40.017 PROBABILITY & STATISTICS

Homework 1

Michael Hoon
1006617

February 1, 2024

Question 1

(a)

Let A be the random variable denoting the number of questions chosen from section A. Since the student 'samples' the question without replacement, A follows a hypergeometric distribution $A \sim \text{hypgeo}(12, 9, 9)$. The pmf $f(x)$ is given by:

$$f(x) = \mathbb{P}(X = x) = \frac{\binom{j}{x}\binom{k}{n-x}}{\binom{j+k}{n}}$$

for 6 questions from section A, we have $\mathbb{P}(X = 6)$:

$$\begin{aligned}\mathbb{P}(X = 6) &= \frac{\binom{9}{6}\binom{9}{6}}{\binom{18}{12}} \\ &= 0.380090 \\ &\boxed{\approx 0.380}\end{aligned}$$

(b)

We need to consider two cases, one where 5 questions come from Section A and the other 7 from Section B, and vice versa. i.e.

$$\begin{aligned}\mathbb{P}(A = 5) + \mathbb{P}(A = 7) &= \frac{\binom{9}{5}\binom{9}{7}}{\binom{18}{12}} + \frac{\binom{9}{7}\binom{9}{5}}{\binom{18}{12}} \\ &= 0.4886877 \\ &\boxed{\approx 0.489}\end{aligned}$$

Question 2

(a)

To find the probability that at least 1 man receives his own hat, we consider the complementary case of no man receiving his own hat, given by D_6. Thus:

$$\begin{aligned}\mathbb{P}(\text{'at least 1 man receives his own hat'}) &= 1 - \frac{D_6}{6!} \\ &= \\ &\boxed{\approx}\end{aligned}$$

(b)

Again we consider the complementary case that no man receives his own hat, or only 1 man receives his own hat. Thus:

$$\mathbb{P}(\text{'at least 2 men receives their own hats'}) = 1 - \underbrace{\frac{D_6}{6!}}_{\text{0 man receives his own hat}} - \underbrace{\frac{6 \times D_5}{6!}}_{\text{1 man receives his own hat}}$$

$$=$$

$$\boxed{\approx}$$

0.1 (c)

The probability that at least 5 men receive their own hats can be interpreted as every man receiving their own hats, since if 5 man receive their own hats, the last man will automatically also receive his own hat. In this case, the probability is thus 1:

$$\mathbb{P}(\text{'at least 5 men receives their own hats'}) = \boxed{1}$$

Question 3

Question 4

Question 5

Let $X \sim \text{negbin}(n, p)$. The MGF of X is:

$$M_X(t) =$$

Question 6

Question 7