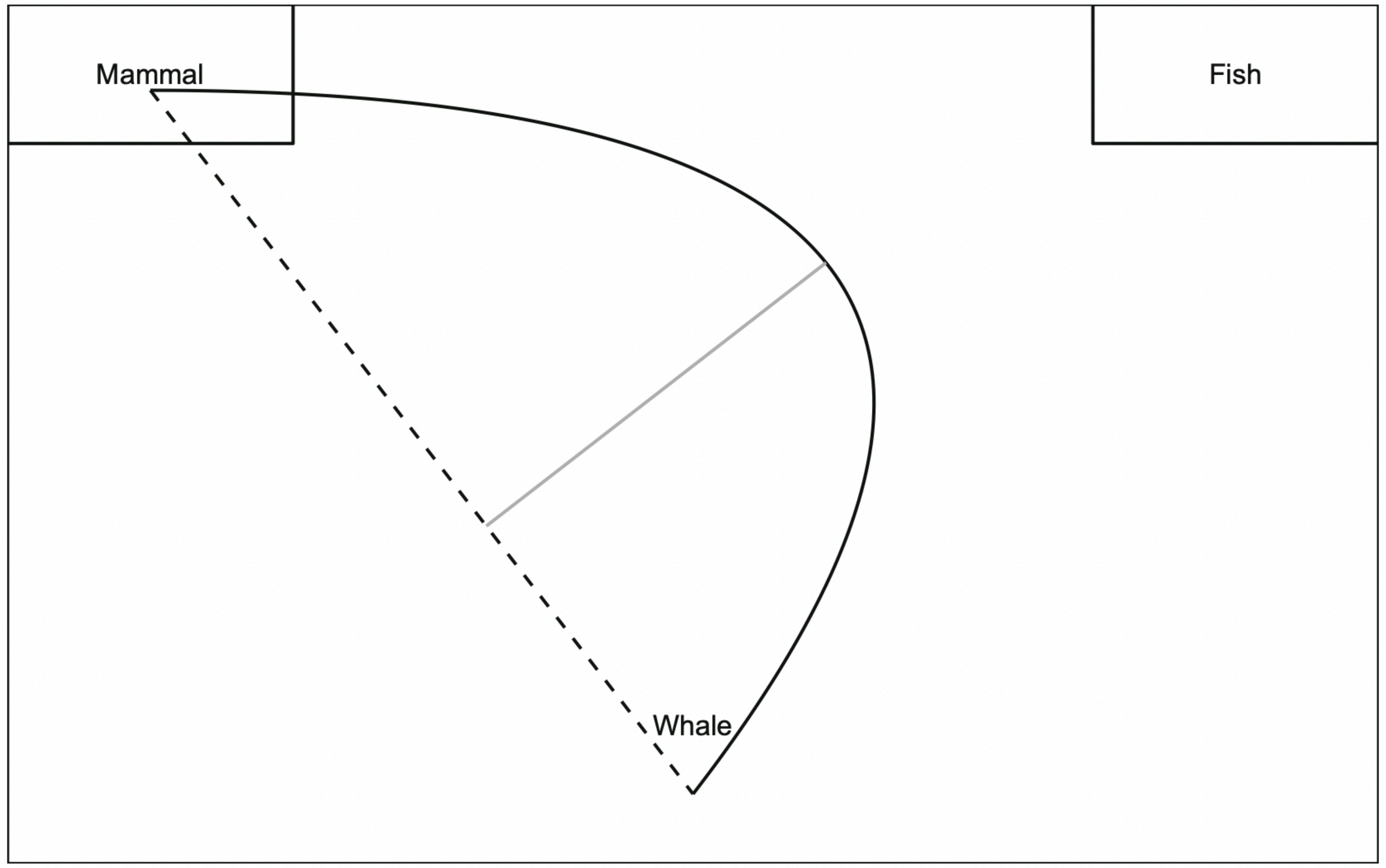
- general idea: motor-execution provides information about the ongoing decision process
  - uncertainty
  - gradual evidence accumulation
  - change-of-mind
  - time-point of decision
  - ...
- many subtle design decisions
  - click vs touch
  - move horizontally or vertically
  - ...

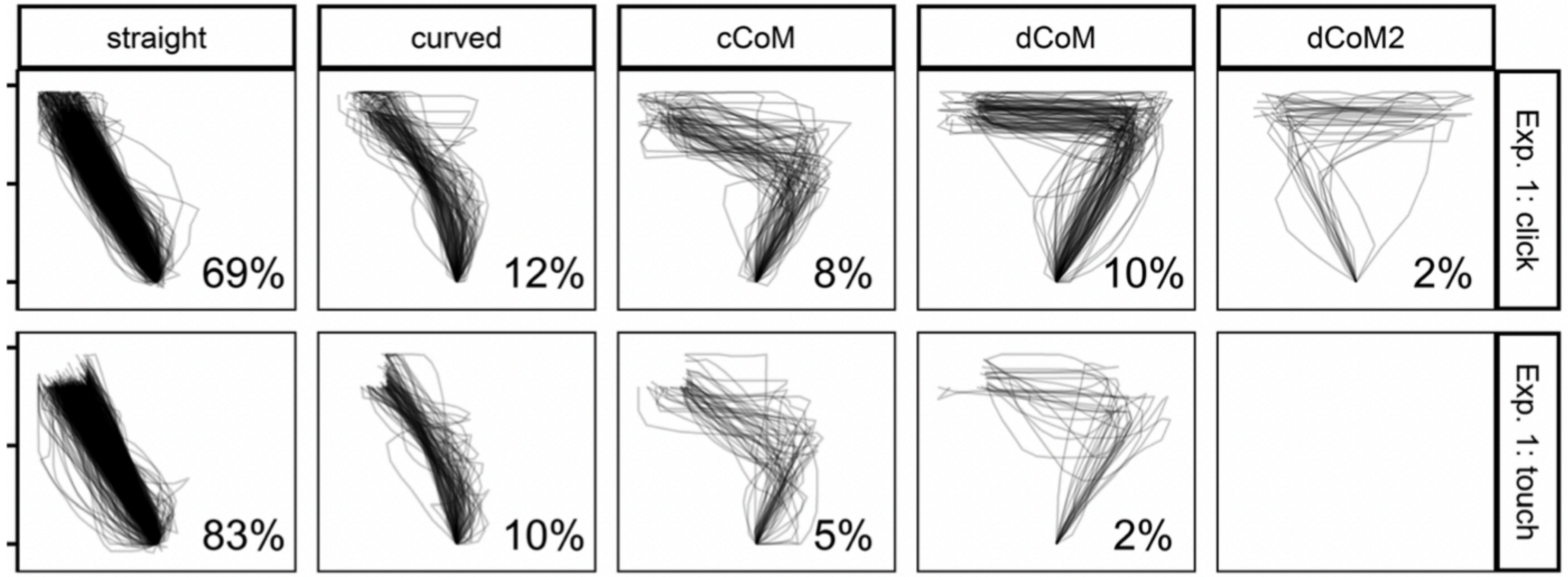


Mouse-tracking

common measures of mouse-trajectories

- raw data are lists of triples
  - (time, x-position, y-position)
- commonly used measures
  - area-under the curve (AUC)
    - area between the mouse trajectory and a straight line from start to selected option
  - maximal deviation (MAD)
    - maximum distance between trajectory and straight line from start to selected option
  - correctness
    - whether choice of option was correct or not
  - reaction time (RT)
    - how long did the movement last in total
  - type of trajectory
    - result of clustering analysis based on shape of the trajectories (usually some 3-5 categories)
  - x-flips
    - number of times the trajectory crossed the vertical middle line (at x = 0)





Running example

category recognition for typical vs atypical exemplars

- materials & procedure
  - participants read an animal name (e.g. ‘dolphin’)
  - they choose the true category the animal belongs to (e.g., ‘fish’ or ‘mammal’)
  - some trigger words are typical others atypical representatives of the true category
- methodological investigation:
  - two groups: **click vs touch** to select category
- **hypothesis:** typical exemplars are easier to categorize than atypical ones
  - fewer mistakes
  - smaller RTs, AUC, MAD
  - less x-flips
  - less “change-of-mind” curve types
- **research question (methods):** any differences between click & touch selection?

