40.012 MANUFACTURING AND SERVICE OPERATIONS

Assignment 2

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Question 1

Holding Costs

Holding costs are the expenses associated with storing unsold goods (having capital tied up in inventory), and they include storage fees, insurance, spoilage, obsolescence, and **capital costs**. To find holding costs, we need to calculate the **storage fees** (cost of renting warehouse space or the depreciation cost of owned storage facilities), **insurance costs** (insurance premiums paid to cover inventory), **spoilage and obsolescence fees** (depreciation of parts that is expected to become obsolete over time), and the **capital cost** (opportunity cost of capital tied up in unsold inventory, which could be invested elsewhere). Then to find the overall holding cost, we can sum the values obtained.

For a production manager in a company producing hobby electric aircrafts, this can include the costs for storing components such as **motors, batteries, and aircraft kits** (and the warehousing, insurance, capital costs tied up in inventory).

Smoothing Costs

Smoothing costs accrue as a result of changing the production levels from one period to the next. They are incurred when adjusting production rates to match demand fluctuations, avoiding large swings in production levels. Examples include costs related to overtime, hiring and training temporary staff (that work on and pack the aircraft kits or designing the aircrafts), and adjusting production schedules. To find smoothing costs, we need to keep track of the **overtime payments** (extra wages paid to workers for working overtime), **temporary staff costs** (hiring and training expenditure), **Schedule Adjustment Costs** (changing production schedules such as machine downtime and setup, used to produce aircraft kits). Furthermore, Firms that hire and fire frequently develop a poor public image. This could adversely affect sales and discourage potential employees from joining the company. Figure below shows the cost of changing the size of the workforce:

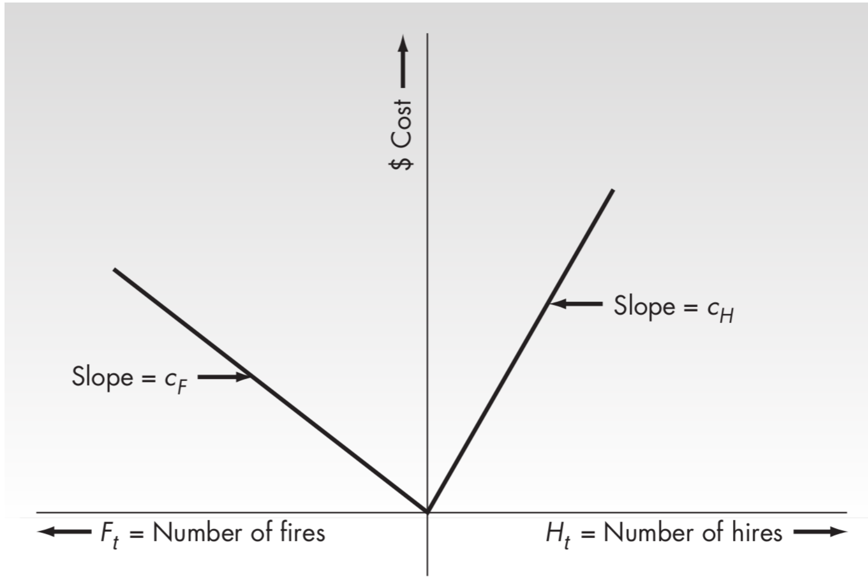

Figure 1: Cost of changing the size of the workforce

Backordering Costs

Backordering costs are expenses incurred when fulfilling orders that **cannot be immediately satisfied from current inventory** (including admin expenses, expedited shipping, and penalties for late deliveries). In the context of the company, this can be measured via additional costs of managing backorders, rush shipping for delayed orders, and any penalties for late deliveries (of raw materials or delivering to customers).

