Substitution and income effects of labor income taxation*

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Abstract

This paper provides sufficient and necessary conditions for when the elasticity of taxable income (ETI) estimand can be given a causal interpretation as a positively weighted average of heterogeneous individual elasticities of taxable income. We show how the ETI estimand can be used to learn about compensated and uncompensated elasticities by constructing bounds, or obtaining point estimates by either imposing homogeneity assumptions on elasticities or using external estimates of income effects. We apply our results to analyze a reform of the Norwegian tax system that reduced the marginal tax rates on middle and high incomes. Our results show small elasticities of taxable income for middle-income individuals that increase rapidly with income. By combining the ETI estimates with estimates of income effects obtained using lottery winners, we find that (un)compensated elasticities are small for middle incomes but increase steadily with income. Notably, our estimates imply that the Norwegian top-income tax rates exceed the revenue-maximizing one.

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1 Introduction

The elasticity of taxable income (ETI) measures how taxable income changes in response to a change in the marginal tax rate. It is a key parameter in tax policy for assessing how exogenous changes in tax rates will causally affect income and tax revenue (Auten and Carroll, 1999). It is also often economically interpreted as a compensated elasticity of labor supply (Saez et al., 2012), which is crucial to assess the excess burden of taxation (Feldstein, 1995). The goal of our paper is to examine when a class of ETI estimands can and cannot be given a causal interpretation as an average elasticity of taxable income, and an economic interpretation as an average labor supply elasticity.

In section 2, we begin by providing sufficient and necessary conditions for when the ETI estimand can be given a causal interpretation as a positively weighted average of heterogeneous individual elasticities of taxable income. We formalize and extend the ETI framework to allow for heterogeneity in individual ETIs, and show that two key conditions characterize causal ETI estimands. First, they compare earnings changes only between individuals affected by the reform and those not affected by it. Comparing earnings changes across groups affected by the reform but with different intensities would lead to negative weights on individual elasticities if there is heterogeneity. Second, they control for initial earnings nonparametrically. This ensures the estimands will always capture the differential earnings growth across the income distribution (under the maintained assumptions). It also ensures the specification is "saturated" in covariates, which is necessary and sufficient for two-stage least squares specifications to be interpreted causally (Blandhol et al., 2022) when there is heterogeneity in individual treatment effects.

This identification result is constructive, leading to specific empirical specifications and estimators that can be easily implemented. These specifications differ from the ETI estimators commonly used, which fail to satisfy the conditions for a causal interpretation with heterogeneous elasticities. We also suggest an alternative *local* ETI estimand that, under the same assumptions as the ETI estimand, allows one to recover positively weighted averages of individual ETIs across the income distribution.

We apply these results to analyze a reform of the Norwegian tax system that reduced the marginal tax rates on middle and high incomes, and find that commonly used ETI estimators understate the average elasticity of taxable income. The specifications that can be given a causal interpretation produce estimates of the weighted-average ETI around 0.23, meaning that a ten-percentage increase in the net-of-tax rate increases taxable incomes by 2.3 percent. Our local ETI estimates rise steadily from 0.1 for middle incomes to 0.7 for the highest incomes. This variation in ETIs across the income distribution is significant, both statistically and substantively, and means that the (weighted-) average ETIs is far from sufficient to characterize how taxes affect earnings.

In Section 4, we show how the (local) ETI estimand can be used to learn about compensated and uncompensated elasticities across the income distribution. Using a static labor supply model, we first show that the ETI estimates recover an average of (un)compensated elasticities plus a bias term that would be observed if the income effects were known. We use this insight to bound or point-identify (un)compensated elasticities with and without additional assumptions. Our results allow us to conclude that the compensated (uncompensated) labor supply elasticity of middle (high) income individuals is bounded between 0.1 and 0.3 (0.5 and 0.7). We next show how imposing constant substitution and income effects across all individuals, as in Gruber and Saez (2002), allows for point identification of both compensated and uncompensated elasticities. We find that this assumption is at odds with the data and conclude that it poorly approximates individual labor supply behavior, at least in our context.

The conclusion above means that to draw stronger conclusions about income and substitution effects, it is necessary to estimate income effects directly by using external data. One possible way to estimate income effects is to use lottery winnings, which may allow one to isolate the impact of changes in unearned income, holding fixed all other determinants of behavior, such as preferences and wages. Following a set of empirical studies that have used lottery winnings to estimate wealth and income effects in the US (Golosov et al., 2024; Imbens et al., 2001), Sweden (Cesarini et al., 2017), and the Netherlands (Bulman et al., 2021; Picchio et al., 2018), we use Norwegian data on lottery winnings to estimate income effects. By combining these estimates with our local ETI estimates, we learn that the compensated elasticity is small and equal to 0.1 for low incomes, and increases gradually to around 0.8 for the top earners. Moreover, uncompensated elasticities tend to be positive, implying that substitution effects are larger than the income effects, especially for higher incomes, where they exceed 0.5.

We conclude our empirical analysis in Section 5, where we use our estimated elasticities to (i) quantify the efficiency cost of taxation and (ii) estimate the revenue-maximizing top-income tax rate. Our results show that the efficiency cost of taxation is substantial, with an excess burden that exceeds 0.9. They also show that the actual top-income tax rate exceeds the revenue-maximizing one, meaning that decreasing top-income tax rates would increase tax revenue. Lastly, we consider how assuming homogenous ETIs and no income effects would affect these conclusions, and find it would substantially reduce the estimated excess burden to around 0.25, and give estimates of revenue-maximizing tax rates that are ten percentage points larger than the actual ones.

Our paper contributes to the literature that analyzes labor supply responses to tax reforms by formalizing and extending the identification arguments to allow for heterogeneity in individuals' ETIs.[1] The ETI estimands commonly used in the literature typically fail to sat-

[1]Notable contributions to this literature includes Auten and Carroll (1999), Burns and Ziliak (2017), Feldstein

isfy the conditions for a causal interpretation while allowing for heterogeneous elasticities. However, our results shows that by making minor and easily implementable adjustments allow these estimatands to be interpreted causally.

We also contribute to existing work, which has noted that the ETI estimates could be biased if the individual ETIs vary across income groups (Kumar and Liang, 2020; Saez et al., 2012). Kumar and Liang (2020) considers the causal interpretation of the ETI estimand without covariates, under the assumption that the tax system is randomly assigned across individuals. They show that this estimand is generally not equal to *particular* weighted average of individual ETIs, unless the instrument is valid and the individual ETIs are homogenous. Our results about the ETI estimand differ in several ways. First, we consider the causal interpretation of the ETI estimand with covariates and do not assume that the tax system is randomly assigned. The inclusion of covariates in the theoretical results is important, since empirical work tries to flexibly control for individual characteristics such as initial income. Second, we provide sufficient and necessary conditions for the ETI estimand to recover *any* positively weighted average of individual elasticities of taxable income. This is arguably a minimal requirement for the ETI estimand to be an interesting quantity, but it is not sufficient. We therefore strengthen our result by showing the causal interpretation of the ETI estimand as a specific positively weighted average of individual ETI. Third, our identification results are constructive, leading to specific empirical specifications and estimators that can be implemented with standard statistical software. Fourth, we consider the bias of the ETI estimand due to income effects and how it can be corrected for by constructing bounds, invoking auxiliary assumptions, or using additional data.

Our paper is also related to a set of empirical studies that have used lottery winnings to estimate wealth and income effects (Bulman et al., 2021; Cesarini et al., 2017; Golosov et al., 2024; Imbens et al., 2001; Picchio et al., 2018). To measure how lottery winnings are allocated over time, most of these studies rely on either the capitalization or the annuitization approach. In contrast, our rich administrative data allows for imputing consumption and savings over time (Eika et al., 2020), which means we can measure unearned income in each period without relying on additional assumptions. Thus, our paper offers evidence on income effects, including how they vary across the income distribution, without relying on assumptions on how households allocate their wealth over time.

Lastly, our paper relates to the public finance literature that uses elasticities and observable data to express the welfare and efficiency consequences of tax policy (Auerbach and Hines, 2002; Bierbrauer et al., 2023; Diamond, 1998; Feldstein, 1999; Harberger, 1964; Saez, 2001). We adjust the well-known expressions of the marginal deadweight loss and optimal top-income tax rates to apply to the Norwegian setting by incorporating payroll and value-

(1995), Gruber and Saez (2002), and Kleven and Schultz (2014). See Saez et al. (2012) for a review and Neisser (2021) for a meta-analysis.

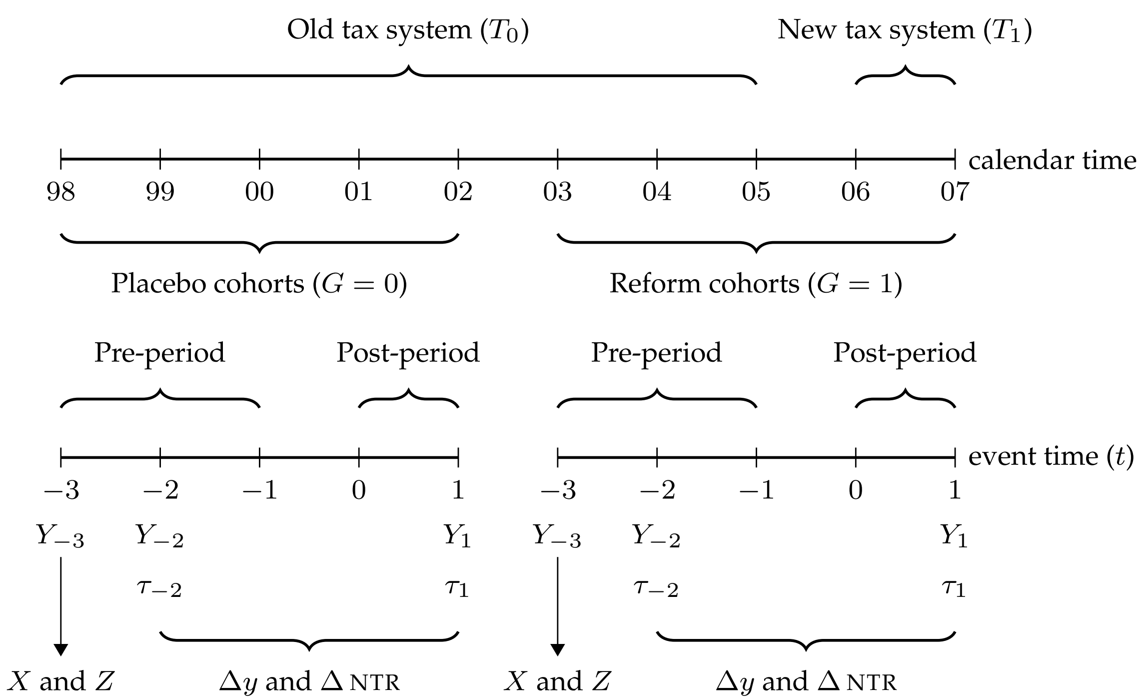
added taxes.

2 Interpretation of ETI estimands

In this section, we introduce and analyze a class of commonly used ETI estimands.

2.1 Research design

Figure 1: Anatomy of the research design.



Notes: The figure presents the research design, illustrates the timing of the reform, and introduces notation.

Figure 1 describes the research design and the data that it will use. This figure illustrates that the tax system changed from T_0 to T_1 in 2006, which affected the marginal tax rates on incomes above $\bar{Y}$. The reform cohorts ($G = 1$), which consist of observations from 2003-2007, experienced the *actual* reform in 2006. The placebo cohorts ($G = 0$), consisting of observations from 1998-2002, experience no change in the tax system. For each cohort type, we divide the data into a pre-period consisting of the first three years (event time $t = -3, -2, -1$) and a post-period covering the last two years ($t = 0, 1$).

We observe the earnings Y and marginal tax rates τ for every individual at each event time:

$$\tau_t = T_0'(Y_t) + \mathbb{1}[t \geq 0] G\Big(T_1'(Y_t) - T_0'(Y_t)\Big),$$

where T_d' denotes the derivative of tax function T_d. Thus, the marginal tax rate of an individual depends on her earnings Y and whether she faces the old (T_0) or the reformed tax system (T_1).

To address the simultaneity between marginal tax rates and earnings, the empirical ETI literature uses *simulated instruments*, defined as the predicted percentage change in net-of-tax rates because of the reform:

$$Z \equiv \log\left(\frac{1 - T_1'(X)}{1 - T_0'(X)}\right), \tag{1}$$

where X is earnings at event time -3. The variable Z can take more than two values. It not only captures that the reform changes the marginal tax rates of some (treatment group with $Z \neq 0$), but not all individuals (control group with $Z = 0$), but also that the magnitude of the change may differ across earnings levels. Hence, treatment intensity can vary across treated individuals.

A possible estimator for the elasticity of taxable income is the difference-in-differences estimator that compares the earnings- and marginal tax rate growth of the treatment group to that of the control group:

$$\beta^{\text{DD}} \equiv \frac{\overbrace{\mathbb{E}[\Delta y \mid G = 1, Z \neq 0] - \mathbb{E}[\Delta y \mid G = 1, Z = 0]}^{\equiv RF \text{ (Earnings DiD)}}}{\underbrace{\mathbb{E}[\Delta \,\text{NTR} \mid G = 1, Z \neq 0] - \mathbb{E}[\Delta \,\text{NTR} \mid G = 1, Z = 0]}_{\equiv FS \text{ (Net-of-tax rate DiD)}}},$$

where $\Delta y \equiv \log Y_1 - \log Y_{-2}$, and $\Delta \,\text{NTR} \equiv \log(1 - \tau_1) - \log(1 - \tau_{-2})$.

A concern with this estimator is that because the reform changed marginal tax rates on incomes above $\bar{Y}$, the treated group has higher initial earnings than the control group. Thus, if individual income growth depends on initial earnings, either due to mean reversion or differential underlying income growth, parallel trends are unlikely to hold, see e.g. Auten and Carroll (1999) and Weber (2014). To address these concerns, the literature uses the placebo cohorts to estimate the difference in earnings and net-of-tax rate growth between the treatment and control groups over the period when no tax reform occurred. The triple difference estimand subtracts these placebo differences from the reform ones:

$$\beta^{\text{DDD}} \equiv \frac{\overbrace{RF}^{\text{Earnings DiD}} - \overbrace{\left(\mathbb{E}[\Delta y \mid G=0,\, Z \neq 0] - \mathbb{E}[\Delta y \mid G=0,\, Z=0]\right)}^{\text{Placebo earnings DiD}}}{\underbrace{FS}_{\text{Net-of-tax rate DiD}} - \underbrace{\left(\mathbb{E}[\Delta\,\text{NTR} \mid G=0,\, Z \neq 0] - \mathbb{E}[\Delta\,\text{NTR} \mid G=0,\, Z=0]\right)}_{\text{Placebo net-of-tax rate DiD}}}. \tag{2}$$

The triple difference estimand in (2) is nested by the following two-stage least squares (TSLS) regression model:

$$\Delta y = \alpha_0^y G + \beta \Delta\,\text{NTR} + f(X; \alpha^y) + u^y, \tag{3}$$

$$\Delta\,\text{NTR} = \alpha_0^{\text{NTR}} G + \alpha G h(Z) + f(X; \alpha^{\text{NTR}}) + u^{\text{NTR}}, \tag{4}$$

where f is a function that is linear in parameters, and h is some function chosen by the researcher. We refer to the coefficient β as the *elasticity of taxable income* (ETI) estimand.

The TSLS model nests the triple differences model above when $h(Z) = \mathbb{1}[Z \neq 0]$ and f is a constant plus the indicator variable $\mathbb{1}[X \geq \bar{Y}]$. However, the TSLS model allows the researcher to choose a rich specification of f, thereby controlling flexibly for differential earnings- and tax rate growth, and to use variation in the intensity of treatment when the simulated instrument takes more than two values. The empirical ETI literature typically chooses $h(Z) = Z$ and specifies f to be a polynomial or spline function of X (see, e.g., Auten and Carroll (1999), Gruber and Saez (2002), and Kleven and Schultz (2014)).

2.2 Potential earnings model

We will provide sufficient and necessary conditions for when the ETI estimand (defined by equations (3) and (4)) can be given a causal interpretation as a positively weighted average of individual elasticities of taxable income. To this end, it is necessary to introduce a potential earnings model that allows us to link the ETI estimand and the data to individual elasticities of taxable income.

Given the tax systems T_0 and T_1, we let $Y_t(d)$ denote period t potential earnings under tax system T_d. Similarly, $\tau_t(d) \equiv T'_d(Y_t(d))$ denotes their potential marginal tax rate and $\text{NTR}_t(d) \equiv \log(1 - T'_d(Y(d)))$ their potential (log of net-of-) marginal tax rate. The potential outcomes map to observed outcomes through,

$$Y_t = \mathbb{1}[Z=0] Y_t(0) + \mathbb{1}[Z \neq 0] \left[(1-G) Y_t(0) + G Y_t(1)\right], \tag{5}$$

$$\text{NTR}_t = \mathbb{1}[Z=0]\, \text{NTR}_t(0) + \mathbb{1}[Z \neq 0] \left[(1-G)\, \text{NTR}_t(0) + G\, \text{NTR}_t(1)\right], \tag{6}$$

for each $t \geq 0$, and $Y_t = Y_t(0), \text{NTR}_t = \text{NTR}_t(0)$ for $t < 0$.

Following Saez et al. (2012), we assume the *potential earnings function* is,

$$\log Y_t(\mathrm{NTR}, d) = \zeta \times \mathrm{NTR} + \nu_t(d), \tag{7}$$

where the parameters $\nu_t(0)$ and $\nu_t(1)$ can vary freely across individuals and ζ is the individual's elasticity of taxable income. Unless otherwise noted, ζ can also vary freely across individuals. The specification in (7) allows the tax system to affect earnings both through marginal tax rates NTR and other channels ν.[2]

It is useful to consider a set of assumptions commonly invoked to give TSLS estimands a causal interpretation, which imposes additional restrictions on the potential outcomes. Throughout the paper, we invoke the following common trends assumption, to recover the reform effects on earnings and marginal tax rates:

Assumption 1 (Common trend assumption)**.** *For each G, X, average earnings and marginal tax rate absent the tax reform changes according to*

$$\mathbb{E}\left[\Delta y(0) \mid G, X\right] = \lambda^y G + f_y(X), \tag{8}$$

$$\mathbb{E}\left[\Delta\, \mathrm{NTR}(0) \mid G, X\right] = \lambda^{\mathrm{NTR}} G + f_{\mathrm{NTR}}(X), \tag{9}$$

where the functions f_y and f_{NTR} are unrestricted.

Assumption 1 states that average growth in earnings (marginal tax rates) conditional on initial income X and cohort G would have changed according to equation (8) (equation (9)) in the absence of the tax reform. The key restriction is that there is no interaction between G and X in the average growth in earnings (or marginal tax rates) in the absence of the reform. It means that the aggregate growth due to calendar-time effects (for example, due to business cycles) is allowed to vary freely over time, and that any idiosyncratic growth can vary freely across individuals depending on their initial income X.

Our second assumption is an exclusion restriction, which implies that individual earnings are affected by the tax reform only through its effects on marginal tax rates.

Assumption 2 (Exclusion restriction)**.** $\nu_t(0) = \nu_t(1)$ *with probability one.*

As shown below, this restriction implies no income effects, an assumption we will relax in Section 4.

Lastly, we consider the assumption that the tax reform either weakly increases (or decreases) the marginal tax rates for all:

Assumption 3 (Monotonicity)**.** $\mathbb{P}(\mathrm{NTR}(1) \geq \mathrm{NTR}(0)) = 1$ *or* $\mathbb{P}(\mathrm{NTR}(1) \leq \mathrm{NTR}(0)) = 1$.

[2]The log specification implicitly assumes no extensive margin responses to the tax reform. In Section 3, we test this assumption empirically and find that the reform we consider did not affect extensive margin employment decisions.