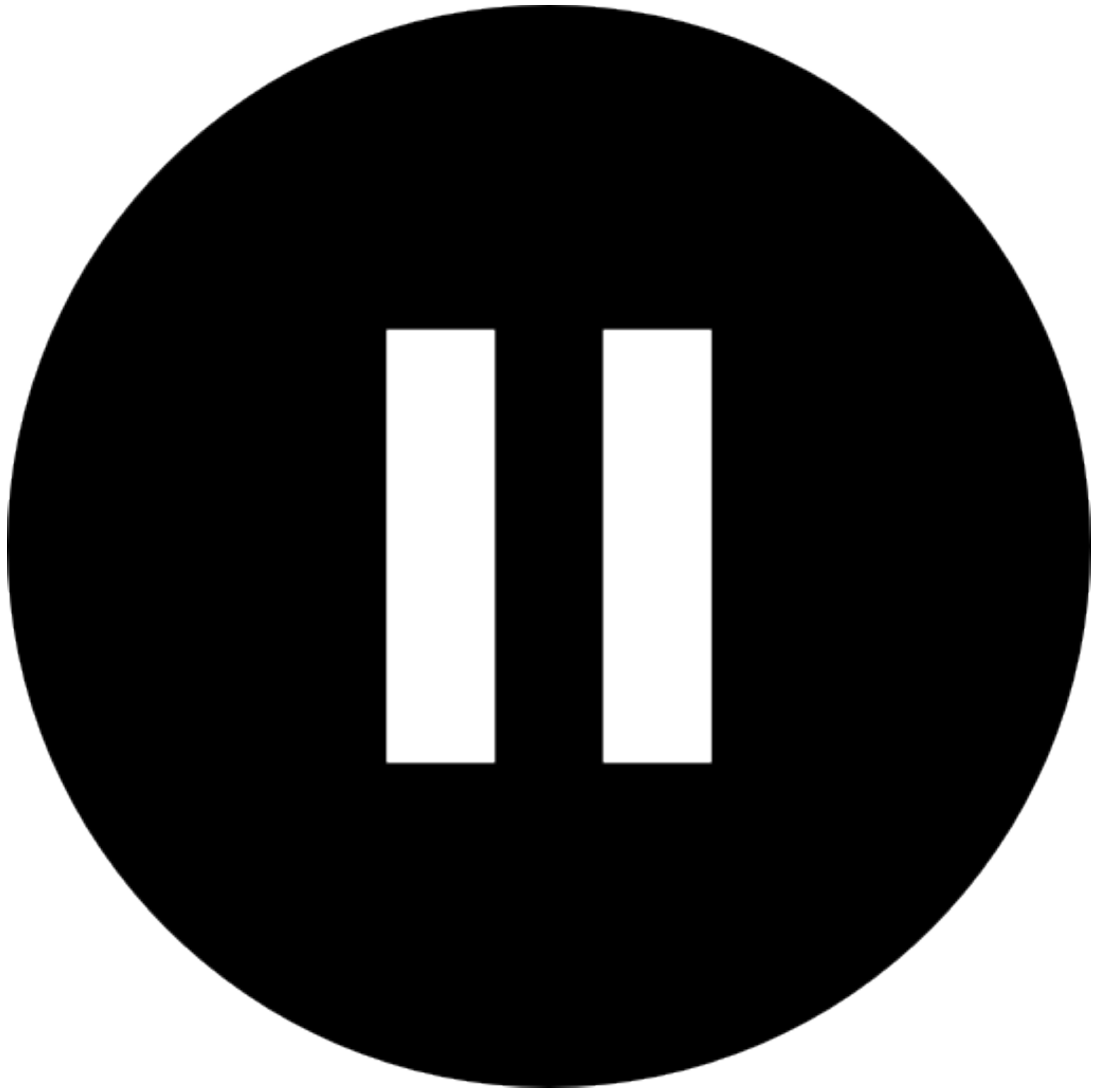
variables used in the data set

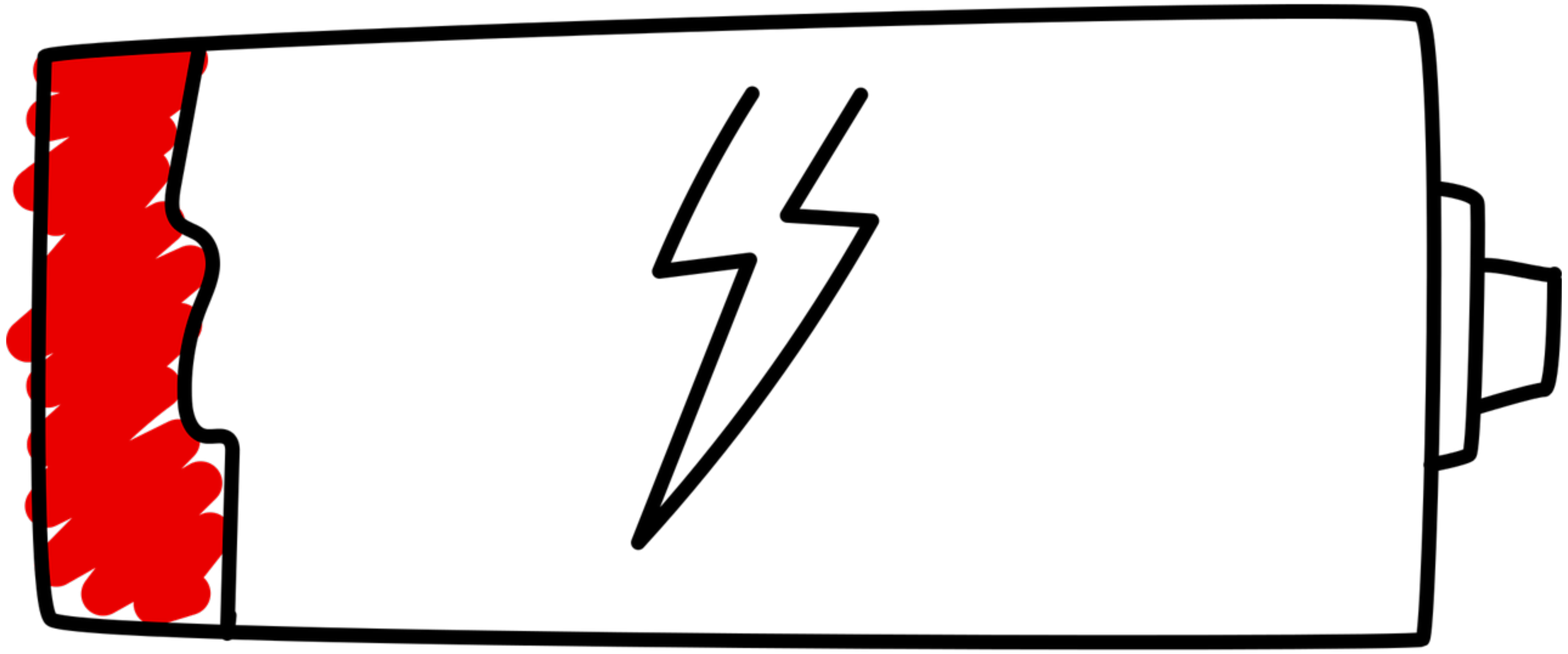
`trial_id` = unique id for individual trials
`MAD` = maximal deviation into competitor space
`AUC` = area under the curve
`xpos_flips` = the amount of horizontal direction changes
`RT` = reaction time in ms
`prototype_label` = different categories of prototypical movement strategies
`subject_id` = unique id for individual participants
`group` = groups differ in the response design (click vs. touch)
`condition` = category membership (Typical vs. Atypical)
`exemplar` = the concrete animal
`category_left` = the category displayed on the left
`category_right` = the category displayed on the right
`category_correct` = the category that is correct
`response` = the selected category
`correct` = whether or not the `response` matches `category_correct`

Kieslich et al.’s (2019) replication of Dale et al.’s (2003) experiment

demo

check the ‘simple linear regression’ tutorial if you want to read up on what we did here





Bayesian parameter estimation

Computing posterior distributions

problem of computational complexity

$$P(\theta \mid D) = \frac{P(D \mid \theta)\, P(\theta)}{\int P(D \mid \theta)\, P(\theta)\, \mathrm{d}\theta}$$

✓fast & easy

✓fast & easy

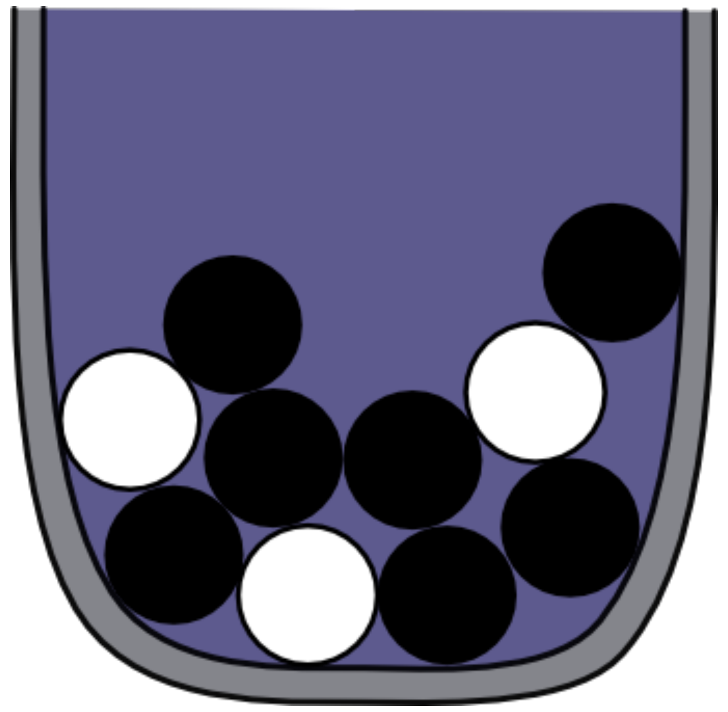
❌☠ possibly intractable ☠❌

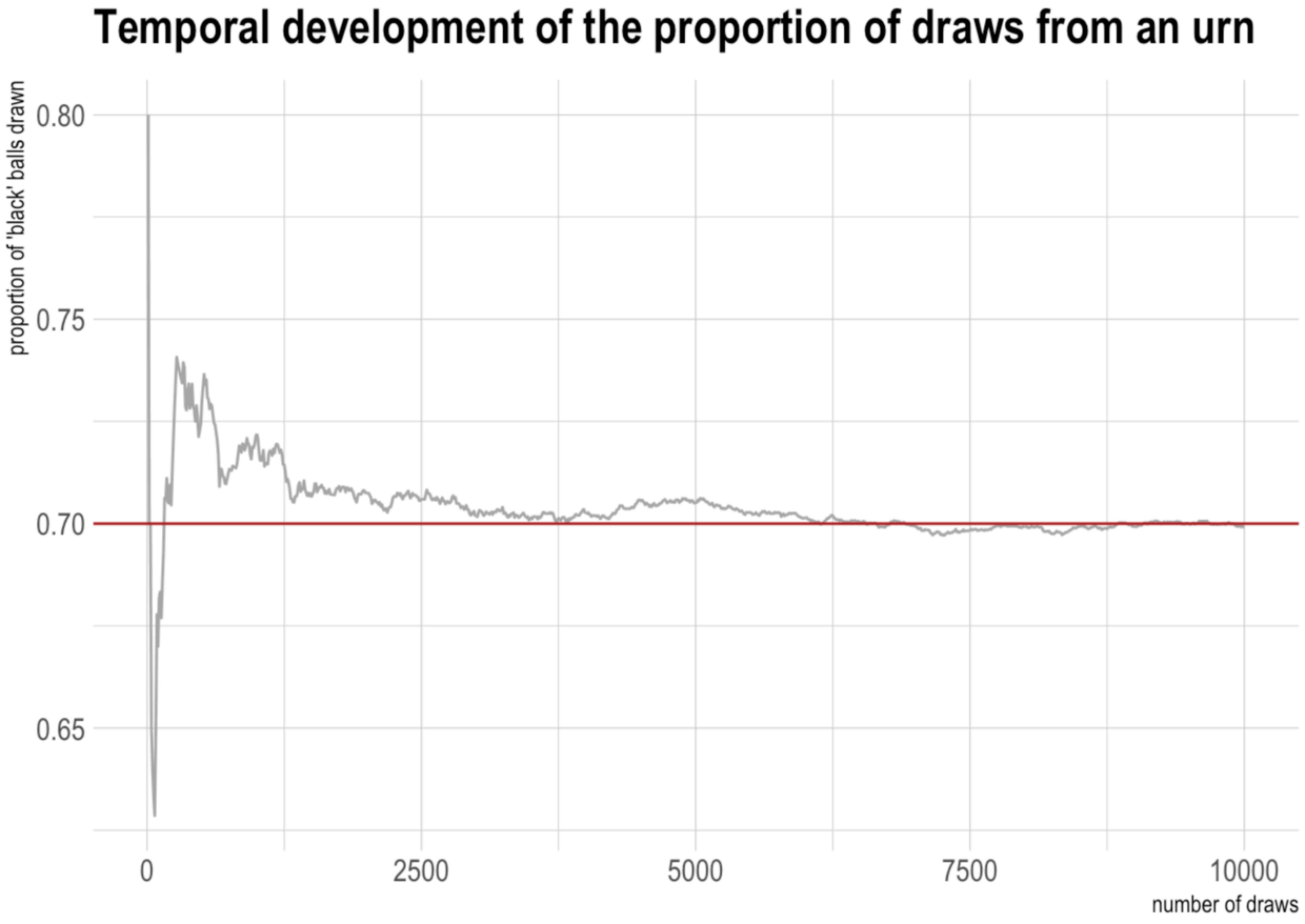
read more here

Approximating distributions via sampling

our go-to solution for approximating posterior distributions beyond conjugacy

- we can approximate any probability distribution by either:
  - a large set of representative samples; or
  - an oracle that returns a sample if needed.





