Question 1

a) Let D and F denote Domestic and Foreign stocks (in millions) respectively. We formulate the Linear Program (LP):

$$
\begin{aligned}
\max_{D,F}\ & 0.11D + 0.17F \\
\text{s.t.}\ & D + F \leq 12 \\
& 0 \leq D \leq 10 \\
& 0 \leq F \leq 7 \\
& D \geq \frac{1}{2}F \\
& F \geq \frac{1}{2}D
\end{aligned}
$$

b) Graphically, we have Figure 1 with corner points labelled and the feasible region shaded:

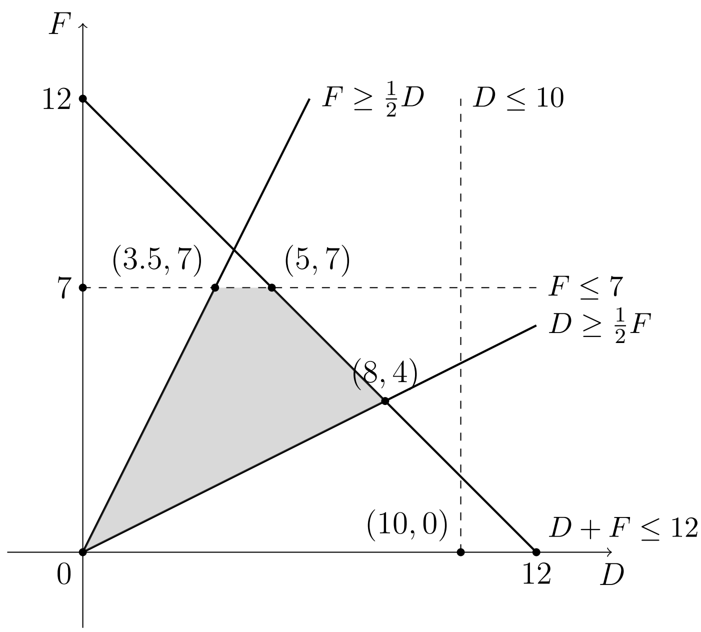

Figure 1: Feasible Region of LP

c) Solving graphically, the optimal value of F and D that maximises returns are $D = 5$, $F = 7$ from Figure 1, for a total (maximum) return of:

$$\boxed{0.11(5) + 0.17(7) = \$1.74 \text{ million}}$$

Question 2

a) Given the LP constraints

$$
\begin{aligned}
-x_1 + x_2 &\geq 1 \\
x_1 &\geq 0
\end{aligned}
$$

We have the feasible region shaded and corner points labelled:

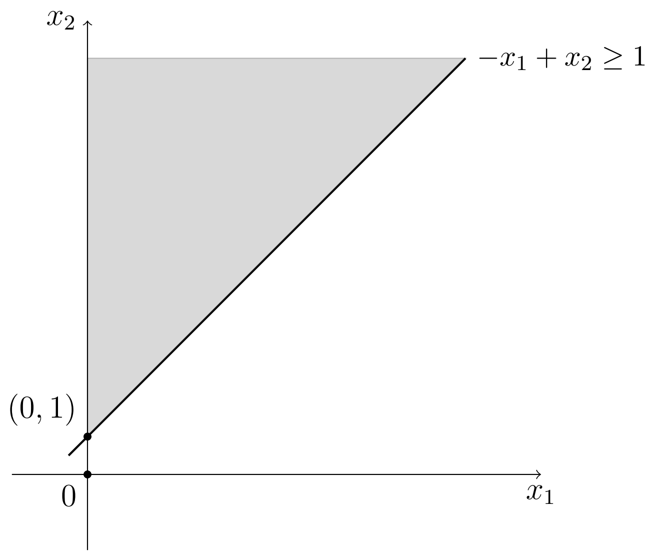

Figure 2: Feasible Region of LP

b) For a minimisation problem, we have the complete LP:

$$
\begin{aligned}
\min_{x_1, x_2} \ & \alpha x_1 + x_2 \\
\text{s.t.} \ & -x_1 + x_2 \geq 1 \\
& x_1 \geq 0
\end{aligned}
$$

i) For the optimal solution to be unique, we need $\alpha = 0$ for a minimum at $x_2 = 1$.

ii) For there to be multiple optimal solutions with infinite optimal objective values, we need $\alpha \in \mathbb{R} \backslash \{-1, 0\}$.

iii) For there to be an unbounded optimal objective value, we need $\alpha = -1$.

Question 3

a) Since Z is a discrete random variable that takes values in the set $\{1, 2, \dots, 10\}$, and $p_i = \text{Prob}(Z = i)\ \forall i \in \{1, 2, \dots, 10\}$, with sum and non-negative constraints, we can formulate the LP:

$$\begin{aligned} \max_{p_i} \quad & \sum_{i=5}^{10} p_i \\ \text{s.t.} \quad & \sum_{i=1}^{10} p_i = 1 \\ & \sum_{i=1}^{10} i \cdot p_i = 4 \\ & p_i \geq 0 \end{aligned}$$

We want to optimise the probability distribution of Z such that the likelihood of Z being 5, 6, 7, 8, 9, or 10 is maximised.

Using the following *JuMP* code, we have the final probability distribution of Z being:

$$\boxed{p = \{0.25, 0, 0, 0, 0.75, 0, 0, 0, 0, 0\}}$$

Question 4

For the minimisation problem:

$$\begin{aligned}
\min\ & x_1 - x_2 \\
\text{s.t.}\quad & 2x_1 + 3x_2 - x_3 + x_4 \leq 0 \\
& 3x_1 + x_2 + 4x_3 - 2x_4 \geq 3 \\
& -x_1 - x_2 + 2x_3 + x_4 = 6 \\
& x_1 \leq 0 \\
& x_2,\ x_3 \geq 0
\end{aligned}$$

We can rearrange the LP into the form:

$$\begin{aligned}
\max\ & \mathbf{c}^\top \mathbf{x} \\
\text{s.t.}\quad & \mathbf{A}\mathbf{x} \leq \mathbf{b}
\end{aligned}$$

For the 3$^{\text{rd}}$ constraint, we separate into two cases:

$$\begin{aligned}
-x_1 - x_2 + 2x_3 + x_4 &\geq 6 \\
x_1 + x_2 - 2x_3 - x_4 &\leq -6
\end{aligned}$$

where:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 2 & 3 & -1 & 4 \\ -3 & -1 & -4 & 2 \\ -1 & -1 & 2 & 1 \\ 1 & 1 & -2 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ -3 \\ 6 \\ -6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Question 5

Question 6

Question 7