This assumption is common in the program evaluation literature when allowing for treatment effect heterogeneity. It is typically necessary to ensure that standard IV estimands reflect positively weighted averages of individual treatment effects: see, e.g., Imbens and Angrist (1994).

2.3 Necessary and sufficient conditions for the ETI estimand to be causal

We now provide a characterization of the ETI estimand in terms of individual ETIs ζ under different choices of f and h, while maintaining the IV assumptions 1 - 3. A key goal is to understand when the ETI estimand β can(not) be given a causal interpretation as a positively weighted average of individual elasticities of taxable incomes ζ:

Definition 1 (Causal ETI estimand)**.** *The ETI estimand β is* causal *if, for any distribution of individual ETIs ζ and initial income X, the maintained assumptions ensure that $\beta = \mathbb{E}[\omega \times \zeta]$ for some ω that satisfies $\mathbb{E}[\omega] = 1$ and $\mathbb{P}(\omega \geq 0) = 1$.*

The requirement that the weights sum to 1 ensures that a causal ETI estimand recovers ζ when ζ is constant across individuals, while the non-negative weights are necessary to ensure that a causal ETI estimand is contained in the support of ζ. For example, allowing for negative weights could mean that $\mathbb{E}[\omega \times \zeta]$ is negative even if ζ is always positive.

Proposition 1 provides necessary and sufficient conditions for the ETI estimand to be causal:

Proposition 1. *Suppose Assumptions 1 - 3 hold. Then, the ETI estimand β is causal if and only if $h(Z)$ is binary and f is unrestricted over the support of X.*

While we refer to Appendix A for a formal proof, it is useful to observe the three distinct reasons why an ETI estimand may fail to be causal. First, if $h(Z)$ is non-binary, the ETI estimand will compare the earnings changes of individuals who are all treated by the reform, but with different intensity, as some experience larger changes in marginal tax rates than others. These comparisons could lead to negative weights if there is heterogeneity in the elasticities of taxable income.[3] In contrast, if $h(Z)$ is binary, the ETI estimand only compares earnings changes between individuals treated by the reform ($Z \neq 0$) and untreated individuals ($Z = 0$), who do not experience any change in marginal tax rates. These comparisons will not produce negative weights, even if the elasticities are heterogeneous.

Second, if f is insufficiently flexible to capture how counterfactual earnings and marginal tax rate growth vary with X given G, then the excluded instrument $Gh(Z)$ in the first stage

[3] Callaway et al. (2025) formalizes the problem of variable treatment intensity in difference-in-differences, and shows that strong auxiliary assumptions are needed (e.g., constant effects) to give the estimate a causal interpretation.

(4) may be correlated with the error term u^y in the outcome equation (3). As a result, the ETI estimand cannot be given a causal interpretation, even when ζ is homogeneous.

If ζ is heterogeneous, the specification of f must also be flexible enough to ensure that it reproduces the conditional mean of the instrument, $\mathbb{E}[Gh(Z) \mid G = 1, X] = f(X)$. Specifications of f that are unrestricted are saturated in X and will always satisfy this condition.[4]

2.4 *The* causal interpretation of the ETI estimand

It is important to emphasize that the criterion of an estimand being causal in Definition 1 is a weak one. Thus, being causal may be necessary for the ETI estimand to be an interesting quantity, but it is not sufficient. For example, the definition does not preclude that all the weight is assigned to a single person with a negative ζ, while the rest of the population receiving zero weight have positive ζ. The following corollary strengthens the result in Proposition 1 by showing *the* causal interpretation of the ETI estimand as a specific positively weighted average of individual ETIs ζ.

Corollary 1. *Suppose Assumptions 1 - 3 hold, that f is flexible and let $h(Z) = \mathbb{1}[Z \neq 0]$. Then, the ETI estimand β is causal and equals:*

$$\beta = \sum_{k=k_0}^{K} \sum_{j=1}^{J} \omega_{k,j} \times \underbrace{\mathbb{E}[\zeta \mid G = 1, X = x_k, \phi = \phi_j]}_{\textit{group-specific average ETI}}. \tag{10}$$

where $\phi \equiv \text{NTR}(1) - \text{NTR}(0)$, $\{\phi_1, ..., \phi_J\}$ is the support of $\phi \mid \phi \neq 0$, $\{x_{k_0}, ..., x_K\}$ is the support of $X \mid X \geq \bar{Y}$, and the weights $\omega_{k,j}$ equal,

$$\omega_{k,j} = \frac{Var(G \mid X = x_k)\, \mathbb{P}(X = x_k, \phi = \phi_j \mid G = 1)\phi_j}{\sum_{l=k_0}^{K} Var(G \mid X = x_l) \sum_{m=1}^{J} \mathbb{P}(X = x_l, \phi = \phi_m \mid G = 1)\phi_m} \geq 0, \tag{11}$$

are positive, and sum to one.

The corollary shows that the ETI estimand recovers a specific positively weighted average of group-specific averages of ζ. The groups are mutually exclusive and defined by initial income X and the reform's effect on their marginal tax change ϕ. Conditional on initial income $X = x$, groups are weighted according to their size and how large changes in the marginal tax rate ϕ they experience.

The ETI estimand aggregates the groups' average ETIs *across* initital income in proportion to how dispersed observations with $X = x$ are across cohorts G, as measured by $Var(G \mid X)$. These weights resemble how linear regression aggregates average treatment effects across covariates, and can be viewed as efficiency weights, see e.g., Angrist (1998).

[4] Blandhol et al. (2022) show that the only specifications of the TSLS estimand that have a LATE interpretation are saturated specifications that control for covariates non-parametrically.

We now consider the special case when ζ is assumed to be homogeneous across individuals. The following result shows that in this case, the ETI estimand recovers ζ provided f is unrestricted, thereby formalizing the identification argument implicit in the existing ETI literature:

Corollary 2. *Suppose Assumptions 1 and 2 are true, that ζ is constant across individuals, and that f is unrestricted over the support of X. Then, $\beta = \zeta$.*

The result is similar to Proposition 6 in Blandhol et al. (2022). Their linearity Assumption (LIN) is satisfied because of our Assumption 1, which ensures that the conditional mean of $\Delta y(0)$ is linear in X.

2.5 Quantifying how the elasticities vary across the income distribution

Corollary 1 shows that the ETI estimand β recovers a positively weighted average of individual elasticities of taxable income ζ across groups defined by initial income X and marginal tax rate response ϕ. An important question for tax policy is how the ETI varies across the income distribution X. To analyze this question, we introduce the *local ETI estimand* $\beta(x)$, as defined by

$$\beta(x) \equiv \frac{\overbrace{\mathbb{E}[\Delta y \mid G=1, X=x]}^{\text{actual income growth}} - \overbrace{\left(\hat{\lambda}_0^y + \sum_{k=k_0}^{K} \hat{\lambda}_k^y \mathbb{1}[x_k = x]\right)}^{\text{counterfactual income growth}}}{\underbrace{\mathbb{E}[\Delta\,\mathrm{NTR} \mid G=1, X=x]}_{\text{actual tax rate growth}} - \underbrace{\left(\hat{\lambda}_0^{\mathrm{NTR}} + \sum_{k=k_0}^{K} \hat{\lambda}_k^{\mathrm{NTR}} \mathbb{1}[x_k = x]\right)}_{\text{counterfactual tax rate growth}}}, \tag{12}$$

for each x in the support of $X \mid X \geq \bar{Y}$, where the coefficient vector $\hat{\lambda}^y$ ($\hat{\lambda}^{\mathrm{NTR}}$) is obtained by regressing Δy ($\Delta\,\mathrm{NTR}$) on G and a set of dummies for each value of X in the sample with $GZ = 0$.

Intuitively, the estimand exploits that Assumption 1 implies the earnings and marginal tax rate growth among untreated ($GZ = 0$) individuals can be used to recover the counterfactual growth of the treated ($GZ \neq 0$) individuals. Subtracting the counterfactual growth from the treated individual's actual growth yields their earnings response and the changes in their marginal tax rate due to the tax reform. Our next result shows that the local ETI estimand $\beta(x)$ is causal under the same assumptions that were necessary to give the ETI estimand β a causal interpretation:

Proposition 2. *Suppose Assumptions 1 - 3 hold. Then, the local ETI estimand $\beta(x)$ is causal and* *equals:*

$$\beta(x) = \sum_{j=1}^{J} \overbrace{\frac{\mathbb{P}(\phi = \phi_j \mid G = 1, X = x)\phi_j}{\sum_{l=1}^{J} \mathbb{P}(\phi = \phi_l \mid G = 1, X = x)\phi_l}}^{\text{weights reflecting group size and marginal tax rate effects}} \times \underbrace{\mathbb{E}[\zeta \mid G = 1, X = x, \phi = \phi_j]}_{\text{group-specific average ETI}}, \tag{13}$$

for each $x \in \{x_{k_0}, ..., x_K\}$. The weights are positive and sum to one.

Proposition 2 shows the local ETI estimand $\beta(x)$ recovers a positively weighted average across the same group-specific averages of ζ as in Corollary 1.[5] However, only groups with initial income $X = x$ receive positive weights. Among the groups with $X = x$, the local ETI estimand weights the groups according to their size $\mathbb{P}(\phi = \phi_k \mid G = 1, X = x)$ and how much their marginal tax rates are affected by the reform ϕ.

3 Estimating elasticities of taxable income

We now use our theoretical results to analyze a Norwegian tax reform from the mid-2000s that decreased the marginal tax rate on the top 50 percent of wage earners.

3.1 The Norwegian tax system and the tax reform

Taxation of labor income The Norwegian personal income tax combines a flat tax on general income with a progressive surtax on personal income. General income includes both labor and capital income and is taxed at a flat rate on net income after deductions. Deductions such as the standard wage-earner deduction, the personal allowance, and certain pension and interest deductions apply only to this base. Personal income, defined as gross labor and pension income before deductions, is subject to a progressive surtax consisting of several income brackets with increasing marginal tax rates. Labor income comprises wages, salaries, and most employer-provided benefits.

Labor income is also subject to social security contributions. Employers pay a payroll tax on gross wages, with rates varying across geographical zones, while employees contribute a 7.8 percent social security tax on personal income from employment, which finances health and pension entitlements.

The 2006 tax reform. We exploit the major Norwegian tax reform of 2006 to estimate the elasticity of taxable income. The reform replaced the previous top-tax structure with a redesigned progressive surtax. While maintaining the two-bracket system, it substantially

[5]The expression is similar to equation 23 in Mogstad and Torgovitsky (2024) with the additional restriction that the potential earnings functions are linear. See their discussion of how it relates to the *average causal response* from Angrist and Imbens (1995).

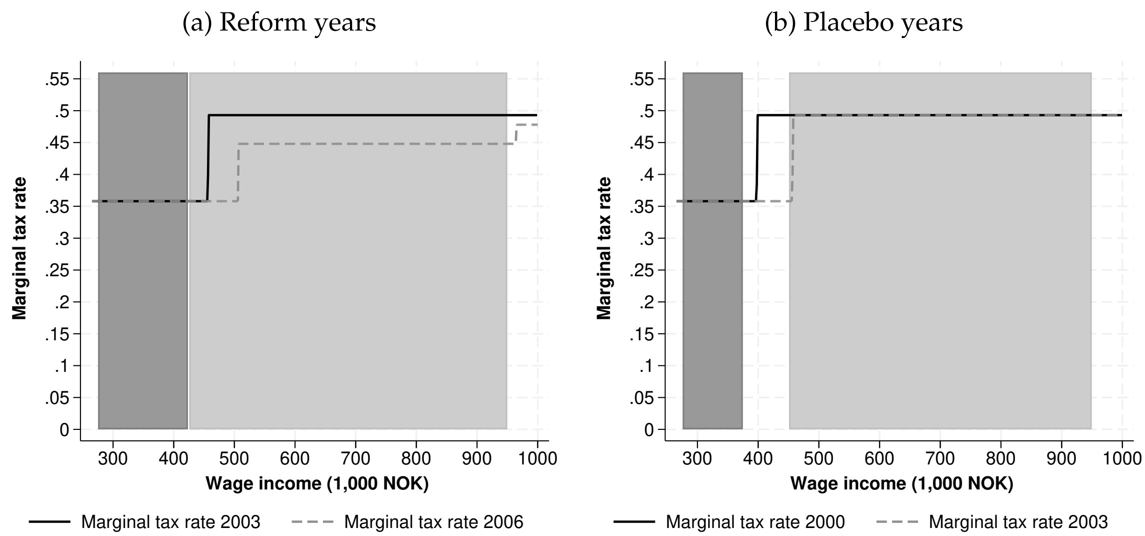


Figure 2: Changes in tax schedules – reform vs placebo years

Notes: This figure shows the marginal tax rates that apply to different income tax brackets for selected reform and placebo years. Shaded areas denote treatment and control income intervals.

reduced marginal tax rates on medium and high labor incomes: the surtax rates fell from 13.5 and 19.5 percent to 9 and 12 percent, respectively. This means that from 2004 to 2006, the marginal tax rate in the (second) highest bracket was reduced from 55.3 to 47.8 (49.3 to 44.8) percent. The sizable rate cuts meant that most workers in the upper half of the income distribution experienced large reductions in their marginal tax rates.

An advantage of the 2006 reform is that it was preceded by a long period of relative stability in the Norwegian tax system, with only minor adjustments and none that materially affected the income ranges we analyze. This long stretch of pre-reform stability makes the placebo exercise particularly informative. Figure 2 illustrates the changes in marginal tax rates associated with the 2006 reform, as well as the corresponding placebo period, highlighting the stability of the tax schedule before the reform. The dark (light) grey areas plot the range of initial income that determines whether individuals are assigned to the control group $Z = 0$ (treatment group $Z \neq 0$).

3.2 Data and sample

Our empirical analyses are based on several administrative data sources that we link together using unique identifiers for individuals and households. This results in a matched panel dataset covering the full Norwegian population in the period 1995–2018. The dataset includes detailed information from income tax returns as well as individual characteristics such as age, sex, educational attainment, marital status, and number of children. Tax rates are not directly observed in tax return data. We therefore simulate tax rates for each taxpayer based on information from tax returns and a detailed tax simulation model of the Norwegian income tax system, building on Vattø (2020).

Our baseline sample includes wage earners between 25 and 61 years old, with wage earners being defined as individuals for whom wage income is the main source of labor income. We exclude students and individuals receiving pensions or unemployment benefits.

When explaining our research design, we only had one reform and one placebo cohort. In estimation, however, we exploit that there are multiple reform and placebo cohorts in the data. Specifically, we classify observations between 2002 and 2004 as placebo cohorts and observations between 2005 and 2007 as reform cohorts. Within the reform cohorts, we restrict the sample to individuals with initial income X between 275,000 and 950,000 NOKs, illustrated by the grey areas in Panel (a) of Figure 2, to ensure they are not affected by the small year-to-year changes in the tax system outside the reform period. We impose the symmetric sample restrictions also within the placebo cohorts, as illustrated by the grey areas in Panel (b) of Figure 2. Finally, we exclude a small number of observations with positive predicted marginal tax rate changes to ensure monotonicity holds.

When estimating the IV model in equations (3) and (4), we pool multiple placebo and reform cohorts and include separate time dummies for each year.

With these restrictions, our baseline estimation sample contains about 4 million observations and 1.7 million individual wage earners. Summary statistics for our baseline estimation sample are provided in Table 3.

3.3 Results

Table 1 reports estimates (multiplied by 100) under different specifications of the ETI estimand. The first column of Panel A reports estimates when the instrument $h(Z)$ is binary and the income controls f are specified as dummies for initial income X in each percentile, and are thus practically unrestricted. The first row shows that the F-statistic on the instrument is above $70,000$, indicating that the first stage is very strong. The first stage estimates reported in the second row show that the reform increases the marginal net-of-tax rate by around 5.3 percent, while the reduced form estimates reported in the third row show that earnings increase by 1.2 percent. The ETI estimate, reported in the fourth row, is given by the ratio between the reduced form and the first stage and equals 0.233. Proposition 1 showed that this ETI estimate can be given a causal interpretation as a positively weighted average of individual elasticities of taxable income.

In contrast, this is not true for the ETI estimates reported in the second and third columns. The second column reports the ETI estimate with a binary indicator for having initial income above the tax reform cut-off $\bar{Y}$, corresponding to the triple difference estimand β^{DDD} in equa-

Table 1: Main results

Panel A. Specification of income controls f: dummies for X in bins.

Instrument	Binary: $h(Z) = \mathbb{1}[Z \neq 0]$		Continuous: $h(Z) = Z$
Income bins	Percentiles	Above/below $\bar{Y}$	Percentiles
F-stat on excl. instr.	71,329	79,588	33,448
First stage	5.26	5.03	30.45
	(0.02)	(0.02)	(0.17)
Reduced form	1.22	1.00	4.90
	(0.06)	(0.05)	(0.39)
ETI	23.25	19.96	16.09
	(1.21)	(1.09)	(1.33)

Panel B. Specification of income controls f: linear splines.

Instrument	Binary: $h(Z) = \mathbb{1}[Z \neq 0]$		Continuous: $h(Z) = Z$
Spline knots	Deciles	Above/below $\bar{Y}$	Deciles
F-stat on excl. instr.	72,480	168,684	33,892
First stage	5.27	6.72	30.09
	(0.02)	(0.02)	(0.16)
Reduced form	1.22	1.11	4.81
	(0.06)	(0.05)	(0.38)
ETI	23.06	16.56	15.99
	(1.18)	(0.76)	(1.31)

[a] All coefficients and standard errors have been multiplied by 100. F-statistics are unchanged.

tion (2). It shows that reduced flexibility in f lowers the ETI estimate from 0.233 to 0.2. Maintaining the flexible specification of f and changing the instrument to $h(Z) = Z$ has a more substantial effect on the estimated ETI, reducing it from 0.233 to 0.162.

Panel B of Table 1 reports the corresponding estimates, controlling for initial income through splines instead of dummies for income bins. Our conclusions from the discussion above remain true. In particular, the point estimate of the ETI with a binary instrument and a spline with knots at each decile barely moves compared to the one in Panel A. We conclude that when the instrument is binary and the income controls f are reasonably flexible, the ETI estimates are robust to the exact specification of f.

One possible concern is that the common trends from Assumption 1 do not hold. To address this concern, Panel A of Table 2 reports placebo estimates (multiplied by 100) obtained by exploiting the fact that we have three placebo cohorts. Specifically, we estimate the

Table 2: Robustness

Panel A. Specification of income controls f: dummies for X in bins.			
Instrument	Binary: $h(Z) = \mathbb{1}[Z \neq 0]$		Continuous: $h(Z) = Z$
Income bins	Percentiles	Above/below $\bar{Y}$	Percentiles
Placebo reduced form	0.0022	-0.0024	-6.19
	(0.0006)	(0.0006)	(2.20)
Placebo ETI	0.0423	-0.0470	-20.34
	(0.0113)	(0.0111)	(7.22)

Panel B. Specification of income controls f: dummies for X in bins.			
Instrument	Binary: $h(Z) = \mathbb{1}[Z \neq 0]$		Continuous: $h(Z) = Z$
Income bins	Percentiles	Above/below $\bar{Y}$	Percentiles
Employment reduced form	0.06	0.001	0.27
	(0.01)	(0.01)	(0.07)

[a] All coefficients and standard errors have been multiplied by 100.

reduced form of the regression model in equations (3) and (4) on the placebo cohorts only, sequentially treating each cohort as the reform cohort using the remaining two as controls, and averaging the resulting estimates. The resulting estimate reported in the first column is indistinguishable from zero. To understand how potential violations of Assumption 1 would affect ETI estimates, the second row reports the placebo reduced form divided by the actual first stage from Table 1. It is reassuring to see that the resulting estimate is almost two orders of magnitude smaller than the actual ETI estimate.

One final concern is that the tax reform induces extensive margin responses. Panel B of Table 2 reports the reduced form estimates of the regression model in equations (3) and (4) estimated on a sample that includes the non-employed, replacing the outcome by a binary variable equal to one if the individual works and zero otherwise. Our results show that the tax reform had no meaningful impact on employment, increasing it by 0.06 percentage points.