Excursion: Posteriors from conjugacy

closed-form posteriors from clever choice of priors

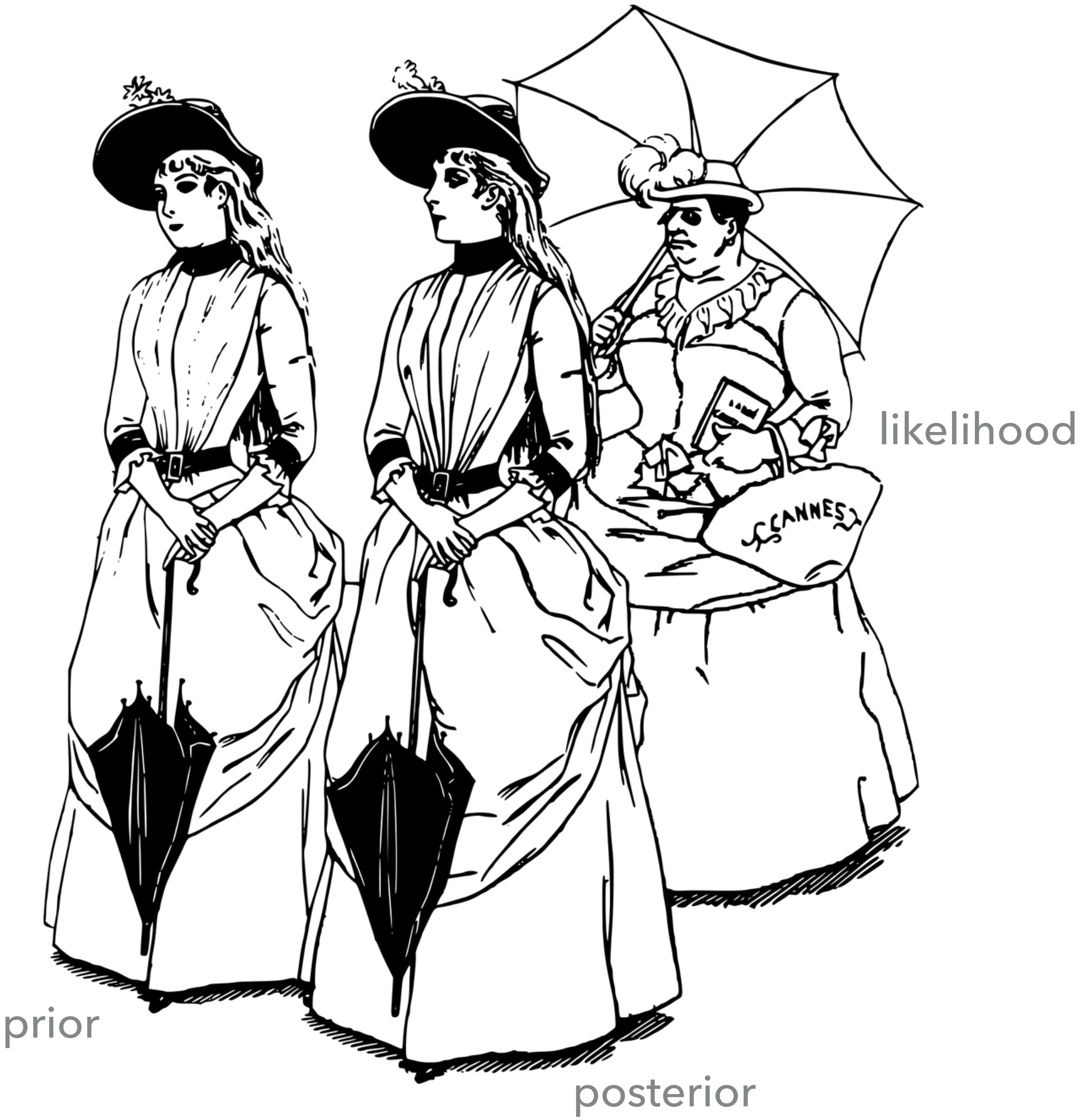
- prior $P(\theta)$ is a **conjugate prior** for likelihood $P(D \mid \theta)$ iff prior $P(\theta)$ and posterior $P(\theta \mid D)$ are the same kind of probability distribution, e.g.:
  - prior: $\theta \sim \text{Beta}(1,1)$
  - posterior: $\theta \mid D \sim \text{Beta}(8,18)$
- **claim:** the beta distribution is a conjugate prior for the binomial likelihood function
  - proof:

$$P(\theta \mid k, N) \propto \text{Binomial}(k; N, \theta)\ \text{Beta}(\theta \mid a, b)$$

$$P(\theta \mid k, N) \propto \theta^k (1-\theta)^{N-k}\, \theta^{a-1} (1-\theta)^{b-1}$$

$$P(\theta \mid k, N) \propto \theta^{k+a-1} (1-\theta)^{N-k+b-1}$$

$$P(\theta \mid k, N) = \text{Beta}(\theta \mid k+a, N-k+b)$$

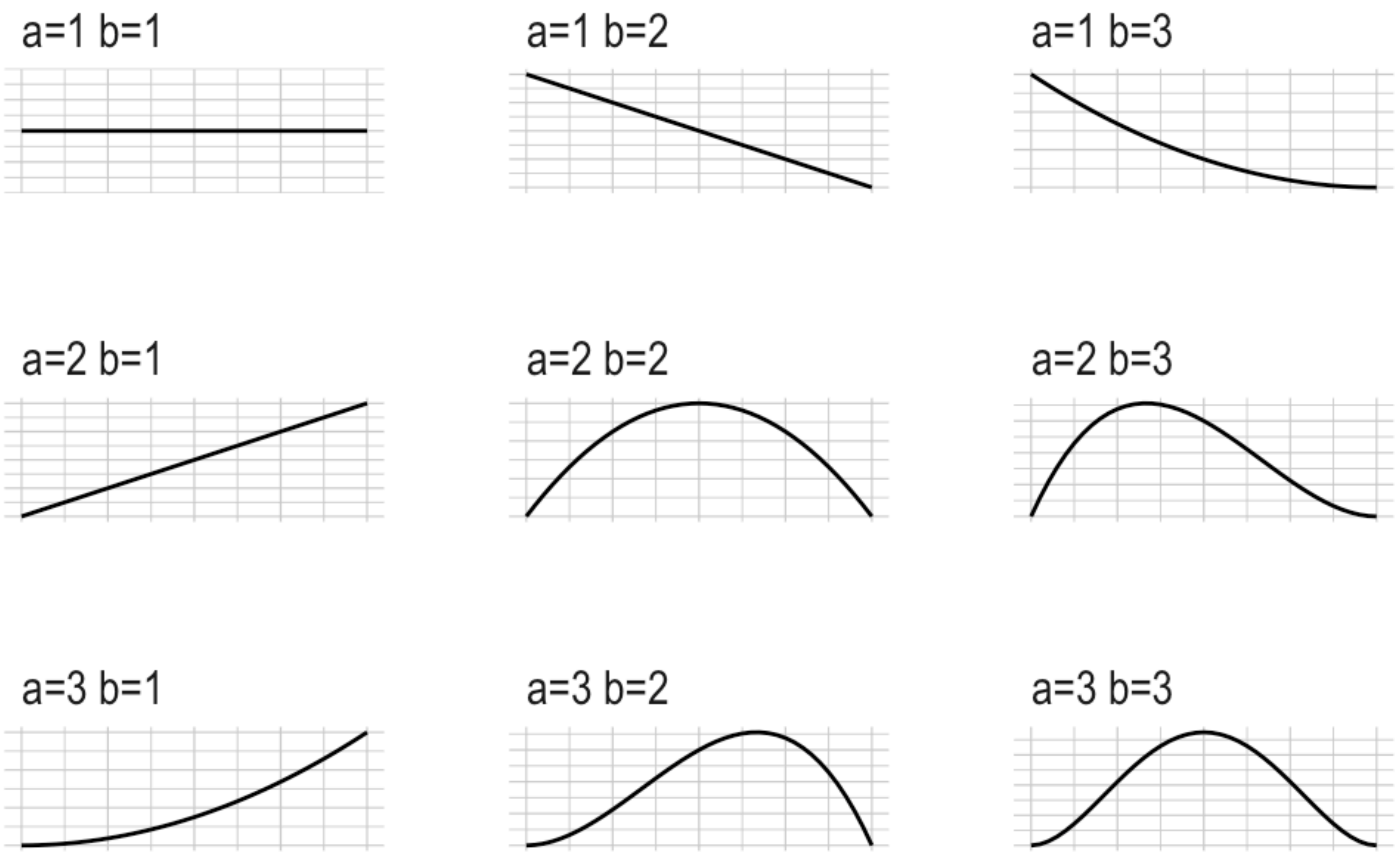


read more here

Excursion: Sequential updating

for the beta-binomial model

- sequence of updating does not matter
  - any order of single-observation updates
  - any 'chunking': whole data set, different subsets in whatever sequence (as long as disjoined)
- "today's posterior is tomorrow's prior"



