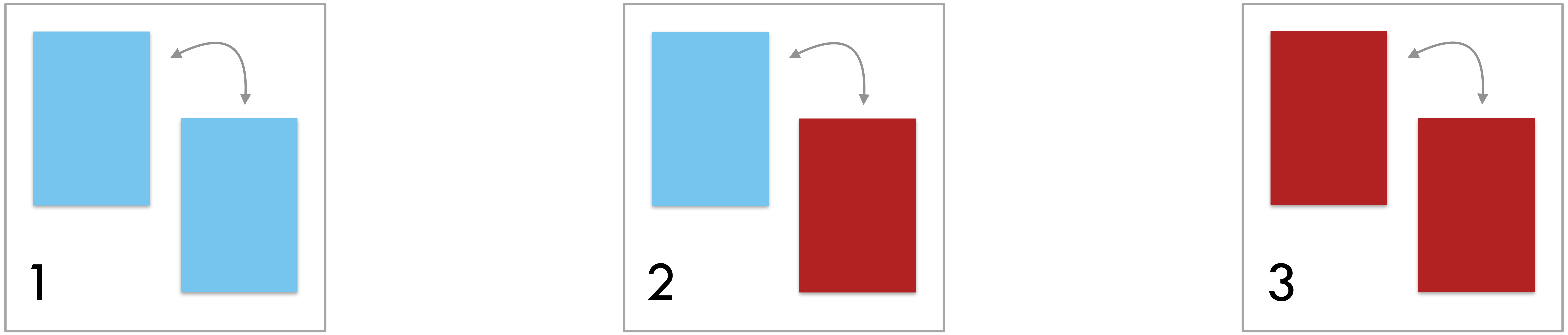
Computational Pragmatics

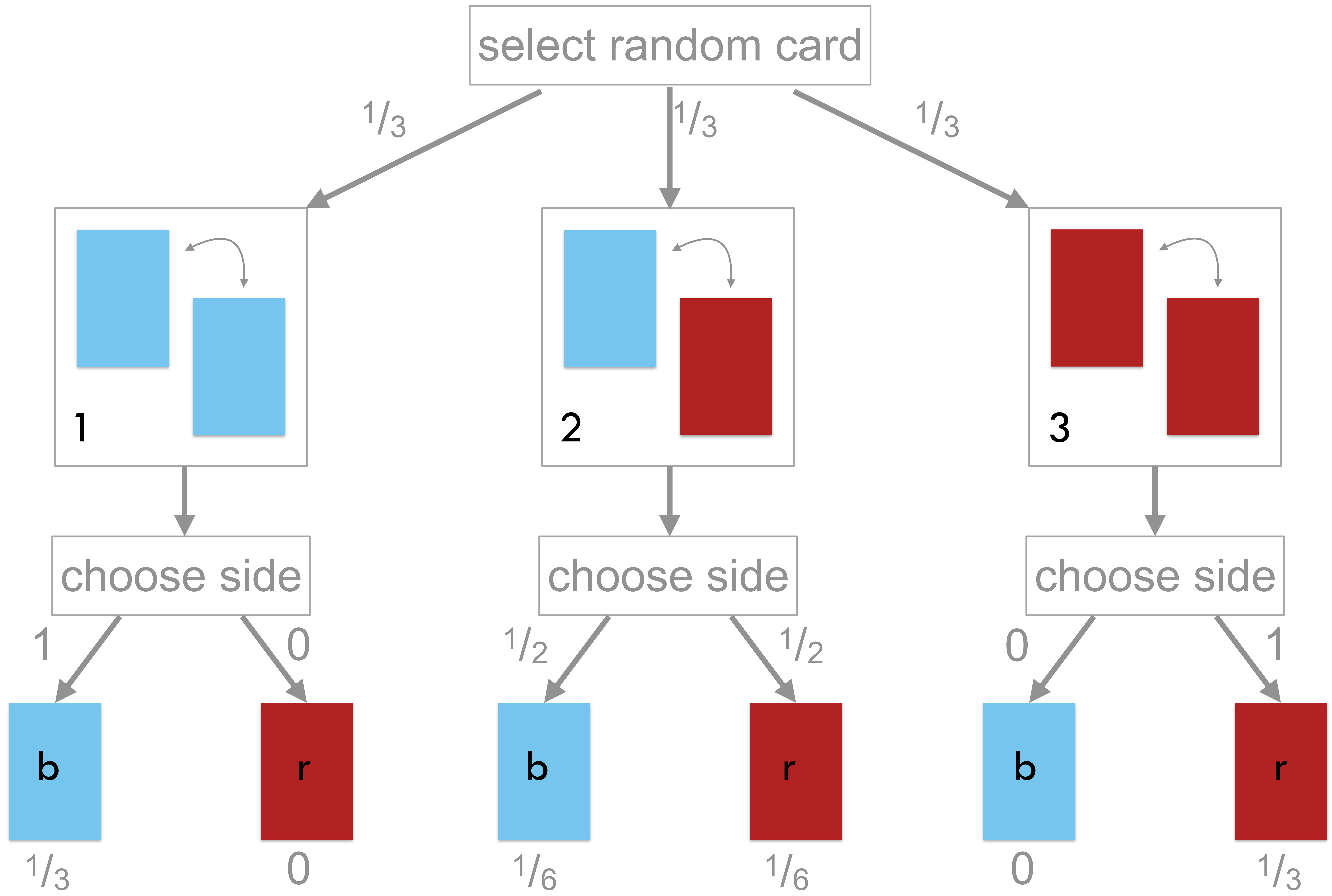
Probability Basics

Session 2

Three-card problem



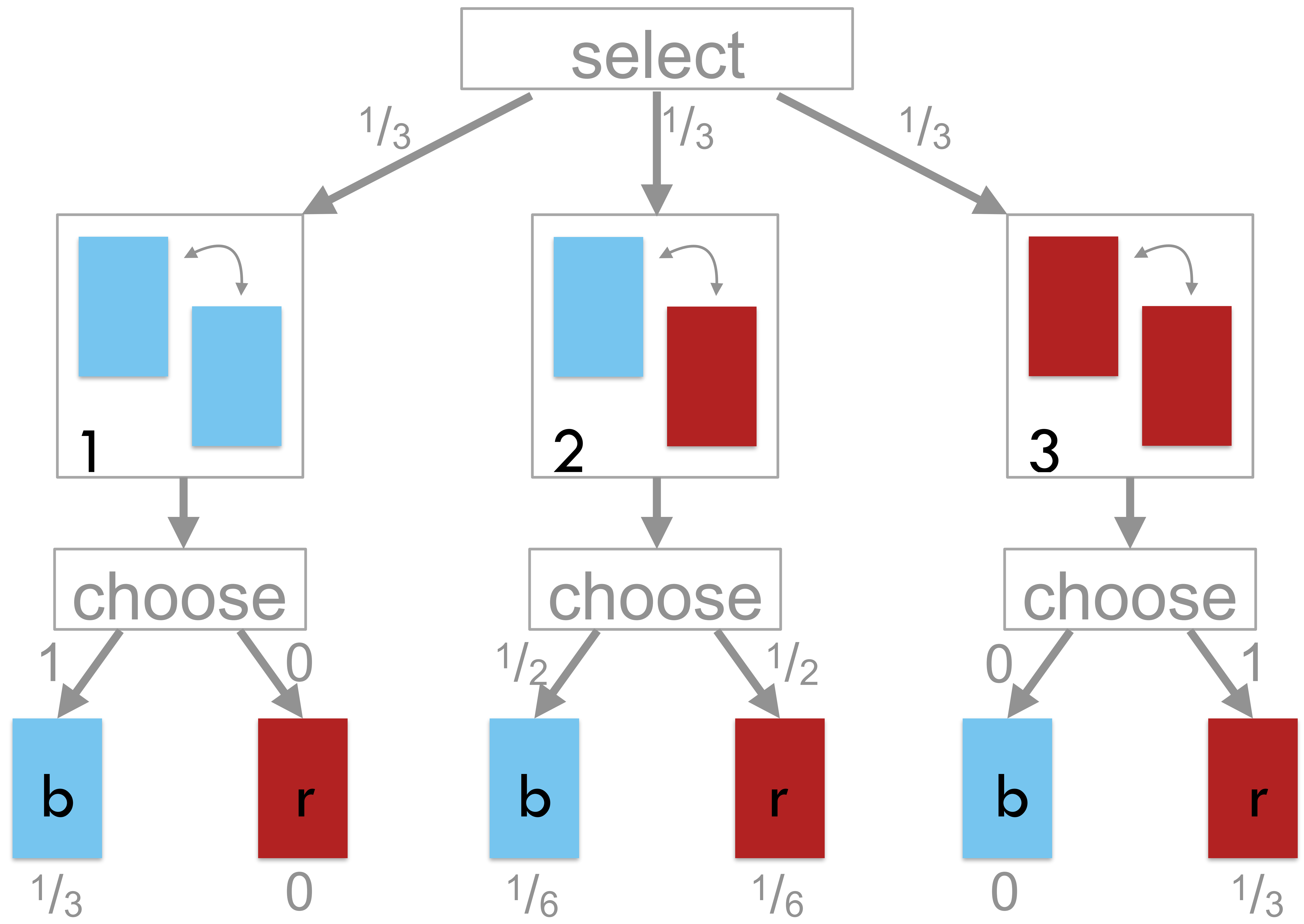
Sample a card (uniformly at random).
Choose a side of that card to reveal (uniformly at random).
What's the probability that the side you did not see is **BLUE**, given that the side you saw was **BLUE**?



select random card
1/3
1/3
1/3
1
2
3
choose side
choose side
choose side
1
0
1/2
1/2
0
1
b
r
b
r
b
r
1/3
0
1/6
1/6
0
1/3

Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



Bayes rule

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

$$P(\text{blue-blue} \mid \text{obs. blue})$$

$$= \frac{P(\text{obs. blue} \mid \text{blue-blue})\, P(\text{blue-blue})}{P(\text{obs. blue})}$$

$$= \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Notation & computation

$$P(A \mid B) \propto \underbrace{P(B \mid A)\ P(A)}_{= P(A \cap B)}$$

```
// three cards; with blue or red on either side
var cards = [["blue", "blue"],
             ["blue", "red"],
             ["red", "red"]]

var model = function() {
  var card  = uniformDraw(cards)
  var color = uniformDraw(card)
  condition(color == "blue")
  return card.join("-")
}

viz.table(Infer({method: "enumerate",
                 model: model
                }))
```

WebPPL code (preview)

any (conditional) probability distribution can be approximated by a large set of samples