

NTS:  $P(s_c | \bar{V}) > P(s_g | \bar{V})$   
 [deaccented "has" is a cue for competitor]

$$\Leftrightarrow \tau_c q_{\bar{V}} > \tau_g p_{\bar{V}}$$

$$\Leftrightarrow \frac{q_{\bar{V}}}{p_{\bar{V}}} > \frac{\tau_g}{\tau_c}$$

any bias towards competitor must be counterbalanced by the evidence  $q_{\bar{V}}/p_{\bar{V}} > 1$ .

[we could naturally expect this to be the case] even if  $\tau_g \neq \tau_c$  because of  $[q_{\bar{V}} > p_{\bar{V}}]$

So assume:  
 $\tau_g = \tau_c + \epsilon_{\tau}$   
 $q_{\bar{V}} = p_{\bar{V}} + \epsilon_{\bar{V}}$  with  $0 \leq \epsilon_{\tau} < \epsilon_{\bar{V}}$

Then:

$$\frac{p_{\bar{V}} + \epsilon_{\bar{V}}}{p_{\bar{V}}} > \frac{\tau_c + \epsilon_{\tau}}{\tau_c}$$

$$\Leftrightarrow (p_{\bar{V}} + \epsilon_{\bar{V}}) \cdot \tau_c > p_{\bar{V}} (\tau_c + \epsilon_{\tau})$$

$$\Leftrightarrow p_{\bar{V}} \tau_c + \epsilon_{\bar{V}} \tau_c > p_{\bar{V}} \tau_c + p_{\bar{V}} \epsilon_{\tau}$$

$$\Leftrightarrow \epsilon_{\bar{V}} \tau_c > p_{\bar{V}} \epsilon_{\tau}$$

$$\Leftrightarrow \frac{\epsilon_{\bar{V}}}{\epsilon_{\tau}} > \frac{p_{\bar{V}}}{\tau_c}$$

true if  $\epsilon_{\tau} < \epsilon_{\bar{V}}$  &  $p_{\bar{V}} < \tau_c$   
 both are natural assumptions  
 also assumed here:  $q_{\bar{V}} > p_{\bar{V}}$  &  $\tau_g \geq \tau_c$  ✓

maybe assume that:  $\tau_g = \tau_c + \epsilon_{\tau}$   
 with  $0 \leq \epsilon_{\tau}$  small &  $p_{\bar{V}} + \epsilon_{\bar{V}} = q_{\bar{V}}$   
 with  $0 \leq \epsilon_{\bar{V}} < \epsilon_{\tau}$ .

NTS:  $P(s_g | V) > P(s_c | \bar{V})$   
 [proof exists (?) for flat prior  $\tau_g = \tau_c$   
 and identical string likelihood  $p_{\bar{V}} = q_{\bar{V}}$ ]

$$\Leftrightarrow \frac{\tau_g p_V}{\tau_g p_V + \tau_c q_V} > \frac{\tau_c q_{\bar{V}}}{\tau_g p_{\bar{V}} + \tau_c q_{\bar{V}}}$$

$$\Leftrightarrow \frac{\tau_g p_V}{\tau_g p_V + \tau_c q_V} > \frac{\tau_c (1 - q_V - q_{\bar{V}})}{\tau_g (1 - p_V - p_{\bar{V}}) + \tau_c (1 - q_V - q_{\bar{V}})}$$

$$\Leftrightarrow \dots > \frac{\tau_c - \tau_c q_V - \tau_c q_{\bar{V}}}{\tau_g - \tau_g p_V - \tau_g p_{\bar{V}} + \tau_c - \tau_c q_V - \tau_c q_{\bar{V}}}$$

$$\Leftrightarrow \tau_g p_V (\tau_g - \tau_g p_V - \tau_g p_{\bar{V}} + \tau_c - \tau_c q_V - \tau_c q_{\bar{V}}) > (\tau_c - \tau_c q_V - \tau_c q_{\bar{V}}) \cdot (\tau_g p_V + \tau_c q_V)$$

