

Unemployment and Vacancy Dynamics with Imperfect Financial Markets *

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Abstract

This paper proposes a simple general equilibrium model with labour market frictions and an imperfect financial market. The aim of the paper is to analyse the transitional dynamics of unemployment and vacancies when financial constraints are in place. We model the financial sector as a monopolistically competitive banking sector that intermediates financial capital between firms. This structure implies a per period financial resource constraint which has a closed form solution and describes the transition path of unemployment and vacancies to their steady state values. We show that the transition path crucially depends on the degree of wage flexibility. When wages are bargained sequentially the transition path is always downward sloping. This implies unemployment and vacancies adjust in opposite directions as observed in the data. When calibrating the model to the Great Recession and its aftermath we find that the lack of an improvement in the financial sector's effectiveness to intermediate resources played a crucial role in the slow recovery of the labour market.

Keywords: Job search, unemployment, financial markets.

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1 Introduction

The Great Recession has highlighted the importance financial markets can have on the performance of the labour market. Most countries that were affected by the 2007/2008 financial crisis saw a burst of layoffs that made their unemployment rates increase dramatically and stay stubbornly high during the recovery period. Since the financial crises particularly affected the ability of firms to expand and create new vacancies due to the lack of available investment funds, it has been argued that the credit crunch also played an important role in slowing down the recovery of unemployment. In this paper we investigate to what extent the adjustment of unemployment and vacancies depends on the effectiveness of financial markets to intermediate resources.

We construct a simple general equilibrium model with labour market frictions and an imperfect financial market. Our focus is to analyse the transitional dynamics of unemployment and vacancies in respond to job displacement shocks and financial shocks. We are particularly interested in assessing the effects of these shocks on unemployment and vacancies when financial constraints are in place.

Our framework extends the canonical search and matching model (as described in Pissarides, 2000) by adding a monopolistically competitive banking sector that firms must visit in order to finance job creation. Once jobs are filled and firms become productive, they service their debts over time until the job is exogenously destroyed. This simple structure implies that at any point in time firms flow profits must be used to cover the cost of posting vacancies. The resulting per period financial resource constraint is the key element of our analysis. It shows that the number of vacancies is positively related to firms' flow profits. Since in the search and matching framework, the latter is directly related to the level of unemployment, the resource constraint then describes the relation unemployment and vacancies must satisfy to guarantee equilibrium in the banking sector.¹ Furthermore, we show that this constraint has a closed form solution and describes the transition path of the economy to its steady state.

These features allow us to characterise the equilibrium adjustment path of unemployment and vacancies and to analyse how it depends on the variables of interest. Changes in aggregate productivity or changes in the parameters governing the banking sector generate shifts of the transition path, affecting its slope and hence the speed of adjustment of unemployment and vacancies towards the new steady state. A decrease in labour productivity or in the productivity of the banking sector, for example, shifts the transition path downwards, decreasing the speed of adjustment towards the new steady state. Changes in the rate at which employed workers become unemployed or changes in the parameters governing the matching technology generate movements along the transition path.

We show that the relation between unemployment and vacancies along the transition path crucially depends on the degree of wage flexibility. In particular, if wages are determined through Nash bargaining, unemployment and vacancies adjust following a non-monotonic relationship. For low levels of unemployment and vacancies, they both adjust in the same direction. For larger values, however, they adjust in opposite directions. The resource constraint implies that as unemployment increases the number of productive firms decreases, reducing the available funds for job

¹Benmelech, Bergman and Seru (2012) document, using data for the US and Japan, a negative relation between firms' cash flows and employment as well as a negative relation between the extent of credit availability to firms and local unemployment rates.

creation and generating a negative relation between unemployment and vacancies. However, the non-monotonicity arises due to the feedback effect of the job finding rate on wages and ultimately profits. Under Nash Bargaining an increase in unemployment reduces workers' outside options and increases firms' flow profits, which in turn implies there are more funds per productive firms to finance vacancies, generating a positive relation between unemployment and vacancies. At low levels of unemployment and vacancies the latter force dominates, while for larger values the former force dominates. As wages become less dependent on the unemployment rate due, for example, to some form of wage rigidity, this feedback effect diminishes. We show that when wages are bargained following the sequential protocol proposed by Hall and Milgrom (2008) one always obtain a negative relation between unemployment and vacancies along the transition path.

Being able to generate such an adjustment process is important as one of the main features of the canonical search and matching model is that unemployment and vacancies always adjust in the same direction along the transition path. Blanchard and Diamond (1989) document, however, that unemployment and vacancies adjust in opposite directions after a shock to output or to the rate at which workers and firms separate. Furthermore, Shimer (2005) shows that when the canonical search and matching model is calibrated to the US, shocks to the job destruction rate generate a counterfactual positive correlation between unemployment and vacancies. Given that we are able to solve for the transition path in close form, we provide an analytical characterisation of how changes to aggregate output, the job destruction rate or to the banking sector parameters affect the speed of adjustment of unemployment and vacancies.

In the quantitative section of the paper we calibrate the model to match the transitional dynamics of unemployment and vacancies observed during the Great Recession and its aftermath for the US. We find that the calibration favours the Hall and Milgrom (2008) sequential bargaining setting with respect to Nash Bargaining, delivering a downward sloping transition path that replicates the observed transitional dynamics very well. The main message of this exercise is that observed slow recovery in the labour market was due to the lack of a significant improvement in the effectiveness of the banking sector in intermediating resources to fund job creation.

A related paper is that of Petrosky-Nadeau and Wasmer (2013). These authors extend the work by Wasmer and Weil (2004) to analyse the effects of financial markets on unemployment and vacancies using a stochastic version of the search and matching model (see Shimer, 2005, and Mortensen and Nagypal, 2007, among others). They emphasise the role financial frictions have in increasing the cyclical volatility of unemployment and vacancies in response to productivity shocks. Furthermore, these authors model the financial market as a frictional market governed by a meeting function that brings together banks and vacant firms, taking as given the interest rate.² Although we also investigate how financial frictions affect the behaviour of unemployment and vacancies, our focus and approach are very different. For one we are interested in analysing the transitional dynamics of unemployment and vacancies that arise from real and financial shocks in a general equilibrium setting in which the interest rate becomes an endogenous variable. Financial markets

²Petrosky-Nadeau (2014) analyse the extent to which financial frictions, modelled as an agency problem between banks and firms with vacant jobs, can improve the performance of the search and matching model in matching on the cyclical volatility and persistence of unemployment and vacancies in response to productivity shocks. Petrosky-Nadeau and Wasmer (2015) analysis, in a similar setting to Petrosky-Nadeau and Wasmer (2013), the interaction between frictional financial, labour and goods markets to also analyse the cyclical volatility and persistence of unemployment and vacancies.

in our framework are imperfect not due to search frictions but due to monopolistic competition. We emphasize the importance of the resource constraint, imposed by the financial sector, as an important determinant of the relationship between unemployment and vacancies in an economy's adjustment process, and provide a cross-country comparison of the implications of our model.

Uren (2012) also considers the relationship between financial markets and a labour market with search and matching frictions (see also Krusell, Muyokama and Sahin, 2010). He constructs a general equilibrium model in which financial markets are incomplete, agents heterogenous and the labour market follows the Diamond-Mortensen-Pissarides structure. In this model, investments take the form of financing the cost of vacancies using aggregate savings. He shows that, in the steady state, allowing savings to finance job creation lowers the equilibrium unemployment rate with respect to the complete market setting. In contrast we consider a complete market environment as our aim is to provide a tractable analysis of the transitional dynamics of the economy and not of its distributional aspects across agents.

The rest of the paper is outlined as follows. In the next section we describe the search and matching model that describes the aggregate labour market and the monopolistically competitive banking model describing the financial sector. In section 3 we characterise the equilibrium and discuss the steady state and transitional dynamics. Here we analyse the main implications of the financial resource constraint on the transition path of vacancies and unemployment and show how parameters governing the financial market affect this transition path. Section 4 presents the calibration procedure and describes the main results. Section 5 concludes discussing briefly the main results. All proofs and tedious derivations are relegated to a technical Appendix.

2 The Model

2.1 Basic Framework

The labour market setup follows Pissarides (2000, ch.1). Since our objective is to understand the transitional dynamics of the economy we consider out-of-steady-state analysis. Time is continuous with infinite horizon. There is a unit mass of workers and a mass of firms. Both agents discount the future at a potentially time-dependent interest rate $r(t)$. Workers can be either employed or unemployed. Unemployed workers receive constant benefits z per unit of time. An employed worker receives a wage rate of $w(t)$ per unit of time. Each firm has only one job that can be either vacant or filled. A filled job generates a constant flow of output $p > z$. A firm with a vacant job pays a cost measured in terms of productivity units of $k > 0$ per unit of time. Jobs are destroyed at an exogenous Poisson rate $s > 0$. Once destroyed, the firm's job becomes vacant and the worker becomes unemployed.

Agents must search for each other to find a match. The search process is sequential and random and we assume that only vacancies and unemployed workers search. Meetings are governed by a meeting or "matching function" $m(u(t), v(t))$ which gives the number of meetings that take place per unit time as a function of the number of unemployed workers $u(t)$ and the number of vacancies $v(t)$. Assume that $m(\cdot)$ is increasing and concave in both arguments and exhibits constant returns to scale. Let $\theta(t) \equiv v(t)/u(t)$ denote the labour market tightness. Constant returns to scale then implies the job filling rate is given by $q(\theta) \equiv m(u, v)/v$, while the job finding rate is then

$\lambda(\theta) = \theta q(\theta)$. These rates govern the Poisson processes by which agents meet in this labour market.

