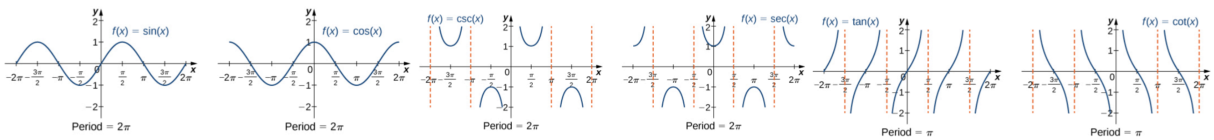
**Trigonometry (prob can be quick, in my experience last year we don't want to dwell too long on parts that are generally known pretty well)**

Trig functions:

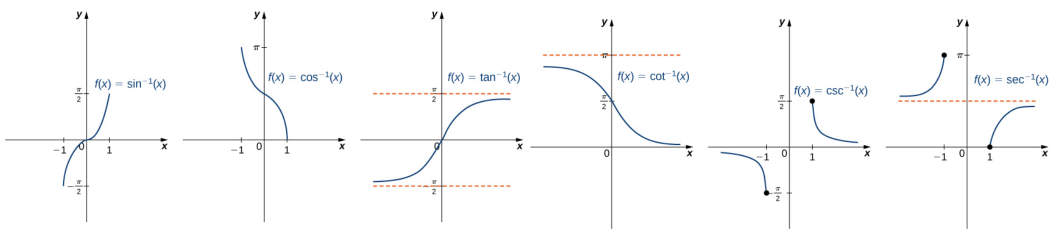



$secx = \frac{1}{cosx}$

$cscx = \frac{1}{sinx}$

$cotx = \frac{1}{tanx}$

All of these have inverses, denoted with a -1, such as $sin^{-1}x$ or arcsinx, which mean the same thing. DO NOT confuse this with the reciprocal of sinx, so $sin^{-1}x \neq \frac{1}{sinx}$, that is cscx.

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**Inverse functions**

You can find the inverse of a function by swapping the x and y's and resolving for y. We can denote this by f(y) = x when we swap the x and y and then $f^{-1}(x) = y$ when resolving for y (since we like using x and y in their traditional format).

Invertible functions must be bijective, that is both:

- Injective or one-to-one, meaning all x values have a **unique** y value (no two x values can map to the same y value)
- Surjective or onto, meaning all y values exist in the codomain (remember that the codomain can be restricted, so it doesn't necessarily mean all y-values)

Remember the horizontal line test which you can apply on the original function to tell you whether its inverse is also a function. You can restrict the domain to make it a function.

When graphed, inverses are a reflection along the line y = x of the original function

**Logarithms and Exponents**

Exponents are of the form $a^x$ where a is a constant. Remember that polynomials are NOT exponents. Logarithms are the inverse of exponents. If we say $y = a^x$, then using logarithms gives $log_a y = x$ where the variables mean the same thing in both cases. Logarithms and exponents are inverses of each other. Remember that for $log_a x$, a>0 and a=0, and x>0, basically both the base and the x value must be bigger than 0 and the base cannot be 1 (as otherwise 1 raised to any power is 1, so that is not very useful)

Practice problem: Which of the intervals below represents the entire domain of $f(x) = \frac{ln(ln(x))}{x-4}$

a. $(-\infty, 4) \cup (4, \infty)$
b. $(0, 4) \cup (4, \infty)$
c. $(1, 4) \cup (4, \infty)$
d. $(e, 4) \cup (4, \infty)$
e. $(4, \infty)$

Properties of Exponents: *a, b, m, n* are real numbers

- Products of powers: $a^m a^n = a^{m+n}$
- Power of a power: $a^{mn} = (a^m)^n$
- Power of a product: $(ab)^m = a^m b^m$
- Negative exponent: $a^{-m} = \frac{1}{a^m},\ a \neq 0$
- $a^0 = 1$

Logarithms: The inverse of exponents

- $log_b(MN) = log_b(M) + log_b(N)$
- $log_b(\frac{M}{N}) = log_b(M) - log_b(N)$
- $log_b(M^k) = klog_b(M)$
- $log_b(1) = 0$

**Estimations, Absolute Values, and Inequalities**

For absolute values, the main trick is to split into a piecewise function, which gets rid of the absolute values. Then, you can perform other operations on that as needed. For each absolute value, find where each of them equal 0 and do separate functions for them (like in your PCE).

For inequalities, remember that the inequality flips when you multiply both sides by a negative number (same for division, but you could think of that as the same as multiplication by the reciprocal). Otherwise you can do anything to both sides like a regular equation without flipping the sign.