Turning to how the ETIs vary across the distribution of initial income, Panel (a) of Figure 3 plots estimates of the local ETI $\beta(x)$ from equation (12).[6] It shows that the ETI estimates considered above mask considerable heterogeneity. The variation in ETIs across the income distribution is significant, both statistically and substantively, and means that the (weighted-

[6]We implement the estimator for $\beta(x)$ in two steps. The first step estimates the counterfactual income and tax rate growth using 50 quantile bins of X, while the second step estimates the numerator and denominator separately using a local regression.

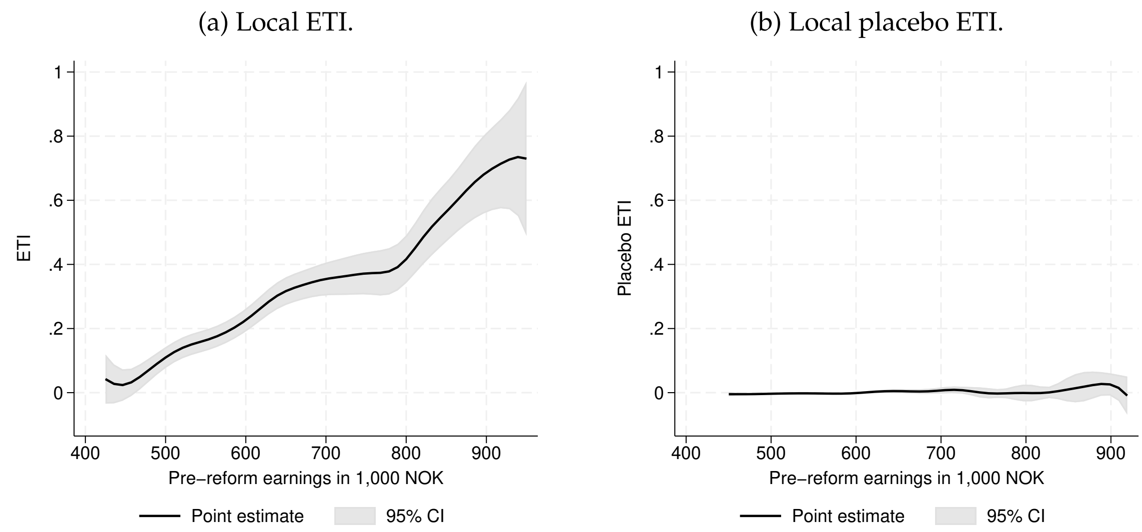


Figure 3: Local ETI estimates.

Notes: The figure plots actual and placebo estimates of the local ETI across the income distribution. Panel (a) plots the actual local ETI obtained by first estimating the counterfactual income and tax rate growth using 50 quantile bins of X, then estimating the numerator and denominator of equation (12) separately using a local regression. Panel (b) reports local placebo ETIs, obtained by estimating the numerator of equation (12) on the placebo cohorts only, sequentially treating each cohort as "treated," using the remaining two as controls, and averaging the resulting estimates. The local placebo ETI estimates are obtained by dividing these placebo reduced forms by the actual first stage from Panel (a). 95 percent confidence intervals are shown, with standard errors obtained by bootstrapping the entire estimation procedure using 500 replications.

) average ETIs is far from sufficient to characterize how taxes affect earnings.[7] The local ETIs are close to zero for incomes around 400,000 NOKs, roughly corresponding to the median, and rise steadily with income, reaching 0.7 for incomes around 900,000 NOKs, corresponding to the 95th percentile.

Given the considerable heterogeneity in ETIs across the income distribution, it is natural to ask if the placebo ETI also masks heterogeneity. Panel (b) of Figure 3 answers this concern by plotting local placebo ETI estimates, which are estimated analogously to the placebo ETI estimates introduced above. The placebo estimates remain close to zero throughout the income distribution. Similarly, Figure 10 of Appendix B plots the local (placebo) employment ETIs, which also remain indistinguishable from zero throughout the income distribution.

4 Using ETIs to learn about labor supply elasticities

We now use the ETI estimates to learn about labor supply elasticities.

[7]However, it shows that the (non-local) ETI estimand from above actually recovers something close to the unweighted average ETI: aggregating the local ETI estimates using population weights recovers an average of 0.242, compared to the 0.233 of the causal ETI estimand.

4.1 A labor supply model

We consider a labor supply model where workers have convex preferences over K consumption goods C_k and L margins of labor supply Y_l. The L margins of labor supply produce pre-tax income Y according to $Y = F(Y_1, ..., Y_L)$, which is concave and strictly increasing in each of its arguments. The multidimensional labor supply margins capture the idea that individuals can affect their earnings through many different choices, including, but not limited to, their hours worked, effort on the job, and firm and occupation choices.

Given a linear tax system with marginal tax rate τ and transfer R, their utility maximization problem is given by,

$$\max_{C_1,...,C_K,Y_1,...,Y_L} U(C_1, ..., C_K, Y_1, ..., Y_L) \qquad \text{subject to } \sum_{k=1}^{K}(1+\tau_k)p_k C_k \leq I, \\ I = (1-\tau)Y + R + B, \text{ and } Y = F(Y_1, ..., Y_L), \tag{14}$$

where B is unearned income, I is disposable income or consumption expenditure, and p_k and τ_k is the price of and tax on consumption good k respectively.

As noted by Feldstein (1999), the answer to many questions related to the revenue- and efficiency effects of taxation depends on how tax policy affects earnings, but not through which margins it does so. We thus focus on the earnings choice, which, by suppressing its dependence on prices and consumption tax rates, can be written as,

$$Y^u(\tau, R+B) \equiv F(Y_1^u(\tau, R+B), ..., Y_L^u(\tau, R+B)), \tag{15}$$

where $Y_l^u(\tau, R+B)$ is the optimally chosen l-th labor supply component. This earnings function $Y^u(\tau, R+B)$ allows for defining the standard labor supply elasticities,

$$\varepsilon^u \equiv \frac{\partial Y^u}{\partial 1-\tau}\frac{1-\tau}{Y^u}, \qquad \eta \equiv (1-\tau)\frac{\partial Y^u}{\partial(R+B)} \in [-1, 0], \qquad \varepsilon^c \equiv \varepsilon^u - \eta \geq 0, \tag{16}$$

where ε^u and ε^c denote the uncompensated and compensated earnings elasticity, respectively, η denotes the income effect, and the relationship between (un)compensated elasticities and the income effect is given by the Slutsky equation. The restriction that $\eta \in [-1, 0]$ follows by assuming that consumption and leisure are normal goods, in the sense that consumption expenditure I (pre-tax earnings Y) increases (decreases) in unearned income B.

In reality, the Norwegian tax system is piecewise linear. To accommodate this, we follow the argument of Hall (1973): convex preferences ensure that individuals behave as if they were facing the following budget constraint,

$$I = (1-\tau(d))Y + R(d) + B \text{ for } d = 0, 1, \tag{17}$$

where

$$\tau(d) = T_d'(Y(d)), \qquad\qquad R(d) = T_d'(Y(d))Y(d) - T_d(Y(d)), \tag{18}$$

even if the actual tax system T_d is non-linear. This argument implies the potential earnings function $Y(\tau(d), d)$ in equation (3) can be viewed as the solution to the worker's labor supply problem subject to the linear budget constraint defined in equation (17):

$$Y(\tau(d), d) \; = \; Y^u(\tau(d), R(d) + B) \text{ for } d = 0, 1.$$

4.2 Recovering labor supply elasticities

The previous subsection forged a tight link between the potential earnings function and the model of labor supply. Having established this link, we now use ETI estimates (and other specific features of the data) to draw inferences about labor supply elasticities and functionals of these elasticities, such as the excess burden of taxation.

We first consider the case with no income effects, a restriction typically imposed in the models used to interpret estimates of the ETI. Our next result shows that this model restriction ensures that Assumption 2 (exclusion restriction) is satisfied and, therefore, lets us interpret the individual elasticities of taxable income ζ as compensated earnings elasticities ε^c:

Proposition 3. *If there are no income effects ($\eta = 0$ across all individuals), then Assumption 2 is satisfied and $\zeta = \varepsilon^c$ for each individual.*

An immediate implication of the result is that the ETI estimands β and $\beta(x)$ in Section 2 recover positively weighted averages of compensated earnings elasticities, provided the instrument is binary and the specification of the income controls f is unrestricted.

Allowing for income effects violates Assumption 2 (exclusion restriction), since the tax reform can then affect labor supply both through changes in the marginal *and* average tax rates. The following proposition clarifies the identification problem that arises due to income effects. It expresses the compensated elasticities ε^c and the uncompensated elasticities ε^u in terms of the local ETI estimands $\beta(x)$ and a bias term that would be observable in data if the income effects were known or could be estimated:

Proposition 4. *Suppose that Assumption 1 holds, there is no bracket switching, and that ε^c and η are homogeneous among individuals with the same X. Then, for any $x \geq \bar{Y}$:*

$$\varepsilon^c(x) = \beta(x) + B(x)\eta(x), \tag{19}$$

$$\varepsilon^u(x) = \beta(x) + (B(x) - 1)\eta(x). \tag{20}$$

where $B(x)$ is estimable and equal to:

$$B(x) \equiv \frac{\mathbb{E}\left[\frac{T_1(Y_t)-T_0(Y_t)}{(1-T_1'(Y_t))Y_t} \mid G=1, X=x\right]}{\mathbb{E}[\Delta\,\mathrm{NTR} \mid G=1, X=x] - \left(\hat{\lambda}_0^{\mathrm{NTR}} + \sum_{k=k_0}^{K} \hat{\lambda}_k^{\mathrm{NTR}} \mathbb{1}[x_k = x]\right)}, \tag{21}$$

and $\hat{\lambda}^{\mathrm{NTR}}$ is defined as in Equation (12)*.*

A key insight from Proposition 4 is that the local ETI estimand neither recovers the compensated elasticity nor the uncompensated elasticity if one allows for income effects, even if one assumes no bracket switching and that elasticities are homogeneous conditional on initial income X.[8] However, it also shows that the bias term is a multiplicatively separable function of the income effects $\eta(x)$ and an observable term $B(x)$. Therefore, if $\eta(x)$ were known or could be estimated, one could recover $\varepsilon^c(x)$ and $\varepsilon^u(x)$ from observed data. This observation motivates and guides the analysis in the next subsections, where we consider different approaches to point identify or bound the compensated and uncompensated earnings elasticities.

4.3 No additional assumption bounds on labor supply elasticities

In order to use Proposition 4 to construct bounds on labor supply elasticities, it is useful to recall that $\eta(x)$ is theoretically bounded between -1 and 0. Thus, evaluating equations (19) and (20) for $\eta(x) \in \{-1, 0\}$ produces bounds on compensated and uncompensated earnings elasticities without imposing assumptions other than those stated in Proposition 4.

The light grey areas of Figure 4 plot these bounds using the estimates of $\beta(x)$ reported in Figure 3 and estimates of $B(x)$ from the cross-sectional earnings distribution. Panel (a) provides two insights about compensated elasticities. First, the compensated elasticities for individuals with incomes equal to 600,000 (85th percentile), 750,000 (90th percentile), and 900,000 (95th percentile) are at least 0.2, 0.4, and 0.7, respectively. Second, the bounds rule out larger than modest compensated elasticities for low-income individuals: the upper bound for individuals with income 450,000 equals 0.3. Taken together, these insights imply that compensated elasticities more than double as income increases from the median to the 95th percentile.

Panel (b) plots the corresponding bounds for the uncompensated elasticities. A key result is that the bounds are highly informative for high-income individuals, implying a relatively large uncompensated elasticity between 0.5 and 0.7 for individuals with incomes around 900,000. A second result is that we can rule out that income effects dominate substitution

[8] The result in Proposition 4 does not invoke Assumption 3. It is no longer needed since it is assumed that elasticities are homogenous among individuals with the same initial income X. For tractability, Proposition 4 also imposes no bracket switching, which will necessarily hold if the reform under consideration is only changing the top income tax.

effects for all individuals with income above 700,000. Finally, the bounds rule out large uncompensated elasticities for individuals with incomes below 500,000.

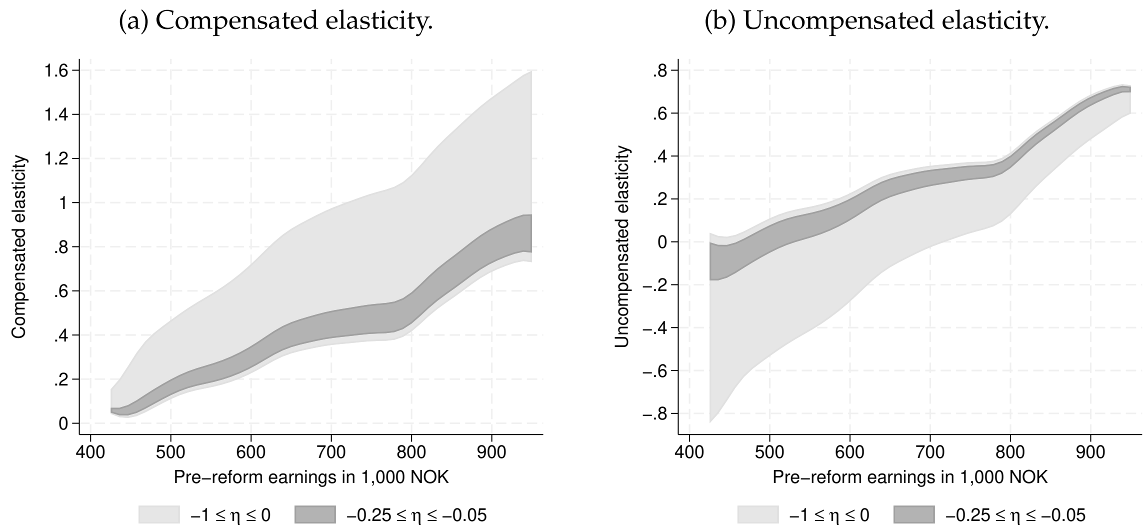


Figure 4: Average (un)compensated earnings elasticities across initial income X.

Notes: The figure plots bounds on the compensated and uncompensated labor supply elasticities over the income distribution. The light grey area in Panel (a) plots the bounds on $\varepsilon^c(x)$ using that $\eta \in [-1, 0]$, while the darker grey area imposes that $\eta \in [-0.25, 0.05]$. The light grey area in Panel (b) plots the bounds on $\varepsilon^u(x)$ using that $\eta \in [-1, 0]$, while the darker grey area imposes that $\eta \in [-0.25, 0.05]$. 95 percent confidence intervals are shown, with standard errors obtained by bootstrapping the entire estimation procedure using 500 replications.

The conclusions from the no assumption bounds can be strengthened by ruling out implausibly small and large income effects such that $\eta \in [-0.25, -0.05]$. The dark grey areas in Figure 4 represent the resulting bounds and show that this additional assumption allows for drawing firm conclusions about the magnitudes of compensated and uncompensated earnings elasticities across the income distribution.

There are also alternative ways to use the local ETI estimates to learn even more about compensated and uncompensated elasticities. One possibility, which we consider in subsection 4.4, is to assume constant income and substitution effects across the income distribution. Another possibility, which we consider in subsection 4.5, is to use more data to estimate income effects directly.

4.4 Labor supply elasticities assuming constant income and substitution effects

As shown by Gruber and Saez (2002), it is possible to use variation from tax reforms to jointly estimate income and substitution effects under the assumption that the compensated elasticity ε^c and the income effect η are constant across all individuals.Under this homogeneity assumption, Proposition 4 implies that the local ETI estimands $\beta(x)$ relates to the constant ε^c

and η through,

$$\beta(x) = \varepsilon^c + \eta B(x). \tag{22}$$

A natural way to estimate (ε^c, η) is the following least squares estimator

$$(\hat{\varepsilon}^c, \hat{\eta}) = \arg\min_{\varepsilon^c, \eta} \sum_{k=k_0}^{K} p_k \left(\hat{\beta}(x_k) - \varepsilon^c - \eta \hat{B}(x_k)\right)^2 \tag{23}$$

where $p_k \equiv \mathbb{P}(X = x_k \mid G = 1)$ are weights corresponding to each point in the support of $X \mid X \geq \bar{Y}$. An estimator for the uncompensated elasticity is then $\hat{\varepsilon}^u = \hat{\varepsilon}^c + \hat{\eta}$.

Since equation (22) must hold for any x, ε^c and η are (over)identified if $\beta(x)$ and $B(x)$ are observed for (more than) two values of x. Even with only two values of x, the homogeneity assumption can be tested by examining if the labor supply parameters satisfy the theoretical restrictions $\varepsilon^c > 0$ and $\eta \in [-1, 0]$. In the overidentified case, the sharp test would not only use these theoretical restrictions, but also that the sum of squared residuals in (23) is zero.

Using the same estimates of $\beta(x)$ and $B(x)$ as in Figure 4, we solve the problem in (23) and obtain $\hat{\varepsilon}^c = -0.26$ and $\hat{\eta} = 1.04$. Since the estimated compensated elasticity is negative and the income effect is positive, the homogeneity assumption is clearly at odds with the data. To understand why, it is useful to recall that the tax reform we consider decreased the top income tax rate and, as a result, the reduction in average tax rates is increasing in income. The assumption of constant ε^c and η therefore implies that earnings responses should decrease across the income distribution, in sharp contrast with Figure 3. Thus, we conclude that the assumption of constant income and substitution effects poorly approximates individual labor supply behavior, at least in our context.

4.5 Labor supply elasticities with external estimates of income effects

Proposition 4 shows that the compensated $\varepsilon^c(x)$ and uncompensated elasticities $\varepsilon^u(x)$ can be point identified from the local ETI estimands $\beta(x)$ if income effects $\eta(x)$ are available externally for each initial income level x. Motivated by this result, we now use Norwegian data on lottery winnings to estimate income effects across the income distribution. Figure 5 presents the resulting estimates of the compensated and uncompensated elasticities. We find that both elasticities are monotonically increasing in income. Panel (a) shows that the compensated elasticity is small— around 0.1 for incomes near 450,000 — but increases steadily to roughly 0.8 for incomes around 900,000. Panel (b) shows the uncompensated elasticity tends to be positive, implying that substitution effects are larger than the income effects, especially for higher incomes, where it exceeds 0.5.

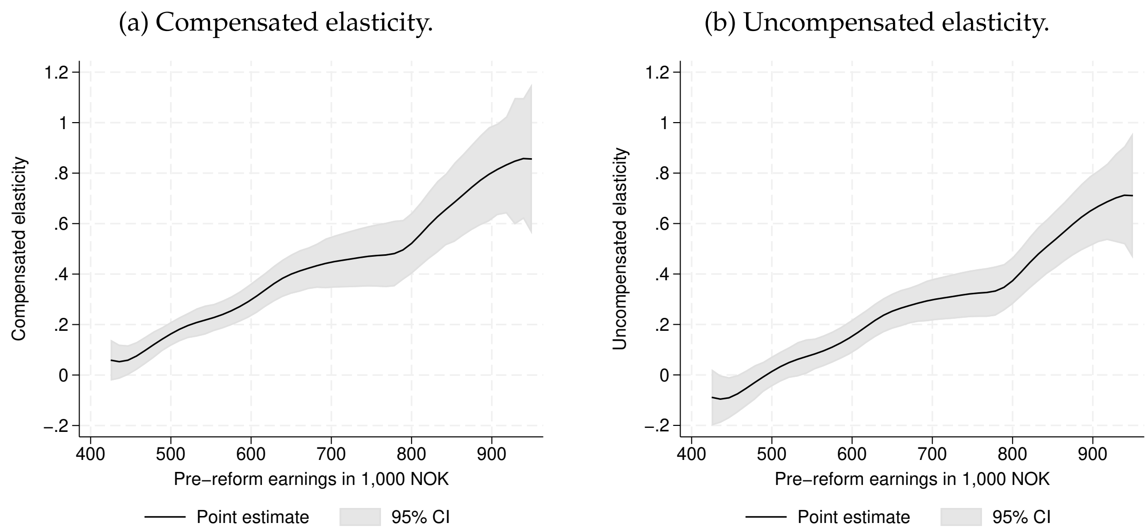


Figure 5: Average (un)compensated earnings elasticities across initial income X.