2.2 Bellman Equations for Workers and Firms

Workers and firms are risk neutral. The workers' objective consists in maximizing the expected present value of their lifetime income $E_0 \int_0^\infty e^{-\int_0^s r(s)ds} y(t) dt$, where individual income $y(t) = \{z, w(t)\}$ at every point in time. Let U denote the expected value of an unemployed worker and let W denote the expected value of a worker employed at some net wage w . Dynamic programming arguments imply the following U and W satisfy the Hamilton-Jacobi-Bellman equations

$$r(t)U(t) = z + \dot{U}(t) + \lambda(\theta)[W(t) - U(t)], \quad (1)$$

$$r(t)W(t) = w(t) + \dot{W}(t) + s[U(t) - W(t)]. \quad (2)$$

The firm's flow profit from a filled job is

$$\pi(t) = p - w(t). \quad (3)$$

Depending on the current state of the firm, flow profits of firms are therefore given by $\Pi(t) = \{-k, \pi(t)\}$. Firms are infinitely lived and their objective is to maximize the expected present value of total profits $E_0 \int_0^\infty e^{-\int_0^s r(s)ds} \Pi(t) dt$. Let V denote the expected value of holding a job vacant. Let J denote the expected value of a filled job paying w . We then obtain the corresponding Hamilton-Jacobi-Bellman equations by dynamic programming arguments,

$$r(t)V(t) = -k + \dot{V}(t) + q(\theta)[J(t) - V(t)], \quad (4)$$

$$r(t)J(t) = \pi(t) + \dot{J}(t) + s[V(t) - J(t)]. \quad (5)$$

The interpretation of the above equations is identical to the canonical DMP model only that discounting takes place at an endogenous interest rate $r(t)$.

2.3 Free Entry and Wage Determination

For a given tightness and wage rate, the number of vacancies is determined by the free entry condition. As long as the value V of opening a vacancy is positive, firms will create vacancies and enter the labour market. Firms will stop entering only when there are no more (inter-temporal) profits to be made; i.e $V = 0$. Using the Bellman equation (4), we obtain that

$$J(t) = \frac{k}{q(\theta(t))}. \quad (6)$$

When an unemployed worker and a vacant firm meet, $p > z$ insures that they immediately form a productive match. There are many ways a firm and a worker can split the surplus of the match. It has been standard to use the generalised Nash Bargaining solution as a way to determine wages. This protocol implies spot, fully flexible wages that are renegotiated every instant. On the other extreme, Hall (2005) proposed an alternative wage determination mechanism motivated by the fact

that wages do not seem to behave as spot market wages in the data. He uses a Nash demand game in which wages are fixed within the bargaining set. In this setup, wages are not renegotiated until they lie outside the bargaining set and hence prevent inefficient separations. Other wage determination mechanisms that lie somewhere in between these two cases have also been studied in the literature. For example, the staggered wage setting protocol proposed by Gertler and Trigari (2006) and the sequential bargaining protocol proposed by Hall and Milgrom (2008). All these wage setting mechanism are able to somewhat isolate wages from external (to the match) labour market condition and hence generate some form of wage rigidity. In this paper we follow an agnostic approach and use a wage equation that relies on a linear sharing rule that has as special cases the Nash bargaining wage and the one proposed by Hall and Milgrom (2008).

$$w(t) = (1 - \beta) z + \beta [p + \tau\theta(t)k]. \quad (7)$$

In this wage equation β is the worker's exogenous bargaining power standard in the Nash Bargaining protocol. The crucial parameter, however, is $\tau \in [0, 1]$ which captures, in reduced form, the extend to which labour market tightness affects wages. When $\tau = 1$, for example, we are back to the Nash Bargaining solution of fully flexible wages. When $\tau = 0$ we have the outcome of a simplified version of the sequential bargaining game proposed by Hall and Milgrom (2008).³ The main reason for the choice of this functional form is that we are interested in understanding the role wage rigidity plays in the interaction between the financial and labour markets. Equation (7) presents a specification that allows us to do this in a simple and tractable way. In the quantitative section we recover τ and β from our calibration procedure.

2.4 Equilibrium Without a Financial Sector

Given a time-dependent interest rate, equilibrium can then be described by the evolution of the unemployment rate $\dot{u}(t)$ and the evolution of labour market tightness $\dot{\theta}(t)$. For a path of labour market tightness $\theta(t)$, employment is derived by equating inflows and outflows. Inflows to unemployment amount to $s[1 - u(t)]$, while $\lambda(\theta(t))u(t)$ unemployed individuals find a job at each instant. Equating inflows and outflows yields the familiar equation describing the evolution of the unemployment rate $u(t)$ in this economy.

$$\dot{u}(t) = s[1 - u(t)] - \lambda(\theta(t))u(t). \quad (8)$$

The evolution of labour market tightness can be determined by the value of a filled vacancy (6) and by an equation describing its evolution over time. After some steps (see app. A.1), we obtain a differential equation describing the evolution of labour market tightness

$$\dot{\theta}(t) = \left[\frac{q(\theta(t))}{q'(\theta(t))} \right] \left[\frac{(1 - \beta)}{k} (p - z) q(\theta(t)) - r(t) - s - \tau\beta\lambda(\theta(t)) \right]. \quad (9)$$

Equations (8) and (9) determine the paths of $u(t)$ and $\theta(t)$ as a function of $r(t)$.

³See Mortensen and Nagypal (2007) who implicitly use the same wage equation to capture the Nash and the sequential bargaining solutions.

2.5 The Financial Sector

We now close our matching model by including a financial sector. We consider a banking sector that is the only source for financing the vacancy costs and to which all profits of productive firms flow. In this environment, a potential market entrant that wants to finance a vacancy must visit a bank and ask for a flow of resources allowing to cover the vacancy costs k to be paid at each point in time until a worker is found. In order to get these resources, the firm needs to sign a contract that says that the entrant commits to repay the bank by the flow of profits it makes once the vacancy is filled and until the next separation takes place. The bank bears all the risk and diversifies across all entrants and productive firms such that the bank behaves as if the world was deterministic.

Suppose that the banking sector consists of $n(t)$ different types of banks offering each one single banking service i at any time t . Banks operate under monopolistic competition. Financial services are aggregated to one big “financing package for opening a vacancy” by a technology of the Dixit-Stiglitz type

$$Y(t) = \left[\int_0^{n(t)} x(i, t)^\gamma di \right]^{1/\gamma}, \quad (10)$$

where $x(i, t)$ is the amount of services provided by bank i at time t . The elasticity of substitution between these services is given by $(1 - \gamma)^{-1}$.

Banking service i is produced by the technology

$$x(i, t) = by(i, t) - \phi \quad (11)$$

where b is a productivity parameter, $y(i, t)$ is the input of the final good produced and consumed in this economy and ϕ describes the fixed costs to be paid by monopolistic competitors. Just as with vacancy costs, fixed costs in the banking sector are measured in units of the output good.

Service providers maximize profits by choosing output $x(i, t)$ optimally at each point in time. As all firms use the same technology, service provision will be symmetric and the usual steps (see app. A.2.1) imply aggregate output of the banking sector (10) to amount to

$$Y(t) = n(t)^{1/\gamma} x(t) = n(t)^{1/\gamma} \left[b \frac{\pi(t) [1 - u(t)]}{n(t)} - \phi \right]. \quad (12)$$

A crucial assumption here is that we require that all resources available for financing vacancies must actually be used for financing vacancies. Resources must not be lost or are allowed to enter the model. Making such a market-clearing assumption for the banking sector implies that the aggregate banking services (12) equal the total costs for financing vacancies. The latter are given by the costs k per vacancy times the number of vacancies, $\theta(t)u(t)$,

$$n(t)^{1/\gamma} \left[b \frac{\pi(t) [1 - u(t)]}{n(t)} - \phi \right] = k\theta(t)u(t). \quad (13)$$

Economically speaking, market clearing for financial services (13) determines the number of vacancies $v(t)$. Technically, as vacancies are already determined in (9), this additional market fixes the endogenous interest rate $r(t)$.

While the resource constraint described in (13) makes sure that resources used for financing

vacancies can only come from profits made by firms, it does not guarantee that there are no resources left unused. As monopolistic service providers make a profit, this profit needs to go somewhere. It can actually not be ruled out at this point that firms would even make negative profits, given that there are fixed costs ϕ to be paid per period. To guarantee that all resources supplied by firms making a profit are used either for covering fixed costs for the provision of services or for financing vacancies, we apply the standard assumption here as well and assume that there is free entry and exit into the banking sector. This implies (see app. A.2.1) that the number of services is given by

$$n(t) = (1 - \gamma) b \frac{\pi(t)[1 - u(t)]}{\phi}. \quad (14)$$

After some more algebra (see app. A.2.1), we obtain

$$\frac{\theta(t)^\gamma}{(1 - \beta) \frac{p-z}{k} - \tau\beta\theta(t)} = b \left[(1 - \gamma) \frac{k}{\phi} \right]^{1-\gamma} \frac{1 - u(t)}{u^\gamma(t)} \gamma^\gamma. \quad (15)$$

Equation (15) describes the resource constraint that is consistent with free entry in the banking sector. Under this specification, all profits made by firms are used for financing vacancies and all costs of vacancies are financed by profits by firms. Profits made by banks are used to pay their fixed costs. This therefore makes sure that the financial market is in equilibrium, no resources leave or enter the model and we have specified a general equilibrium matching model.

3 General Equilibrium

Equations (8) and (9) provide the basis to understand the goods and labour markets dynamics by describing \dot{u} and $\dot{\theta}$. Equation (15) describes equilibrium in the financial market. These three equations simultaneously solve for $u(t)$, $\theta(t)$ and $r(t)$. Before we describe the transitional dynamics of this system, we analyse its steady state.

3.1 Zero-motion lines and steady state

From equation (8) we obtain that the zero-motion line for u is given by

$$\lambda(\theta) = s \frac{1 - u}{u} \Leftrightarrow u = \frac{s}{s + \lambda(\theta)}, \quad (16)$$

which describes a negative relationship between u and θ . The zero-motion line for θ is implicitly given, from (9), by

$$(1 - \beta) \frac{p - z}{k} q(\theta) - s - r(t) - \tau\beta\lambda(\theta) = 0. \quad (17)$$

What is special about this zero motion line is that the interest rate is a function of time which means that the zero-motion line moves in the $(u - \theta)$ space. What is standard is that the zero motion line for θ is not a function of u , i.e. it is horizontal in the $(u - \theta)$ space.

In standard descriptions of phase diagrams, the equilibrium path is to be inferred from the zero motion lines and laws of motions subsequently. In our system, however, the equilibrium path towards the steady state is described in closed-form by (15). There, it is easy to verify θ falls with

u : The right-hand side unambiguously falls in u while the left-hand side rises in θ . Our general equilibrium matching model therefore also provides an explicit expression for the transition path in terms of unemployment and vacancies, which we discuss below.