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Excursion: Sequential updating

general proof

- **claim:** if $\{D_1, D_2\}$ is a partition of D, then $P(\theta \mid D) \propto P(\theta \mid D_1)\ P(D_2 \mid \theta)$
- **sketch of proof:**

$$P(\theta \mid D) = \frac{P(\theta)\ P(D \mid \theta)}{\int P(\theta')\ P(D \mid \theta') \mathrm{d}\theta'}$$

$$= \frac{P(\theta)\ P(D_1 \mid \theta)\ P(D_2 \mid \theta)}{\int P(\theta')\ P(D_1 \mid \theta')\ P(D_2 \mid \theta') \mathrm{d}\theta'}$$ [from multiplicativity of likelihood]

$$= \frac{P(\theta)\ P(D_1 \mid \theta)\ P(D_2 \mid \theta)}{\frac{k}{k} \int P(\theta')\ P(D_1 \mid \theta')\ P(D_2 \mid \theta') \mathrm{d}\theta'}$$ [for random positive k]

$$= \frac{\frac{P(\theta)\ P(D_1 \mid \theta)}{k}\ P(D_2 \mid \theta)}{\int \frac{P(\theta')\ P(D_1 \mid \theta')}{k}\ P(D_2 \mid \theta') \mathrm{d}\theta'}$$ [rules of integration; basic calculus]

$$= \frac{P(\theta \mid D_1)\ P(D_2 \mid \theta)}{\int P(\theta' \mid D_1)\ P(D_2 \mid \theta') \mathrm{d}\theta'}$$ [Bayes rule with $k = \int P(\theta) P(D_1 \mid \theta) \mathrm{d}\theta$]

read more here

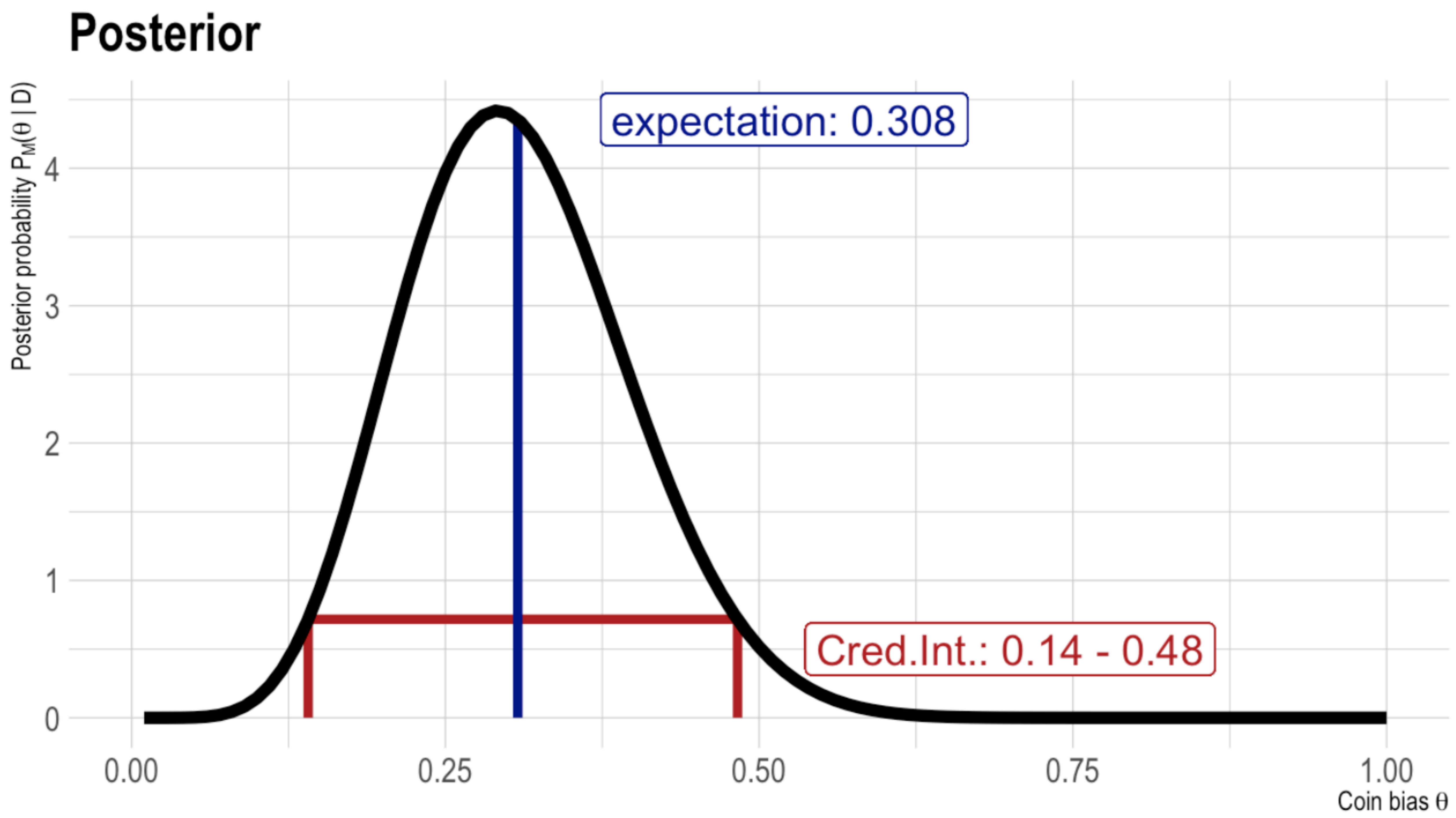
Parameter estimation

point- and interval-valued estimates

- Bayes' rule for parameter estimation:

$$P(\theta \mid D) = \frac{P(D \mid \theta)\ P(\theta)}{\int P(D \mid \theta)\ P(\theta)\ \mathrm{d}\theta}$$

- common point estimates ("best" values):
  - maximum likelihood estimate (MLE)
  - maximum a posteriori (MAP)
  - **posterior mean / expected value**
- common interval estimates (range of "good" values):
  - confidence intervals
  - inner quantile ranges
  - **credible intervals**



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Point-valued estimates

MLE, MAP and (posterior) expected value

- **MLE:**

  $$\arg\max_{\theta} P(D \mid \theta)$$

  - doesn't take prior into account (not Bayesian)
  - not necessarily unique

- **MAP:**

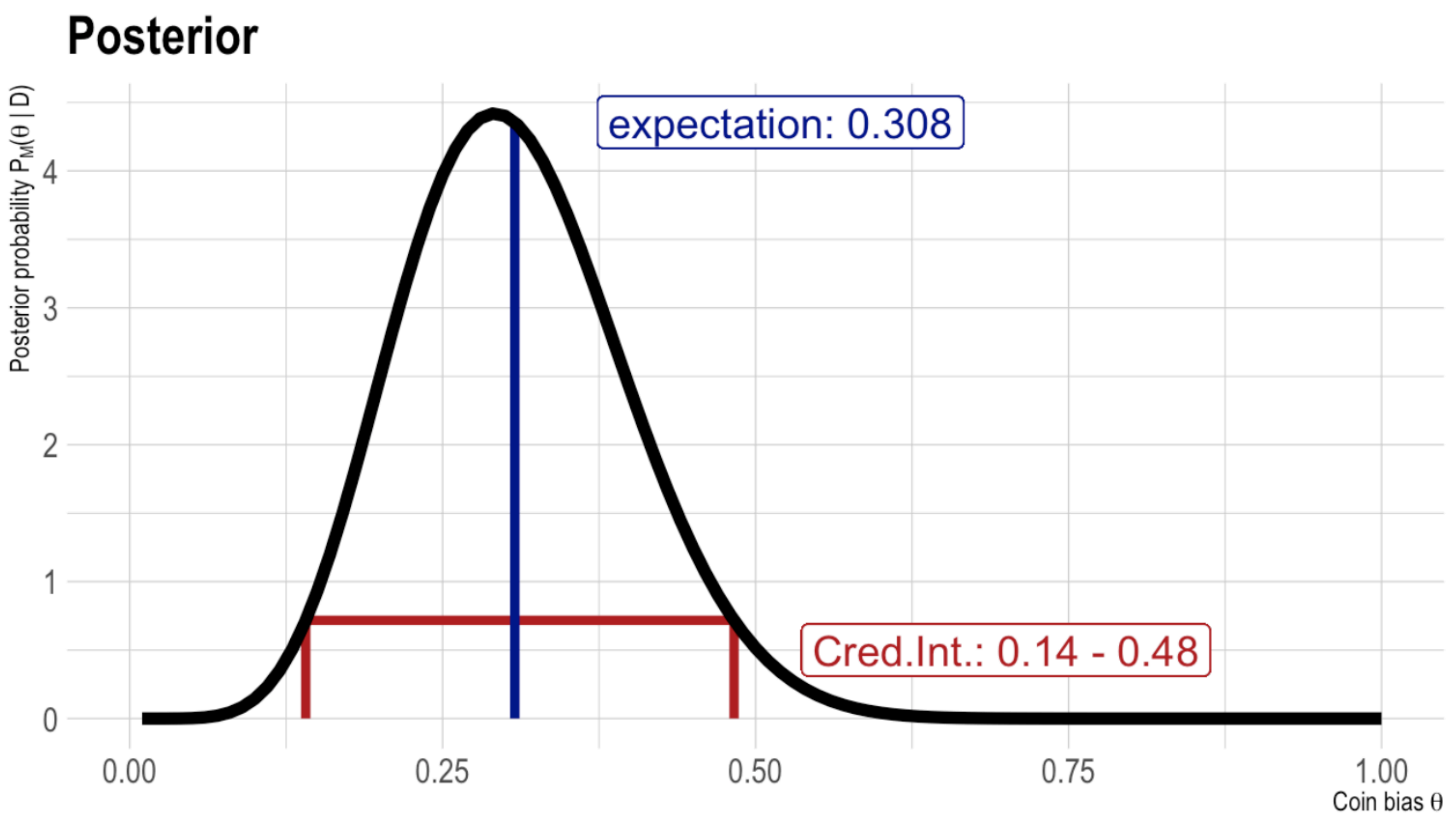
  $$\arg\max_{\theta} P(\theta \mid D)$$