Notes: The figure plots point estimates of the compensated (Panel (a)) and uncompensated (Panel (b)) labor supply elasticities over the income distribution. 95 percent confidence intervals are shown, with standard errors obtained from assuming lottery-based income effects $\eta(x)$ and local ETIs $\beta(x)$ are uncorrelated.

We recover the income effects used to produce Figure 5 in three steps. In **step 1**, we estimate the earnings effect of winning the lottery conditional on initial earnings X by exploiting random variation in the timing of the lottery wins. In **step 2**, we study how winning the lottery changes the unearned income — defined as consumption in excess of labor income - that households allocate to consumption and leisure across years. Dividing the earnings effects from step 1 by the unearned income effects from step 2 gives a measure of the earnings response per additional NOK of unearned income. **Step 3** decomposes the earnings response per dollar of unearned income into its intensive- and extensive margin components, and recovers the intensive-margin income effect $\eta(x)$.

Data and sample. Before detailing each of the three steps, we introduce our data and sample and describe how we construct our measure of unearned income. Our empirical analysis combines multiple administrative data sources linked through unique personal and household identifiers. We supplement the tax records on wealth with measures of market values of real estate, using data on transactions in real estate, information on the characteristics of each property, and detailed housing price indices. The resulting panel covers the entire Norwegian population from 1995 to 2018 and includes demographic characteristics, detailed tax records on income and wealth, and information on lottery prizes and asset values.

We restrict our sample to winners aged 24–61 in the year before the win who were neither pensioners nor students and were the primary or secondary earner in their household, ensuring stable labor-market attachment. Since individuals are only required to report winnings of NOK 100,000 or more, we restrict our sample to those who won at least NOK 100,000.[9] Lastly, to separate gambling wins that resemble income from an entrepreneurial activity (e.g., professional poker) from windfall gains due to lottery winnings, we also exclude repeated winners with prizes over the reporting threshold. The final sample includes more than 14,000 unique winners with a median prize of 343,000 NOKs across 23 win-year cohorts. Table 4 in Appendix B compares working-age lottery winners with the general population. It shows that winners are somewhat older, more often male, and have higher earnings, but are otherwise similar to the working-age population.

Measuring unearned income. To understand how we measure period-by-period unearned income, it is useful to start with the household's intertemporal budget constraint,

$$C_t = Y_t - T(Y_t) + \underbrace{\Big((1+r)A_{t-1} - A_t\Big)}_{\text{unearned income } \equiv B_t}, \tag{24}$$

where C_t is period t consumption expenditure, A_t is assets held at time t, r is the net-of-tax return rate on the asset. B_t is the total amount of unearned income used by the household in period t, or unearned income for short.[10] Lottery winnings provide an exogenous increase in unearned income, and, as we show in Appendix E, the earnings responses to this variation are directly linked to the individuals' income effects.

Following Eika et al. (2020), we use our detailed data to construct household-level measures of consumption and savings. These measures allow us to compute unearned income directly using equation (24), which we then convert to a per-adult measure for consistent comparison between single and married households. Thus, our rich data allow us to *observe* how winners allocate their wealth over time, eliminating the need to rely on the annuitization or capitalization approaches used in Imbens et al. (2001) and Golosov et al. (2024).

Step 1: the earnings effect of winning the lottery. To recover the earnings effect of winning the lottery, we employ a difference-in-differences design that compares the evolution of earnings over time for individuals who have already won the lottery with those who have yet to win. Let $G = g$ denote the calendar year in which a cohort receives the lottery prize, and use $g - 1$ as the pre-win reference year. For each event time $t \geq 0$, we compare changes between years $g - 1$ and $g + t$ for cohort g to changes over the same two years for cohorts that will receive the prize at a later date, so that the comparison group remains untreated at

[9] This threshold has been in place since 2007. Before 2007, the threshold was NOK 10,000.

[10] This definition of unearned income is consistent both with intertemporal two-stage budgeting in the absence of liquidity constraints and with the presence of liquidity constraints (Arellano and Meghir, 1992; Blundell and Macurdy, 1999; Blundell and Walker, 1986; MaCurdy, 1983).

both points in time. Conditioning on pre-win earnings $X \equiv Y_{g-1}$, we define

$$\text{RF}_{g,t}(x) \equiv \underbrace{\mathbb{E}\left[Y_{g+t} - Y_{g-1} \middle| G = g, X = x\right]}_{\text{earnings change for year } g \text{ winners}} - \overbrace{\mathbb{E}\left[Y_{g+t} - Y_{g-1} \middle| G > g + t, X = x\right]}^{\text{earnings change for later-than } g+t\text{-winners}}. \tag{25}$$

This estimand captures how earnings change for lottery winners relative to individuals with the same pre-win earnings who have not yet won. We show in Appendix C that $\text{RF}_{g,t}(x)$ recovers the earnings effect of winning the lottery under a standard parallel trends assumption.

In estimation, we implement the conditioning on X using kernel weights and estimate the parameters separately for each cohort g and event time t. The parameters $\text{RF}_{g,t}(x)$ are then aggregated across cohorts using cohort-size weights.[11] Panel (a) of Figure 6 reports the unconditional estimates of the impact of winning on the winners' earnings for each event time t. It shows there is no evidence of systematically different time trends between current and later winners. Moreover, it shows that earnings decrease sharply in the first two years after winning before stabilizing at a lower level.

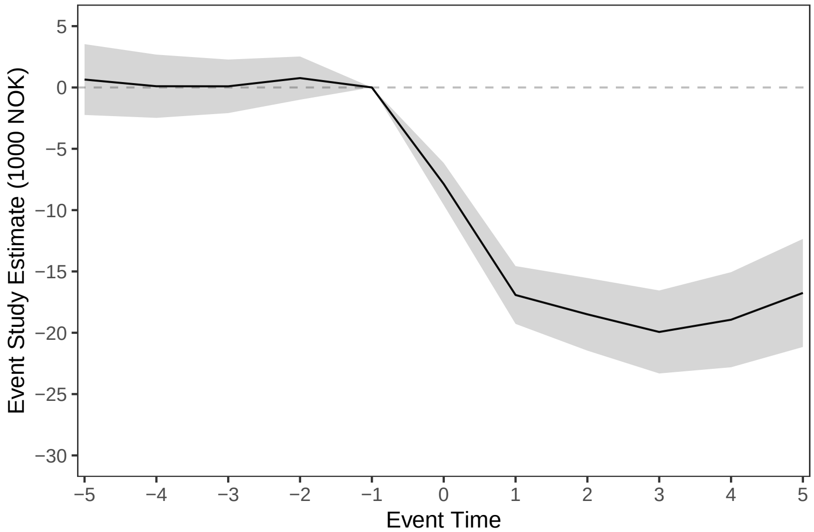
The average effects in Panel (a) could mask considerable heterogeneity across the earnings distribution. To understand how these averages vary across the income distribution, Panel (b) shows pre- and post-event estimates across the distribution of pre-win labor earnings. It is reassuring to see that the pre-trends are flat throughout the distribution. Following the win, we observe a substantial earnings reduction of around 15,000-20,000 NOKs that increases in magnitude with pre-win income. Figure 11 in Appendix B plots the corresponding figure for the employment response. It shows that winning the lottery decreases employment over time, and that employment responses are more pronounced for low levels of pre-win income.

Step 2: the earnings effect of unearned income We now turn to understanding the earnings effect of an additional NOK of unearned income. Using the same difference-in-differences design as above, we recover the effect of winning the lottery on unearned income by estimating the following equation:

$$\text{FS}_{g,t}(x) \equiv \underbrace{\mathbb{E}\left[B_{g+t} - B_{g-1} \middle| G = g, X = x\right]}_{\text{unearned income change for year } g \text{ winners}} - \overbrace{\mathbb{E}\left[B_{g+t} - B_{g-1} \middle| G > g + t, X = x\right]}^{\text{unearned income change for later-than } g+t\text{-winners}}. \tag{26}$$

[11]We estimate all cohort-by-event-time parameters in a single, fully interacted specification, which allows us to construct the joint variance-covariance matrix and compute standard errors for aggregated effects using the delta method. All specifications include flexible controls for age to account for systematic age differences between earlier and later winners.

(a) Unconditional effect on earnings



(b) Conditional effect on earnings

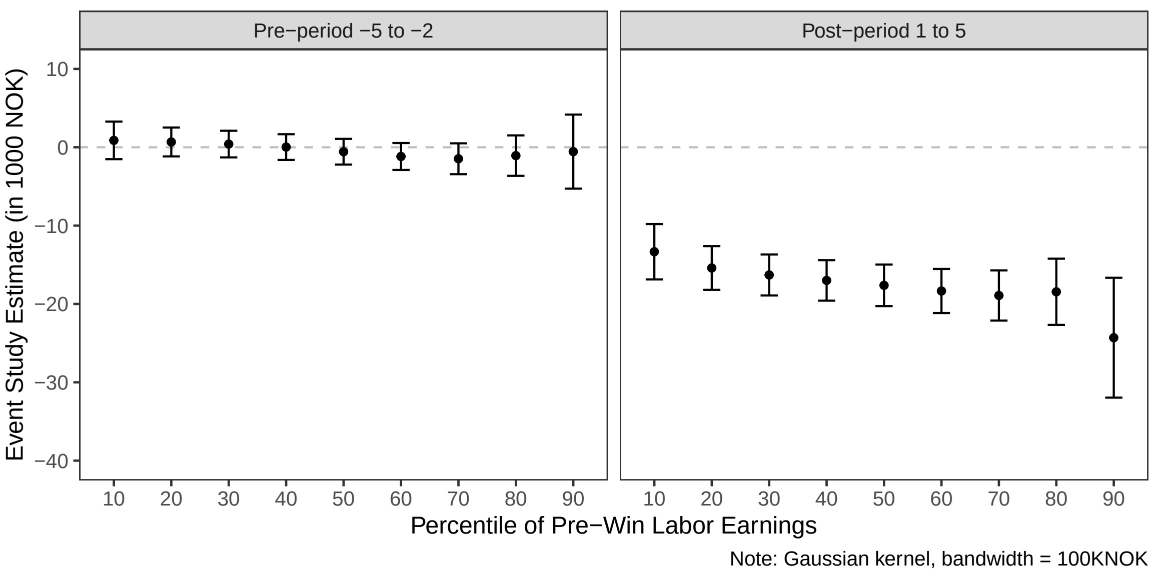
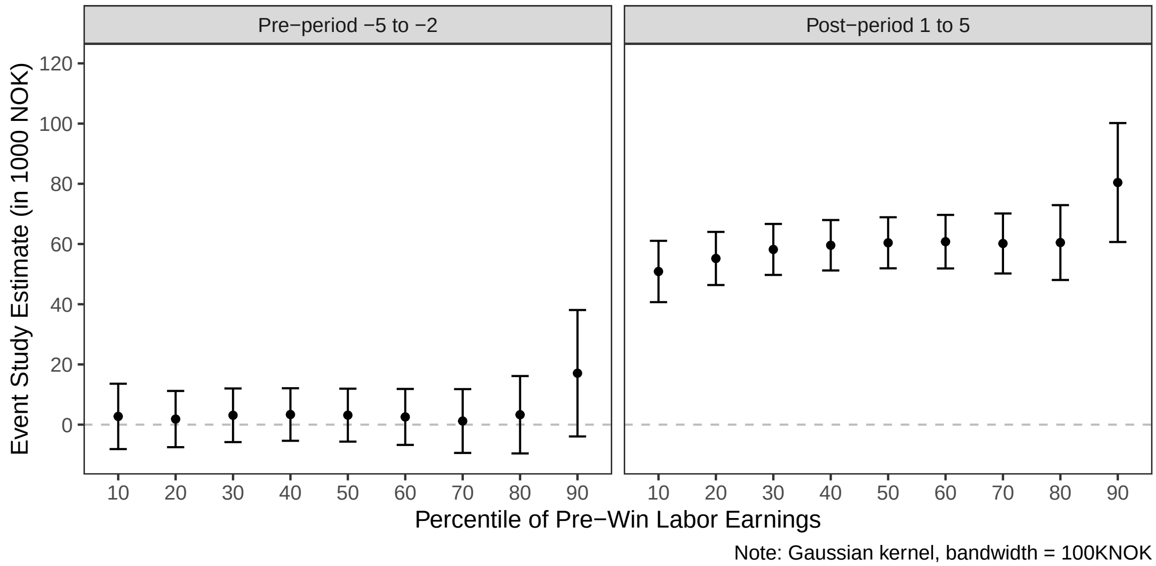


Figure 6: Earnings effects of winning

Notes: This figure presents estimates of the effect of winning the lottery on winners' earnings. Panel (a) presents unconditional estimates of the effect of winning the lottery on winners' earnings for each event time t. The estimates correspond to the sample analogue of equation (25), evaluated without conditioning on X, controlling for age, and averaged across cohorts using cohort-size weights. The estimates in Panel (b) correspond to the sample analogs of equation (25), evaluated at different values of X. We use a Gaussian kernel with a bandwidth of 100,000 NOK, where the evaluation points $\{x\}$ correspond to the cohort-specific deciles of pre-win earnings. For each decile, we compute cohort-size weighted averages across cohorts and report equally weighted averages across event times $t = -5$ to -2 and $t = +1$ to $+5$. 90 percent confidence intervals are shown, with standard errors clustered at the winner level. Throughout, we use $g - 1$ as the omitted event time.

(a) Effect on unearned income



(b) Earnings effect of unearned income

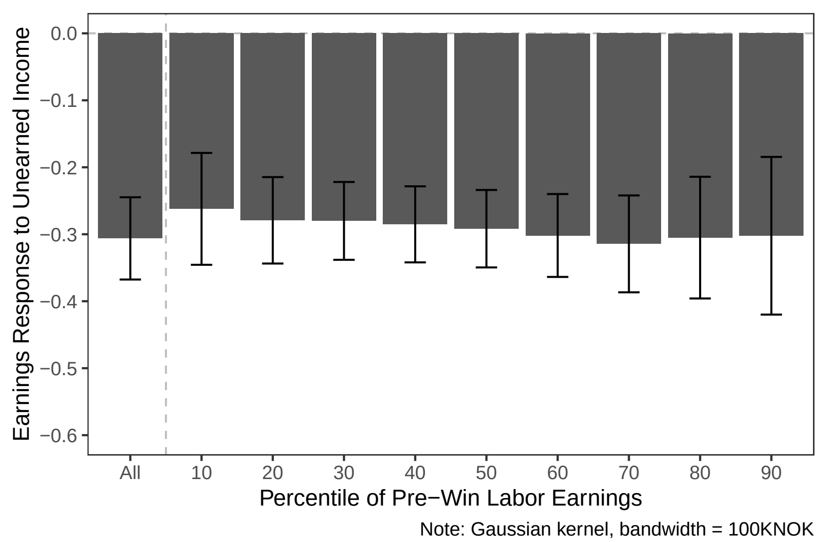


Figure 7: Unearned income effects of winning and IV estimates

Notes: This figure presents estimates of the effect of winning the lottery on winners' unearned income and the earnings response per unearned income. The estimates in Panel (a) correspond to the sample analogs of equation (26), evaluated at different values of X. We use a a Gaussian kernel with a bandwidth of 100,000 NOK, where the evaluation points $\{x\}$ correspond to the cohort-specific deciles of pre-win earnings. For each decile, we compute cohort-size weighted averages across cohorts and report equally weighted averages across event times $t = -5$ to -2 and $t = +1$ to $+5$. The estimates in Panel (b) correspond to the ratio between the aggregated values of $\text{RF}_{t,g}(x)$ and $\text{FS}_{t,g}(x)$ evaluated at different values of x. 90 percent confidence intervals are shown, with standard errors clustered at the winner level. Throughout, we use $g-1$ as the omitted event time.

In estimation, we condition on pre-win earnings and aggregate across cohorts and event time as in Figure 6. Panel (a) of Figure 7 presents the resulting estimates and shows that winning the lottery increases unearned income throughout the income distribution. The effect equals around 50,000 NOKs for the lowest decile and gradually increases to around 80,000 NOKs in the highest decile. The figure also shows no pre-trends throughout the distribution.

To measure the earnings response per additional NOK of unearned income, we take the ratio between the reduced form in equation (25) and the first stage in equation (26). This parameter captures how earnings change for lottery winners relative to individuals with the same pre-win earnings who have not yet won, scaled by the corresponding change in unearned income induced by the prize.

Panel (b) of Figure 7 reports these IV estimates across the distribution of pre-win earnings. It shows that an additional NOK of unearned income reduces earnings by approximately 0.3 NOK throughout the pre-win distribution. This number aligns closely with the estimates for the bottom quartile from Golosov et al. (2024), but is lower than their reported average of 50 cents, mainly because higher-income US households are more responsive.

Step 3: recovering the intensive-margin income effect $\eta(x)$. The estimates in Figure 7 reflect a combination of intensive and extensive-margin labor supply responses, whereas our target parameter $\eta(x)$ is the intensive-margin labor supply response to changes in unearned income. To recover intensive-margin income effects, we first decompose the total earnings response into its intensive- and extensive-margin components.[12] The decomposition relies on the additional assumption that, conditional on pre-win earnings, the earnings of individuals who continue working after winning would have evolved in parallel with the earnings of working individuals who have not yet won.

Panel (a) of Figure 8 plots the results from the decomposition. It shows that the intensive-margin component is by far the most important one, typically accounting for more than 75 percent of the total response. The intensive-margin contribution equals the share of individuals who respond on the margin times their response. To obtain the intensive-margin income response, we must therefore divide the intensive-margin contribution by the share that responds on the intensive margin, i.e., the share that continues to work after winning.

Panel (b) plots the resulting intensive-margin responses. It shows that, although adjusting for extensive-margin behavior reduces the magnitude of the estimates slightly, they remain close to a 0.25 NOK reduction in earnings per NOK of unearned income and continue to exhibit stability across the income distribution. Finally, to arrive at our estimate of $\eta(x)$, we multiply the income effect reported in Panel (b) by the net-of-tax rate at the relevant earnings level.

[12] See Appendix C for the formal derivation of the decomposition.

(a) Unearned income response decomposition. (b) Income effect.

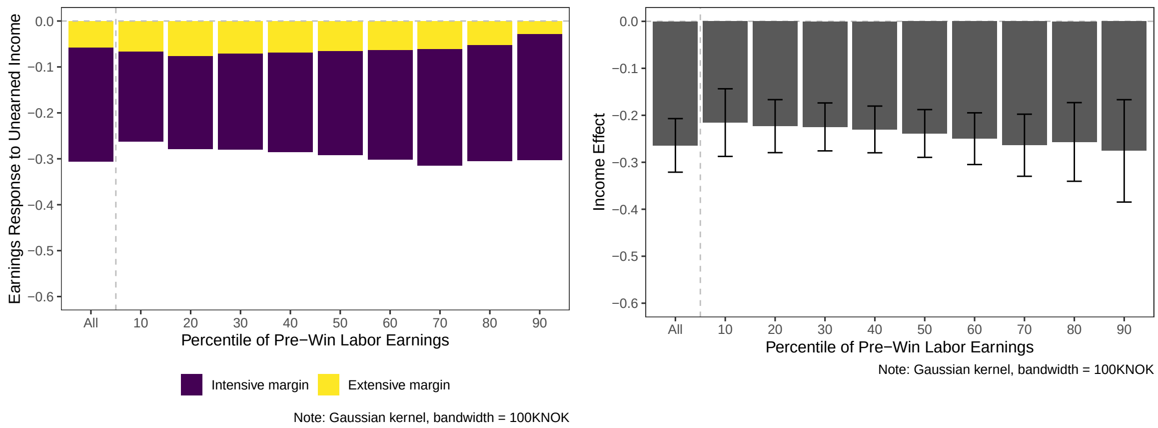


Figure 8: Total and intensive-margin income effects.