In a steady state, the unemployment rate, labour market tightness and the interest rate are constant. Denote their steady state values as u^* , θ^* and r^* . To show the existence of a steady state note from (16) that as u goes to zero, θ grows unboundedly; and while as u goes to one, θ goes to zero. From (15), however, we have that as u goes to zero, θ goes to $(1 - \beta)(p - z)/\tau\beta k$; while as u goes to one, θ goes to zero. Hence these functions intersect at $u = 1$ and $\theta = 0$. Further, since these functions are continuous and decrease monotonically, they can intersect at most once at some $u \in (0, 1)$ and $\theta \in (0, \infty)$. Given the steady state values of u and θ , the interest rate r then adjusts such that (17) holds. Since in the case in which $u = 1$ and $\theta = 0$ (17) implies r is undetermined, in what follows we focus on characterising the transition dynamics towards the interior steady state, (u^*, θ^*, r^*) , given that one exists.

In app. A.3 we provide a sufficient condition under which a unique interior steady state exists for any CRS matching function. Further, we show that under a Cobb-Douglas matching function $M(u, v) = Au^{1-\alpha}v^\alpha$, the parametric restriction $\gamma \geq \alpha$ is sufficient (but not necessary) to guarantee existence of an interior steady state equilibrium. In the quantitative section of the paper we show that a unique interior steady state always exists in our calibration.

3.2 Transitional Dynamics

An insightful way to analyse the equilibrium path described in (15) is to consider it in the Beveridge space; i.e. $v - u$ space. It is well documented that unemployment and vacancies move in opposite directions. Further, earlier work by Blanchard and Diamond (1989) and, more recently, by Shimer (2005) show that unemployment and vacancies move in opposite directions during the adjustment process of the US economy; and similar results have been obtained for European countries (see Elsby et al., 2013). It is of interest to understand the conditions under which the interaction between the labour market and the financial sector, as modelled in this paper, has the potential to generate such a negative relation.

Re-writing equation (15) in $v - u$ space using an implicit formulation, we obtain that the sign of the slope of the equilibrium path is determined by (see app. A.4)

$$\text{sign} \left[\frac{dv}{du} \right] = \text{sign} \left[\frac{\partial \pi}{\partial u} (1 - u) - \pi \right].$$

To understand this condition, note that the resource constraint implies that aggregate firms' profits and the number of vacancies must move in the same direction along the equilibrium path. Aggregate firms' profits, however, depend positively on (i) the number of jobs filled and negatively on (ii) the wage paid to workers; and both are inversely related to the unemployment rate. The slope of the transition path then depends on how responsive are wages to changes in the unemployment rate. Using the expression for $\partial \pi / \partial u$, we find that

$$\frac{dv}{du} < 0 \Leftrightarrow u^2 > \frac{\tau\beta kv}{(1 - \beta)(p - z)}. \quad (18)$$

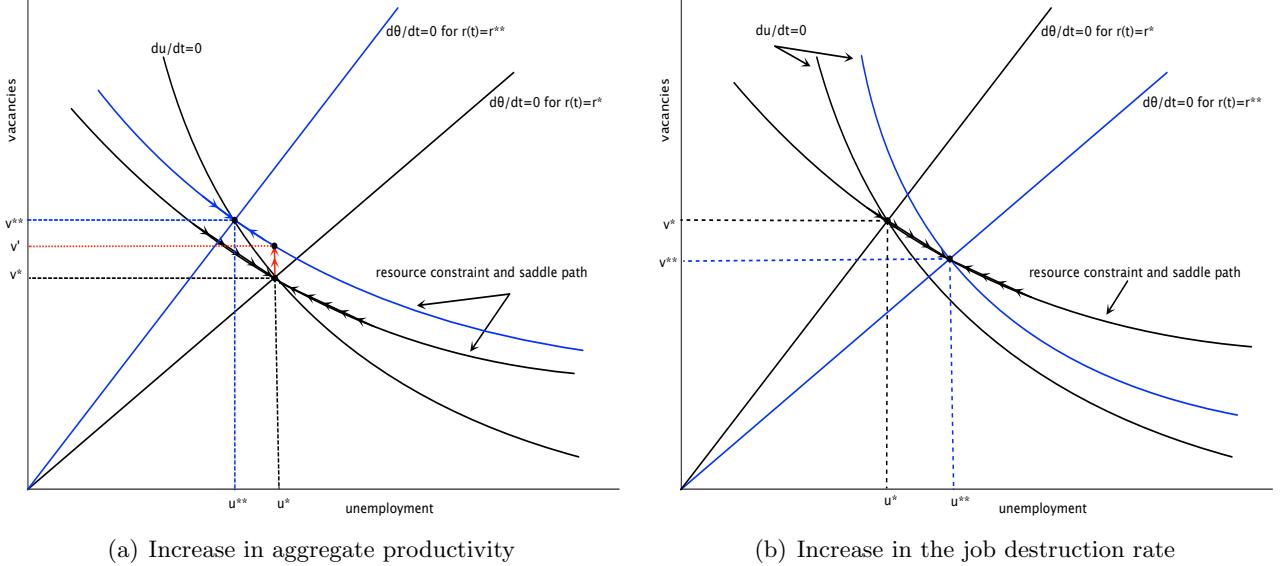


Figure 1: Transitional dynamics after a one time unexpected shock to p and s

Under Nash bargaining, $\tau = 1$, an increase in the unemployment rate decreases a worker's outside option in the bargaining game through a decrease in the job finding rate. This in turn leads to a decrease in wages and ultimately to an increase in profits. In this case, (18) implies that the transition path is non-monotonic. It is increasing when $u^2 < \beta kv/(1 - \beta)(p - z)$ and decreasing when $u^2 > \beta kv/(1 - \beta)(p - z)$. The point of inflexion, however, depends on the parameters affecting the resource constraint. When wages are bargained as in Hall and Milgrom (2008), a worker's disagreement point is no longer the same as his outside option and the unemployment rate has a much reduced effect on wages. In the version of the sequential bargaining game we consider in this paper, this implies that wages are independent of unemployment and $\tau = 0$. The feedback effect between unemployment and profits disappears, $\partial\pi/\partial u = 0$, and the transition path is always downward sloping.

This feature incorporates an important dimension to the canonical search and matching model. In the latter, with a constant interest rate, the equilibrium path towards the steady state is given by the zero-motion line for θ for any value of τ . This implies that during adjustment, vacancies and unemployment move in the same direction irrespectively of the degree of wage rigidity as modelled in (7). Here, however, the above arguments imply that during adjustment vacancies and unemployment can move in *opposite* directions.

3.3 Changes in Output and the Job destruction rate

To illustrate these differences consider a one time unexpected increase in aggregate productivity p . Figure 1.a depicts this exercise in $v - u$ space assuming the existence of an interior steady state and a range of values of v and u in which the equilibrium path is downward sloping. Here we want to show the qualitative workings of the model under the latter conditions. In Section 4, we provide a quantitative analysis of the model and show in our calibration the conditions under which the equilibrium path is downward sloping.

In both models, the zero-motion line for θ rotates upwards and, in our model, the resource constraint will shift outwards. In the figure, these are depicted in blue. The increase in p makes labour market tightness jump upwards as firms create new vacancies up to the point in which the economy is on the equilibrium path for the original unemployment rate. Note that in our model the initial jump of vacancies (shown by the red arrowed line from v^* to v') is smaller as the equilibrium path lies below the zero-motion line for θ , over the relevant range. Further, along this path the unemployment rate decreases, while the vacancy rate increases; which contrasts with the canonical model in which *both* unemployment and vacancies decrease. Also note that in both cases the transitional dynamics yield counter-clockwise movements of u and v , and the new steady states involve a higher vacancy rate and a lower unemployment rate. Hence our model can generate transitional dynamics consistent with the evidence in Blanchard and Diamond (1989) who show that the counter-clockwise movements of u and v around the zero-motion line for u after a productivity shock involve these rates moving in opposite directions.

As a second example consider a one time unexpected increase in the job destruction rate, s . Figure 1.b shows this exercise. Here the difference between the two models is starker. After an increase in s , both models imply that the zero-motion line for u shifts to the right, while the zero-motion line for θ rotates downwards (in $v - u$ space). Once again, in the figure, these are depicted in blue. In the canonical model, however, v jumps downwards, while u stays constant immediately after impact. As the economy adjust, both variables then increase along the new zero-motion line for θ until the new steady state is achieved. In our model, the equilibrium path does not depend on s (see (15)), which implies that v does not jump. Instead v decreases and u increases smoothly along the equilibrium path until the new steady state is achieved. Furthermore, an increase in the job destruction rate will always imply a new steady state with a lower vacancy rate and a higher unemployment rate, while in the canonical model the new steady state can be characterised by a higher vacancy and unemployment rate. This implications suggests that our model can also be consistent with the evidence presented in Blanchard and Diamond (1989) and Shimer (2005) on the effects of reallocations shocks on u and v better than the canonical model.

Note that changes in matching function parameters will have similar effects as changes in the job destruction rate, although in the opposite direction. For example, consider the Cobb-Douglas matching function, $M(u, v) = Au^{1-\alpha}v^\alpha$, changes in A and α will shift the zero motion line for unemployment along the equilibrium transition path of the economy. This feature is also very different from the standard search and matching model in the same way as describe under changes in s .

In addition since in our model changes in p affect both the position and the slope of the equilibrium transition path, output affects the speed by which unemployment and vacancies adjust from one steady state to another. On the contrary, since the job destruction rate and the matching function parameters do not determine the equilibrium transition path, the speed of adjustment of unemployment and vacancies is independent of these variables.

