

$$NTS: P(s_c | \bar{V}) > P(s_q | \bar{V})$$

[deaccented "has" is a cue for competitor]

$$\Leftrightarrow \tau_c q_{\bar{V}} > \tau_q p_{\bar{V}}$$

$$\Leftrightarrow \frac{q_{\bar{V}}}{p_{\bar{V}}} > \frac{\tau_q}{\tau_c}$$

any bias towards competitor must be counterbalanced by the evidence

$$q_{\bar{V}}/p_{\bar{V}} > 1$$

[we could naturally expect this to be the case] even if  $\tau_q \neq \tau_c$  because of  $[q_{\bar{V}} > p_{\bar{V}}]$

So assume:

$$\left. \begin{array}{l} \tau_q = \tau_c + \epsilon_{\tau} \\ q_{\bar{V}} = p_{\bar{V}} + \epsilon_{\bar{V}} \end{array} \right\} \text{with } 0 \leq \epsilon_{\tau} < \epsilon_{\bar{V}}$$

Then:

$$\frac{p_{\bar{V}} + \epsilon_{\bar{V}}}{p_{\bar{V}}} > \frac{\tau_c + \epsilon_{\tau}}{\tau_c}$$

$$\Leftrightarrow (p_{\bar{V}} + \epsilon_{\bar{V}}) \cdot \tau_c > p_{\bar{V}} (\tau_c + \epsilon_{\tau})$$

$$\Leftrightarrow p_{\bar{V}} \tau_c + \epsilon_{\bar{V}} \tau_c > p_{\bar{V}} \tau_c + p_{\bar{V}} \epsilon_{\tau}$$

$$\Leftrightarrow \epsilon_{\bar{V}} \tau_c > p_{\bar{V}} \epsilon_{\tau}$$

$$\Leftrightarrow \frac{\epsilon_{\bar{V}}}{\epsilon_{\tau}} > \frac{p_{\bar{V}}}{\tau_c}$$

true if  $\epsilon_{\tau} < \epsilon_{\bar{V}}$  &  $p_{\bar{V}} < \tau_c$

both are natural assumptions

also assumed here:  $q_{\bar{V}} > p_{\bar{V}}$  &

$$\tau_q \geq \tau_c \quad \checkmark$$

maybe assume that:  $\tau_q = \tau_c + \epsilon_{\tau}$   
with  $0 \leq \epsilon_{\tau}$  small &  $p_{\bar{V}} + \epsilon_{\bar{V}} = q_{\bar{V}}$   
with  $0 \leq \epsilon_{\bar{V}} < \epsilon_{\tau}$

$$NTS: P(s_q | V) > P(s_c | \bar{V})$$

[proof exists (?) for flat prior  $\tau_q = \tau_c$   
and identical string likelihood  $p_{\bar{V}} = q_{\bar{V}}$ ]

$$\Leftrightarrow \frac{\tau_q p_V}{\tau_q p_V + \tau_c q_V} > \frac{\tau_c q_{\bar{V}}}{\tau_q p_{\bar{V}} + \tau_c q_{\bar{V}}}$$

$$\Leftrightarrow \frac{\tau_q p_V}{\tau_q p_V + \tau_c q_V} > \frac{\tau_c (1 - q_V - q_{\bar{V}})}{\tau_q (1 - p_V - p_{\bar{V}}) + \tau_c (1 - q_V - q_{\bar{V}})}$$

$$\Leftrightarrow \frac{\tau_q p_V}{\tau_q p_V + \tau_c q_V} > \frac{\tau_c - \tau_c q_V - \tau_c q_{\bar{V}}}{\tau_q - \tau_q p_V - \tau_q p_{\bar{V}} + \tau_c - \tau_c q_V - \tau_c q_{\bar{V}}}$$

$$\Leftrightarrow \tau_q p_V (\tau_q - \tau_q p_V - \tau_q p_{\bar{V}} + \tau_c - \tau_c q_V - \tau_c q_{\bar{V}}) > (\tau_c - \tau_c q_V - \tau_c q_{\bar{V}}) \cdot (\tau_q p_V + \tau_c q_V)$$

$$\Leftrightarrow \tau_q^2 p_V - \tau_q^2 p_V^2 - \tau_q^2 p_V p_{\bar{V}} + \tau_q \tau_c p_V - \tau_q \tau_c p_V q_V - \tau_q \tau_c p_V q_{\bar{V}} > \tau_q \tau_c p_V + \tau_q \tau_c q_V - \tau_q \tau_c p_V q_V - \tau_c^2 q_V^2 - \tau_q \tau_c p_V q_{\bar{V}} - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow \tau_q^2 p_V - \tau_q^2 p_V^2 - \tau_q^2 p_V p_{\bar{V}} > \tau_q \tau_c p_V - \tau_c^2 q_V^2 - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow (\tau_q^2 - \tau_q \tau_c) p_V - \tau_q^2 p_V^2 - \tau_q^2 p_V p_{\bar{V}} > -\tau_c^2 q_V^2 - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow (\tau_q^2 - \tau_q \tau_c) p_V - \tau_q^2 p_V^2 - \tau_q^2 p_V p_{\bar{V}} > -\tau_c^2 q_V^2 - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow (\tau_q^2 - \tau_q \tau_c) p_V - \tau_q^2 p_V^2 - \tau_q^2 p_V p_{\bar{V}} > -\tau_c^2 q_V^2 - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow (\tau_q^2 - \tau_q \tau_c) p_V - \tau_q^2 p_V^2 - \tau_q^2 p_V p_{\bar{V}} > -\tau_c^2 q_V^2 - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow (\tau_q^2 - \tau_q \tau_c + \tau_q \epsilon_{\tau}) p_V - \tau_q^2 p_V^2 - \tau_q^2 p_V p_{\bar{V}} > -(\tau_q^2 - 2\tau_q \epsilon_{\tau} + \epsilon_{\tau}^2) (q_V^2 - q_V q_{\bar{V}})$$