Notes: Panel (a) decomposes the earnings response per extra NOK of unearned income into its intensive- and extensive- margin components according to Equation 68. Panel (b) plots the intensive-margin income effects conditional on pre-win labor earnings. 90 percent confidence intervals are shown, with standard errors clustered at the winner level. Throughout, we use $g-1$ as the omitted event time.

5 Revenue-maximizing tax rates and excess burden

In this section, we use our estimates of income and substitution effects to (a) calculate the revenue-maximizing top-income tax rate, and (b) quantify the efficiency cost of taxation.

Tax revenue. To answer questions (a) and (b), it is necessary to model the government's budget constraint. In addition to taxing labor income and consumption, the Norwegian tax system imposes a proportional payroll tax τ_w. This implies the tax revenue collected from an individual with consumption bundle $(C_1, ..., C_K)$ and earnings Y equals,

$$TR(C_1, ..., C_K, Y) \equiv \underbrace{\sum_{k=1}^{K} \tau_k p_k C_k}_{\text{consumption tax}} + \overbrace{T(Y)}^{\text{income tax}} + \underbrace{\tau_w Y}_{\text{payroll tax}}, \qquad (27)$$

where it is useful to recall that τ_k is the tax rate on consumption good k. Since the consumption tax can vary across consumption goods, the revenue effect of changes in the income tax system will depend on the response of each of the K consumption goods.

To solve this identification problem, we specialize our model from Section 4 by assuming that individual preferences are separable between the consumption and labor supply components:

$$U(C_1, ..., C_K, Y_1, ..., Y_L) = \underbrace{U_c(C_1, ..., C_K)}_{\text{utility from consumption}} + \overbrace{U_y(Y_1, ..., Y_L)}^{\text{disutility from labor}}. \tag{28}$$

By standard two-stage budgeting arguments, this means uncompensated demand for the k-th consumption good can be written as,

$$C_k = C_k^u\Big(\tau_1, ..., \tau_K, \underbrace{I^u(\tau_1, ..., \tau_K, \tau, R + B)}_{\equiv(1-\tau)Y^u(\tau_1,...,\tau_K,\tau,R+B)+R+B}\Big), \tag{29}$$

which means that the income tax system τ and R only affect the consumption bundle through changes in disposable income I. Using these demand functions, the average marginal-propensity-to-spend-weighted consumption tax rate, defined as

$$\tilde{\tau} \equiv \frac{\sum_{k=1}^K \tau_k p_k \frac{\partial C_k^u}{\partial I}}{\sum_{k=1}^K p_k \frac{\partial C_k^u}{\partial I}}, \tag{30}$$

allows us to summarize how changes in disposable income affect the revenue from consumption taxes. If each consumption good is normal, then $\tilde{\tau}$ is bounded between the lowest and largest consumption tax rate, which corresponds to 0 and 0.25 in the Norwegian tax system. In our application, we assume that the marginal-propensity-to-spend-weighted consumption tax rate is unknown but bounded $\tilde{\tau} \in [0, 0.25]$, and that it does not vary across individuals.[13]

5.1 Efficiency cost of taxation

We now turn to quantifying the efficiency cost of income taxation as measured by the marginal deadweight loss. It is defined as the government's cost of increasing the marginal tax rate slightly while using lump-sum transfers to ensure the consumer is as well off as before.

It is useful to start by defining the deadweight loss. We introduce the consumers' compensated demand functions and expenditure function E, obtained through solving

$$\begin{aligned} E(\tau_1, ..., \tau_K, \tau, \bar{V}) \equiv \min_{C_1,...,C_K,Y_1,...,Y_L} & \sum_{k=1} (1 + \tau_k) p_k C_k - (1 - \tau) F(Y_1, ..., Y_L) \\ & \text{subject to } U(C_1, ..., C_K, Y_1, ..., Y_L) \geq \bar{V}, \end{aligned} \tag{31}$$

where $\bar{V}$ is individual utility after after the 2006 tax reform. We write their resulting compen-

[13] This is true if, for example, the consumption component of the utility function is homothetic and homogeneous across individuals.

sated demand functions as $C_k^c(\tau_1, ..., \tau_K, \tau, \bar{V})$ and the earnings function as $Y^c(\tau_1, ..., \tau_K, \tau, \bar{V}) \equiv F(Y_1^c(\tau_1, ..., \tau_K, \tau, \bar{V}), ..., Y_L^c(\tau_1, ..., \tau_K, \tau, \bar{V}))$. Using this, the deadweight loss of taxation, defined as the expenditure needed to keep an individual at utility level $\bar{V}$, net of the revenue raised, can be written as,

$$DWL(\tau_1, ..., \tau_K, \tau, \bar{V}) \equiv E(\tau_1, ..., \tau_K, \tau, \bar{V}) - TR^c(\tau_1, ..., \tau_K, \tau, \bar{V}),$$

where the tax revenue is given by,

$$TR^c(\tau_1, ..., \tau_K, \tau, \bar{V}) \equiv \sum_{k=1}^{K} \tau_k p_k C_k^c(\tau_1, ..., \tau_K, \tau, \bar{V}) + (\tau + \tau_w) Y^c(\tau_1, ..., \tau_K, \tau, \bar{V}).$$

The marginal deadweight loss is obtained by taking the derivative of DWL with respect to τ. It is often normalized by the marginal tax revenue, i.e., the derivative of TR^c with respect to τ. The resulting measure is called the excess burden EB and measures the economic cost of increasing the marginal tax rate slightly per additional dollar of revenue raised:

$$EB \equiv \frac{\mathbb{E}\left[\frac{\partial DWL}{\partial \tau} \mid Y \geq \bar{Y}\right]}{\mathbb{E}\left[\frac{\partial TR^c}{\partial \tau} \mid Y \geq \bar{Y}\right]}. \tag{32}$$

As we show in Appendix D.1, the excess burden can be written as

$$EB = \frac{\mathbb{E}\left[\left(\frac{\tilde{\tau}}{1+\tilde{\tau}} + \frac{\tau+\tau_w}{1-\tau}\right) \varepsilon^c Y \mid Y \geq \bar{Y}\right]}{\mathbb{E}\left[\left(1 - \left(\frac{\tilde{\tau}}{1+\tilde{\tau}} + \frac{\tau+\tau_w}{1-\tau}\right) \varepsilon^c\right) Y \mid Y \geq \bar{Y}\right]}. \tag{33}$$

In words, the excess burden EB can be interpreted as the economic cost of a marginal increase in the marginal tax rate while ensuring the individuals are equally well off as before, relative to the additional revenue raised.

Results. Using our data and estimates of ε^c from Section 4, it is straightforward to calculate the excess burden in equation (33). We refer to the excess burden obtained by using our estimates of compensated elasticities from Panel (a) in Figure 5 as our preferred specification. For comparison, we also calculate excess burden under the conventional specification, which assumes homogeneous ETIs and no income effects. Under these assumptions, $\varepsilon^c = 0.16$ and $\eta = 0$ for all individuals.[14]

Panel (a) of Figure 9 plots the excess burden implied by these two specifications as functions of the marginal-propensity-to-spend-weighted consumption tax rate $\tilde{\tau}$.[15] The solid

[14] To see why, note that under homogeneous ETIs and no income effects, Corollary 2 and Proposition 3 together imply that the ETI estimand with $h(Z) = Z$ recovers the compensated elasticity. Hence $\beta = 0.16 = \zeta = \varepsilon^c$.

[15] We consider this range because Norwegian VAT rates vary between 0 and 0.25 depending on the good.

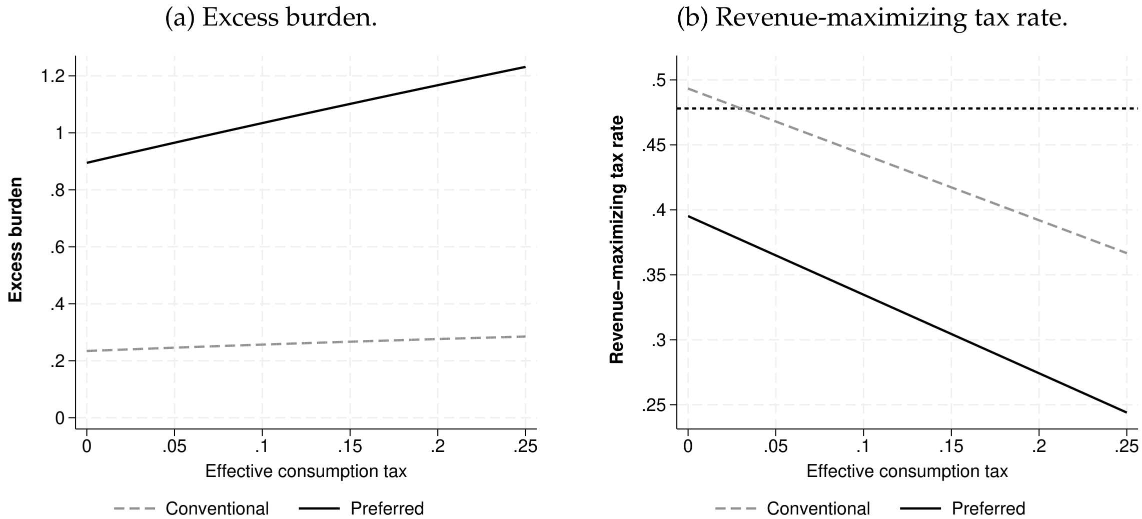


Figure 9: Top-income tax rates and excess burden.

Notes: Panel (a) plots the revenue-maximizing top-income tax rates as a function of $\tilde{\tau}$ using our preferred estimates (the solid black line) and using the estimates obtained under the assumption of homogenous uncompensated elasticities and no income effects (dashed grey line). The dotted line shows the actual top-income tax rate in Norway after the 2006 reform. Panel (b) plots the excess burden of taxation as a function of $\tilde{\tau}$ using the same two sets of estimates.

black line shows the results using our preferred specification: the excess burden is at least 0.9, meaning that at least 90 cents are lost per dollar of additional tax revenue. In contrast, the gray dashed line uses the conventional elasticities and implies a more modest excess burden of 0.2-0.25, depending on the exact value of $\tilde{\tau}$. We conclude that ignoring income effects and heterogeneity in elasticities could lead to severely downwards-biased estimates of the excess burden. In our setting, actual efficiency costs of taxation are 4-5 times larger than suggested by the conventional specification.

5.2 Revenue-maximizing tax rate

We now turn to deriving the revenue-maximizing tax rate. It is useful to start by considering the tax revenue collected from individuals with earnings above $\bar{Y}$:

$$\begin{aligned}\mathbb{E}[TR \mid Y \geq \bar{Y}] = \overbrace{\sum_{k=1}^{K} \tau_k p_k \, \mathbb{E}[C_k \mid Y \geq \bar{Y}]}^{\text{consumption tax}} + \overbrace{\tau \, \mathbb{E}[Y - \bar{Y} \mid Y \geq \bar{Y}] + T(\bar{Y})}^{\text{income tax}} \\ + \underbrace{\tau_w \, \mathbb{E}[Y \mid Y \geq \bar{Y}]}_{\text{payroll tax}}.\end{aligned} \tag{34}$$

The first-order condition to the revenue-maximization problem is obtained by taking the derivative of equation (34) with respect to the top-income tax rate. As we show in Appendix D.2, this first-order condition implies that the following equation is satisfied when the top-income tax rate maximizes revenue,

$$\tau_{\text{TOP}} = \frac{\alpha - 1 - (\tilde{\tau} + \tau_w + \tau_w \tilde{\tau})\,(\alpha \bar{\varepsilon}^u - \bar{\eta})}{\alpha - 1 + (\alpha \bar{\varepsilon}^u - \bar{\eta})}, \tag{35}$$

with

$$\bar{\varepsilon}^u \equiv \frac{\mathbb{E}[Y^* \varepsilon^u \mid Y^u \geq \bar{Y}]}{\mathbb{E}[Y^u \mid Y^u \geq \bar{Y}],} \qquad \bar{\eta} \equiv \mathbb{E}[\eta \mid Y^u \geq \bar{Y}], \qquad \alpha \equiv \frac{\mathbb{E}[Y^y \mid Y^u \geq \bar{Y}]}{\bar{Y}}, \tag{36}$$

where the expectations are taken across individuals in the top-income tax bracket, and $Y^u, \varepsilon^u, \eta, \alpha$, and $\tilde{\tau}$ are evaluated at the revenue-maximizing tax system.

Results. We consider the revenue-maximizing top-income tax rate for income above $\bar{Y} = 450{,}000$ NOK. Following Saez (2001) and Saez and Stantcheva (2016), we assume that the weighted elasticities $\bar{\varepsilon}^u$, $\bar{\eta}$, and the Pareto parameter α are unaffected by the top tax rate. Using the estimates in Section 4, we obtain $\alpha = 1.46$, $\bar{\varepsilon}^u = 0.25$, and $\bar{\eta} = -0.15$. We refer to this set of (weighted-average) elasticities as our preferred specification. For comparison, we also examine a conventional specification that assumes homogeneous ETIs and no income effects. Under these assumptions, $\bar{\varepsilon}^u = 0.16$ and $\bar{\eta} = 0$.[16]

Panel (b) of Figure 9 plots the revenue-maximizing top-income tax rates implied by these two specifications as functions of the marginal-propensity-to-spend-weighted consumption tax rate $\tilde{\tau}$.[17] The solid black line shows the results using our preferred specification: the revenue-maximizing top rate declines with $\tilde{\tau}$ and lies between 0.25 and 0.40 for the relevant range. The gray dashed line uses the conventional elasticities and yields revenue-maximizing rates roughly ten percentage points higher throughout.

Interestingly, our estimates imply that the actual top-income tax rate after the 2006 reform, illustrated by the dotted black line, exceeds the revenue-maximizing level for any $\tilde{\tau}$. This means that the government could increase revenue by *reducing* top-income tax rates. Notably, the conclusion remains true even under the conventional specification, provided $\tilde{\tau}$ is positive.

[16] Under homogeneous ETIs and no income effects, Corollary 2 and Proposition 3 imply that the ETI estimand with $h(Z) = Z$ recovers the compensated elasticity. Hence $\beta = 0.16 = \zeta = \varepsilon^c = \varepsilon^u = \bar{\varepsilon}^u$.

[17] We consider this range because Norwegian VAT rates vary between 0 and 0.25 depending on the good.

6 Conclusions

This paper examined both the causal and the economic interpretation of the ETI estimand. We provided sufficient and necessary conditions for when the ETI estimand can be given a causal interpretation as a positively weighted average of heterogeneous individual elasticities of taxable income. We showed that the specifications used in practice fail to satisfy these conditions. Next, we showed how the ETI estimand can be used to learn about compensated and uncompensated elasticities across the income distribution. By using model-implied restrictions, we showed how to construct bounds on these elasticities. We also showed that point identification could be achieved by either assuming homogeneous income and substitution effects or by using additional data to estimate income effects directly.

We then used these results to analyze a reform of the Norwegian tax system that reduced the marginal tax rates on middle and high-income earners. We found that commonly used ETI estimators understate the average elasticity of taxable income and that specifications that can be given a causal interpretation produce estimates that indicate substantial heterogeneity across the income distribution. Using these estimates, we constructed bounds that showed that the compensated and uncompensated earnings elasticities of high-income individuals are larger than 0.7 and 0.5, respectively. We then showed that imposing constant substitution and income effects across all individuals is at odds with the data and conclude that the assumption poorly approximates individual labor supply behavior, at least in our context.

To do so, we used our Norwegian data on lottery winners to estimate income effects. By combining these estimates of income effects with our ETI estimates, we learned that the compensated elasticity is small for middle incomes but increases rapidly with income. Uncompensated elasticities tend to be positive, implying that substitution effects are larger than the income effects, especially for higher incomes. We applied our estimates to measure revenue-maximizing top-income tax rates and the excess burden, and found that the actual top-income tax rate exceeds the revenue-maximizing one and that the excess burden of taxation is substantial.

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A Proofs

Many of our results in Section 2 will build on the following two lemmas, which maps the ETI estimand β to the potential outcomes for any h provided f is unrestricted.

Lemma 1. *Suppose Assumptions 1 and 2 are true and let f be unrestricted over the support of X. Then, the ETI estimand equals a weighted average conditional-on-X elasticities of taxable income,*

$$\beta = \sum_{j=k_0}^{K} \omega_j^h \zeta(x_j), \tag{37}$$

where,

$$\omega_j^h \equiv \frac{p_j \phi(x_j)\, \mathbb{E}[D^h \mid G = 1, X = x_j]}{\sum_{l=k_0}^{K} p_l \phi(x_l)\, \mathbb{E}[D^h \mid G = 1, X = x_l]},$$

where $p_j \equiv \mathbb{P}(X = x_j \mid G = 1)$, and D^h is the predicted residuals resulting from regressing $Gh(Z)$ on G and $f(X;.)$ and

$$\begin{aligned}
\phi(x) &\equiv \mathbb{E}[\text{NTR}(1) - \text{NTR}(0) \mid G = 1, X = x], \\
\zeta(x) &\equiv \mathbb{E}\left[\frac{\text{NTR}(1) - \text{NTR}(0)}{\mathbb{E}[\text{NTR}(1) - \text{NTR}(0) \mid G = 1, X = x]} \times \zeta \mid G = 1, X = x\right].
\end{aligned}$$

Proof of Lemma 1. We start by deriving the expression for the ETI estimand. This requires some additional notation.

For some fixed function h, the Frisch-Waugh-Lovell theorem allows for expressing the ETI estimand as,

$$\beta \equiv \frac{\mathbb{E}\left[\Delta y D^h\right]}{\mathbb{E}\left[\Delta\, \text{NTR}\, D^h\right]}, \tag{38}$$

where

$$D^h \equiv Gh(Z) - \theta_0 G - f(X;\theta), \tag{39}$$

is predicted residuals from the linear projection of $Gh(Z)$ on $(G, f(X;.))$ where the coefficients are given as the solution to,

$$(\theta_0, \theta) \equiv \arg\min_{\theta_g, \theta_x} \mathbb{E}\left[\left(Gh(Z) - \theta_g G - f(X;\theta_x)\right)^2\right]. \tag{40}$$

Since f is unrestricted, it can be written as:

$$f(x;\theta) = \sum_{k=1}^{K} \theta_k \mathbb{1}[x = x_k], \tag{41}$$

where $\{x_1, ..., x_K\}$ is the support of X.

It is useful to denote by $\rho \equiv y(1) - y(0)$ the earnings response to the tax reform. Then, by recalling equation (5), we get that $y = y(0) + \rho \mathbb{1}[Z \neq 0]G$, which implies that $\Delta y = \Delta y(0) + \rho \mathbb{1}[Z \neq 0]G$. Using equation (8) from Assumption 1, we can write,

$$\Delta y = \lambda^y G + \rho \mathbb{1}[Z \neq 0]G + f_y(X) + u, \tag{42}$$

with $\mathbb{E}[u \mid G, X] = 0$. Plugging (42) into the numerator of the ETI estimand in equation (38) gives,

$$\begin{aligned}
\mathbb{E}[\Delta y D^h] &= \mathbb{E}\left[\left(\lambda^y G + \rho \mathbb{1}[Z \neq 0]G + f_y(X) + u\right) D^h\right], \\
&= \mathbb{E}\left[\rho \mathbb{1}[Z \neq 0] G D^h\right], \\
&= \mathbb{P}(GZ \neq 0)\, \mathbb{E}[\rho D^h \mid GZ \neq 0],
\end{aligned}$$

where the second equality follows from the fact that $\mathbb{E}[u \mid G, X] = 0$ and that the moment conditions that determine D^h since the unrestricted specification of f ensures that $f(\cdot;\theta) = f_y(\cdot)$ for some θ. The third equality follows from the law of total expectations.