3.4 The Role of the Financial Sector

We now consider how the market power of banks, measured by the price mark-up $1/\gamma$, the productivity of each bank, b and the fixed cost, ϕ , affect the transitional dynamics of this economy and

the steady state equilibrium. For this purpose, re-write (15) in implicit form as

$$\Psi(\theta, u) \equiv \theta u - \frac{\gamma}{(1-\gamma)} \frac{\phi}{k} \left[\frac{b(1-\gamma)[(1-\beta)(p-z) - \tau\beta k\theta][1-u]}{\phi} \right]^{\frac{1}{\gamma}} = 0. \quad (19)$$

Note that its slope is given by

$$\frac{d\theta}{du} = -\frac{n[\theta(1-\gamma)k(1-u) + \phi n^{\frac{1}{\gamma}}]}{k(1-\gamma)(1-u)[nu + n^{\frac{1}{\gamma}}\tau(1-u)b\beta]} < 0. \quad (20)$$

Note that when $u = 0$, $\theta = (1-\beta)(p-z)/\tau\beta k$ and that when $u = 1$, $\theta = 0$. Since the intersections of Ψ with the axis are independent of γ , ϕ or b , to analyse the impact of these variables on the equilibrium path it is sufficient to analyse their impact on (20).

First consider an increase on the bank's fixed cost, ϕ . Differentiation of (20) implies that the equilibrium path experiences a leftward expansion and becomes flatter for all values of u when $(1-\beta)(p-z)u > \tau\beta\theta k$. Note that this is the same condition required to guarantee that the transition path is downward sloping in the Beveridge space. Hence this condition becomes easier to satisfy by reducing the value of τ , such that when $\tau = 0$ this condition is always guaranteed. Since the zero-motion line for unemployment is independent of ϕ , in those cases in which the above condition is satisfied, the new steady state unemployment rate increase, while labour market tightness decreases which, by virtue of (17), implies that the interest rate increases. Because the equilibrium path becomes flatter, the speed at which the economy arrives to the new steady state decreases.

Now consider an increase in bank's productivity, b . Once again differentiation of (20) implies that the equilibrium path experiences a rightward expansion and becomes steeper for all values of u when $(p-z)(1-\beta)u > \tau\beta\theta k$. In these cases and given that the zero-motion line for unemployment is independent of b , the new steady state is characterised by a lower level of unemployment, a higher labour market tightness and, by virtue of (17), a lower interest rate. Furthermore, the speed at which the economy arrives to the new steady state increases.

Finally consider an increase in γ , such that the elasticity of substitution between financial products increase. In this case differentiation of (20) shows that there is an ambiguous impact of γ on the slope of the equilibrium path. In app. A.5 we show conditions under which an increase in γ has the same effects as an increase in b , at least for the cases of $\gamma \rightarrow 1$ and $\gamma \rightarrow 0$. We will turn to these comparative statics in more detail in the next section, where we quantitatively evaluate the model.

4 Quantitative analysis

The objective of this section is to analyse whether the transition path implied by our model can replicate the dynamics of the unemployment and vacancy rate in the US economy for the period 2007-2015. To do so, we consider two sub-periods: (i) The Great Recession (November 2007 – August 2009) and (ii) the Recovery (September 2009 – December 2014). Through the lenses of our model, we interpret a sequence of (u, v) points within a given sub-period as movements along a saddle-path towards a new steady state. This section proceeds by calibrating the model to match

the unemployment and vacancy dynamics during the Great Recession. That is, we calibrate the model to match the transition from the beginning of the Great Recession period to the end of the Great Recession. We then explore changes in vacancy costs and changes in the financial sector that can account for the observed unemployment and vacancy dynamics during the recovery period.

4.1 Parametrization

The length of a period in the model is set to one month. We use information on the number of vacancies from the Job Openings and Labor Turnover Survey (JOLTS) and seasonally adjusted monthly series on the stock of employed, unemployed and short-term unemployed workers provided by the Bureau of Labor Statistics.⁴ From these series we construct monthly series of job-finding-, unemployment- and vacancy rates (see Figure 4 and Figure 6 in app. A.7).⁵

We use a Cobb-Douglas specification for the matching function, $M(u_t, v_t) = Au_t^\alpha v_t^{1-\alpha}$, which implies a job finding rate of $\lambda(\theta_t) = A\theta_t^{1-\alpha}$. The parameters A and α are then obtained from regressing the (log) job finding rates on a constant and (log) labour market tightness using data for the pre-crisis period December 2000 – October 2007. Given our estimates of A and α , we set the separation rate such that we match the steady-state unemployment rate u^* at the end of the Great Recession, i.e. $s = \frac{u^* \lambda(\theta^*)}{1-u^*}$. Furthermore we set the interest rate $r^* = 0.0027$ such that it corresponds to the (annual) bank prime loan rate of 3.25% at end of the Great Recession.⁶

After normalising the productivity parameters to unity, $p = b = 1$, we are then left with $\mathbf{x} = \{k, z, \beta, \tau, \phi, \gamma\}$ parameters to recover. For this we exploit the variation in the observed values of u and v during the Great Recession. In particular, we minimise the squared relative distance between the observed vacancy rates and the ones implied by our transition path taking the observed unemployment rates as given. That is, we choose

$$\mathbf{x} = \arg \min \sum_t \left(\frac{v_t - \hat{v}(\mathbf{x}; u_t)}{v_t} \right)^2 \quad \text{subject to (17) and } v^* = \hat{v}(\mathbf{x}; u^*), \quad (21)$$

where $\hat{v}(\mathbf{x}; u_t)$ denotes the vacancy rate that solves (15) given the vector of parameters \mathbf{x} and the observed unemployment rate u_t . The first restriction is given by the zero-motion line for θ , while the second restriction requires that the transition path must go through the steady state (u^*, v^*) at the end of the Great Recession. In addition, we impose two further restrictions to pin down \mathbf{x} . First, we interpret z to represent unemployment benefits and set the replacement ratio to one half. Second, we follow Silva and Toledo (2007) and Petrosky-Nadeau and Wasmer (2013) and require that the total vacancy cost amount to 3.6% of the wage rate.⁷

Table 1 shows the parameter values obtained from our calibration procedure as well as the steady state targets. Note that the elasticity of the matching function is close to Shimer (2005)

⁴We use the BLS series LNS13000000, LNS13008396 and LNS12000000.

⁵Let U_t denote the number of unemployed in month t , and let U_{t+1}^s correspond to the number of short-term unemployed with unemployment durations of less than 5 weeks in month $t+1$. Following standard practice in the literature, the job-finding rate in month t is then given by $U2E_t = 1 - (U_{t+1} - U_{t+1}^s)/U_t$. Similarly, the job-destruction rate in month t is given by $E2U_t = U_{t+1}^s/E_t$, where E_t denotes the number of employed workers in month t (see Figure 5 in app. A.7).

⁶The data on bank prime loan rates are taken from the online data base of the Federal Reserve Bank St. Louis: <https://research.stlouisfed.org/fred2>.

⁷Details on the implementation of the optimisation problem can be found in app. A.6.

| Labour Market Parameters | | Financial Sector Parameters | | Steady state (8/2009) | |
|--|-------|---|-------|-----------------------|--------|
| Matching efficiency A | 0.430 | Fixed costs financial sector ϕ | 0.028 | u^* | 0.096 |
| Elasticity of matching function α | 0.709 | Elasticity of substitution $\frac{1}{1-\gamma}$ | 1.097 | θ^* | 0.155 |
| Vacancy cost k | 2.320 | Banks' productivity b | 1.000 | r^* | 0.0027 |
| Worker's bargaining weight β | 0.919 | | | s^* | 0.027 |
| Wage rigidity τ | 0.000 | | | $\lambda(\theta^*)$ | 0.250 |
| Unemployment benefits z | 0.479 | | | | |
| Labour productivity p | 1.000 | | | | |

Table 1: Calibrated Parameters

and Hall (2005). Also note that to generate a sufficiently downward sloping resource constraint, the calibration procedure yields a $\tau = 0$. That is, to replicate the dynamics of unemployment and vacancies as observed during the Great Recession, the model favours the Hall and Milgrom (2008) wage determination protocol. Since the calibration gives workers a high bargaining power, the implied wage equals to 0.96 similar to the one obtained by Hall (2005). Although the value of k seems high, this value is calculated as $v^*k = 0.036w = 0.0346$ and hence the low vacancy rate observed in August 2009 ($v^* = 0.015$) implies a $k = 2.32$.

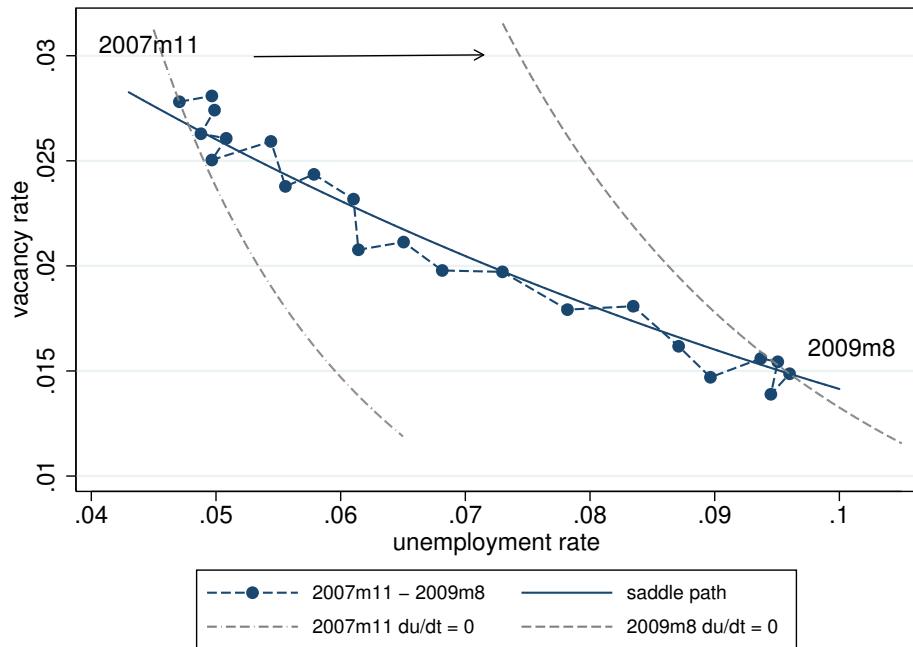


Figure 2: Unemployment and vacancy dynamics

Figure 2 shows the model implications for the observed unemployment and vacancy dynamics during the Great Recession. The dots depict the unemployment and vacancy rate pairs observed during this period, which moved from low unemployment-high vacancy rates to high unemployment-low vacancy rates. Immediately before the crisis in November 2007, the US economy experienced an unemployment rate of 4.7% and a labor market tightness of 0.59. We assume that this was the steady state of the economy before the Great Recession with a job destruction rate of 0.018 that is consistent with the observed unemployment rate at that time. As discussed by Elsby and Smith (2010), the Great Recession in the US was characterised by a sharp increase in the job destruction

rate. In our model the increase in s shifts the zero motion line of unemployment such that the post-crisis steady state implies a higher unemployment level. As one can observe from Figure 2, v decreased and u increased sharply during the Great Recession and our saddle-path tracks these movements closely. Given the unexpected nature of the financial crises (see Caballero and Kurlat, 2009), Figure 2 can then be interpreted as the empirical counterpart to Figure 1.b shown in section 3.3 when discussing the effects of an unexpected shock in s .