  - local / does not consider full distribution (not fully Bayesian)
  - increasingly uninformative in larger parameter spaces
  - not necessarily unique

- **posterior mean / expected valued**

  $$\mathbb{E}_{P(\theta|D)} = \int \theta \ P(\theta \mid D) \ \mathrm{d}\theta$$

  - holistic / depends on full distribution ("genuinely Bayesian")
  - always unique (for proper priors/posteriors)



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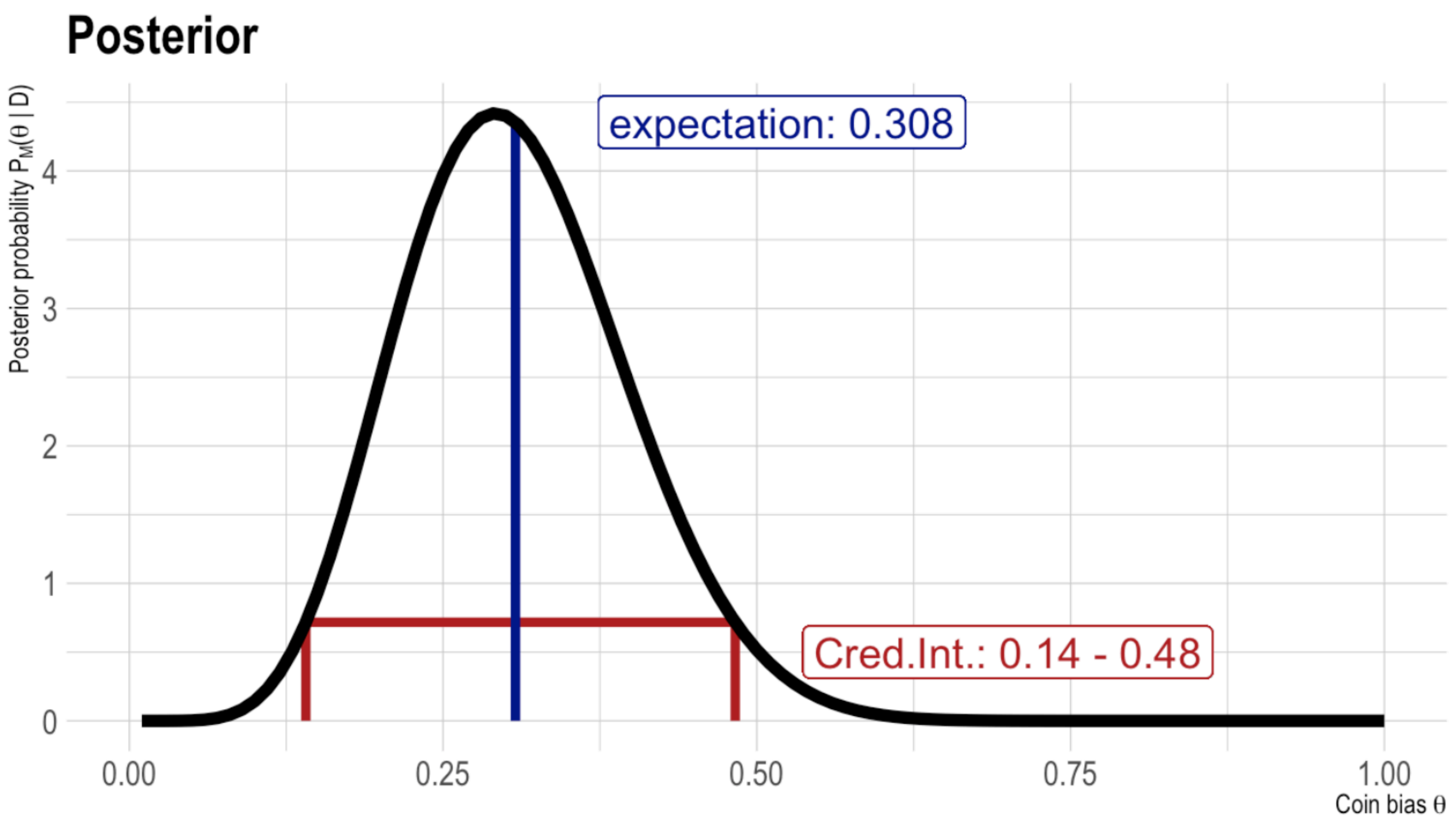
Bayesian credible intervals

a.k.a.: highest density interval …

An interval I is a γ% credible interval, if:

1. the probability that $\theta \in I$ is $\frac{\gamma}{100}$, and
2. no value outside of I is more likely than any point inside of I.



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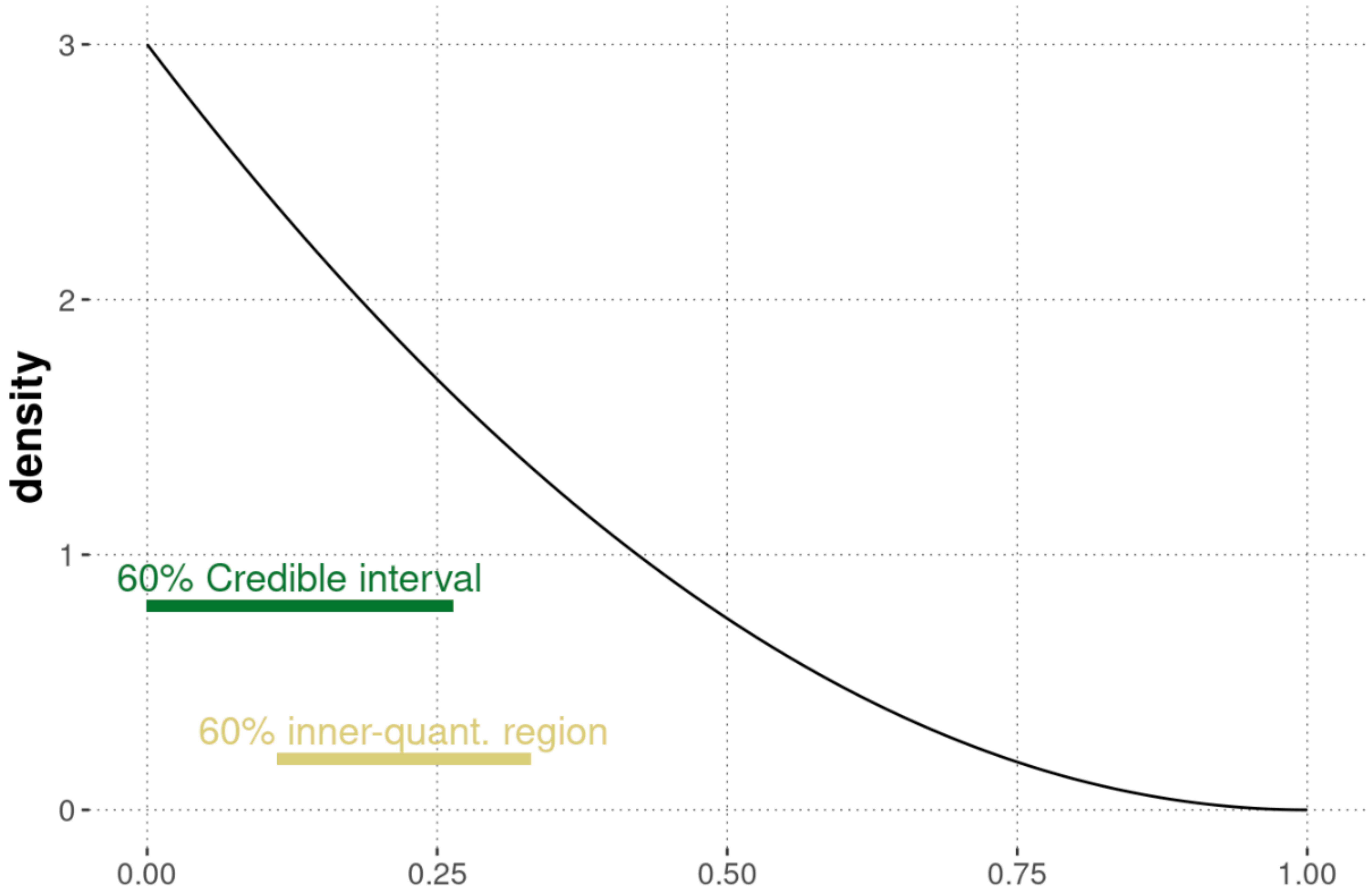
Inner quantile regions

!!!are not credible intervals!!!



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hypotheses & evidence

Bayesian hypothesis testing /w posterior credible intervals

!!! caveat: it is controversial whether this is the best (Bayesian) approach to hypothesis testing !!!

- consider an interval-based hypothesis: $\theta \in I$
  - e.g., inequality-based: "slope is positive" $\beta_1 = \theta > 0$
  - e.g. a **region of practical equivalence [ROPE]:** an ϵ-region around some θ^*: $I = [\theta^* - \epsilon, \theta^* + \epsilon]$
- if $[l; u]$ is a posterior credible interval for θ, we consider this:
  - **reason to accept** hypothesis I if $[l; u]$ is contained entirely in I;
  - **reason to reject** hypothesis I if $[l; u]$ and I have no overlap;
  - **withhold judgement** otherwise.
- this approach is "categorical" (accept, reject, suspend) and not quantitative