Inserting for the linear projection D^h,

$$\mathbb{E}\left[\Delta y D^h\right] = \mathbb{P}(GZ \neq 0)\, \mathbb{E}\left[\rho D^h \mid GZ \neq 0\right],$$

Similar reasoning yields the following expression for the denominator of equation (38),

$$\mathbb{E}\left[\Delta \, \mathrm{NTR}\, D^h\right] = \mathbb{P}(GZ \neq 0)\, \mathbb{E}\left[\phi D^h \mid GZ \neq 0\right].$$

By combining the two terms, we obtain,

$$\begin{aligned}
\beta &= \frac{\mathbb{E}\left[\phi D^h \times \zeta \mid GZ \neq 0\right]}{\mathbb{E}\left[\phi D^h \mid GZ \neq 0\right]}, \\
&= \frac{\mathbb{E}\left[\phi D^h \times \zeta \mid G = 1, X \geq x_{k_0}\right]}{\mathbb{E}\left[\phi D^h \mid G = 1, X \geq x_{k_0}\right]}
\end{aligned}$$

where we have used that $\mathbb{P}(GZ \neq 0)$ cancels and that Assumption 2 implies that $\rho = \phi \times \zeta$. From the definition of the simulated instrument Z, it is clear that X deterministically determines Z. Thus, D^h is a deterministic function of G and X, meaning we can write the realization of D^h for an individual with $G = g$ and $X = x$ as $D^h(g, x)$. We obtain,

$$\begin{aligned}
\beta &= \frac{\sum_{j=k_0}^{K} p_j D^h(1, x_j)\, \mathbb{E}\left[\phi \times \zeta \mid G = 1, X = x_j\right]}{\sum_{j=k_0}^{K} p_j D^h(1, x_j)\, \mathbb{E}\left[\phi \mid G = 1, X = x_j\right]} \\
&= \frac{\sum_{j=k_0}^{K} p_j D^h(1, x_j) \phi(x_j) \zeta(x_j)}{\sum_{j=k_0}^{K} p_j D^h(1, x_j) \phi(x_j)}, \\
&= \sum_{j=k_0}^{K} \frac{p_j D^h(1, x_j) \phi(x_j)}{\sum_{m=k_0}^{K} p_m D^h(1, x_m) \phi(x_m)} \times \zeta(x_j).
\end{aligned}$$

The result follows from noting that $D^h(1, x_k) = \mathbb{E}[D^h \mid G = 1, X = x_k]$ and using the definition of ω_j^h.

□

Lemma 2. *Suppose Assumptions 1 and 2 are true and let f be flexible. Then,*

$$D^h(1,x_k) = \mathbb{P}(G=0 \mid X=x_k)\left(\mathbb{E}[h(Z) \mid X=x_k] - \frac{\sum_{j=1}^K p_j\, \mathbb{P}(G=0 \mid X=x_j)\, \mathbb{E}[h(Z) \mid X=x_j]}{\sum_{j=1}^K p_j\, \mathbb{P}(G=0 \mid X=x_j)}\right).$$

Proof of Lemma 2. When f is unrestricted, the first-order conditions to the minimization problem in equation (40) yield the following moment conditions

$$\mathbb{E}\left[G\left(Gh(Z) - \theta_0 - \sum_{k=1}^K \theta_k \mathbb{1}[X=x_k]\right)\right] = 0, \tag{43}$$

$$\mathbb{E}\left[\mathbb{1}[X=x_k]\left(Gh(Z) - \theta_0 G - \sum_{j=1}^K \theta_j \mathbb{1}[X=x_j]\right)\right] = 0, \tag{44}$$

for $j = 1, ...K$. From (44), we obtain

$$\mathbb{E}\left[Gh(Z) - \theta_0 G - \theta_k \mid X=x_k\right] = 0.$$

Since Z is a deterministic function of X, this can be rewritten as

$$\theta_k = \mathbb{P}(G=1 \mid X=x_k)\left(\mathbb{E}[h(Z) \mid X=x_k] - \theta_0\right), \tag{45}$$

Next, we use equation (43) which implies that,

$$\mathbb{E}[h(Z) \mid G=1] = \theta_0 + \sum_{k=1}^K p_k \theta_k, \tag{46}$$

where $p_k \equiv \mathbb{P}(X=x_k \mid G=1)$. Substituting the expression of θ_k from equation (45) into the expression above yields,

$$\mathbb{E}[h(Z) \mid G=1] = \theta_0 + \sum_{k=1}^K p_k\, \mathbb{P}(G=1 \mid X=x_k)\left(\mathbb{E}[h(Z) \mid X=x_k] - \theta_0\right),$$

solving for θ_0 gives,

$$\begin{aligned}
\theta_0 &= \frac{\mathbb{E}[h(Z) \mid G=1] - \sum_{k=1}^K p_k\, \mathbb{P}(G=1 \mid X=x_k)\, \mathbb{E}[h(Z) \mid X=x_k]}{1 - \sum_{k=1}^K p_k\, \mathbb{P}(G=1 \mid X=x_k)}, \\
&= \frac{\sum_{k=1}^K p_k\left(1 - \mathbb{P}(G=1 \mid X=x_k)\right)\mathbb{E}[h(Z) \mid X=x_k]}{1 - \sum_{k=1}^K p_k\, \mathbb{P}(G=1 \mid X=x_k)}, \\
&= \frac{\sum_{k=1}^K p_k\left(1 - \mathbb{P}(G=1 \mid X=x_k)\right)\mathbb{E}[h(Z) \mid X=x_k]}{\sum_{k=1}^K p_k (1 - \mathbb{P}(G=1 \mid X=x_k))}.
\end{aligned}$$

$D^h(1, x_k)$ can then be rewritten as,

$$
\begin{aligned}
D^h(1, x_k) &= \mathbb{E}[h(Z) \mid X = x_k] - \theta_0 - \mathbb{P}(G = 1 \mid X = x_k) \left(\mathbb{E}[h(Z) \mid X = x_k] - \theta_0\right), \\
&= \left(1 - \mathbb{P}(G = 1 \mid X = x_k)\right) \left(\mathbb{E}[h(Z) \mid X = x_k] - \theta_0\right), \\
&= \left(1 - \mathbb{P}(G = 1 \mid X = x_k)\right) \\
&\times \left(\mathbb{E}[h(Z) \mid X = x_k] - \frac{\sum_{j=1}^{K} p_j \left(1 - \mathbb{P}(G = 1 \mid X = x_j)\right) \mathbb{E}[h(Z) \mid X = x_j]}{\sum_{j=1}^{K} p_j (1 - \mathbb{P}(G = 1 \mid X = x_j))} \right), \\
&= \mathbb{P}(G = 0 \mid X = x_k) \\
&\times \left(\mathbb{E}[h(Z) \mid X = x_k] - \frac{\sum_{j=1}^{K} p_j \, \mathbb{P}(G = 0 \mid X = x_j) \, \mathbb{E}[h(Z) \mid X = x_j]}{\sum_{j=1}^{K} p_j \, \mathbb{P}(G = 0 \mid X = x_j)} \right),
\end{aligned}
$$

where the first equality uses equation (45) and the second substituties in the expression for θ_0.

□

Proof of Proposition 1. We start by proving the if-part of the statement.

If $h(Z)$ is binary and f is flexible then β is causal: Since $h(Z)$ is binary we can write $h(Z) = h_0 \mathbb{1}[Z = 0] + h_1 \mathbb{1}[Z \neq 0]$. Lemma 2 then implies that

$$
\begin{aligned}
D^h(1, x_k) =& \, \mathbb{P}(G = 0 \mid X = x_k) \\
&\times \left(h_1 - \frac{h_0 \sum_{j=1}^{k_0 - 1} p_j \, \mathbb{P}(G = 0 \mid X = x_j) + h_1 \sum_{j=k_0}^{K} p_j \, \mathbb{P}(G = 0 \mid X = x_j)}{\sum_{j=1}^{K} p_j \, \mathbb{P}(G = 0 \mid X = x_j)} \right), \\
=& \, \mathbb{P}(G = 0 \mid X = x_k) \\
&\times \left(h_0 + (h_1 - h_0) - \frac{(h_1 - h_0) \sum_{j=k_0}^{K} p_j \, \mathbb{P}(G = 0 \mid X = x_j)}{\sum_{j=1}^{K} p_j \, \mathbb{P}(G = 0 \mid X = x_j)} - h_0 \right), \\
=& \, \mathbb{P}(G = 0 \mid X = x_k) \times (h_1 - h_0) \times \left(1 - \frac{\sum_{j=k_0}^{K} p_j \, \mathbb{P}(G = 0 \mid X = x_j)}{\sum_{j=1}^{K} p_j \, \mathbb{P}(G = 0 \mid X = x_j)} \right).
\end{aligned}
$$

for any $x_k \in \{x_{k_0}, ..., x_K\}$. According to Lemma 1, $D^h(1, x_k)$ appears both in the numerator and denominator of the expression for β. Since the two last components of $D^h(1, x_k)$ does not vary with x_k, they cancel and we obtain:

$$
\beta = \sum_{j=k_0}^{K} \frac{\mathbb{P}(X = x_j \mid G = 1) \phi(x_j) \, \mathbb{P}(G = 0 \mid X = x_j)}{\sum_{l=k_0}^{K} \mathbb{P}(X = x_l \mid G = 1) \phi(x_l) \, \mathbb{P}(G = 0 \mid X = x_l)} \zeta(x_j). \tag{47}
$$

By Bayes law: $\mathbb{P}(X = x_j \mid G = 1) = \mathbb{P}(G = 1 \mid X = x_j)\,\mathbb{P}(X = x_j)$, so we obtain,

$$\beta = \sum_{j=k_0}^{K} \frac{\mathbb{P}(X = x_j)\,\mathbb{P}(G = 1 \mid X = x_j)\,\mathbb{P}(G = 0 \mid X = x_j)\phi(x_j)}{\sum_{l=k_0}^{K} \mathbb{P}(X = x_l)\,\mathbb{P}(G = 1 \mid X = x_l)\,\mathbb{P}(G = 0 \mid X = x_l)\phi(x_l)} \zeta(x_j), \tag{48}$$

$$= \sum_{j=k_0}^{K} \frac{\mathbb{P}(X = x_j) Var(G \mid X = x_j)\phi(x_j)}{\sum_{l=k_0}^{K} \mathbb{P}(X = x_l) Var(G \mid X = x_l)\phi(x_l)} \zeta(x_j), \tag{49}$$

where the second equality uses that G follows a Bernoulli distribution, implying that $\mathbb{P}(G = 1 \mid X = x_j)\,\mathbb{P}(G = 0 \mid X = x_j) = Var(G \mid X = x_j)$.

Assumption 3, ensures that $\zeta(x_k)$ is a positively weighted average ζ and that the sign of $\phi(x_k)$ does not change with k. Since probabilities and variances are non-negative, it follows that the ETI estimand recovers a positively weighted average of individual elasticities.

If $h(Z)$ is not binary and f is flexible then β is not causal: For β to be causal, it must be the case that the sign of,

$$\left(\mathbb{E}[h(Z) \mid X = x_k] - \frac{\sum_{j=1}^{K} p_j\,\mathbb{P}(G = 0 \mid X = x_j)\,\mathbb{E}[h(Z) \mid X = x_j]}{\sum_{j=1}^{K} p_j\,\mathbb{P}(G = 0 \mid X = x_j)} \right) \tag{50}$$

does not change with $k_0 \geq k \leq K$ for any distribution of X. We prove that it is always possible to construct a distribution of X where these weights change sign in the case when $h(Z)$ takes on more than two values. Let,

$$h(Z) = \begin{cases} h_0 & \text{if } X < \bar{Y}, \\ \sum_{k=k_0}^{K} \mathbb{1}[X = x_k] h_k & \text{if } X = \bar{Y}. \end{cases} \tag{51}$$

for $(h_0, h_{k_0}, ..., h_K)$ is the support of $h(Z)$. Using this, we can write the weights as

$$\sum_{k=k_0}^{K} h_k \mathbb{1}[X = x_k] - \frac{\sum_{j=1}^{K} \mathbb{P}(X = x_j) Var(G \mid X = x_j) h_k}{\sum_{j=1}^{K} \mathbb{P}(X = x_j) Var(G \mid X = x_j)}.$$

The last term is a convex combination of the points in the support of $h(Z)$. This means that, depending on the distribution of X, it can take on any value between the smallest and largest value of h_k. Thus, it is always possible to construct a distribution of X such that the convex combination is between any two values $h_{k'}, h_{k''}$ with $k', k'' \geq k_0$, meaning negative weights will emerge.

If $h(Z)$ is binary and f is not flexible then β is not causal: Suppose that ζ is constant across individuals and consider the IV regression model:

$$\Delta y = \alpha_0^y G + \beta \Delta\,\text{NTR} + f(X; \alpha^y) + u^y, \tag{52}$$

$$\Delta\,\text{NTR} = \alpha_0^{\text{NTR}} G + \alpha G \mathbb{1}[Z \neq 0] + f(X; \alpha^{\text{NTR}}) + u^{\text{NTR}}. \tag{53}$$

Assumptions 1 and 2 then implies that

$$\Delta y = \lambda^y G + f(X;\alpha^y) + \zeta\Delta\,\text{NTR} + u^y. \tag{54}$$

with $u^y \equiv f_y(X) - f(X;\theta) + \Delta y(0) - \mathbb{E}[\Delta y(0) \mid X, G]$. The standard exogeneity assumption requires that $Cov(u^y, G\mathbb{1}[Z \neq 0]) = 0$. Note that,

$$\begin{aligned}
Cov(u^y, G\mathbb{1}[Z \neq 0]) &= Cov(f_y(X) - f(X;\theta) + \Delta y(0) - \mathbb{E}[\Delta y(0) \mid X, G], G\mathbb{1}[Z \neq 0]),\\
&= Cov(f_y(X) - f(X;\theta), G\mathbb{1}[Z \neq 0])\\
&= \mathbb{P}(GZ \neq 0)\Big(\mathbb{E}[f_y(X) - f(X;\theta) \mid GZ \neq 0]\\
&\qquad\qquad - \mathbb{E}[f_y(X) - f(X;\theta)]\Big).
\end{aligned}$$

Unless f is unrestricted so that $f_y(X) = f(X;\theta)$ for all X, $\mathbb{E}[f_y(X) - f(X;\theta) \mid GZ \neq 0] \neq \mathbb{E}[f_y(X) - f(X;\theta)]$ meaning that exogeneity generally fails. $\square$

Proof of Corollary 1. From the proof of Proposition 1, we have that

$$\beta = \sum_{j=k_0}^{K} \frac{\mathbb{P}(X = x_j)Var(G \mid X = x_j)\phi(x_j)}{\sum_{l=k_0}^{K} \mathbb{P}(X = x_l)Var(G \mid X = x_l)\phi(x_l)}\zeta(x_j). \tag{55}$$

Note that by the law of iterated expectaions we obtain

$$\begin{aligned}
\zeta(x_k) &= \sum_{j=1}^{J} \frac{\mathbb{P}(\phi = \phi_j \mid G = 1, X = x_k)\phi_j}{\sum_{l=1}^{J} \mathbb{P}(\phi = \phi_l \mid G = 1, X = x_k)\phi_l}\,\mathbb{E}[\zeta \mid G = 1, X = x_k, \phi = \phi_j],\\
\phi(x_k) &= \sum_{j=1}^{J} \mathbb{P}(\phi = \phi_j \mid G = 1, X = x_k)\phi_j.
\end{aligned}$$

Letting $\zeta_{k,j} \equiv \mathbb{E}[\zeta \mid G=1, X=x_k, \phi=\phi_j]$ and substituting these equations into (55), we obtain

$$\begin{aligned}
\beta &= \sum_{j=k_0}^{K} \frac{\mathbb{P}(X=x_j)Var(G \mid X=x_j)\phi(x_j)}{\sum_{l=k_0}^{K}\mathbb{P}(X=x_l)Var(G \mid X=x_l)\phi(x_l)} \frac{\sum_{j=1}^{J}\mathbb{P}(\phi=\phi_j \mid G=1, X=x_k)\phi_j}{\phi(x_k)}\zeta_{k,l}, \\
&= \frac{\sum_{j=k_0}^{K}\sum_{j=1}^{J}\mathbb{P}(X=x_j)\,\mathbb{P}(\phi=\phi_j \mid G=1, X=x_k)Var(G \mid X=x_j)\phi_j\zeta_{k,l}}{\sum_{l=k_0}^{K}\mathbb{P}(X=x_l)Var(G \mid X=x_l)\phi(x_l)}, \\
&= \frac{\sum_{j=k_0}^{K}\sum_{j=1}^{J}\mathbb{P}(X=x_j, \phi=\phi_j \mid G=1)Var(G \mid X=x_j)\phi_j\zeta_{k,l}}{\sum_{l=k_0}^{K}\mathbb{P}(X=x_l)Var(G \mid X=x_l)\phi(x_l)}, \\
&= \frac{\sum_{j=k_0}^{K}\sum_{j=1}^{J}\mathbb{P}(X=x_j, \phi=\phi_j \mid G=1)Var(G \mid X=x_j)\phi_j\zeta_{k,l}}{\sum_{l=k_0}^{K}\mathbb{P}(X=x_l)Var(G \mid X=x_l)\sum_{j=1}^{J}\mathbb{P}(\phi=\phi_j \mid G=1, X=x_l)\phi_j}, \\
&= \frac{\sum_{j=k_0}^{K}\sum_{j=1}^{J}\mathbb{P}(X=x_j, \phi=\phi_j \mid G=1)Var(G \mid X=x_j)\phi_j\zeta_{k,l}}{\sum_{l=k_0}^{K}\sum_{j=1}^{J}Var(G \mid X=x_l)\,\mathbb{P}(X=x_l, \phi=\phi_j \mid G=1, X=x_l)\phi_j}, \\
&= \sum_{j=k_0}^{K}\sum_{j=1}^{J} \frac{\mathbb{P}(X=x_j, \phi=\phi_j \mid G=1)Var(G \mid X=x_j)\phi_j}{\sum_{l=k_0}^{K}\sum_{j=1}^{J}Var(G \mid X=x_l)\,\mathbb{P}(X=x_l, \phi=\phi_j \mid G=1, X=x_l)\phi_j} \times \zeta_{k,l}.
\end{aligned}$$

□

Proof of Corollary 2. When ζ is constant across individuals, then $\zeta(x_k) = \zeta(x_l) = \zeta$ for any k, l. Thus, by Lemma 1:

$$\begin{aligned}
\beta &= \sum_{j=k_0}^{K} \frac{p_j D^h(1, x_j)\phi_j}{\sum_{m=k_0}^{K} p_m D^h(1, x_m)\phi_m} \times \zeta, \\
&= \zeta \frac{\sum_{j=k_0}^{K} p_j D^h(1, x_j)\phi_j}{\sum_{m=k_0}^{K} p_m D^h(1, x_m)\phi_m} \\
&= \zeta.
\end{aligned}$$

□

Proof of Proposition 2. Start by noting that equation (5) and the definition of k_0 implies

$$\mathbb{E}[\Delta y \mid G, X] = \begin{cases} \mathbb{E}[\Delta y(0) + \rho \mid G, X] & \text{if } G=1 \text{ and } X \geq x_{k_0}, \\ \mathbb{E}[\Delta y(0) \mid G, X] & \text{otherwise.} \end{cases}$$