$$\Leftrightarrow \tau_g^2 p_V - \tau_g^2 p_V^2 - \tau_g^2 p_V p_{\bar{V}} + \tau_g \tau_c p_V - \tau_g \tau_c p_V q_V - \tau_g \tau_c p_V q_{\bar{V}} > \tau_g \tau_c p_V + \tau_g \tau_c q_V - \tau_g \tau_c p_V q_V - \tau_c^2 q_V^2 - \tau_g \tau_c p_V q_{\bar{V}} - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow \tau_g^2 p_V - \tau_g^2 p_V^2 - \tau_g^2 p_V p_{\bar{V}} > \tau_g \tau_c p_V - \tau_c^2 q_V^2 - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow (\tau_g^2 - \tau_g \tau_c) p_V - \tau_g^2 p_V^2 - \tau_g^2 p_V p_{\bar{V}} > -\tau_c^2 q_V^2 - \tau_c^2 q_V q_{\bar{V}}$$

$$\Leftrightarrow (\tau_g^2 - \tau_g \tau_c + \tau_g \epsilon_{\tau}) p_V - \tau_g^2 p_V^2 - \tau_g^2 p_V p_{\bar{V}} > -(\tau_g^2 - 2\tau_g \epsilon_{\tau} + \epsilon_{\tau}^2) (q_V^2 - q_V q_{\bar{V}})$$

$$\Leftrightarrow \tau_g \epsilon_{\tau} p_V - \tau_g^2 p_V^2 - \tau_g^2 p_V p_{\bar{V}} > \dots$$

okay, it is clear that any prior bias for  $s_g$  will pull closer towards  $s_g$ ; shows that, by mere likelihood, the same result is expected; so set:  $\epsilon_{\tau} = 0$

$$\frac{p_V}{p_V + q_V} > \frac{q_{\bar{V}}}{p_{\bar{V}} + q_{\bar{V}}}$$

$$\Leftrightarrow p_V p_{\bar{V}} + p_V q_{\bar{V}} > p_V q_V + q_V q_{\bar{V}}$$

