Exercise 3 – A Quantum Game

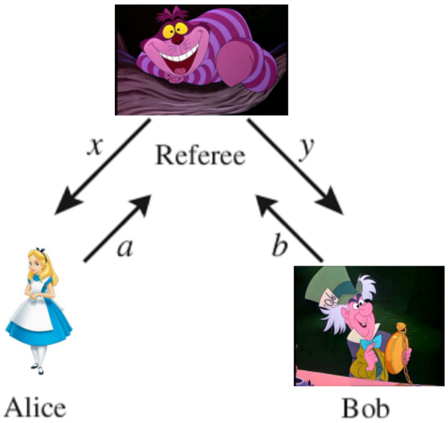

Figure 1: The referee distributes the bits x and y to Alice and Bob in the first round. Alice and Bob return the bits a and b to the referee.

Alice and Bob play a game. The game begins with a referee selecting two bits x and y uniformly at random. He then sends x to Alice and y to Bob. Alice and Bob are not allowed to communicate in any way at this point. Alice sends back to the referee a bit a, and Bob sends back a bit b (see Fig. 1). Since they are spatially separated, Alice's bit a can only depend on x, and similarly, Bob's bit b can only depend on y. The referee then determines if the `AND` of x and y is equal to the `XOR` of a and b. If so, Alice and Bob win the game. That is, the winning condition is

$$x \wedge y = a \oplus b. \tag{1}$$

a) Make a table that presents the winning conditions depending on the various different values of x, y, a and b.

b) Remember that $a = a(x)$ and $b = b(y)$. What is an optimal strategy and what is the maximal winning propability?

c) Let's assume Alice and Bob share a two-qubit system wich is initialized in the Bell state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|00\rangle + |11\rangle\right). \tag{2}$$

Alice and Bob agree to use the following strategy:

1. Alice takes the first qubit and Bob takes the second qubit from the quantum system.
2. If Alice receives $x = 0$, she does not apply any operation on her qubit. If Alice receives $x = 1$, she applies a rotation by $\pi/4$.
3. If Bob receives $y = 0$, he applies a rotation by $\pi/8$. If Bob receives $y = 1$, he applies a rotation by $-\pi/8$.
4. Alice and Bob meassure their qubits and output the values obtained as their answers a and b.

What is the winning propability?