read more here

Posterior plausibility of interval-based hypotheses

this is NOT a testing approach, just one way of quantifying support

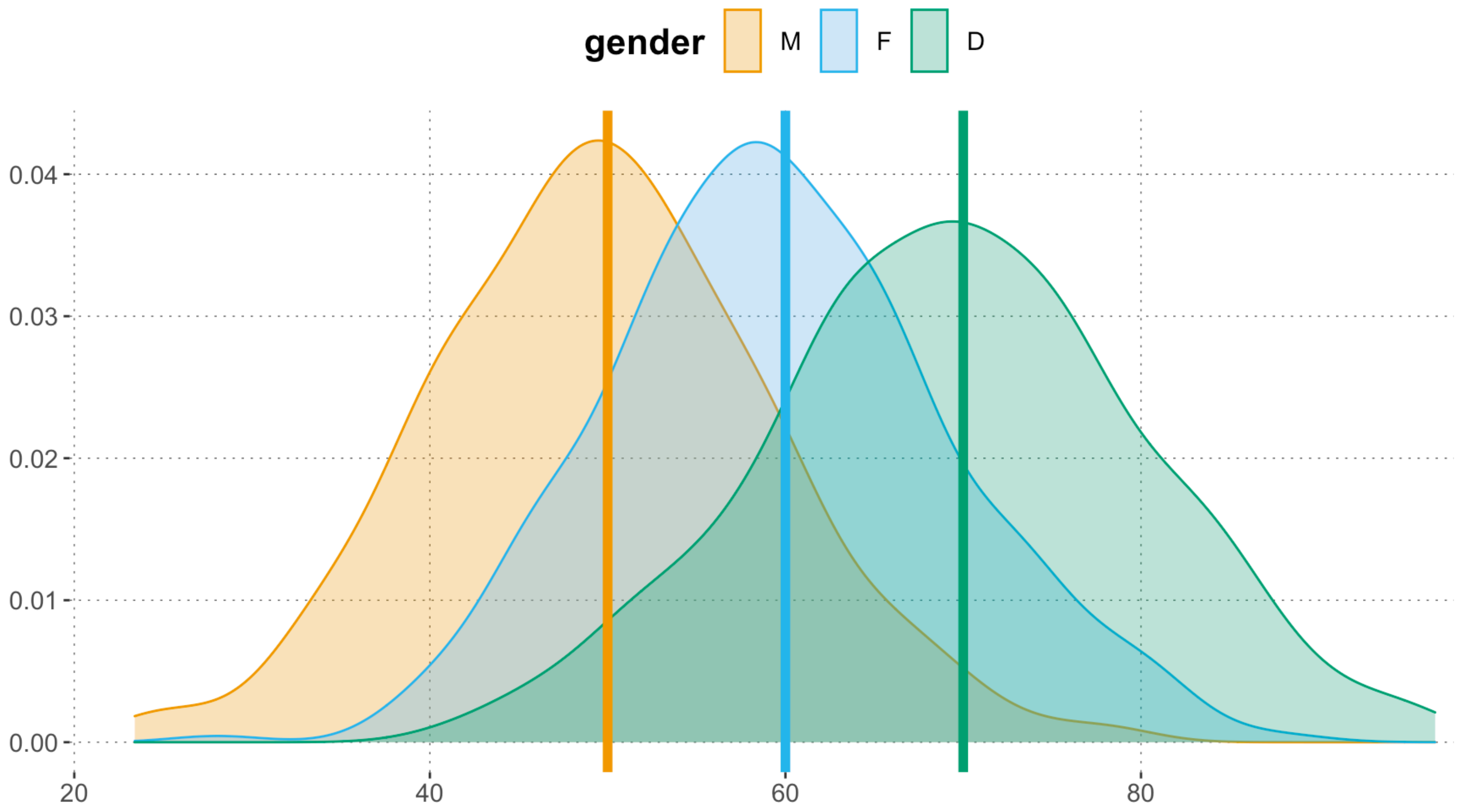
- consider an interval-based hypothesis $\theta \in I$ as before
- the **posterior plausibility** of I given a model M and the data D is just the posterior probability: $P(\theta \in I \mid D)$
- **not** a notion of observational evidence:
  - if prior is high for I and data is uninformative, posterior plausibility can be high
- good-enough first heuristic when priors are "unbiased" regarding I
- more on hypothesis testing later (if you want)

read more here

contrast coding

Some fake data

three-way categorical variable



Finding numbers for categories

metric predictors

y	x_0	x_1	x_2
42	1	4	163
19	1	7	128
38	1	2	99
⋮	⋮	⋮	⋮

categorical predictor

y	gender
51	M
59	F
73	D
⋮	⋮

Treatment coding

comparing against a reference category

y	gender	x0	x1	x2
51	M	1	0	0
59	F	1	1	0
73	D	1	0	1
⋮	⋮	⋮	⋮	⋮

$$y_i = \sum_{j=0}^{k} \beta_j x_{ij} + \epsilon_i$$

$$\hat{\mu}_M = \beta_0 1 + \beta_1 0 + \beta_2 0$$
$$\hat{\mu}_F = \beta_0 1 + \beta_1 1 + \beta_2 0$$
$$\hat{\mu}_D = \beta_0 1 + \beta_1 0 + \beta_2 1$$

hypotheses

$$\beta_0 = \hat{\mu}_M$$
$$\beta_1 = \hat{\mu}_F - \hat{\mu}_M$$
$$\beta_2 = \hat{\mu}_D - \hat{\mu}_M$$

Treatment coding

comparing against a reference category

y	gender	x0	x1	x2
51	M	1	0	0
59	F	1	1	0
73	D	1	0	1
⋮	⋮	⋮	⋮	⋮

hypotheses

$$\beta_0 = \hat{\mu}_M$$
$$\beta_1 = \hat{\mu}_F - \hat{\mu}_M$$
$$\beta_2 = \hat{\mu}_D - \hat{\mu}_M$$

$\mu_M = 0$	μ_F	μ_D

Cell means coding

estimating a mean for each cell

y	gender	x_0	x_1	x_2
51	**M**	1	0	0
59	**F**	0	1	0
73	**D**	0	0	1
⋮	⋮	⋮	⋮	⋮

hypotheses

$$\beta_0 = \hat{\mu}_M$$
$$\beta_1 = \hat{\mu}_F$$
$$\beta_2 = \hat{\mu}_D$$

$$\mu_M = 0 \qquad \mu_F = 0 \qquad \mu_D = 0$$

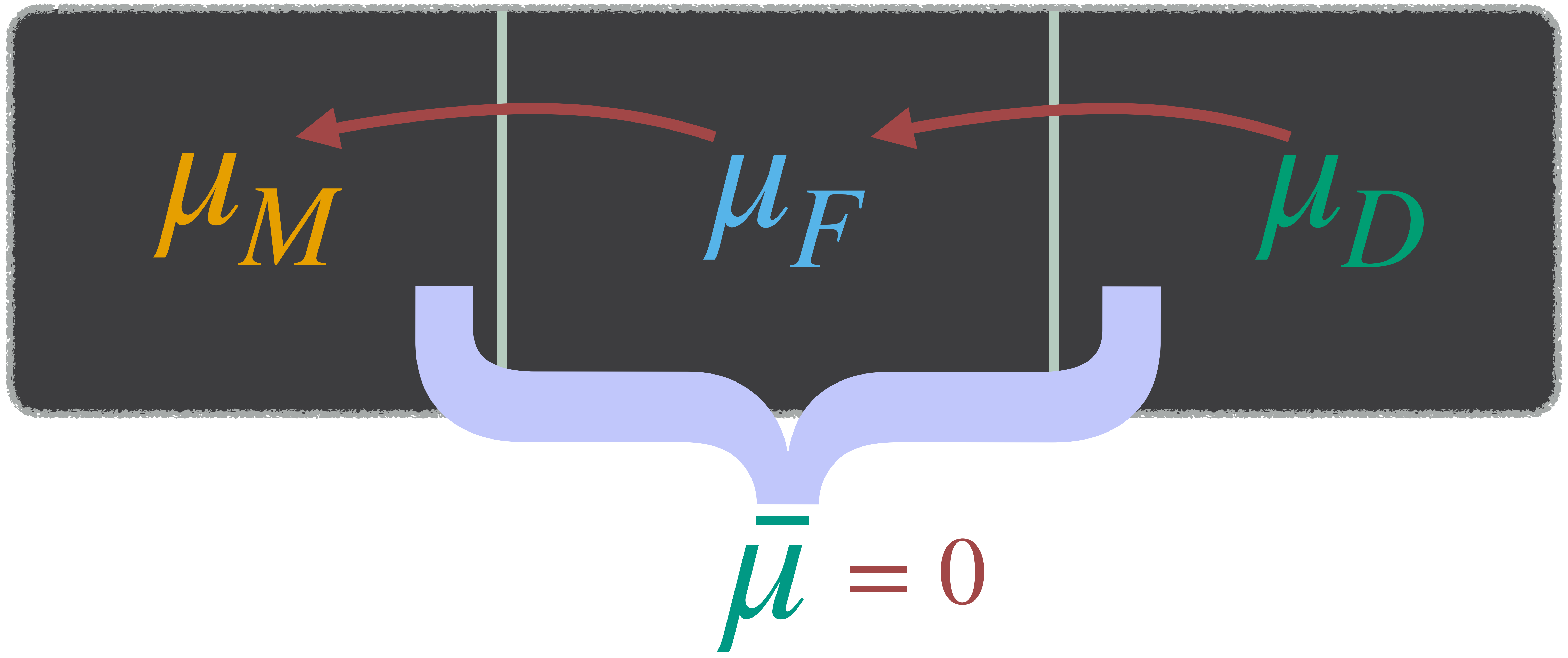
Simple difference coding

estimating a mean for each cell

y	gender	x_0	x_1	x_2
51	M	1	$-^2/_3$	$-^1/_3$
59	F	1	$^1/_3$	$-^1/_3$
73	D	1	$^1/_3$	$^2/_3$
⋮	⋮	⋮	⋮	⋮