$$\Leftrightarrow p_V p_{\bar{V}} > q_V q_{\bar{V}}$$

$$\Leftrightarrow \frac{p_{\bar{V}}}{q_V} > \frac{q_{\bar{V}}}{p_V}$$

$$(p_V + \epsilon_p) p_{\bar{V}} > q_V (q_V + \epsilon_q)$$

$$p_V^2 + \epsilon_p p_V > q_V^2 + \epsilon_q q_V$$

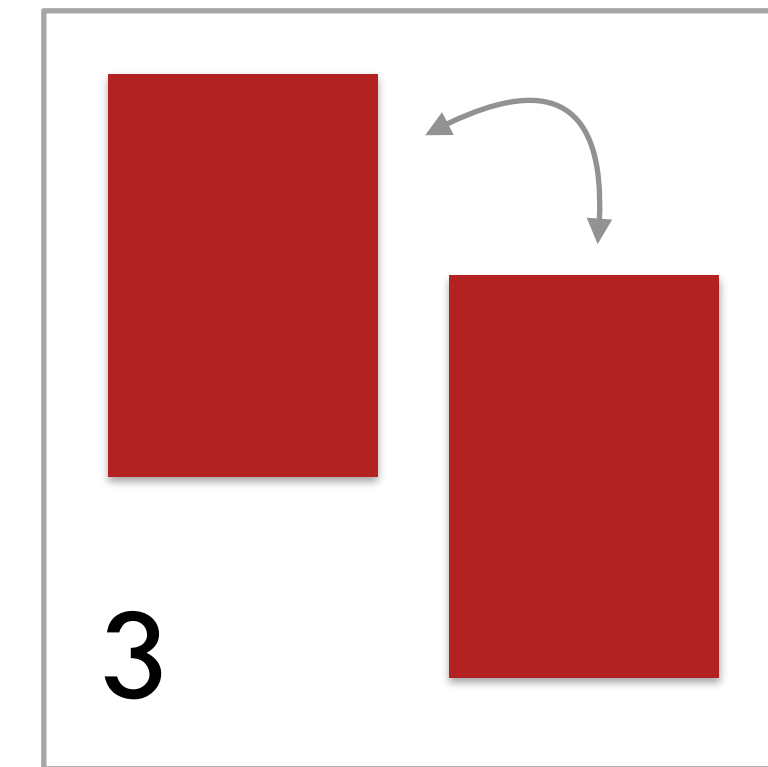
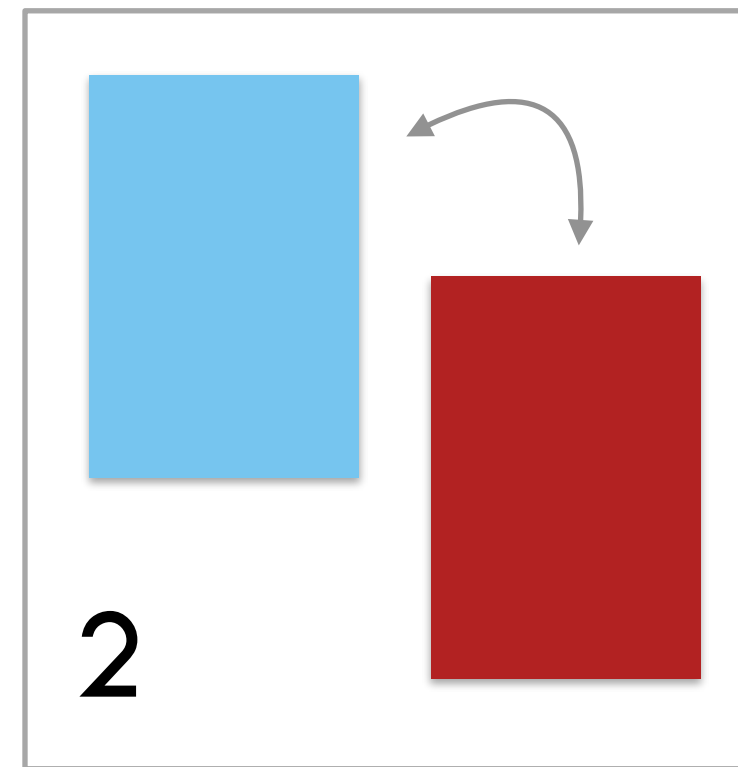
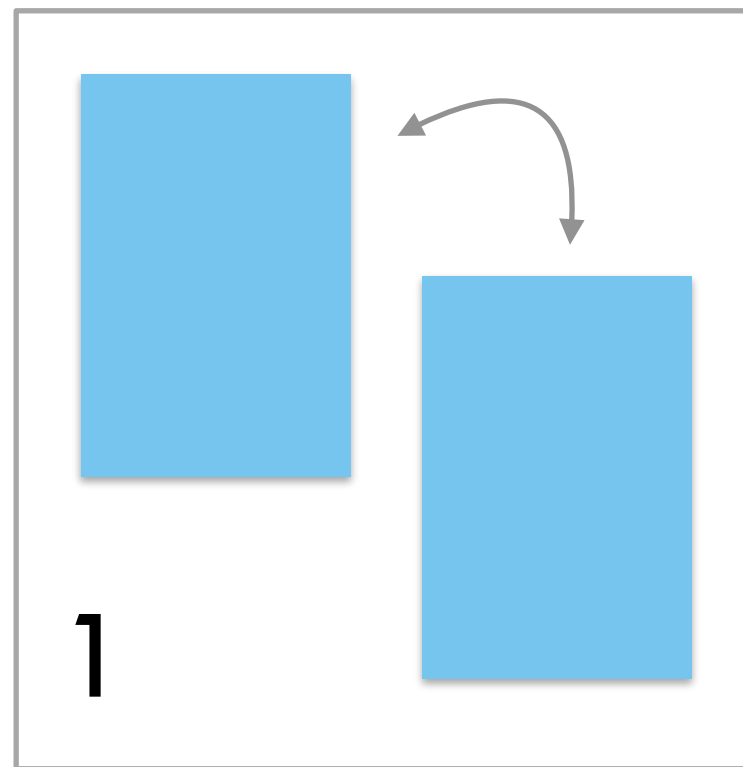
$$p_V^2 + \epsilon_p p_V > q_V^2 + \epsilon_q q_V + \epsilon_q q_V$$

assume:  
 $q_{\bar{V}} = q_V + \epsilon_q$   $\epsilon_q > \epsilon_p$   
 $p_V = p_{\bar{V}} + \epsilon_p$   
 [producing  $V$  when adequate is less likely than producing  $\bar{V}$  when adequate]  $p_V \leq q_{\bar{V}}$   
 [producing  $V$  when inadequate is less likely than producing  $\bar{V}$  when inadequate]  $p_{\bar{V}} > q_V$