**Limits**

Limits basically ask: What does a function's value approach as you get closer and closer to it? This problem arises often when the function doesn't exist at the point itself, has an asymptote, or has a discontinuity. You could take a limit of a function that exists at the point (and is continuous there) but that would just be plugging in the value into the function which is no different than just evaluating the function at that point itself.

For a limit to exist, it must exist on both sides and those sides need to be equal to each other. So, if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ and these limits exist, then the two-sided limit exists. In other words: $\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$.

Remember that if the function's limit equals something at a point, it doesn't necessarily have to mean that the function must exist there. In other words, $\lim_{x \to a} f(x) = f(a)$ doesn't have to be true.

A couple strategies for limits:

- Factor polynomials and cancel out as much as you can (try to remove the divide by 0 problem)
- Splitting fractions could make things easier
- For radicals, multiply by conjugate
- Remember that $\lim_{x \to 0} \frac{sinx}{x} = 1$ and $\lim_{x \to 0} \frac{1-cosx}{x} = 0$ for trigonometric limits
- Remember squeeze theorem

Practice problem:

6. $\lim_{x \to \infty} \left( \sqrt{x + \sqrt{x}} - \sqrt{x - \sqrt{x}} \right) =$ _____? [2 marks]

For limits at infinity, we want to look at end behaviour, that is, what the graph tends towards as we keep going farther out in any direction. When were dealing with limits with x approaching infinity, if they tend towards infinity, such as in the case of polynomials, the limit then approaches infinity. In other words, $\lim_{x \to \infty} f(x) = \pm\ \infty$ when f(x) is a polynomial (as an example, many other functions do this too). If the limit as x approaches infinity tends towards a specific value, then the function has a horizontal asymptote where that value is the asymptote. For example, $\lim_{x \to \infty} arctan(x) = \frac{\pi}{2}$, meaning that $\frac{\pi}{2}$ is a horizontal asymptote of $arctan(x)$.

We can think of this in the other direction as well. If a limit as x approaches a specific value is infinity, there is a vertical asymptote at that location. For example, $\lim_{x \to 0} \frac{1}{x^2} = \infty$.

Oscillating or repeating functions do not have limits as x approaches infinity. A function's end behaviour must do one specific thing, either go to infinity, or tend towards a specific value, it cannot rotate between different values as it would not "agree" on a certain value.

Limit laws basically allow you to distribute limits among addition, subtraction, multiplication, and division, and move constants outside of the limit. In other words,
$\lim_{x \to a} f(x) \pm g(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$, $\lim_{x \to a} f(x)g(x) = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$,
$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, and $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

**Squeeze Theorem**

The squeeze theorem basically squeezes a function whose limit is hard to evaluate between two functions, one that is always equal to or larger than that function, and another that is always equal to or smaller than that function. Both of these squeezing functions should have limits that are easy to evaluate. Then, at the point where the limit is evaluated, we need the upper and lower function's limits to equal each other, which would therefore mean that the original function's limit must also equal given that the function is sandwiched in between the other two functions.

In other words, given a function f(x) and bounding functions g(x) and h(x) where $g(x) \leq f(x) \leq h(x)$ at all points, then if $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = g(a) = h(a)$, then $\lim_{x \to a} f(x) = f(a) = g(a) = h(a)$. Basically once the bounded function's limits equal each other, the squeezed original function must be in between each other, and since the bounded function's limits equal each other, the limit of the original function must be equal to that too.

This is very often used with sinx and cosx as for both, they are bounded by -1 and 1. When encountering a difficult limit question using sinx or cosx, very often the answer uses squeeze theorem. For example, if you have xsinx, you can bound it by -x and x (multiplying the original bounds by x as done for sinx).

**Continuity and IVT**

Continuity, put very simply, is the ability to draw a graph without lifting the pen from the paper. For a function to be continuous **at a point**, it must satisfy 3 conditions:

1. f(x) at a must exist
2. $\lim_{x \to a} f(x)$ must exist
3. $\lim_{x \to a} f(x) = f(a)$

In other words, we can say the function must exist at the point, the limit must exist at the point (meaning both one-sided limits must exist and be equal), and the limit must equal the function at the point.

Remember that [] closed brackets means the interval including the end points, and () open brackets means the interval not including the points.