4.2 The Recovery

During the period September 2009 – December 2014 the economy underwent a slow recovery, where the unemployment and vacancy rates slowly reverted to their pre-recession levels (see Figure 6 in app. A.7). We now analyse what change in the parameters p, k, γ, b are required to match the observed transition to the new steady state during the recovery.⁸ This exercise informs us whether the model requires drastic changes to the parameters governing the financial sector to explain the transitional dynamics of unemployment and vacancies in the aftermath of the Great Recession. This exercise also inform us about the magnitude of the change the model requires in output per worker and vacancy costs to fuel firm entry and converge to the new steady state.

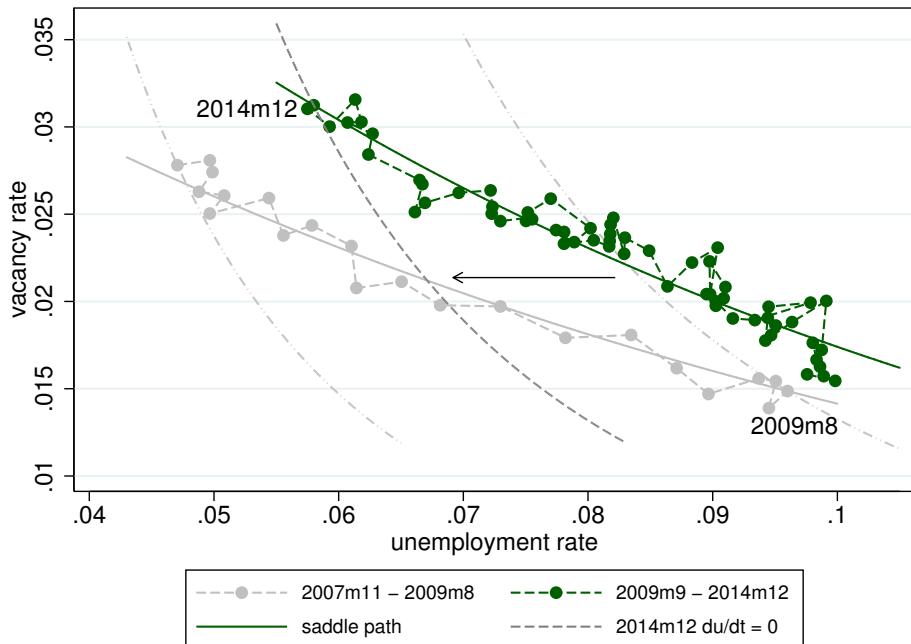


Figure 3: Unemployment and vacancy dynamics

We take this new steady state to be December 2014 (the end period of our window of observation). This steady state is characterised by $u^{**} = 0.058$, $\theta^{**} = 0.54$ and $r^{**} = 0.0027$. Since by December 2014 the job destruction rate decreased relative to August 2009 (see Figure 5 in app. A.7), the zero motion line for the unemployment rate shifted to the left. As before, we set the separation

⁸We keep the banks' fixed cost of entry, ϕ , constant for this exercise as a joint minimisation with respect to b and ϕ exhibits many local minima. However, holding ϕ constant is consistent with the evolution of the ratio between banks operational expenses and the employment in the financial sector series we obtained from the OECD Banking Statistics: Financial Statements of Banks. This series show basically no change during the period 2007-2014.

rate such that we match the steady-state unemployment rate u^{**} , i.e. $s = \frac{u^{**}\lambda(\theta^{**})}{1-u^{**}} = 0.22$. We then calibrate $\{p, k, \gamma, b\}$ by solving the optimisation problem in (21) using data on vacancies and unemployment during the Recovery. All other parameters are held fixed. As before we require that the transition path goes through the steady state (u^{**}, v^{**}) .

Figure 3 shows the model implications for the observed unemployment and vacancy dynamics during the Recovery under this exercise. Relative to the Great Recession, the model's transition path shifted to the right with a slight upward rotation. However, note that the transition path during the Recovery period is still relatively flat, implying that the unemployment and vacancy rates converge slowly to the new steady state. The new values for the calibrated parameters are $p = 0.98$, $k = 1.08$, $b = 0.97$ and $\gamma = 0.078$, where the latter implies an elasticity of substitution between financial products of 1.085.

The first implication of this exercise is that, from the lenses of our model, the observed recovery in the labour market during the 2010-2014 period was not due to improvements in the effectiveness of banks to intermediate financial resources, b , or due to an increase in the degree of competition among banks γ . When compared to the values in Table 1, the value of these parameters hardly changed. Indeed, b only dropped by 3 percentage points and the elasticity of substitution dropped by about one percentage point. Compared to the Great Recession period the number of banks in the financial sector decreased from $n_{GR} = 1.25$ to $n_R = 1.23$ and the aggregate output of banks remained unchanged $Y_{GR} = Y_R = 0.034$. This implications seems to have some support in the data. For example, Figure 7 in the app. A.7 shows that the money multiplier, a measure of related to the productivity of banks, had a large drop during 2008 and then stayed essentially flat through the rest of the period.

The second implication of this exercise is that convergences to the new steady state was propelled by a lower vacancy cost and job destruction rate, which led to an increase in firm entry.⁹ However, as opposed to the canonical search and matching model firm entry is not a jump variable. In our model job creation needs to be financed by the profit of existing firms using the banking sector to intermediate the financial resources. Given that the effectiveness of the banking sector to undertake such a task hardly changed during this period, the increase in firm entry developed slowly over time and hence produced a slow recovery in the unemployment and vacancy rates.

5 Conclusion

In this paper we have constructed a simple general equilibrium matching model with an imperfect financial market in the form a monopolistically competitive banking sector. The role of the financial sector is to fund job creation through the firms' profits. The critical element of our model is the per period financial resource constraint that determines the transitional dynamics of vacancies and unemployment towards the steady state. The resource constraint adds a new dimension to the canonical search and matching model. It makes it potentially consistent with the fact that vacancies and unemployment adjust in opposite directions. We show that this feature is readily obtained when wages are bargaining using the protocol proposed by Hall and Milgrom (2008). To

⁹As an alternative calibration we restricted the value of p to stay constant at one and obtained that $k = 1.13$, $\gamma = 0.078$ and $b = 0.93$, confirming that changes in p are of second order importance for our results.

illustrate some of the quantitative implications of our model we calibrated to match the transitional dynamics of the Great Recession and its aftermath. We find that observed slow recovery in the labour market was due to the lack of a significant improvement in the effectiveness of the banking sector in intermediating resources to fund job creation.

The model we developed is very parsimonious as our goal was to understand its main mechanism using analytical solutions, rather than numerical simulations. Clearly this comes at the cost of presenting a perhaps too simplistic model. In particular, an important assumption made here is that firms always required external funding to finance job creation. This assumption might be reasonable among small firms, but it is somewhat more difficult to defend among bigger firms with large internal financial reserves. Indeed it has been argued that some firms where not short of funds but where just reluctant to spend some it to finance investment and hence job creation. Adding this feature is an important extension to the model developed here. However, we leave this extension for future research.

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A Appendix

A.1 Deriving equation (9)

The equation describing the evolution of J over time results from (5) with $V = 0$,

$$\dot{J}(t) = [r(t) + s] J(t) - \pi(t).$$

With profits and the wage being substituted out from the profit equation (3) and wage equation (7), i.e. with

$$\begin{aligned}\pi(t) &= p - w(t) = p - ((1 - \beta)z + \beta[p + \tau\theta(t)k]) \\ &= (1 - \beta)(p - z) - \beta\tau\theta(t)k\end{aligned}\tag{22}$$

we get

$$\dot{J}(t) = [r(t) + s] J(t) - (1 - \beta)[p - z] + \tau\beta\theta(t)k.\tag{23}$$

Using (6) to compute $\dot{J}(t) = -\frac{k}{q(\theta(t))^2}q'(\theta(t))\dot{\theta}(t)$ and substitute $J(t) = \frac{k}{q(\theta(t))}$ into (23), we find (9).

A.2 The financial sector

In this section we derive in more detail the financial sector. We consider two versions, one assuming that the banking sector is described by a monopolistically competitive industry and the other assuming the banking sector is described by a perfectly competitive industry.

A.2.1 Monopolistic Competition

A service provider i maximizes

$$\pi_s(i, t) = \hat{p}(i, t)x(i, t) - c(x(i, t)).$$

Given the parameter γ , this implies mark-up pricing of

$$\hat{p}(i, t) = \frac{c'(x(i, t))}{\gamma}.\tag{24}$$

We now consider the costs of providing $x(i, t)$. Given the technology (11), $x(i, t) = by(i, t) - \phi$, the cost to produce output $x(i, t)$ is given by

$$c(x(i, t)) = y(i, t)$$

given that $y(i, t)$ is input of the final good whose price is normalized to one. The cost function therefore reads with (11)

$$c(x(i, t)) = \frac{\phi + x(i, t)}{b}.$$

Profits can therefore be computed to amount to (suppressing i and t for simplicity)

$$\pi_s = \hat{p}x - \frac{\phi + x}{b} = \hat{p}x - \frac{\hat{p}\gamma}{b^{-1}} \frac{\phi + x}{b},$$

where the last equality used (25). Hence,

$$\pi_s = \hat{p}x - \hat{p}\gamma\phi - \hat{p}\gamma x = \hat{p}[(1 - \gamma)x - \gamma\phi].$$

From (24), this implies that the price $\hat{p}(i, t)$ of one unit of service is given by the usual mark-up pricing rule

$$\hat{p}(i, t) = \frac{b^{-1}}{\gamma}. \quad (25)$$

Marginal costs to provide one unit of $x(i, t)$ are given by the price of the output good (which we normalized to one) divided by the productivity parameter b from (11). The mark-up $1/\gamma$ is determined by the price-elasticity of demand for services $x(i, t)$ implied by (10).

