Exercise 7 – Interaction-free Measurement

We usually think of a measurement as an interaction between an object and a measuring device. However, there is also another way to measure – without interaction. Consider a

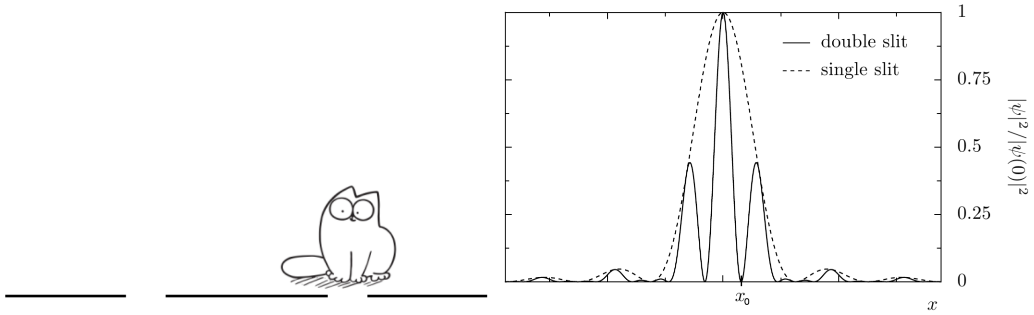

Figure 1: left: a cat is blocking a slit in a double-slit experiment, right: probability of measuring a photon dependent on position for the single and double-slit experiment

double-slit experiment as seen in Fig. 1. Let x_0 be one minimum of the interference pattern. If we place a detector at $x = x_0$, and let the source emit photon by photon, then no photon will reach the detector. Now, consider a cat blocking one of the slits. There is no more interference, hence, there is a possibility for a photon to reach the detector. If a photon reaches the detector, the photon has not interacted with the object, yet we know, that the cat is blocking the slit. We've seen the cat *in the dark*.

In the following we will consider interaction-free measurements with Mach-Zehnder interferometers.

a) The Mach-Zehnder interferometer in Fig. 2 consists of two beam splitters (B1 and B2) and two mirrors (M). The path lengths in the upper and lower arms of the interferometer are the same.

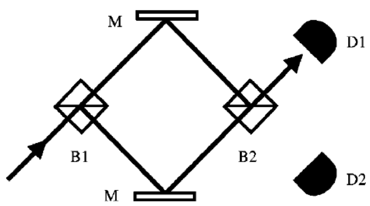

Figure 2: Schematic of a Mach-Zehnder interferometer.

Show that if R_1 (reflectivity of the first beam splitter) is the same as T_2 (transmittivity of the second beam splitter), then a photon entering the interferometer from below will always reach the first detector (D1).

Hint: The relative phase shift between reflected and transmitted waves is $\pi/2$, not π.

b) Suppose a perfectly absorbing object is placed in the lower arm of the interferometer as shown in Fig. 3. A photon may reach the first detector as before, but it also has a chance of either being absorbed by the object or reaching the second detector. The final possibility is a measurement of the object's presence without interaction, because light never reaches the second detector when the object is not persistent.

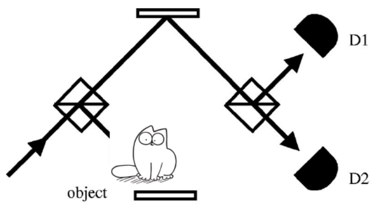

Figure 3: The interferometer with an object in one arm.

Determine the efficiency of the interaction-free measurement, which is defined as the ratio of the probability of interaction-free measurement to the probability of either measurement or absorption as a function of R_1. What is the upper limit of the efficiency?

c) In an improved arrangement, light is reflected through a series of beam splitters as shown in Fig. 4. If the reflectivity of each beam splitter in the series is $R = \cos^2(\pi/(2N))$, where N is the number of beam splitters, show that light will always reach the first detector (D1).

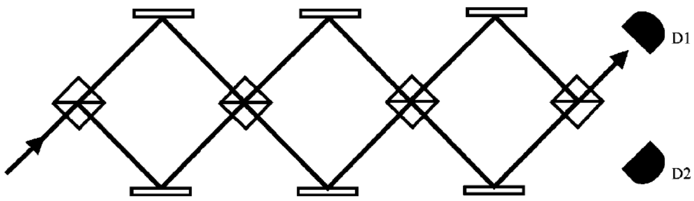

Figure 4: The interferometer described in c) for $N = 4$.

d) If an object blocks all of the upper arms of the interferometer in Fig. 4, a photon can also be absorbed or reach the second detector. Determine the efficiency of interaction-free measurement as a function of the number of beam splitters.