Exercise 1)

1) 1. $J(E)$ is a function only of the probability of the event E.

$$\Rightarrow \quad J(E) = J(p)$$

2. Information is a non-negative quantity.

$$\Rightarrow \quad J(E) = J(p) \geq 0$$

3. Information due to independent events is ~~positive~~ additive.

$$\Rightarrow \quad J(p_1 p_2) = J(p_1) + J(p_2)$$

a) $J(p) = k \log(p) = \tilde{k} \log\left(\frac{1}{p}\right)$ $\qquad k < 0$, $\tilde{k} > 0$

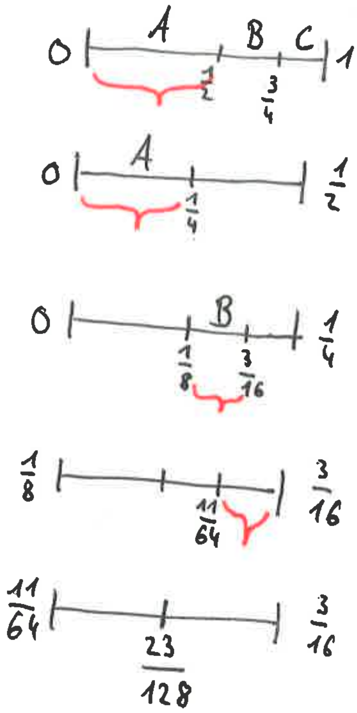
b) $H(X) = \sum_i p_i J(p_i) = k \sum_i p_i \log(p_i)$

Shannon-Entropy : $k = -\frac{1}{\log 2}$

$$\Rightarrow \quad H(X) = -\sum_i p_i \log_2(p_i) = \sum_i p_i \log_2\left(\frac{1}{p_i}\right)$$

2)

message	interval	length of interval
A	$[0, \frac{1}{2}]$	$\frac{1}{2}$
AA	$[0, \frac{1}{4}]$	$\frac{1}{4}$
AAB	$[\frac{1}{8}, \frac{3}{16}]$	$\frac{1}{16}$
AABC	$[\frac{11}{64}, \frac{3}{16}]$	$\frac{1}{64}$



$$\frac{23}{128} = 0.\underline{\overset{\frac{1}{2}}{0}\,\overset{\frac{1}{4}}{0}\,\overset{\frac{1}{8}}{1}\,\overset{\frac{1}{16}}{0}\,\overset{\frac{1}{32}}{1}\,\overset{\frac{1}{64}}{1}\,\overset{\frac{1}{128}}{1}}$$

$H \approx 0{,}985$, length: 7 bits

4) Suppose length of input is 1bit.

Apply algorithm: compressed text has length 0bit. ↯

3)