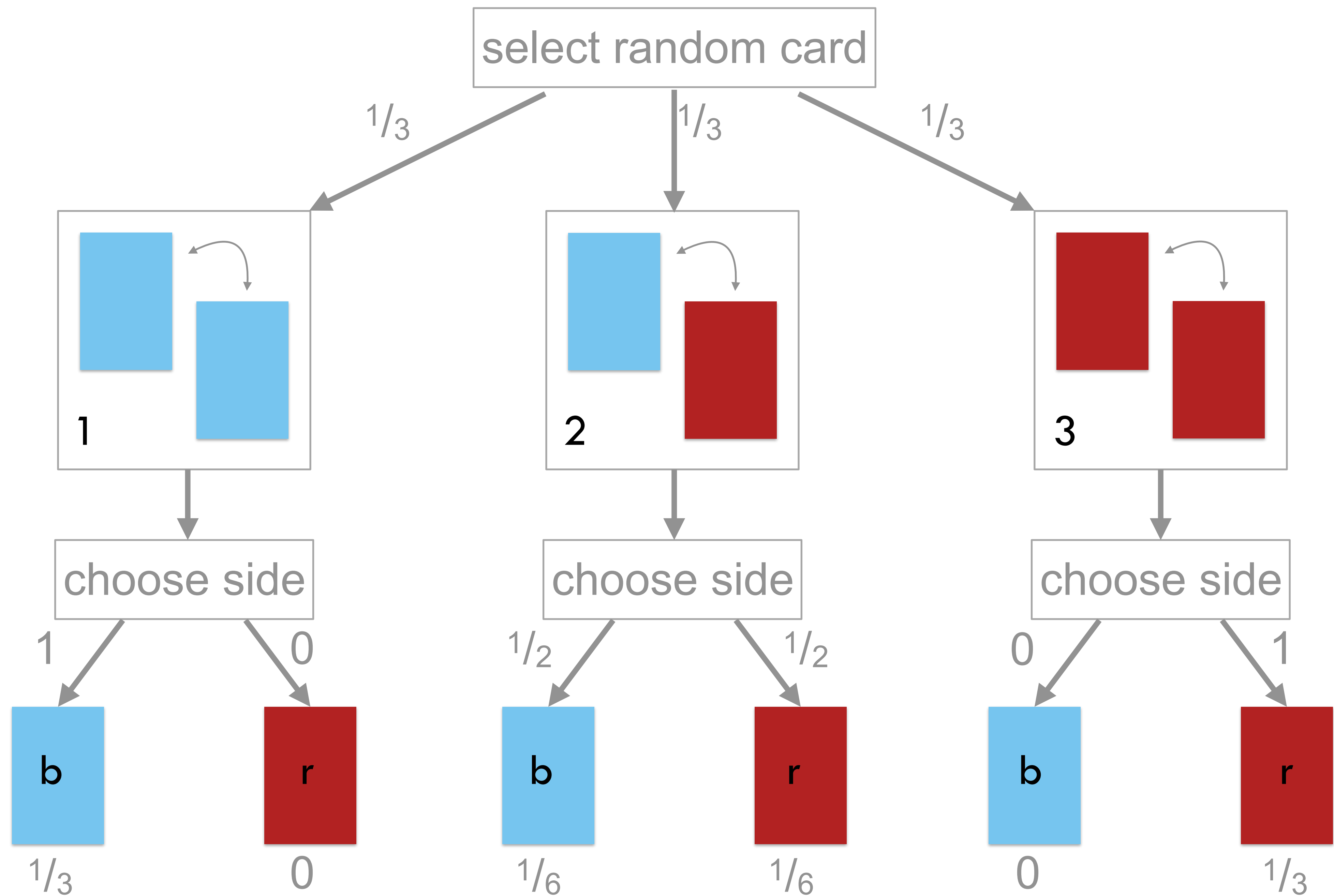
# Computational Pragmatics

Probability Basics  
 Session 2

# Three-card problem

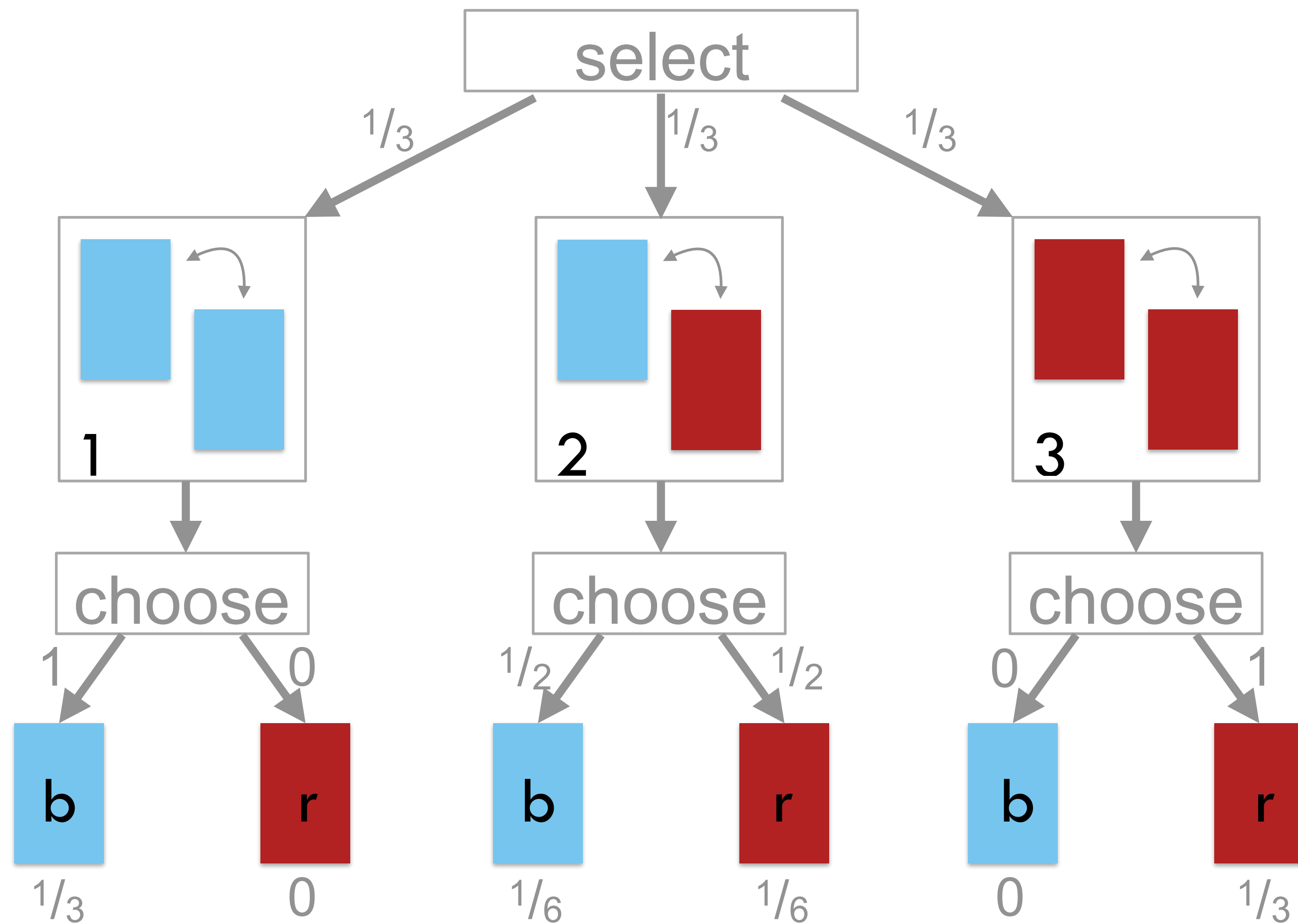


Sample a card (uniformly at random).  
Choose a side of that card to reveal (uniformly at random).  
What's the probability that the side you did not see is **BLUE**,  
given that the side you saw was **BLUE**?



## Conditional probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



## Bayes rule

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

$$P(\text{blue-blue} \mid \text{obs. blue})$$

$$= \frac{P(\text{obs. blue} \mid \text{blue-blue}) P(\text{blue-blue})}{P(\text{obs. blue})}$$

$$= \frac{1 \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

## Notation & computation

$$P(A \mid B) \propto \underbrace{P(B \mid A) P(A)}_{=P(A \cap B)}$$

```
// three cards; with blue or red on either side
var cards = [
  ["blue", "blue"],
  ["blue", "red"],
  ["red", "red"]
]

var model = function() {
  var card = uniformDraw(cards)
  var color = uniformDraw(card)
  condition(color == "blue")
  return card.join("-")
}

viz.table(Infer({method: "enumerate",
  model: model
}))
```

WebPPL code (preview)

any (conditional) probability distribution can be approximated by a large set of samples



I can you a sample give!