Thus, under assumption 1, the regression of δy on G and dummies for each value of X on the subsample with $GZ = 0$ recovers

$$\mathbb{E}[\Delta y(0) \mid G=g, X=x] = \hat{\lambda}_0^y g + \sum_{k=1}^{K} \hat{\lambda}_k^y \mathbb{1}[x = x_k],$$

for each x in the support of X. Thus, for any $x \geq x_{k_0}$, the numerator of $\beta(x)$ equals,

$$\begin{aligned}
&\mathbb{E}[\Delta y \mid G=1, X=x] - \left(\hat{\lambda}_0^y g + \sum_{k=k_0}^{K} \hat{\lambda}_k^y \mathbb{1}[x = x_k]\right), \\
=&\mathbb{E}[\Delta y(0) + \rho \mid G=1, X=x] - \mathbb{E}[\Delta y(0) \mid G=1, X=x], \\
=&\mathbb{E}[\phi \times \zeta \mid G=1, X=x], \\
=&\phi(x) \times \zeta(x),
\end{aligned}$$

where the second equality follows from Assumption 2. Corresponding arguments show that the denominator of the $\beta(x)$ equals,

$$\mathbb{E}[\Delta\,\text{NTR} \mid G=1, X=x] - \left(\hat{\lambda}_0^{\text{NTR}} g + \sum_{k=k_0}^{K} \hat{\lambda}_k^{\text{NTR}} \mathbb{1}[x = x_k]\right) = \phi(x).$$

Thus, the local ETI estimand can be written as,

$$\begin{aligned}
\beta(x) &= \frac{\phi(x)\zeta(x)}{\phi(x)} = \zeta(x) \\
&= \sum_{j=1}^{J} \frac{\mathbb{P}(X=x, \phi=\phi_j \mid G=1)\phi_j}{\sum_{l=1}^{J} \mathbb{P}(X=x, \phi=\phi_l \mid G=1)\phi_l} \times \mathbb{E}[\zeta \mid G=1, X=x, \phi=\phi_j],
\end{aligned}$$

where the second line uses the law of iterated expectations, completing the proof. □

Proof of Proposition 3. The result follows from the proof of Proposition 4. Specifically, by noting that with no income effects ($\eta = 0$),

$$\log Y(\tau, R) \approx y_t + \varepsilon^u \log\left(\frac{1-\tau}{1-\tau_t}\right) + \frac{\eta(R - R_t)}{(1-\tau_t)Y_t} = y_t + \varepsilon^c \log\left(\frac{1-\tau}{1-\tau_t}\right),$$

Thus,

$$\log Y(\tau(d), R(d)) = y_t + \varepsilon^c \log\left(\frac{1-\tau(d)}{1-\tau_t}\right) = \zeta \log\left(\frac{1-\tau(d)}{1-\tau_t}\right) + \nu(d),$$

meaning that $\zeta = \varepsilon^c$ and $y_t = \nu(0) = \nu(1)$. □

Proof of Proposition 4. We start by deriving the first-order approximation to the (log-)earnings function $\log Y(\tau, R)$ with respect to changes in $\log(1-\tau)$ and R around the observed virtual income tax system (τ_t, R_t). Start by noting that $\log Y(\tau, R) = \log Y\left(1 - \exp(\log(1-\tau)), R\right)$. Taking the derivative with respect to $\log(1-\tau)$ then gives,

$$\frac{\partial \log Y}{\partial \log(1-\tau)} = \frac{\partial Y}{\partial 1-\tau} \frac{1-\tau}{Y} = \varepsilon^u.$$

Taking the derivative with respect to R,

$$\frac{\partial \log Y}{\partial R} = \frac{1}{Y}\frac{\partial Y}{\partial R} = \frac{\eta}{(1-\tau)Y}.$$

This means the first-order approximation to $\log Y(\tau, R)$ around τ_t, R_t equals,

$$\log Y(\tau, R) \approx y_t + \varepsilon^u \log\left(\frac{1-\tau}{1-\tau_t}\right) + \frac{\eta(R-R_t)}{(1-\tau_t)Y_t}.$$

Substituting in the potential marginal tax rates and virtual incomes defined in equation (18) gives,

$$\begin{aligned} y(\tau(1), R(1)) - y(\tau(0), R(0)) &= \varepsilon^u \log\left(\frac{1-\tau(1)}{1-\tau(0)}\right) + \frac{\eta(R(1)-R(0))}{(1-\tau_t)Y_t}, \\ &= \varepsilon^c \phi + \eta\left(\phi + \frac{R(1)-R(0)}{(1-\tau_t)Y_t}\right), \end{aligned}$$

where the second equality uses the Slutsky equation and the definition of ϕ. Now, consider a treated individual (i.e., with $GZ \neq 0$). Focusing on the last term, we can write,

$$\begin{aligned} \phi + \frac{R(1)-R(0)}{(1-\tau_t)Y_t} &= \log\left(\frac{1-\tau(1)}{1-\tau(0)}\right) + \frac{\tau(1)Y(1) - T_1(Y(1)) - (\tau(0)Y(0) - T_0(Y(0)))}{(1-\tau_t)Y_t}, \\ &\approx -\frac{\tau(1)-\tau(0)}{1-\tau(1)} + \frac{\tau(1)Y(1) - T_1(Y(1)) - (\tau(0)Y(1) - T_0(Y(1)))}{(1-\tau_t)Y_t}, \\ &= -\frac{\tau(1)-\tau(0)}{1-\tau(1)} + \frac{(\tau(1)-\tau(0))Y(1)}{(1-\tau(1))Y(1)} + \frac{-T_1(Y(1)) + T_0(Y(1))}{(1-\tau_t)Y_t}, \\ &= -\frac{T_1(Y_t) - T_0(Y_t)}{(1-\tau_t)Y_t}, \end{aligned}$$

where the first equality uses the definitions of $\tau(d)$ and $R(d)$. The second eqality is obtained from taking a first-order approximation of the the log term around $\tau(1) = \tau_t$ and using the no bracket switching assumption, whiile the third and forth line obtains from rearranging and exploiting that $Y_t = Y(1)$ and $\tau_t = \tau(1)$ provided $GZ \neq 0$.

From the proof of Proposition 2, we obtain that,

$$\begin{aligned} \beta(x) &= \frac{\mathbb{E}\left[y(1) - y(0) \mid G = 1, X = x\right]}{\phi(x)}, \\ &= \frac{\mathbb{E}\left[\varepsilon^c \phi + \eta\left(\phi + \frac{R(1)-R(0)}{(1-\tau_t)Y_t}\right) \mid G = 1, X = x\right]}{\phi(x)}, \\ &= \varepsilon^c(x) + \eta(x)\frac{\mathbb{E}\left[\left(\phi + \frac{R(1)-R(0)}{(1-\tau_t)Y_t}\right) \mid G = 1, X = x\right]}{\phi(x)}, \\ &= \varepsilon^c(x) - \eta(x)\frac{\mathbb{E}\left[\frac{T_1(Y_t)-T_0(Y_t)}{(1-\tau_t)Y_t} \mid G = 1, X = x\right]}{\phi(x)}. \end{aligned}$$

Under Assumption 1, we have that

$$\phi(x) = \mathbb{E}[\Delta \, \text{NTR} \mid G = 1, X = x] - \left(\hat{\lambda}_0^{\text{NTR}} g + \sum_{k=k_0}^{K} \hat{\lambda}_k^{\text{NTR}} \mathbb{1}[x = x_k] \right).$$

Re-arranging gives the first part of the result:

$$\varepsilon^c(x) = \beta(x) + \eta(x) \frac{\mathbb{E}\left[\frac{T_1(Y_t) - T_0(Y_t)}{(1-\tau_t)Y_t} \mid G = 1, X = x \right]}{\mathbb{E}[\Delta \, \text{NTR} \mid G = 1, X = x] - \left(\hat{\lambda}_0^{\text{NTR}} g + \sum_{k=k_0}^{K} \hat{\lambda}_k^{\text{NTR}} \mathbb{1}[x = x_k] \right)}.$$

The second part of the result follows by adding $\eta(x)$ to both sides of the equation and using the Slutsky equation: $\varepsilon^u(x) = \varepsilon^c(x) + \eta(x)$.

□

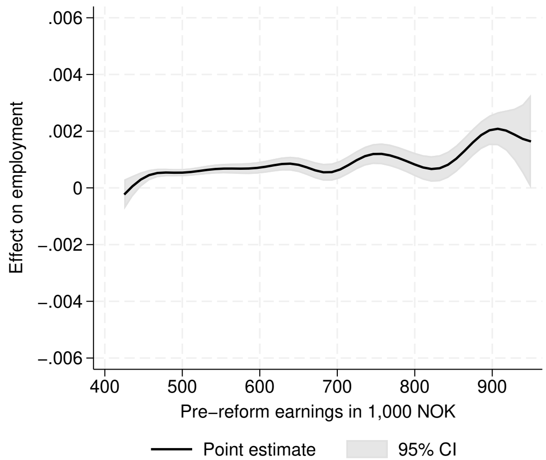
B Appendix figures and tables

Table 3: Summary statistics for ETI sample

	Full sample	Estimation sample	
		Reform cohorts	Placebo cohorts
		Demographics	
Age	40.70	40.86	40.66
Education	12.14	12.31	12.29
Male	57.20	62.80	64.80
Married	61.60	57.70	61.30
		Income and taxes	
Taxable income X	443	470	468
Income taxes $T(X)$	130	131	139
Marginal tax rate $T'(X)$	0.414	0.424	0.430
Observations	5,702,759	2,333,992	1,677,302
Individuals	1,346,358	961,832	796,533

Notes: This table reports summary statistics for the sample used in the estimation of the elasticity of taxable income (ETI). Monetary values are consumer-price-index adjusted and reported in 1,000 2018 NOKs (8.13 NOK/USD), and binary outcomes are reported in percent. Observations from 2002-2004 comprise the placebo cohorts, while observations from 2005-2007 comprise the reform cohorts.

(a) Employment effects.



(b) Placebo employment effects.

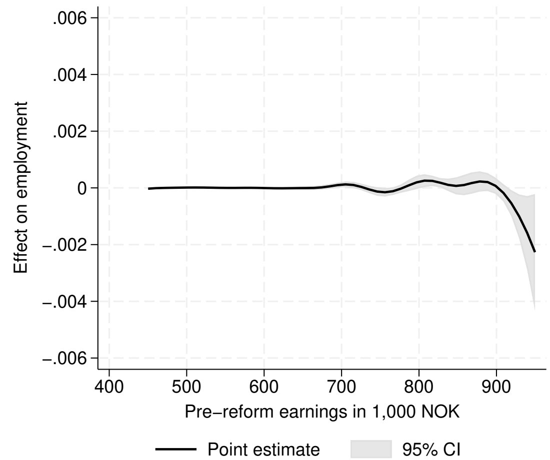


Figure 10: Local employment effects across initial income X.

Notes: The figure plots actual and placebo estimates of the local employment effects across the income distribution. Panel (a) plots the actual local employment effects obtained by first estimating the counterfactual change in employment using 50 quantile bins of X, then estimating the numerator of equation (12) using a local regression, with the change in employment replacing the change in log incomes. Panel (b) reports local placebo employment effects, obtained by estimating the numerator of equation (12) on the placebo cohorts only, sequentially treating each cohort as “treated,” using the remaining two as controls, and averaging the resulting estimates. 95 percent confidence intervals are shown, with standard errors obtained by bootstraping the entire estimation procedure using 500 replications.

Table 4: Summary statistics of individual characteristics and labor market outcomes for lottery sample

	Winners	Population
Labor earnings	489,213	447,720
Employment	0.98	0.96
Age	45.01	42.20
Male	0.61	0.52
Married	0.55	0.55
Years of schooling	11.93	12.17
Household size	2.82	2.95
Q1 share	0.18	0.25
Q2 share	0.25	0.25
Q3 share	0.27	0.25
Q4 share	0.30	0.25

Notes: Monetary values are consumer-price-index adjusted and reported in 2018 NOK (8.13 NOK/USD). Winners' values are measured one year prior to the win and cohort-size weighted. Population statistics cover ages 25–61 from 1996–2017. Net worth is at the household level, normalized by the number of adults. The second panel reports the share of winners in each quartile of the annual earnings distribution of the working-age population; for the population, each share is mechanically 0.25.

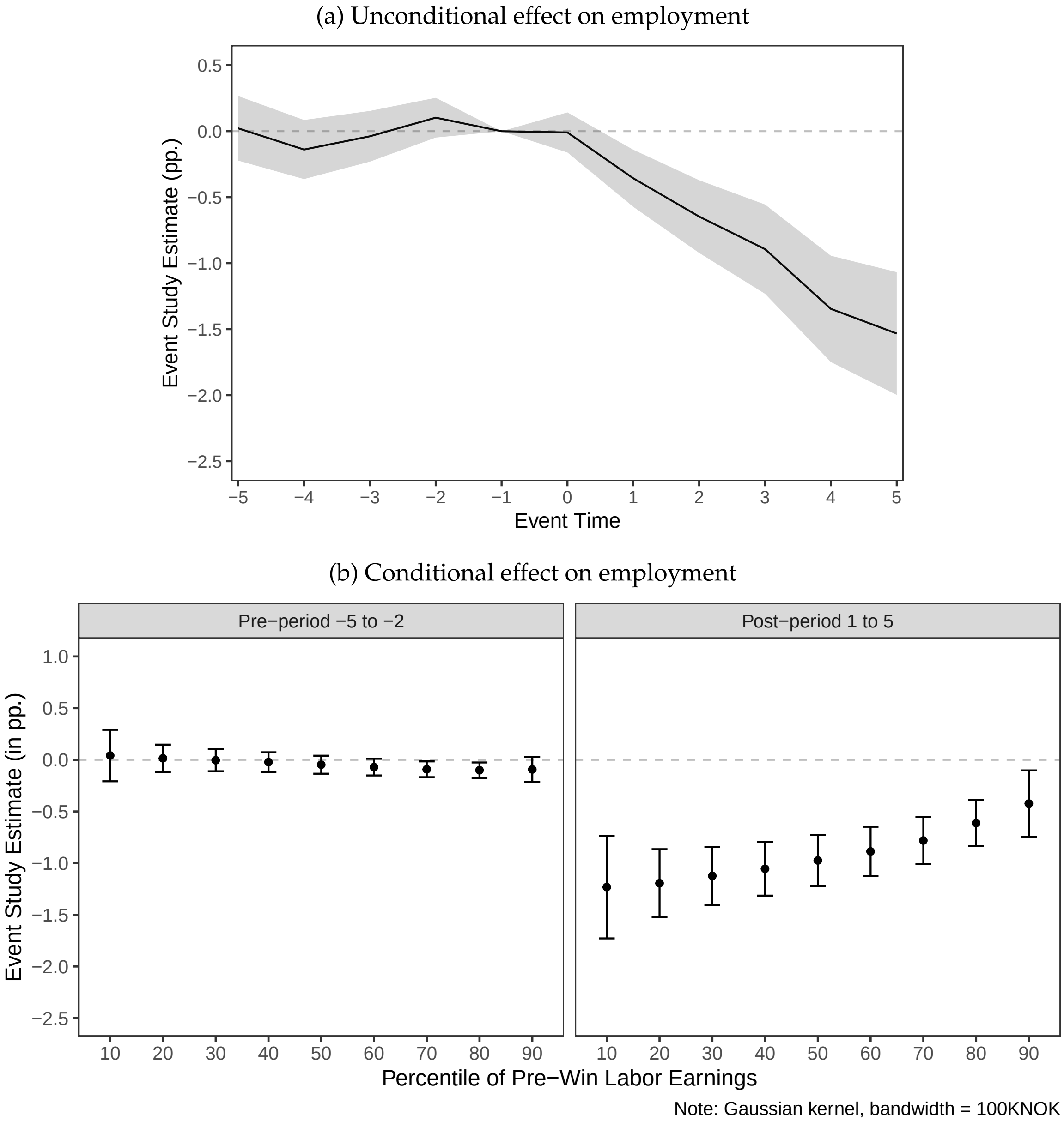


Figure 11: Employment effects of winning

Notes: This figure presents estimates of the effect of winning the lottery on winners' employment. Panel (a) presents unconditional estimates of the effect of winning the lottery on winners' employment for each event time t. The estimates correspond to the sample analogue of equation (25) using employment as an outcome instead if earnings, evaluated without conditioning on X, controlling for age, and averaged across cohorts using cohort-size weights. The estimates in Panel (b) correspond to the sample analogs of equation (25) using employment as an outcome instead if earnings, evaluated at different values of X. We use a Gaussian kernel with a bandwidth of 100,000 NOK, where the evaluation points $\{x\}$ correspond to the cohort-specific deciles of pre-win earnings. For each decile, we compute cohort-size weighted averages across cohorts and report equally weighted averages across event times $t = -5$ to -2 and $t = +1$ to $+5$. 90 percent confidence intervals are shown, with standard errors clustered at the winner level. Throughout, we use $g - 1$ as the omitted event time.

C Formal Identification Results for the Lottery Design

This appendix derives the formal identification results underlying our empirical strategy for the lottery design. We first map potential outcomes to observed data, show that the reduced-form estimand identifies the causal earnings effect under a conditional parallel-trends assumption, and then establish the decomposition of the total earnings response into intensive- and extensive-margin components.

Mapping potential outcomes to observed data. Let $W_{g,t}(d)$ denote the potential outcome W t years after winning ($d = 1$) or not winning ($d = 0$) the lottery in year g, for $W = Y, B$. Potential outcomes map to the observed data through

$$W_{g+t} = \mathbb{1}[G \leq g + t]\, W_{G,t}(1) + \mathbb{1}[G > g + t]\, W_{g,t}(0), \tag{56}$$

Earnings, employment and unearned income effects of winning the lottery. We start by stating the parallell trends assumption we rely on for obtaining the effect of winning the lottery on earnings and unearned income. It says that conditional on pre-win earnings $X = x$, average earnings and unearned income for winners and later-winners would have evolved in paralell:

$$\begin{aligned} &\mathbb{E}[W_{g,t}(0) - W_{g,-1}(0) \mid X = x,\, G = g] \\ &\qquad\qquad = \mathbb{E}\left[W_{g,t}(0) - W_{g,-1}(0) \mid X = x,\, G > g + t\right], \end{aligned} \tag{57}$$

for all $t \geq 0$ for $W = Y, B$. Under this assumption, the reduced form estimand $\text{RF}_{g,t}(x)$ ($\text{FS}_{g,t}(x)$) recovers the average earnings (unearned income) effect of winning the lottery for cohort g. To see why, note that

$$\begin{aligned} \text{RF}_{g,t}(x) &\equiv \mathbb{E}[Y_{g+t} - Y_{g-1} \mid X = x, G = g] - \mathbb{E}[Y_{g+t} - Y_{g-1} \mid X = x, G > g + t] \\ &= \mathbb{E}[Y_{G,t}(1) - Y_{g,-1}(0) \mid X = x, G = g] - \mathbb{E}[Y_{g,t}(0) - Y_{g,-1}(0) \mid X = x, G > g + t] \\ &= \mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0) \mid X = x, G = g] \\ &\qquad + \Big(\mathbb{E}[Y_{g,t}(0) - Y_{g,-1}(0) \mid X = x, G = g] - \mathbb{E}[Y_{g,t}(0) - Y_{g,-1}(0) \mid X = x, G > g + t] \Big). \end{aligned}$$

By the parallel-trends assumption (57),

$$\text{RF}_{g,t}(x) = \mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0) \mid X = x,\, G = g]\,.$$

Analogous arguments apply to unearned income and employment.

Aggregating across cohorts and event times. We aggregate the cohort and event-time specific reduced form and first stage estimands according to

$$\mathrm{RF}(x) \equiv \sum_{g \in \mathcal{G}} \mathbb{P}(G = g \mid X = x)\, \frac{1}{5} \sum_{t=1}^{5} \mathrm{RF}_{g,t}(x), \tag{58}$$

$$\mathrm{FS}(x) \equiv \sum_{g \in \mathcal{G}} \mathbb{P}(G = g \mid X = x)\, \frac{1}{5} \sum_{t=1}^{5} \mathrm{FS}_{g,t}(x). \tag{59}$$

The ratio between the two estimands then recovers

$$\frac{\mathrm{RF}(x)}{\mathrm{FS}(x)} = \sum_{g \in \mathcal{G}} \sum_{t=1}^{5} \omega_{g,t}(x)\, \frac{\mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0) \mid X = x, G = g]}{\mathbb{E}[B_{g,t}(1) - B_{g,t}(0) \mid X = x, G = g]}, \tag{60}$$

with weights

$$\omega_{g,t}(x) = \frac{\mathbb{P}(G = g \mid X = x)\ \mathbb{E}[B_{g,t}(1) - B_{g,t}(0) \mid X = x, G = g]}{\sum_{j \in \mathcal{G}} \sum_{k=1}^{5} \mathbb{P}(G = j \mid X = x)\ \mathbb{E}[B_{j,k}(1) - B_{j,k}(0) \mid X = x, G = j]}. \tag{61}$$

The weights are positive and sum to one provided unearned income increases with the prize.