As all firms use the same technology, the banking sector is symmetric and input per banking service is given by

$$y(i, t) = y(t) = \frac{\pi(t)[1 - u(t)]}{n(t)}. \quad (26)$$

The second equality shows that the input is given by total real profits of active firms divided by the number of banking services. Note that the second equality is the first crucial component of our general equilibrium setup. Resources available at each point in time are given by real profits $\pi(t)$ per active firm from (3) times the number of active firms, which is given by the number of employed workers $1 - u(t)$. Equation (11) then implies that output per service provider is given by

$$x(t) = by(t) - \phi = b \frac{\pi(t)[1 - u(t)]}{n(t)} - \phi. \quad (27)$$

Given symmetry and (27), we obtain(12) in the text.

The number of banks Monopolistic service providers i choose output $x(t)$ such that profits are maximized.¹⁰ This yields markup pricing (25) and implies flow profits per service provider is given by $\pi_s(t) = \hat{p}(t)[(1 - \gamma)x(t) - \gamma\phi]$. After substituting output x from (27) in $\pi_s(t) = \hat{p}(t)[(1 - \gamma)x(t) - \gamma\phi]$, we obtain

$$\begin{aligned} \pi_s(t) &= \hat{p}(t) \left[(1 - \gamma) \left(b \frac{\pi(t)[1 - u(t)]}{n(t)} - \phi \right) - \gamma\phi \right] \\ &= \hat{p}(t) \left[(1 - \gamma) b \frac{\pi(t)[1 - u(t)]}{n(t)} - (1 - \gamma)\phi - \gamma\phi \right] \\ &= \hat{p}(t) \left[(1 - \gamma) b \frac{\pi(t)[1 - u(t)]}{n(t)} - \phi \right]. \end{aligned}$$

Given free entry of banks, profits $\pi_s(t)$ are driven to zero and we get (14).

¹⁰We suppress the provider index i as (27) has established symmetry.

Deriving the resource constraint (15) Noting that $b \frac{\pi(t)[1-u(t)]}{n(t)} = \frac{\phi}{1-\gamma}$ and $\frac{\phi}{1-\gamma} - \phi = \frac{\gamma}{1-\gamma}\phi$, the resource constraint (13) can be re-written as

$$\left[(1-\gamma) b \frac{\pi(t)[1-u(t)]}{\phi} \right]^{1/\gamma} \frac{\gamma}{1-\gamma} \phi = k \theta(t) u(t). \quad (28)$$

Use (3) and (7) to substitute out for profits and wages yields (22). Plugging this into (28) yields

$$\begin{aligned} \left[(1-\gamma) b \frac{((1-\beta)(p-z) - \beta\tau\theta(t)k)[1-u(t)]}{\phi} \right]^{1/\gamma} \frac{\gamma}{1-\gamma} \phi &= k \theta(t) u(t) \Leftrightarrow \\ \left[(1-\gamma) b \frac{k}{\phi} \left((1-\beta) \frac{p-z}{k} - \beta\tau\theta(t) \right) [1-u(t)] \right]^{1/\gamma} \gamma &= (1-\gamma) \frac{k}{\phi} \theta(t) u(t) \Leftrightarrow \\ (1-\gamma) b \frac{k}{\phi} \left((1-\beta) \frac{p-z}{k} - \beta\tau\theta(t) \right) [1-u(t)] \gamma^\gamma &= \left((1-\gamma) \frac{k}{\phi} \right)^\gamma \theta^\gamma(t) u^\gamma(t) \Leftrightarrow \\ b \left[(1-\gamma) \frac{k}{\phi} \right]^{1-\gamma} \left((1-\beta) \frac{p-z}{k} - \beta\tau\theta(t) \right) [1-u(t)] \gamma^\gamma &= \theta^\gamma(t) u^\gamma(t) \Leftrightarrow \\ b \left[(1-\gamma) \frac{k}{\phi} \right]^{1-\gamma} \frac{1-u(t)}{u^\gamma(t)} \gamma^\gamma &= \frac{\theta^\gamma(t)}{(1-\beta) \frac{p-z}{k} - \beta\tau\theta(t)}. \end{aligned}$$

The last expression is (15).

The resource constraint A crucial assumption here is that we require that all resources available for financing vacancies must actually be used for financing vacancies. Resources must not be lost or are allowed to enter the model. Making such a market-clearing assumption for the banking sector implies that the aggregate banking services (??) equal the total costs for financing vacancies. The latter are given by the costs k per vacancy times the number of vacancies, $\theta(t)u(t)$,

$$n^{(1-\gamma)/\gamma} b \pi(t) [1-u(t)] = k \theta(t) u(t). \quad (29)$$

Economically speaking, market clearing for financial services (29) determines the number of vacancies $v(t)$. Technically, as vacancies are already determined in (9) as in the textbook matching model, this additional market fixes the endogenous interest rate $r(t)$. This constraint (29) would replace (28) in the text for the competitive banking sector case.

We now analyse the slope of the resource constraint in the Beveridge space. Rewrite (29) as $n^{(1-\gamma)/\gamma} b \pi(t) [1-u(t)] = kv(t)$ and use (22) to get

$$G(v, u) \equiv v(t) - \frac{n^{(1-\gamma)/\gamma} b}{k} \left[(1-\beta)(p-z) - \beta\tau \frac{v}{u} k \right] [1-u] = 0.$$

Now we compute

$$\frac{dv}{du} = -\frac{\frac{\partial G(v,u)}{\partial u}}{\frac{\partial G(v,u)}{\partial v}}. \quad (30)$$

We see that $\frac{\partial G(v,u)}{\partial v} > 0$. Hence, we need the sign of $\frac{\partial G(v,u)}{\partial u}$ which is identical to the sign of the

derivative of $H(u) \equiv -[(1-\beta)(p-z) - \beta\tau\frac{v}{u}k][1-u]$ with respect to u . We compute

$$\begin{aligned}\frac{\partial H(u)}{\partial u} &= -\beta\tau\frac{v}{u^2}k[1-u] + (1-\beta)(p-z) - \beta\tau\frac{v}{u}k > 0 \\ &\Leftrightarrow -\beta\tauvk[1-u] + (1-\beta)(p-z)u^2 - \beta\tauvuk > 0 \Leftrightarrow -\beta\tauvk + (1-\beta)(p-z)u^2 > 0 \\ &\Leftrightarrow (1-\beta)(p-z)u^2 > \beta\tauvk.\end{aligned}$$

Hence, the resource constraint is downward-sloping, noting the minus sign in (30), iff

$$v > \frac{1-\beta}{\beta} \frac{p-z}{\tau k} u^2.$$

A.3 Existence of an interior steady state

We characterise conditions under which a unique interior steady state exists. To do so we analyse how does the equilibrium path (15) behaves with respect to labour market tightness after substituting out for unemployment using (16). Let the left-hand side of (15) by described by

$$T_1(\theta) = \frac{\theta}{[(1-\beta)(p-z) - \tau\beta k\theta]^{1/\gamma}}. \quad (31)$$

Similarly, substituting out for u using (16), the right-hand side of (15) can be described by

$$T_2(\theta) = C \frac{(s + \lambda(\theta))}{s} \left[\frac{\lambda(\theta)}{s + \lambda(\theta)} \right]^{\frac{1}{\gamma}}, \quad (32)$$

where

$$C = \frac{\gamma}{(1-\gamma)^{-\frac{1-\gamma}{\gamma}}} \frac{\phi^{-\frac{1-\gamma}{\gamma}}}{k} b^{\frac{1}{\gamma}} \quad (33)$$

describes a constant. Note that both T_1 and T_2 take the value of zero when $\theta = 0$.

Differentiation of T_1 and T_2 wrt θ implies that both functions are increasing in θ .

$$\frac{dT_1}{d\theta} = \frac{1}{\gamma} \left[\frac{\gamma(1-\beta)(p-z) + \theta\tau\beta k(1-\gamma)}{[(1-\beta)(p-z) - \tau\beta k\theta]^{\frac{1-\gamma}{\gamma}}} \right] > 0,$$

$$\frac{dT_2}{d\theta} = \frac{C\lambda'(\theta)}{s} \left(\frac{\lambda(\theta)}{s + \lambda(\theta)} \right)^{\frac{1}{\gamma}} \left[1 + \frac{s}{\gamma\lambda(\theta)} \right] > 0.$$

Further differentiation implies that $d^2T_1/d\theta^2 > 0$, while

$$\frac{d^2T_2}{d\theta^2} = \frac{C}{s} \left(\frac{\lambda(\theta)}{s + \lambda(\theta)} \right)^{\frac{1}{\gamma}} \frac{1}{(\lambda(\theta)\gamma)^2} \left[\lambda''(\theta)\lambda(\theta)\gamma(\lambda(\theta)\gamma + s) + \frac{\lambda'^2 s^2}{s + \lambda(\theta)}(1-\gamma) \right].$$

The sign of $d^2T_2/d\theta^2$ is then determined by the sign of the term in squared brackets. Note that the first term inside the squared brackets is negative (as the job finding rate is concave in θ), while the second term is positive. Given $d^2T_2/d\theta^2 < 0$ for all θ , continuity of T_1 and T_2 imply that in this case there exists a unique interior steady state equilibrium. If $d^2T_2/d\theta^2 > 0$ or non-monotone, we could also have multiple interior steady state equilibria or no equilibria at all.

Given that in the quantitative section of the paper we focus on a Cobb-Douglas matching function, $M(u, v) = Au^{1-\alpha}v^\alpha$, it is instructive to analyse the conditions for existence under such a parametrisation. Noting that under this matching function the job finding rate and its derivatives are given by: $\lambda(\theta) = A\theta^\alpha$, $\lambda'-(1-\alpha)$ and $\lambda''(\theta) = -\lambda'^{-1}$, the term in the squared bracket in the above expression for $d^2T_2/d\theta^2$ is given by

$$-\frac{A^2\theta^{-2(1-\alpha)}\alpha}{s+A\theta^\alpha}\left[s^2(\gamma-\alpha)+(1-\alpha)\gamma A\theta^\alpha[s(1+\gamma)+A\theta^\alpha\gamma]\right].$$

Inspection shows that $\gamma \geq \alpha$ provides a sufficient (but not necessary) condition for $d^2T_2/d\theta^2 < 0$ and hence for existence of a unique interior steady state equilibrium.