hypotheses

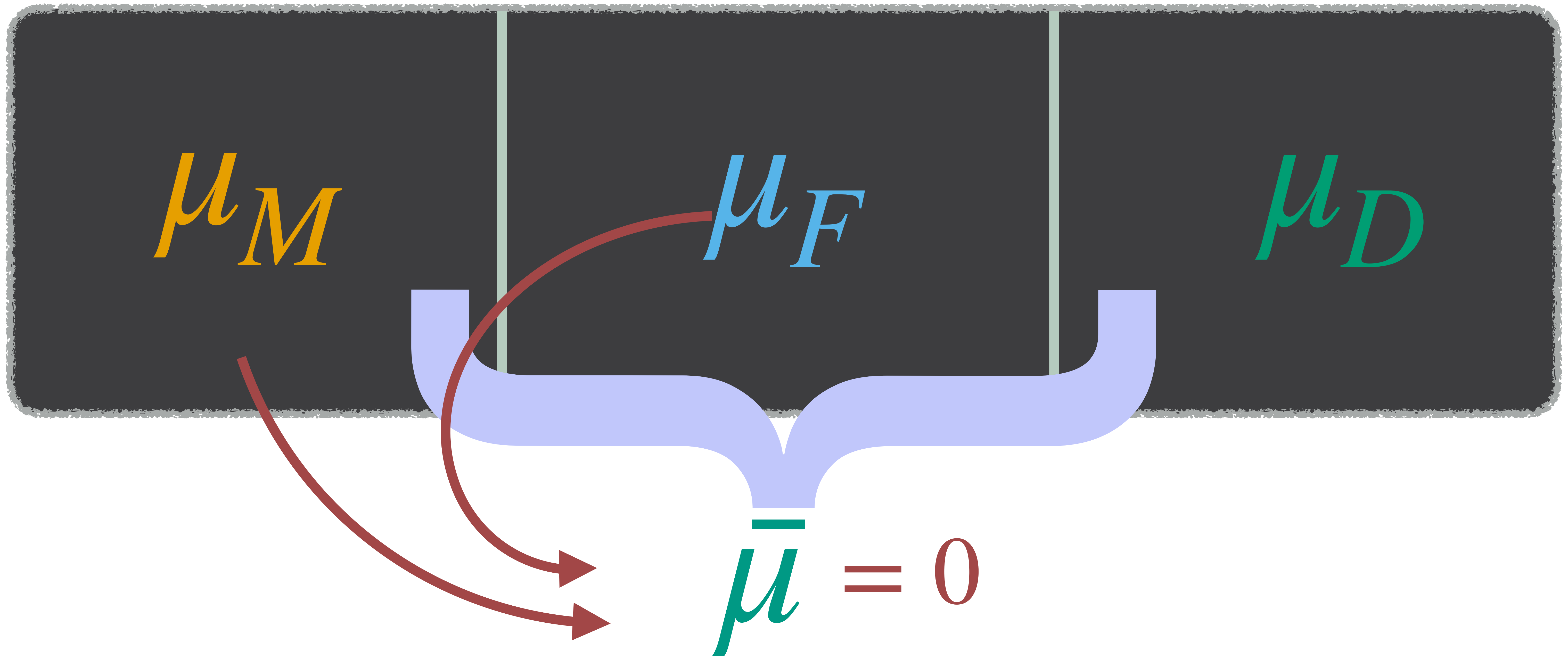
$$\beta_0 = \bar{\mu}$$
$$\beta_1 = \hat{\mu}_F - \hat{\mu}_M$$
$$\beta_2 = \hat{\mu}_D - \hat{\mu}_F$$



Sum coding

comparing $k - 1$ cells to the grand mean

y	gender	x₀	x₁	x₂
51	M	1	1	0
59	F	1	0	1
73	D	1	-1	-1
⋮	⋮	⋮	⋮	⋮



hypotheses

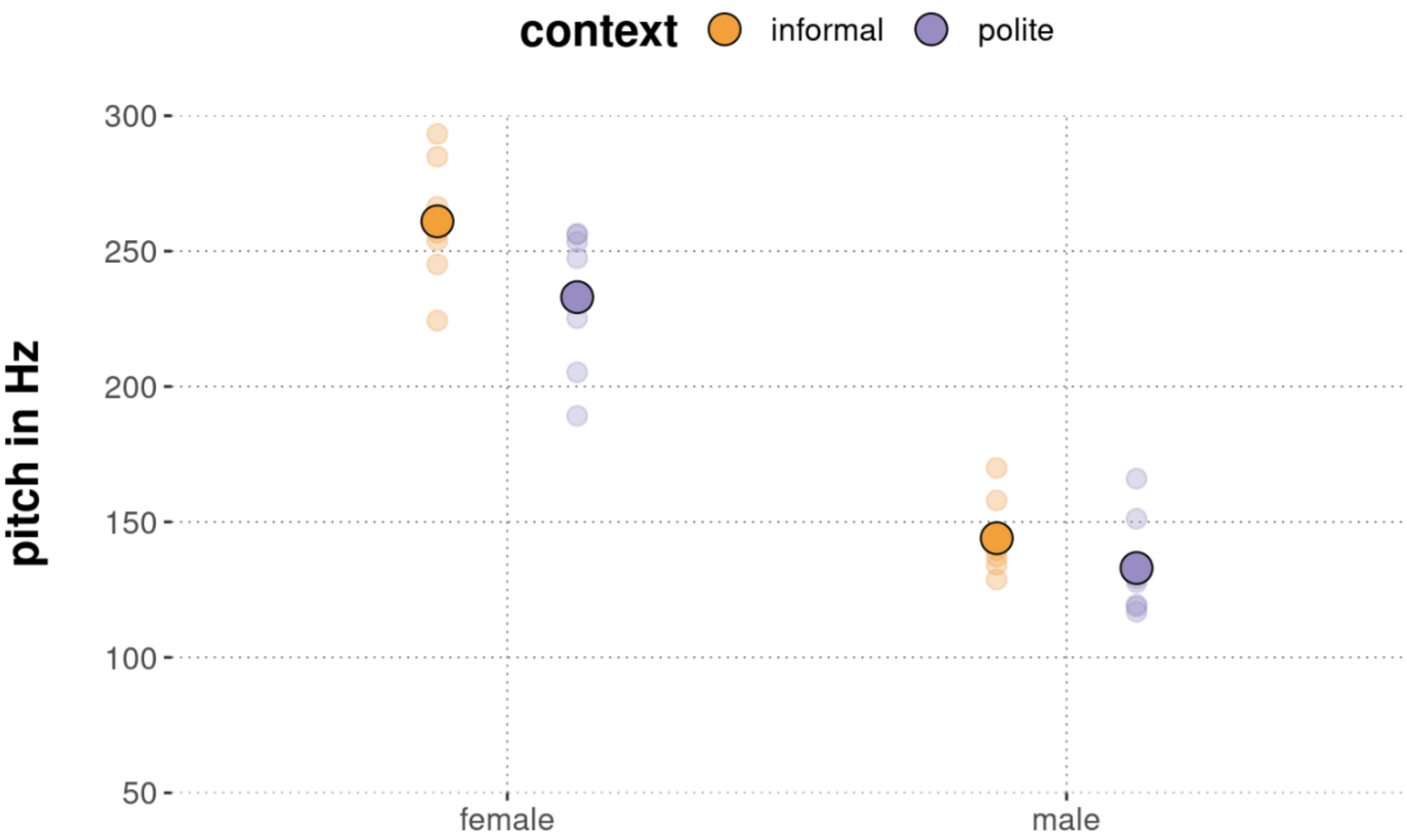
$$\beta_0 = \bar{\mu}$$
$$\beta_1 = \hat{\mu}_M - \bar{\mu}$$
$$\beta_2 = \hat{\mu}_F - \bar{\mu}$$

Case study: pitch in context

data from Winter & Grawunder (2012)

```r
politeness_data <- aida::data_polite
politeness_data %>% head(5)
```

```
## # A tibble: 5 × 5
##   subject gender sentence context pitch
##   <chr>   <chr>  <chr>    <chr>   <dbl>
## 1 F1      F      S1       pol      213.
## 2 F1      F      S1       inf      204.
## 3 F1      F      S2       pol      285.
## 4 F1      F      S2       inf      260.
## 5 F1      F      S3       pol      204.
```



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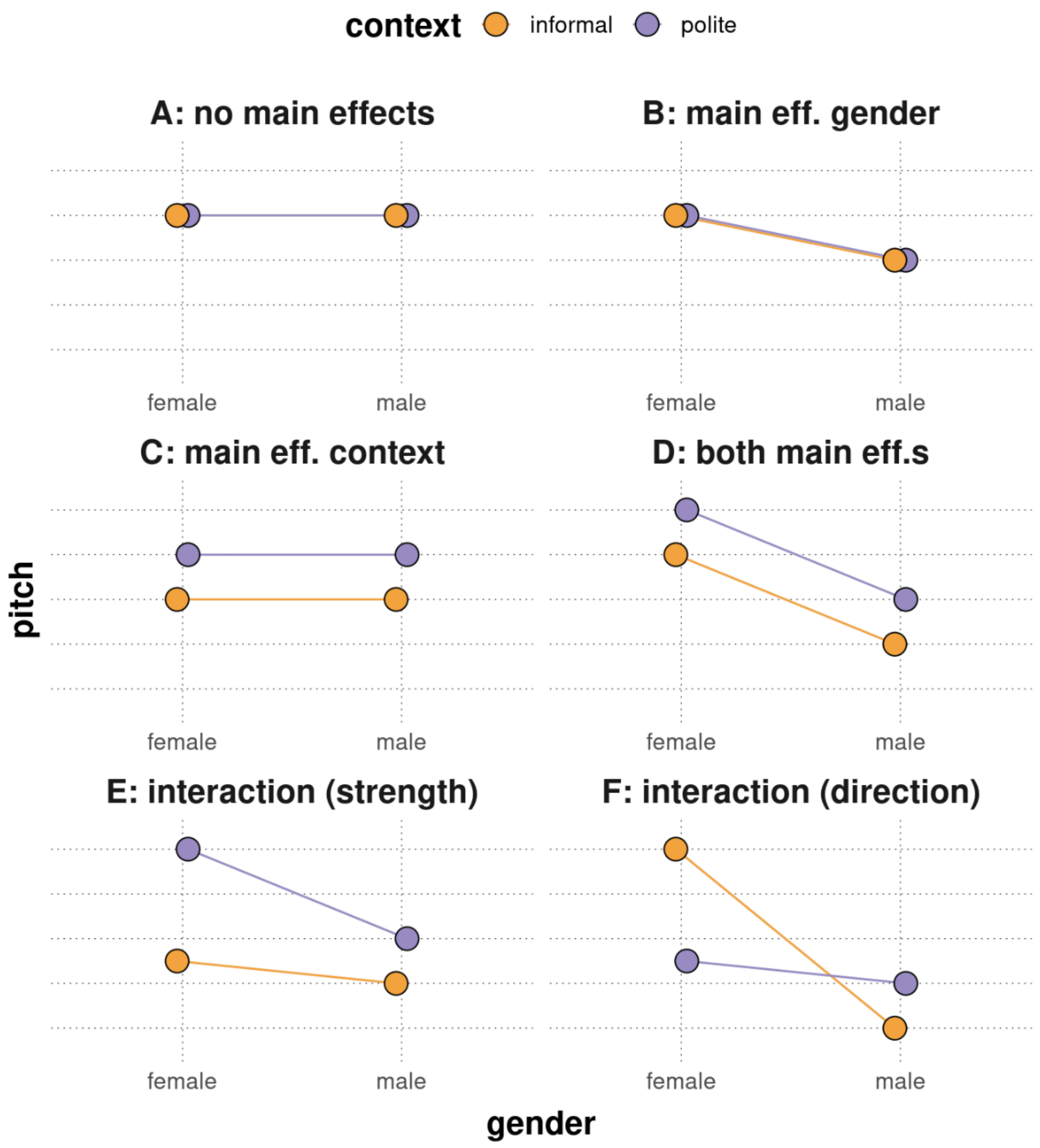
Dummy coding for 2x2 design