character	A	B	C	D	E
#	15	7	6	6	5

length of text: 39

a) $H \approx 2{,}186$

b) length: $39 \cdot 3$ bits $= 117$ bits

bit	0	1
#	87	30

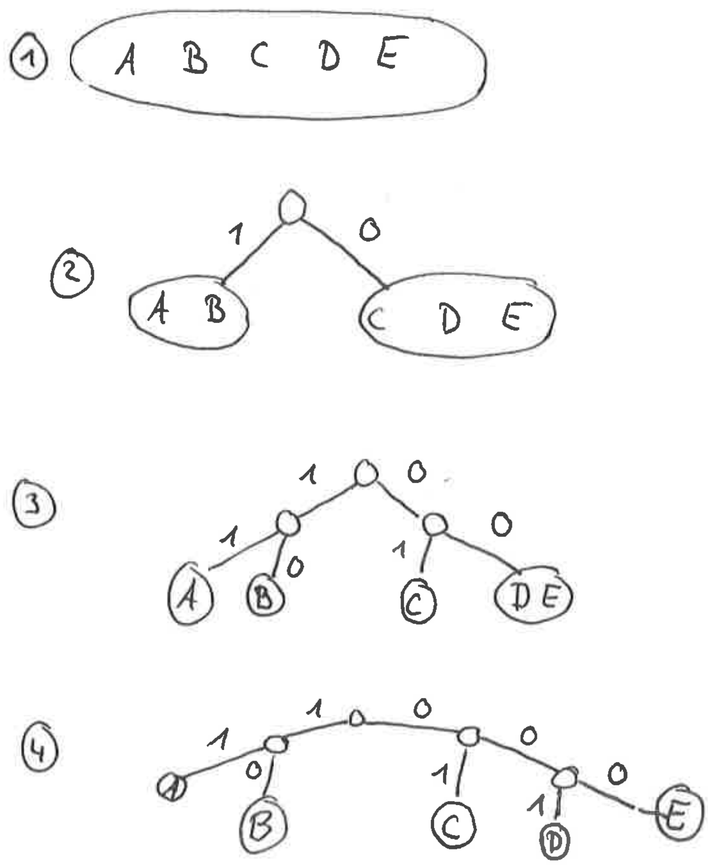
$H \approx 0{,}821$ bit

The length is independent of the choice of the block encoding, the entropy does depend ~~on~~ on the specific choice.

E.g.: $A \mapsto 111$, ...

$H \approx 0{,}942$

c)



$A \mapsto 11$

$B \mapsto 10$

$C \mapsto 01$

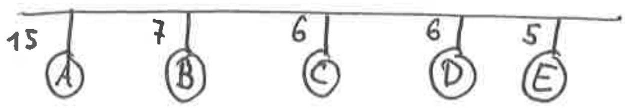
$D \mapsto 001$

$E \mapsto 000$

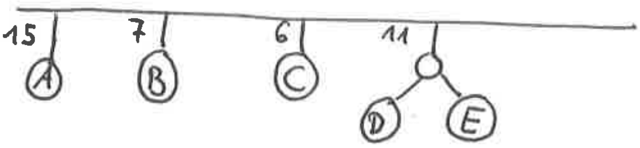
length encoded : 89 bit

entropy $H \approx 0{,}993$ bit

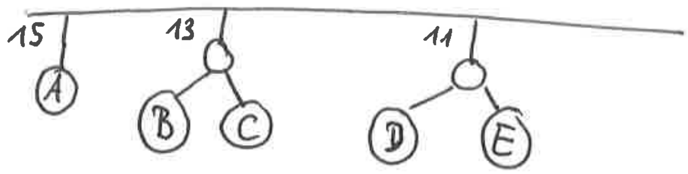
d) ①



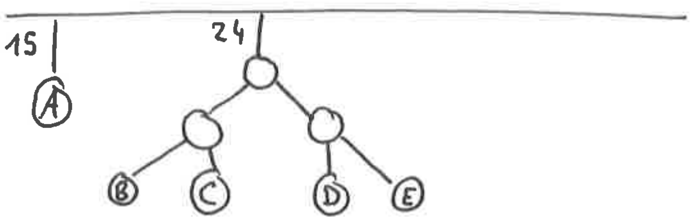
②



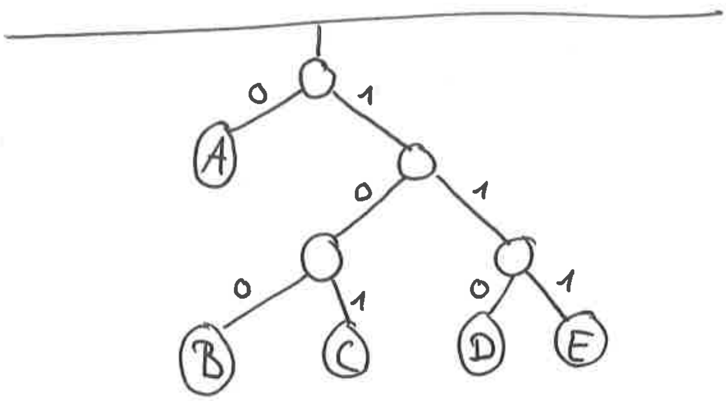
③



④



⑤



$A \longmapsto 0$

$B \longmapsto 100$

$C \longmapsto 101$

$D \longmapsto 110$

$E \longmapsto 111$

length encoded : 87 bit

entropy : $H \approx 0{,}998$ bit