A.4 The slope of the resource constraint

A.4.1 Preliminaries

Re-writing equation (15) in $v - u$ space using an implicit formulation yields

$$G(v, u) \equiv v - \frac{\gamma\phi}{(1-\gamma)k} \left[\frac{(1-\gamma)b[(1-\beta)(p-z) - \tau\beta k \frac{v}{u}][1-u]}{\phi} \right]^{\frac{1}{\gamma}} = 0. \quad (34)$$

It follows from (34) that when all workers are unemployed, $u = 1$, there are no vacancies $v = 0$. In this case no firm is producing and hence firms profits are zero implying that there are no available funds to pay for vacancies.

Now consider the slope of $G(v, u)$. Note that the resource constraint shows that aggregate firms' profits and the number of vacancies must move in the same direction along the equilibrium path. Aggregate firms' profits, however, depend positively on (i) the number of jobs filled and negatively on (ii) the wage paid to workers; and both are inversely related to the unemployment rate. The slope of $G(v, u)$ then depends on how responsive are wages to changes in the unemployment rate. Employing the implicit function theorem, we obtain (see app. A.4.2)

$$\frac{dv}{du} = \frac{\partial n}{\partial u} \left[\left[\frac{\gamma\phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \left(\frac{1}{\gamma n} \right) \right]^{-1} + \frac{(1-\gamma)b\tau\beta k(1-u)}{u\phi} \right]^{-1},$$

where the slope of the equilibrium path in Beveridge space is determined by

$$\text{sign} \left[\frac{\partial n}{\partial u} \right] = \text{sign} \left[\frac{\partial \pi}{\partial u} (1-u) - \pi \right], \quad (35)$$

which in turn depends on how the unemployment rate affects firms flow profits via wages.

A.4.2 Total differentiation of $G(v, u)$

Total differentiation of $G(v, u)$ implies $\frac{dv}{du} = -\frac{\partial G(v, u)}{\partial u}$. Originally, equation (34) reads

$$G(v, u) \equiv v - \frac{\gamma\phi}{(1-\gamma)k} \left[\frac{(1-\gamma)b[(1-\beta)(p-z) - \tau\beta k \frac{v}{u}][1-u]}{\phi} \right]^{\frac{1}{\gamma}}.$$

As from (14) and (22),

$$n = \frac{(1-\gamma)b[(1-\beta)(p-z) - \tau\beta k \frac{v}{u}][1-u]}{\phi}, \quad (36)$$

we can express it more compactly as

$$G(v, u) = v - \frac{\gamma\phi}{(1-\gamma)k} n^{\frac{1}{\gamma}} = 0.$$

It follows that

$$\begin{aligned} \frac{\partial G(v, u)}{\partial u} &= - \left[\frac{\gamma\phi}{(1-\gamma)k} \frac{n^{\frac{1}{\gamma}}}{\gamma n} \frac{\partial n}{\partial u} \right] = - \frac{\partial n}{\partial u} \left[\frac{\gamma\phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \frac{1}{\gamma n} \right] \\ \frac{\partial G(v, u)}{\partial v} &= 1 - \frac{\partial n}{\partial v} \left[\frac{\gamma\phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \frac{1}{\gamma n} \right]. \end{aligned}$$

Thus

$$\frac{dv}{du} = - \frac{-\frac{\partial n}{\partial u} \left[\frac{\gamma\phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \frac{1}{\gamma n} \right]}{1 - \frac{\partial n}{\partial v} \left[\frac{\gamma\phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \frac{1}{\gamma n} \right]} = \frac{\frac{\partial n}{\partial u}}{\left[\frac{\gamma\phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \frac{1}{\gamma n} \right]^{-1} - \frac{\partial n}{\partial v}} = \frac{\partial n}{\partial u} \left[\left[\frac{\gamma\phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \left[\frac{1}{\gamma n} \right] \right]^{-1} - \frac{\partial n}{\partial v} \right]^{-1}.$$

Note that from (36)

$$\frac{\partial n}{\partial v} = - \frac{(1-\gamma)b\tau\beta k(1-u)}{\phi u},$$

so that

$$\frac{dv}{du} = \frac{\partial n}{\partial u} \left[\left[\frac{\gamma\phi n^{\frac{1}{\gamma}}}{(1-\gamma)k} \left[\frac{1}{\gamma n} \right] \right]^{-1} + \frac{(1-\gamma)b\tau\beta k(1-u)}{\phi u} \right].$$

A.4.3 The slope of $G(v, u)$

The resource constraint is falling by (35) iff

$$\frac{dv}{du} < 0 \Leftrightarrow \text{sign} \left[\frac{\partial \pi}{\partial u}(1-u) - \pi \right] < 0.$$

As $\partial \pi / \partial u = \beta \tau v k / u^2 > 0$ from (22), this holds iff, using (22),

$$\begin{aligned} \frac{\beta \tau v k}{u^2}(1-u) - \pi &< 0 \Leftrightarrow \frac{\beta \tau v k}{u^2}(1-u) < \pi \Leftrightarrow \frac{\beta \tau v k}{u^2}(1-u) < (1-\beta)(p-z) - \beta \tau \theta k \\ &\Leftrightarrow \beta \tau \frac{v}{u} \frac{1-u}{u} < (1-\beta) \frac{p-z}{k} - \beta \tau \frac{v}{u} \Leftrightarrow \frac{\beta \tau v}{u^2} < (1-\beta) \frac{p-z}{k} \\ &\Leftrightarrow v < \frac{1-\beta}{\beta \tau} \frac{p-z}{k} u^2. \end{aligned}$$

A.5 Comparative statics for the slope of Ψ

We want to analyse how changes in ϕ and b affect $d\theta/du$, the slope of Ψ , as described in (20). To do this let

$$\begin{aligned}\Psi_1 &= -n[\theta(1-\gamma)k(1-u) + \phi n^{\frac{1}{\gamma}}], \\ \Psi_2 &= k(1-\gamma)(1-u)[nu + n^{\frac{1}{\gamma}}(1-u)\tau b\beta].\end{aligned}$$

Changes in ϕ

Noting that $dn/d\phi = -n/\phi$, we have that

$$\begin{aligned}\frac{\partial\Psi_1}{\partial\phi} &= \frac{n}{\phi\gamma} \left[\theta(1-\gamma)k(1-u)\gamma + \phi n^{\frac{1}{\gamma}} \right], \\ \frac{\partial\Psi_2}{\partial\phi} &= -\frac{k(1-\gamma)(1-u)}{\phi\gamma} \left[nu\gamma + n^{\frac{1}{\gamma}}(1-u)\tau b\beta \right].\end{aligned}$$

Since the

$$\frac{\partial[d\theta/du]}{\partial\phi} = \frac{1}{\Psi_2^2} \left[\frac{\partial\Psi_1}{\partial\phi} \Psi_2 - \frac{\partial\Psi_2}{\partial\phi} \Psi_1 \right],$$

the sign of the change is determined by the expression in the squared bracket. Substituting the corresponding expressions and some algebra establishes that

$$\frac{\partial[d\theta/du]}{\partial\phi} = \frac{1}{\Psi_2^2} \frac{nk(1-\gamma)^2(1-u)n^{\frac{1}{\gamma}}}{\phi\gamma} [nu\phi - \theta(1-\gamma)k(1-u)^2\tau b\beta].$$

Noting that $n = (1-\gamma)b^{\frac{[(1-\beta)(p-z)-\beta\theta k][1-u]}{\phi}}$, the above expression can be simplified to

$$\frac{\partial[d\theta/du]}{\partial\phi} = \frac{1}{\Psi_2^2} \frac{nbk(1-\gamma)^3(1-u)^2n^{\frac{1}{\gamma}}}{\phi\gamma} [(1-\beta)(p-z)u - \tau\beta\theta k].$$

The slope of Ψ increases with ϕ when $(1-\beta)(p-z)u > \tau\beta\theta k$. Since Ψ is downward sloping, an increase in its slope implies it becomes flatter, which in turn imply that Ψ shifts to the left towards the origin.

Changes in b

In this case we have that $dn/db = n/b$ and

$$\begin{aligned}\frac{\partial\Psi_1}{\partial b} &= -\frac{n}{b\gamma} \left[\theta(1-\gamma)k(1-u)\gamma + \phi n^{\frac{1}{\gamma}}(1+\gamma) \right], \\ \frac{\partial\Psi_2}{\partial b} &= \frac{k(1-\gamma)(1-u)}{b\gamma} \left[nu\gamma + n^{\frac{1}{\gamma}}(1-u)\tau b\beta(1+\gamma) \right].\end{aligned}$$

Since the

$$\frac{\partial[d\theta/du]}{\partial b} = \frac{1}{\Psi_2^2} \left[\frac{\partial\Psi_1}{\partial b} \Psi_2 - \frac{\partial\Psi_2}{\partial b} \Psi_1 \right],$$

the sign of the derivative is determined by the expression in the squared bracket. Substituting the corresponding expressions and some algebra establishes that

$$\frac{\partial[d\theta/du]}{\partial b} = -\frac{1}{\Psi_2^2} \frac{nk(1-\gamma)(1-u)n^{\frac{1}{\gamma}}}{b\gamma} [nu\phi - \theta(1-\gamma)k(1-u)^2\tau b\beta],$$

where the term in squared brackets is the same as in the case of changes in ϕ . Using the expression for n we obtain that

$$\frac{\partial[d\theta/du]}{\partial b} = -\frac{1}{\Psi_2^2} \frac{nk(1-\gamma)^2(1-u)^2n^{\frac{1}{\gamma}}}{\gamma} [(1-\beta)(p-z)u - \tau\beta\theta k].$$

The slope of Ψ decreases with b when $(1-\beta)(p-z)u > \tau\beta\theta k$. Since Ψ is downward sloping, a decrease in its slope implies it becomes stepper, which in turn imply that Ψ shifts to the right away from the origin.