	informal	polite
♀	β_0 reference level	$\beta_0 + \beta_{\text{pol}}$
♂	$\beta_0 + \beta_{\text{male}}$	$\beta_0 + \beta_{\text{male}} + \beta_{\text{pol}} + \beta_{\text{pol\&male}}$

read more here

Main effects & interactions

in 2x2 designs



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Bayesian regression

```r
# here, we only use fixed effects
fit_dummy_FE <- brm(
  pitch ~ gender * context,
  data = politeness_df,
  cores = 4,
  iter = 1000
)
```

```
Population-Level Effects:
                   Estimate Est.Error l-95% CI u-95% CI
Intercept            260.56      7.87   244.15   275.21
genderM             -116.16     11.01  -137.31   -94.05
contextpol           -27.23     11.10   -48.38    -5.23
genderM:contextpol    15.77     16.05   -16.54    46.24
```

	informal	polite
♀	β_0 reference level	$\beta_0 + \beta_{\text{pol}}$
♂	$\beta_0 + \beta_{\text{male}}$	$\beta_0 + \beta_{\text{male}} + \beta_{\text{pol}} + \beta_{\text{pol\&male}}$

Which questions about cell mean differences can we address with this information directly?

read more here

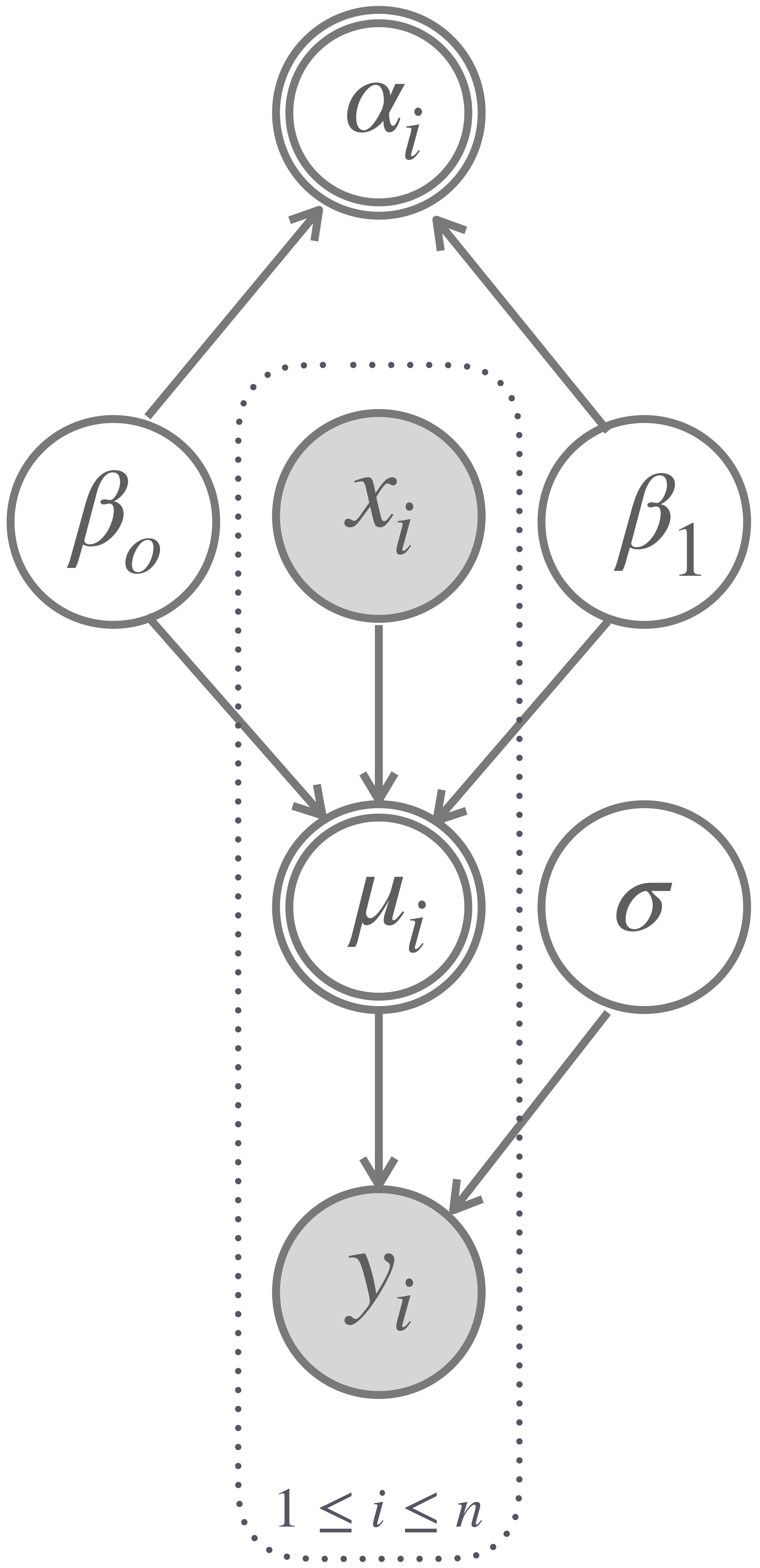
faintR

demo

check the ‘contrast coding’ tutorial if you want to read up on what we did here

Derived variables

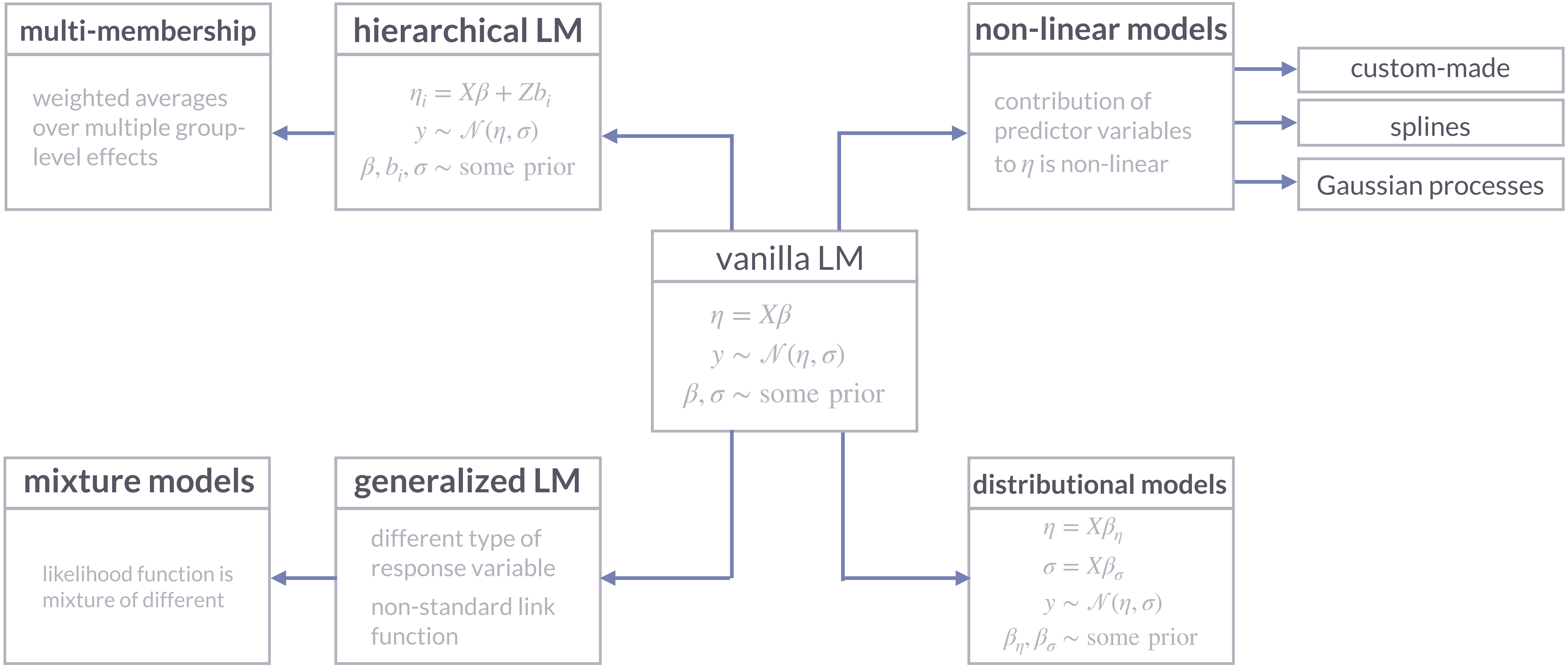
- obtain samples from model parameters
- apply (deterministic) function to each sample
  - to derive (deterministically) a new model variable
- violà: samples from the posterior of a new "derived variable"



outlook

Roadmap "beyond vanilla"

common extensions of linear regression modeling



Three pillars of BDA

1. parameter estimation / inference [which parameter values are credible given data and model?]

$$\underbrace{P(\theta \mid D)}_{\text{posterior}} \propto \underbrace{P(\theta)}_{\text{prior}} \times \underbrace{P(D \mid \theta)}_{\text{likelihood}}$$

2. predictions [which future data observations are likely given my model?]

a. prior

$$P(D_{\text{pred}}) = \int P(\theta)\ P(D_{\text{pred}} \mid \theta)\ \mathrm{d}\theta$$

b. posterior

$$P(D_{\text{pred}} \mid D_{\text{obs}}) = \int P(\theta \mid D_{\text{obs}})\ P(D_{\text{pred}} \mid \theta)\ \mathrm{d}\theta$$

3. model comparison [which model of two models is more likely to have generated the data?]

$$\underbrace{\frac{P(M_1 \mid D)}{P(M_2 \mid D)}}_{\text{posterior odds}} = \underbrace{\frac{P(D \mid M_1)}{P(D \mid M_2)}}_{\text{Bayes factor}}\ \underbrace{\frac{P(M_1)}{P(M_2)}}_{\text{prior odds}}$$