$$\Leftrightarrow \tau_q \epsilon_{\tau} p_V - \tau_q^2 p_V^2 - \tau_q^2 p_V p_{\bar{V}} > (\dots)$$

okay, it is clear that any prior bias for  $s_q$  will pull closer towards  $s_q$ ; shows that, by mere likelihood, the same result is expected; so set:  $\epsilon_{\tau} = 0$

$$\frac{p_V}{p_V + q_V} > \frac{q_{\bar{V}}}{p_{\bar{V}} + q_{\bar{V}}}$$

$$\Leftrightarrow p_V p_{\bar{V}} + p_V q_{\bar{V}} > p_V q_V + q_V q_{\bar{V}}$$

$$\Leftrightarrow p_V p_{\bar{V}} > q_V q_{\bar{V}}$$

$$\Leftrightarrow \frac{p_{\bar{V}}}{q_V} > \frac{q_{\bar{V}}}{p_V}$$

assume:

$$q_{\bar{V}} = q_V + \epsilon_q \quad \epsilon_q > \epsilon_p$$

$$p_V = p_{\bar{V}} + \epsilon_p$$

$$(p_{\bar{V}} + \epsilon_p) p_{\bar{V}} > q_V (q_V + \epsilon_q)$$

$$p_{\bar{V}}^2 + \epsilon_p p_{\bar{V}} > q_V^2 + \epsilon_q q_V$$

$$p_{\bar{V}}^2 + \epsilon_p p_{\bar{V}} > q_V^2 + \epsilon_q q_V + \epsilon_p q_V$$

[producing  $V$  when adequate is less likely than producing  $\bar{V}$  when adequate]  $p_V \leq q_{\bar{V}}$

[producing  $V$  when inadequate is less likely than producing  $\bar{V}$  when inadequate]  $p_{\bar{V}} > q_V$

# Computational Pragmatics

WebPPL

Session 3

# WebPPL

- ▶ probabilistic programming language
- ▶ built on top of JavaScript
  - ▶ executable in the browser
  - ▶ only a part of JS is available (e.g., no loops!, no variable reassignment!)
- ▶ use [webppl.org](http://webppl.org) for development
  - ▶ changes to code in [problang.org](http://problang.org) is not saved!!!
- ▶ read the docs:
  - ▶ <https://webppl.readthedocs.io/en/master/index.html>

# WebPPL: the deterministic basics

- ▶ follow this tutorial now (ca. 20-30 minutes)
  - ▶ <http://probmods.org/chapters/appendix-js-basics.html>
  - ▶ ask questions, poke around!
  - ▶ do exercises at the end if fast and bored
- ▶ make sure you capture the basics of the following:
  - ▶ Boolean operations, in particular the ternary operator `BOOL ? yes : no`
  - ▶ variable declarations with `var`
  - ▶ arrays & objects
  - ▶ function definitions
  - ▶ higher-order functions, in particular `map()` and `repeat()`

## WebPPL: some tips and tricks

- ▶ remark: WebPPL uses the lodash library (formerly “Underscore”)
  - ▶ check <https://lodash.com> for many useful functions
  - ▶ caveat: don't use lodash's higher-order functions!
  - ▶ e.g., use WebPPL's `map()`, not lodash's `_.map()`
- ▶ output from code boxes
  - ▶ the return value of the final line of a code box is shown on screen
  - ▶ use `display()` or `print()` to show intermediate results
    - ▶ `display()` has nicer (higher-level/processed) output
    - ▶ `print()` (but doesn't work the first time it's invoked 😬)
- ▶ keyboard shortcuts in code boxes
  - ▶ TAB indents selected code or current line
  - ▶ CTRL-/ comments or uncomments selection or current line
  - ▶ CTRL-RET runs the code box

# Probabilistic programming

- ▶ probability distributions are first-order objects
  - ▶ finite case: represented as object-value pairs
  - ▶ infinite case: represented as set of samples
- ▶ special functions to interact with probability distributions
  - ▶ construction with `Infer`
  - ▶ sampling with `sample`
  - ▶ fine-tune with `factor`
  - ▶ conditionalize with `condition` or `observe`
  - ▶ visualize with `viz`