Practice problems:

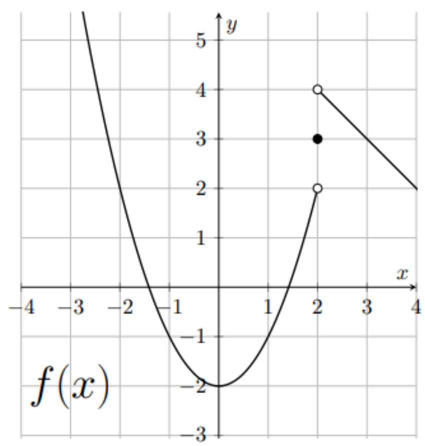
**Short Answer**: Unsupported answers will not receive full credit. Organize your work in a reasonably neat and coherent way. Put your final answer in the boxes provided.

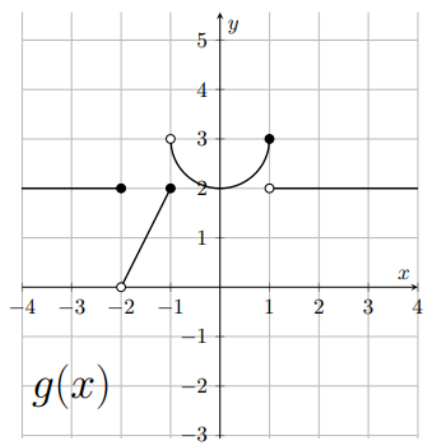
**11.** Let

$$f(x) = \begin{cases} 5^{2x-2} + 2, & x \leq 1 \\ \dfrac{3x^2 + x + 5}{Ax^2 + 2}, & 1 < x < 2 \\ \ln(Bx) + 1, & x \geq 2 \end{cases}$$

where $A$ and $B$ are positive constants.

(a) Find all values of $A$ ad $B$ such that $f(x)$ is continuous at $x = 1$ and $x = 2$. [5 marks]




Which of the following statements are true?

a. $f(x)$ is continuous at -2
b. $\lim\limits_{x \to 0} g(f(x))$ exists and $g(f(0))$ is defined.
c. $\lim\limits_{x \to 0} g(f(x))$ does not exist but $g(f(0))$ is defined
d. $\lim\limits_{x \to 0} g(f(x))$ exists but $g(f(0))$ is undefined.
e. $g(f(x))$ is continuous at 0.

The Intermediate Value Theorem (IVT) is used when the questions asks whether a root exists, or a certain number exists within the codomain. For example, “in this interval, there exists a root” or something like that. Regardless, you generally always want to turn IVT problems into root problems by making everything equal to 0 so that you can just look for a sign change.

It basically states that for a continuous function within a CLOSED interval $[x_1, x_2]$, the function must take on every value between $[f(x_1), f(x_2)]$. In other words, within the closed interval of x-values, every y-value taken from the interval of x-values must exist in that interval.

Practice problems:

1. True or False: Let $f(x) = 2^x - \frac{10}{x}$. Then there exists a $c \in [-\frac{5}{2}, 3]$ such that $f(c) = 0$. Explain your answer.
2. Show that the equation $x^{186} + \frac{188}{x^2+1+\cos^2 x} = 100$ has at least two solutions.

**Most people did poorly on Problem 2 when they did it last year.**

Remember that you don't have to use the interval they give, you can make your own. In this example, you couldn't use this interval since there is an asymptote at x = 0, but you can split into subintervals and make a new interval within that.

Additionally, when testing sign changes, you don't necessarily need to know the value of the function at a point, just whether it is positive or negative, and sometimes you can figure that out just by looking at the function and seeing if it is possible to be positive or negative. This can be used if you don't have a calculator and/or the function is difficult to evaluate, so just finding the sign is sufficient (it is sufficient regardless so either way works).

Steps for IVT problems:

1. Determine whether the function is continuous and determine any areas where the function is discontinuous
2. Turn your problem into a roots problem if it hasn't been done already, usually by moving everything to one side so that it equals 0
3. Find a closed interval that is continuous and test for a sign change within f(x). If you find one with a sign change in a continuous closed interval, that shows that there is at least one solution within that interval which you can state
4. Find new intervals if you need to prove that there is at least more than one interval

Final practice problem:

**11.** On the axes below, sketch a well-labeled graph of a function $f(x)$ defined everywhere that satisfies the given properties below. [6 marks]

- $f(-1) = 1$
- $\lim_{x\to 3+} f(x) \to \infty$
- $f(x)$ is invertible on $(3, 5)$
- $\lim_{x\to\infty} f(x) = 2$
- $f(x)$ is not continuous at $x = -1$
- $\lim_{x\to 3^-} f(x) = 1$
- $\lim_{x\to 5^-} f(x) \neq \lim_{x\to 5^+} f(x)$
- $\lim_{x\to -\infty} f(x)$ does not exist but $\lim_{x\to -\infty} f(x) \not\to \pm\infty$