Recovering intensive-margin income effects To isolate the intensive-margin earnings response, we restrict attention to individuals who would be working t years after winning. We impose that conditional on pre-win earnings $X = x$, individuals in cohort g who would be working at event time t would have experienced the same counterfactual earnings evolution as later winners who are observed to be working in year $g + t$.

$$\begin{aligned} &\mathbb{E}[Y_{g,t}(0) - Y_{g,-1}(0) \,|\, X = x,\, G = g,\, Y_{g,t}(1) > 0] \\ &\qquad = \mathbb{E}\left[Y_{g,t}(0) - Y_{g,-1}(0) \,|\, X = x,\, G > g + t,\, Y_{g,t}(0) > 0\right]. \end{aligned} \tag{62}$$

Two features of our setting makes this assumption plausibile. First, conditioning on pre-win earnings absorbs most systematic differences in earnings capacity and labor-market attachment. Second, the extensive-margin response is small, so conditioning on being employed at $g + t$ selects nearly the same individuals in the treated and control groups. Together, these features substantially mitigate concerns about selection.

Under (62), the intensive-margin reduced form becomes

$$\begin{aligned} \mathrm{RF}^{int}_{g,t}(x) &\equiv \mathbb{E}[Y_{g+t} - Y_{g-1} \mid X = x, G = g, Y_{g+t} > 0] \\ &\quad - \mathbb{E}[Y_{g+t} - Y_{g-1} \mid X = x, G > g + t, Y_{g+t} > 0] \\ &= \mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0) \mid X = x, G = g, Y_{g,t}(1) > 0]. \end{aligned}$$

The corresponding intensive-margin first stage recovers

$$\mathrm{FS}^{int}_{g,t}(x) = \mathbb{E}[B_{g,t}(1) - B_{g,t}(0) \mid X = x, G = g, Y_{g,t}(1) > 0]. \tag{63}$$

Our assumption of constant income effects conditional on X, implies that

$$Y_{g,t}(1) = Y_{g,t}(0) + \frac{\eta(x)}{1-\tau}\left(B_{g,t}(1) - B_{g,t}(0)\right),$$

for individuals who work after winning. Substituting this into the intensive-margin reduced form $\text{RF}^{int}_{g,t}(x)$, we obtain

$$\frac{\text{RF}^{int}_{g,t}(x)}{\text{FS}^{int}_{g,t}(x)} = \frac{\eta(x)}{1-\tau}. \tag{64}$$

Decomposing the total earnings response. To decompose the total earnings effect per additional NOK of unearned income into the intensive- and extensive margin contributions, it is useful to introduce notation for being employed, $E_{g+t} = \mathbb{1}[Y_{g+t} > 0]$ and define the extensive-margin reduced form estimand by

$$\text{RF}^{ext}_{g,t}(x) \equiv \mathbb{E}[E_{g+t} - E_{g-1} \mid X = x, G = g] - \mathbb{E}[E_{g+t} - E_{g-1} \mid X = x, G > g+t]. \tag{65}$$

Under the parallell trends assumption, and the additional assumption that employment decreases in lottery winnings, the extensive-margin reduced form estimand recovers:

$$\text{RF}^{ext}_{g,t}(x) = -\,\mathbb{P}(Y_{g,t}(0) > 0,\, Y_{g,t}(1) = 0 \mid X = x, G = g). \tag{66}$$

By the law of total expectations, the total earnings effect can be written as:

$$\begin{aligned}
&\overbrace{\mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0) \mid X = x, G = g]}^{\text{total effect}=\text{RF}_{g,t}(x)} \\
&= \underbrace{\overbrace{\mathbb{P}(Y_{g,t}(1) > 0 \mid X = x, G = g)}_{\text{employment share }=\mathbb{P}(Y_{g,t}>0|X=x,G=g)} \times \underbrace{\mathbb{E}[Y_{g,t}(1) - Y_{g,t}(0) \mid X = x, G = g, Y_{g,t}(1) > 0]}_{\text{intensive-margin response}=\text{RF}^{int}_{g,t}(x)}}^{\text{intensive margin component}} \\
&\underbrace{\overbrace{-\,\mathbb{P}(Y_{g,t}(1) = 0, Y_{g,t}(0) > 0 \mid X = x, G = g)}^{\text{extensive margin response}=\text{RF}^{ext}_{g,t}(x)} \times \mathbb{E}[Y_{g,t}(0) \mid X = x, G = g, Y_{g,t}(1) = 0, Y_{g,t}(0) > 0]}_{\text{extensive margin component}}.
\end{aligned}$$

Since there is only one unknown quantity in the expression, we can solve for it to obtain the average $Y_{g,t}(0)$ for those who would stop working if they won:

$$\mathbb{E}[Y_{g,t}(0) \mid X = x, G = g, Y_{g,t}(1) = 0, Y_{g,t}(0) > 0] = \frac{\text{RF}_{g,t}(x) - p_{g,t}(x)\text{RF}^{int}_{g,t}(x)}{-\text{RF}^{ext}_{g,t}(x)} \tag{67}$$

where the employment share is denoted by $p_{g,t}(x) \equiv \mathbb{P}(Y_{g,t} > 0 \mid X = x, G = g)$. Dividing the toal effect by the first stage then yields:

$$\frac{\mathrm{RF}_{g,t}(x)}{\mathrm{FS}_{g,t}(x)} = \underbrace{\frac{p_{g,t}(x)}{\mathrm{FS}_{g,t}(x)} \times \mathrm{RF}^{int}_{g,t}(x)}_{\text{intensive-margin component}} + \overbrace{\frac{\mathrm{RF}_{g,t}(x) - p_{g,t}(x)\mathrm{RF}^{int}_{g,t}(x)}{\mathrm{FS}_{g,t}(x)}}^{\text{extensive-margin component}}. \tag{68}$$

Since all the quantities in the expression is either identified or functions of the data, the decomposition is identified.

D Derivation of revenue-maximizing tax rate and marginal deadweight loss of income taxation with consumption- and payroll taxes

D.1 Marginal deadweight loss

It is useful to start by noting that

$$\begin{aligned}\frac{dDWL}{d\tau} &= Y - \sum_{k=1}^{K} \tau_k p_k \frac{\partial C_k^c}{\partial \tau} - Y - (\tau + \tau_w)\frac{\partial Y^c}{\partial \tau}, \\ &= -\left(\sum_{k=1}^{K} \tau_k p_k \frac{\partial C_k^c}{\partial \tau} + (\tau + \tau_w)\frac{\partial Y^c}{\partial \tau}\right),\end{aligned}$$

where the first equality uses the envelope theorem. It is well-known from duality theory that the following equality between the compensated and uncompensated demand functions holds.

$$C_k^c(\tau_1, ..., \tau_K, \tau, \bar{V}) = C_k^u(\tau_1, ..., \tau_K, \tau, E(\tau_1, ..., \tau_K, \tau, \bar{V})),$$

where $E(.)$ is the expenditure function defined in equation (31). Two-step budgeting implies that,

$$C_k^c(\tau_1, ..., \tau_K, \tau, \bar{V}) = C_k(\tau_1, ..., \tau_K, I(\tau_1, ..., \tau_K, \tau, E(\tau_1, ..., \tau_K, \tau, \bar{V}))).$$

Taking the derivative of the compensated demand function with respect to τ thus yields,

$$\frac{\partial C_k^c}{\partial \tau} = \frac{\partial C_k^u}{\partial I}\left(\frac{\partial I}{\partial \tau} + \frac{\partial I}{\partial R}\frac{\partial E}{\partial \tau}\right).$$

By recalling that

$$I(\tau_1, ..., \tau_K, \tau, R) = (1-\tau)Y^u(\tau_1, ..., \tau_K, \tau, R) + R,$$

we obtain

$$\begin{aligned}\frac{\partial I}{\partial \tau} + \frac{\partial I}{\partial R}\frac{\partial E}{\partial \tau} &= -Y + (1-\tau)\frac{\partial Y^u}{\partial \tau} + \left[(1-\tau)\frac{\partial Y^u}{\partial R} + 1\right]\frac{\partial E}{\partial \tau}, \\ &= -Y + (1-\tau)\frac{\partial Y^u}{\partial \tau} + \left[(1-\tau)\frac{\partial Y^u}{\partial R} + 1\right]Y, \\ &= -(1-\tau)\frac{\varepsilon^u Y}{1-\tau} + \eta Y, \\ &= -\varepsilon^c Y,\end{aligned}$$

where the second-to-last equality uses the definition of the uncompensated elasticity and the last equality uses the Slutsky equation.

This implies that we can write the sum in the expression for the MDWL as

$$\sum_{k=1}^{K} \tau_k p_k \frac{\partial C_k^c}{\partial \tau} = -\varepsilon^c Y \sum_{k=1}^{K} \tau_k p_k \frac{\partial C_k}{\partial I},$$
$$= -\varepsilon^c Y \frac{\tilde{\tau}}{1+\tilde{\tau}},$$

where the second equality uses the definition of $\tilde{\tau}$ from equation (30) and equation (69) below.

The marginal deadweight loss can now be written as

$$MDWL = \varepsilon^c Y \frac{\tilde{\tau}}{1+\tilde{\tau}} + (\tau + \tau_w) \frac{\varepsilon^c Y}{1-\tau},$$

which, after rearranging, yields the expression in the numerator of equation (33). To derive the marginal (compensated) tax revenue, we note that

$$\frac{\partial TR^c}{\partial \tau} = \sum_{k=1}^{K} \tau_k p_k \frac{\partial C_k^c}{\partial \tau} + Y + (\tau + \tau_w) \frac{\partial Y^c}{\partial \tau},$$
$$= Y - MDWL.$$

The expression in the numerator of equation (33) follows immediately.

D.2 Revenue-maximizing tax rate

By standard arguments, increasing the top-income tax rate by $d\tau$ is equivalent to the marginal tax rate changing by $d\tau$ and the virtual transfer changing by $d\tau\bar{Y}$ in a linear tax system. The effect on government revenue can, therefore, be expressed as,

$$\frac{dTR}{d\tau_{\text{TOP}}} = \sum_{k=1}^{K} \tau_k p_k \left(\frac{\partial C_k^u}{\partial \tau} + \frac{\partial C_k^u}{\partial R} \bar{Y} \right) + (Y^u - \bar{Y}) + (\tau + \tau_w) \left(\frac{\partial Y^u}{\partial \tau} + \frac{\partial Y^u}{\partial R} \bar{Y} \right).$$

Using the demand functions in equation (29), we obtain,

$$\frac{\partial C_k^u}{\partial \tau} + \frac{\partial C_k^u}{\partial R} \bar{Y} = \frac{\partial C_k^u}{\partial I} \left(-Y^u + \bar{Y} + (1-\tau) \left(\frac{\partial Y^u}{\partial \tau} + \frac{\partial Y^u}{\partial R} \bar{Y} \right) \right).$$

Substituting this back into the expression above, we get that

$$\frac{dTR}{d\tau_{\text{TOP}}} = \left(-(Y^u - \bar{Y}) + (1-\tau) \left(\frac{\partial Y^u}{\partial \tau} + \frac{\partial Y^u}{\partial R} \bar{Y} \right) \right) \sum_{k=1}^{K} \tau_k p_k \frac{\partial C_k^u}{\partial I}$$
$$+ (Y^u - \bar{Y}) + (\tau + \tau_w) \left(\frac{\partial Y^u}{\partial \tau} + \frac{\partial Y^u}{\partial R} \bar{Y} \right).$$

Using the defintion of $\tilde{\tau}$ from equation (30), we have that,

$$1+\tilde{\tau} = \frac{\sum_{k=1}^{K}(1+\tau_k)p_k\frac{\partial C_k}{\partial I}}{\sum_{k=1}^{K}p_k\frac{\partial C_k^u}{\partial I}} = \frac{1}{\sum_{k=1}^{K}p_k\frac{\partial C_k^u}{\partial I}},$$

where the second equality follows since the budget constraint ensures that $\sum_{k=1}^{K}(1+\tau_k)p_k\partial C_k^u/\partial I = 1$. This means that

$$\sum_{k=1}^{K}p_k\frac{\partial C_k^u}{\partial I} = \frac{1}{1+\tilde{\tau}}.$$

which implies,

$$\sum_{k=1}^{K}\tau_k p_k\frac{\partial C_k^u}{\partial I} = \frac{\tilde{\tau}}{1+\tilde{\tau}}. \tag{69}$$

Using this, the revenue effect can be written as,

$$\begin{aligned}
\frac{dTR}{d\tau_{\text{TOP}}} &= \left(-(Y^u-\bar{Y}) + (1-\tau)\left(\frac{\partial Y^u}{\partial \tau} + \frac{\partial Y^u}{\partial R}\bar{Y}\right)\right)\frac{\tilde{\tau}}{1+\tilde{\tau}} \\
&\quad + (Y^u-\bar{Y}) + (\tau+\tau_w)\left(\frac{\partial Y^u}{\partial \tau} + \frac{\partial Y^u}{\partial R}\bar{Y}\right), \\
&= -(Y^u-\bar{Y})\frac{\tilde{\tau}}{1+\tilde{\tau}} + (Y^u-\bar{Y}) \\
&\quad + \left((\tau+\tau_w) + (1-\tau)\frac{\tilde{\tau}}{1+\tilde{\tau}}\right)\left(\frac{\partial Y^u}{\partial \tau} + \frac{\partial Y^u}{\partial R}\bar{Y}\right), \\
&= \frac{Y^u-\bar{Y}}{1+\tilde{\tau}} + \left(\tau_w + \frac{\tau+\tilde{\tau}}{1+\tilde{\tau}}\right)\left(\frac{\partial Y^u}{\partial \tau} + \frac{\partial Y^u}{\partial R}\bar{Y}\right)
\end{aligned}$$

From the definitions of the uncompensated elasticity and income effect, we obtain that,

$$\frac{\partial Y^u}{\partial \tau} = -\frac{Y^u\varepsilon^u}{1-\tau}, \qquad \frac{\partial Y^u}{\partial R} = \frac{\eta}{1-\tau}.$$

Substituting this into the expression above,

$$\frac{dTR}{d\tau_{\text{TOP}}} = \frac{Y^u-\bar{Y}}{1+\tilde{\tau}} + \frac{\tau_w(1+\tilde{\tau})+\tau+\tilde{\tau}}{(1+\tilde{\tau})(1-\tau)}\left(-\varepsilon^u Y^u + \eta\bar{Y}\right)$$

At the revenue-maximizing tax rate, it must be the case that further increases in the top-income tax rate do not affect revenue. Thus,

$$\mathbb{E}\left[\frac{dTR}{d\tau_{\text{TOP}}} \mid Y \geq \bar{Y}\right] = \mathbb{E}\left[\frac{Y^u-\bar{Y}}{1+\tilde{\tau}} + \frac{\tau_w(1+\tilde{\tau})+\tau_{\text{TOP}}+\tilde{\tau}}{(1+\tilde{\tau})(1-\tau_{\text{TOP}})}\left(-\varepsilon^u Y^u + \eta\bar{Y}\right) \mid Y \geq \bar{Y}\right] = 0,$$

where the expectation is taken across individuals in the top-income tax bracket. Using the definitions from equation (36) and the assumption that $\tilde{\tau}$ is constant across individuals, we obtain,

$$\begin{aligned}
\frac{\mathbb{E}[Y^u - \bar{Y} \mid Y^u \geq \bar{Y}]}{1+\tilde{\tau}} + \frac{\tau_w(1+\tilde{\tau}) + \tau_{\text{TOP}} + \tilde{\tau}}{(1+\tilde{\tau})(1-\tau_{\text{TOP}})} \left(-\bar{\varepsilon}^u \, \mathbb{E}[Y^u \mid Y^u \geq \bar{Y}] + \bar{\eta}\bar{y}\right) &= 0, \\
\frac{(\alpha-1)\bar{y}}{1+\tilde{\tau}} + \frac{\tau_w(1+\tilde{\tau}) + \tau_{\text{TOP}} + \tilde{\tau}}{(1+\tilde{\tau})(1-\tau_{\text{TOP}})} \left(-\bar{\varepsilon}^u \alpha\bar{y} + \bar{\eta}\bar{y}\right) &= 0, \\
(\alpha-1) + \frac{\tau_w(1+\tilde{\tau}) + \tau_{\text{TOP}} + \tilde{\tau}}{1-\tau_{\text{TOP}}} \left(-\bar{\varepsilon}^u \alpha + \bar{\eta}\right) &= 0
\end{aligned}$$

where the second line uses the defintion of α the third multiplies both sides by $(1+\tilde{\tau})/\bar{Y}$. Rearranging, we obtain,

$$\frac{\tau_w(1+\tilde{\tau}) + \tau_{\text{TOP}} + \tilde{\tau}}{1-\tau_{\text{TOP}}} = \frac{\alpha-1}{\bar{\varepsilon}^u\alpha - \bar{\eta}}.$$

Solving for τ_{TOP}, we arrive at

$$\tau_{\text{TOP}} = \frac{\alpha - 1 - \left(\tilde{\tau} + \tau_w + \tau_w\tilde{\tau}\right)\left(\alpha\bar{\varepsilon}^u - \bar{\eta}\right)}{\alpha - 1 + \left(\alpha\bar{\varepsilon}^u - \bar{\eta}\right)}.$$

E Dynamic labor supply model

This appendix shows how the static model in Section 4 relates to a standard dynamic life-cycle model following Blomquist (1985), Blundell and Walker (1986), and MaCurdy (1983). For simplicity, we abstract from non-linear labor income taxation, assume that the return on capital is fixed over time and focus on the case where there is only one consumption good and one margin of labor supply per period.

Consider an individual who won ξ dollars at time g with wealth A_{g-1}. They solve:

$$\max_{\{C_t,Y_t,A_t\}_{t=g}^{T}} \sum_{t=g}^{T} \beta^t U(C_t, Y_t), \quad \text{subject to} \quad C_g + A_g = (1-\tau)Y_g + (1+r)A_{g-1} + \xi,$$
$$C_t + A_t = (1-\tau)Y_t + (1+r)A_{t-1}, \quad \text{and} \quad A_T \geq 0 \quad \text{for all } t > g. \tag{70}$$

The solution to the problem depends on the lottery winnings, so we write $C_t(\xi), Y_t(\xi)$ and $A_t(\xi)$ for all $t \geq g$. The following first-order condition determines labor supply in period t:

$$\frac{\partial U}{\partial C}(1-\tau) + \frac{\partial U}{\partial Y} = 0,$$

where the derivatives are evaluated for consumption $C = (1-\tau)Y_t(\xi) + B_t(\xi)$ with $B_t(\xi) \equiv (1 + r)A_{t-1}(\xi) - A_t(\xi) + \mathbb{1}[t = g]\xi$ and earnings $Y = Y_t(\xi)$. This means we can write labor supply as

$$Y_t(\xi) = Y(\tau, B_t(\xi)),$$

The effect at time t of winning an additional dollar is given by

$$\frac{dY_t}{d\xi} \quad \text{and} \quad \frac{dB_t}{d\xi}.$$

Moreover, by the chain rule

$$\frac{dY_t}{d\xi} = \frac{\partial Y}{\partial B}\frac{dB_t}{d\xi} = \frac{\eta}{1-\tau}\frac{dB_t}{d\xi},$$

where η is the same parameter as in the static model in Section 4. Thus, the dynamic model implies the income effect is related to the earnings- and unearned income effect of lottery winnings through,

$$\frac{\partial Y_t/\partial \xi}{\partial B_t/\partial \xi} = \frac{\eta}{1-\tau}.$$