Changes in γ

In this case we have that $dn/d\gamma = -n/(1-\gamma)$ and that

$$\frac{\partial(n^{\frac{1}{\gamma}})}{\partial\gamma} = -\frac{n^{\frac{1}{\gamma}}}{\gamma^2(1-\gamma)} [(1-\gamma)\ln(n) + \gamma].$$

These expressions together imply

$$\begin{aligned} \frac{\partial\Psi_1}{\partial\gamma} &= 2n\theta k(1-u) + \frac{\phi n^{\frac{1}{\gamma}}}{\gamma^2(1-\gamma)} [\gamma(1+\gamma n) + (1-\gamma)\ln(n)], \\ \frac{\partial\Psi_2}{\partial\gamma} &= -2nuk(1-u) - \frac{k(1-u)^2n^{\frac{1}{\gamma}}\tau b\beta}{\gamma^2} [\gamma(1+\gamma) + (1-\gamma)\ln(n)]. \end{aligned}$$

Since the

$$\frac{\partial[d\theta/du]}{\partial\gamma} = \frac{1}{\Psi_2^2} \left[\frac{\partial\Psi_1}{\partial\gamma}\Psi_2 - \frac{\partial\Psi_2}{\partial\gamma}\Psi_1 \right],$$

once again the sign of the change is determined by the expression in the squared bracket. Substituting the corresponding expressions and some algebra establishes that the sign of $\partial[d\theta/du]/\partial\gamma$ equals the sign of

$$\phi u[n\gamma^2 + (1-\gamma)\ln(n)] - \left[\tau b\beta k(1-u)^2(1-\gamma)^2(\gamma + \ln(n)) + \phi u\gamma + \phi n^{\frac{1-\gamma}{\gamma}}(1-u)\tau b\beta(n-1)[\gamma + (1-\gamma)\ln(n)] \right].$$

Consider the sign of $\partial[d\theta/du]/\partial\gamma$ as $\gamma \rightarrow 1$. Since in this limit $n \rightarrow 0$, we find that $\partial[d\theta/du]/\partial\gamma < 0$ when $\tau b\beta\lambda(\theta) - s > 0$. On the other hand, when $\gamma \rightarrow 0$, we find that in this limit $\partial[d\theta/du]/\partial\gamma < 0$ when $n > 1$. In both case a decrease in the slope of Ψ implies it becomes stepper, which in turn imply that Ψ shifts to the right away from the origin.

A.6 Calibration

To calibrate the parameters $\mathbf{x} = \{k, z, \beta, \tau, \phi, \gamma\}$, we minimise the squared relative distance between the observed vacancy rates and the ones implied by our transition path taking the observed unemployment rates as given. That is, we choose

$$\mathbf{x} = \arg \min \sum_t \left(\frac{v_t - \hat{v}(\mathbf{x}; u_t)}{v_t} \right)^2, \quad (37)$$

where $\hat{v}(\mathbf{x}; u_t)$ denotes the vacancy rate that solves (15) given the vector of parameters \mathbf{x} and the observed unemployment rate u_t . The minimisation problem is subject to the following equality constraints:

$$\begin{aligned} (1 - \beta) \frac{p - z}{k} q(\theta^*) - s - r^* - \tau \beta \lambda(\theta^*) &= 0 \\ v^* - \hat{v}(\mathbf{x}; u^*) &= 0 \\ 0.5w - z &= 0 \\ 0.036w - v^* k &= 0 \end{aligned}$$

In practice, we use the `fmincon` function from the Optimization Toolbox (Version 7.2) in Matlab (Version 8.5.0.197613 (R2015a)) designed to find the minimum of a function $f(x)$ with linear and nonlinear inequality and equality constraints. Note that at every evaluation of the objective function, we also have to solve for the series of vacancy rates $\hat{v}(\mathbf{x}; u)$. Given a guess for the parameter vector \mathbf{x} and the observed unemployment rates, we can numerically solve equation (15) to obtain the series of vacancy rates implied by our model. In practice, we use the `lsqnonlin` function from the Optimization Toolbox - a nonlinear least square solver - to perform this step. The advantage of the non-linear least square solver compared to Matlab's built-in solver for nonlinear systems, `fsolve`, is that we can impose a non-negativity constraint ensuring the stability of the optimisation procedure.

A.7 The Impact of the Financial Crises: Time series

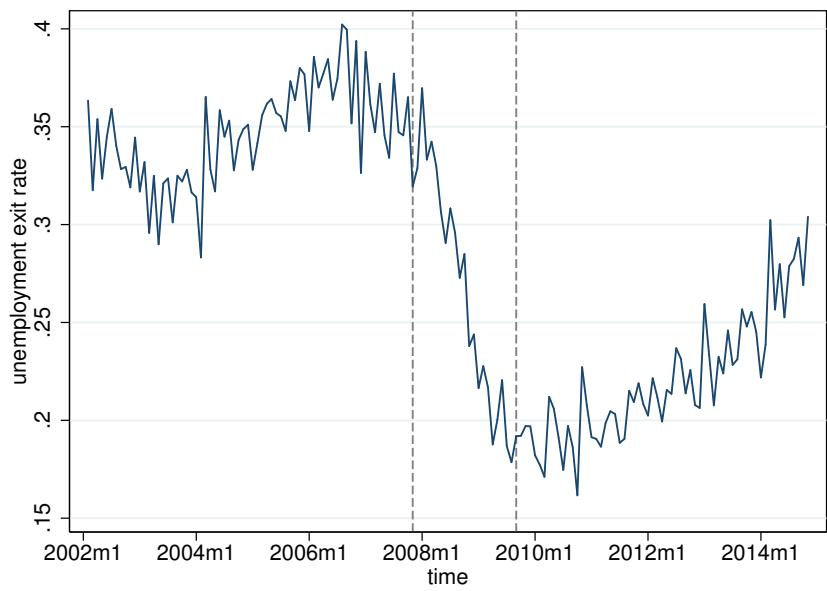


Figure 4: Unemployment exit rate

Notes: This figure shows the monthly unemployment exit rate for the US. The exit rate was calculated following footnote (5) using BLS data on the seasonal adjusted number of unemployed and unemployed with durations less than 5 weeks. The Great Recession is the period between the two dashed lines.

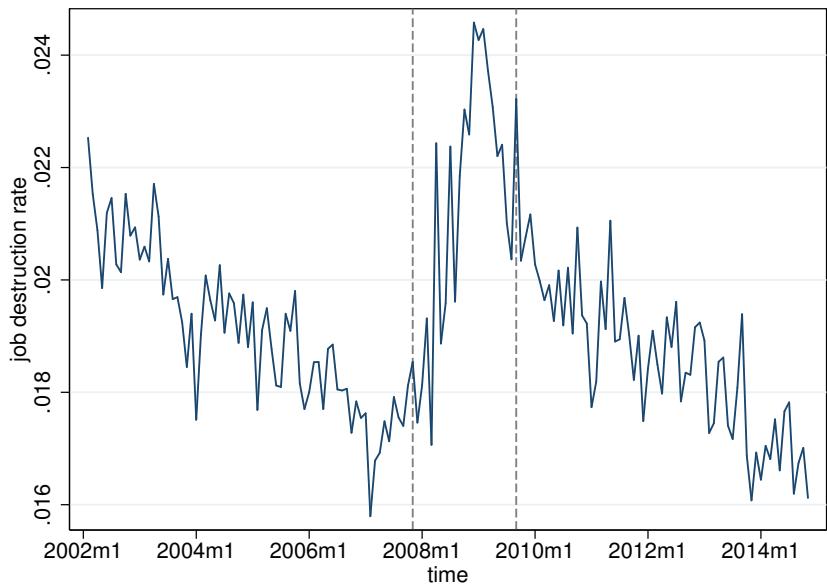


Figure 5: Job destruction rate

Notes: This figure shows the monthly employment exit rate for the US. The exit rate was calculated following footnote (5) using BLS data on the seasonal adjusted number of employed and unemployed with durations less than 5 weeks. The Great Recession is the period between the two dashed lines.



Figure 6: Unemployment and vacancy dynamics

Notes: This figure shows the monthly unemployment and vacancy rate for the US. The data is taken from the BLS and the JOLTS. Rates are calculated by dividing the number of vacancies and unemployed by the sum of the employed and unemployed in a given month. The Great Recession is the period between the two dashed lines.

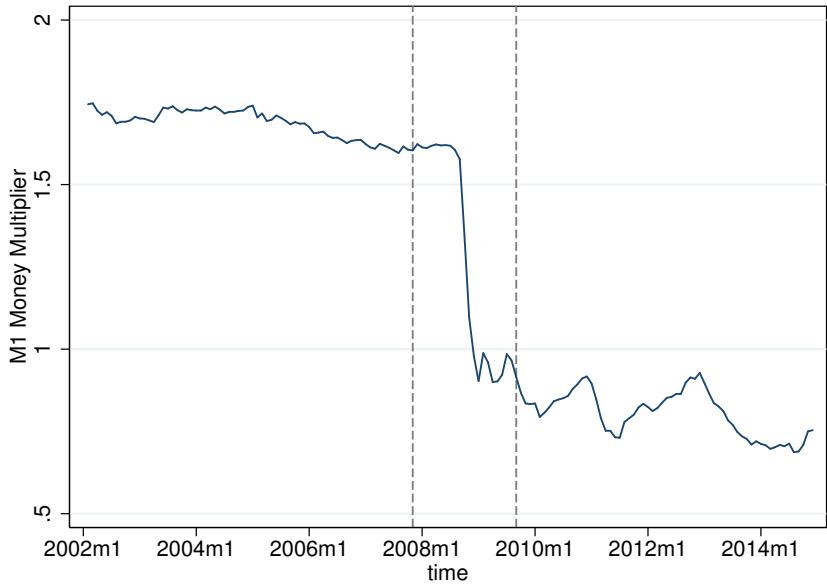


Figure 7: Money multiplier

Notes: This figure shows the evolution of the M1 money multiplier over time. The data is provided by the Federal Reserve Bank of St. Louis (<https://research.stlouisfed.org/fred2/series/MULT>). The Great Recession is the period between the two dashed lines.