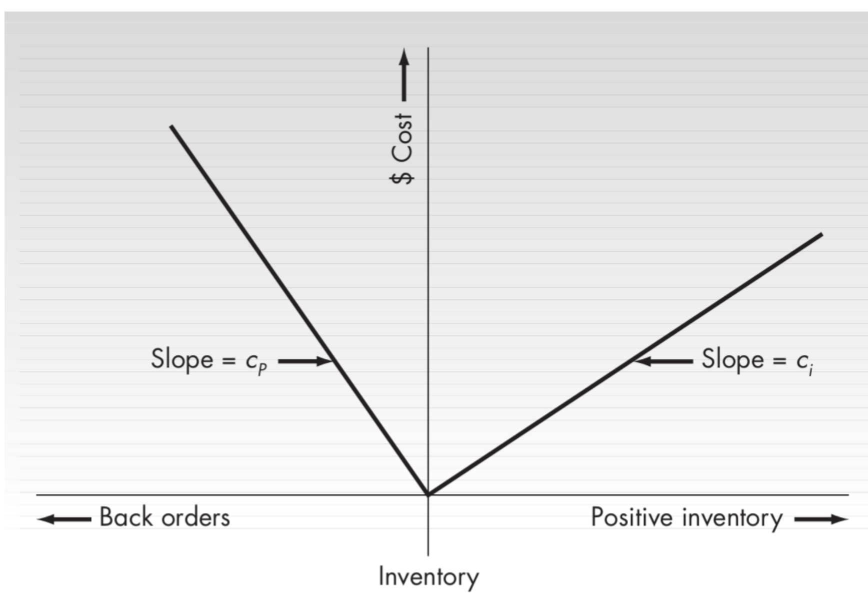

Figure 2: Holding and Backorder Costs

Planning Horizon

The planning horizon is the future time period over which inventory control and production planning are conducted. It determines the timeframe for which forecasts and plans are made. To determine the planning horizon, we can analyse the **typical demand cycles** for the electric aircrafts (identifying seasonal peak periods), **production lead time** (time required to manufacture the components and assemble the aircraft kits), and align the planning horizon with the company's long term strategic objectives, which can usually span up to a year (Annual KPIs etc.).

Planning Period

The planning period is the specific interval within the planning horizon at which plans and forecasts are reviewed and updated (daily, weekly, monthly, etc.) To determine the planning period, we can identify the **review cycle** (how often demand forecasts and inventory levels need reassessment, influenced by demand variability and lead times.), **production cadence** (Coordinate with production schedules to ensure synchronization between planning updates and manufacturing cycles.), and assess based on the **data availability** (data on sales, inventory, and production is available - can include customer data as well).

Question 2

(a)

Since the company has 200 units of inventory at the start of month 1, the net predicted demand for that month is given by $1050 - 200 = 850$. Here, there is no minimum ending inventory constraint. We have defined the following costs for this company:

1. $C_I = \$0.1$ per unit per month (Inventory holding cost)
2. $C_H = \$100$ (Hiring cost for one worker)
3. $C_F = \$200$ (Firing cost for one worker)

and we have that the number of aggregate units produced by one worker in one day $K = 0.308$. Since we want to have a constant workforce plan, we need to implement a level strategy: **capacity is kept constant** during the planning period, and instead inventory is kept between periods; **capacity is set to the minimum possible** to ensure **no shortages** in any period. Table 1 below shows the overall constant workforce production plan, computed using Excel.

A	B	C	D	E	F	G	H	I	J
Month	**Workdays**	**Demand**	**Cum Demand**	**Units / Worker** ($K \times B$)	**Cum Units / Worker**	**Workers** ($\lceil D/F \rceil$)	**Production** ($E \times 156$)	**Cum Production**	**Inventory** ($I - D$)
1	26	850	850	8.008	8.008	107	1249.248	1249.248	399.248
2	24	1260	2110	7.392	15.4	138	1153.152	2402.4	292.4
3	20	510	2620	6.16	21.56	122	960.96	3363.36	743.36
4	18	980	3600	5.544	27.104	133	864.864	4228.224	628.224
5	22	770	4370	6.776	33.88	129	1057.056	5285.28	915.28
6	23	850	5220	7.084	40.964	128	1105.104	6390.384	1170.384
7	14	1050	6270	4.312	45.276	139	672.672	7063.056	793.056
8	21	1550	7820	6.468	51.744	152	1009.008	8072.064	252.064
9	23	1350	9170	7.084	58.828	**156**	1105.104	9177.168	7.168
10	24	1000	10170	7.392	66.22	154	1153.152	10330.32	160.32
11	21	970	11140	6.468	72.688	154	1009.008	11339.328	199.328
12	13	680	11820	4.004	76.692	155	624.624	11963.952	143.952
Total Inventory:									**5704.7840**

Table 1: Constant Workforce Production Plan

We can see that the minimum number of workers needed in this plan without backlogging is 156 (bolded), the values are obtained after rounding with the *ceiling function*. We can then obtain the Production units per month (H) by multiplying the number of units produced per worker (E) by 156. The total inventory is given by the sum of the last column, which is 5704.7840 units. The inventory cost of this plan is given by the number of inventory units multiplied by the holding cost:

$$\begin{aligned} \text{Inventory Cost} &= C_I \times 5704.780 \\ &= \$0.1 \times 5704.7840 \\ &= \boxed{\$570.47480} \end{aligned}$$

Thus, the total cost is the sum of the hiring costs and the holding costs, calculated as:

$$\begin{aligned}\text{Total Cost} &= C_I \times 5704.480 + C_H \times 156 \\ &= \$1.1 \times 5704.480 + \$100 \times 156 \\ &= \$16170.4784 \\ &\approx \boxed{\$16170.48}\end{aligned}$$

(b)

For a zero inventory plan, we need to develop a chase strategy. Under this plan, the workforce is changed each month in order to produce enough units to most closely match the demand pattern. Capacity is adjusted up and down to achieve this matching, with minimal inventory buildup. Similarly, there is no required ending inventory constraint here. Table 2 below shows the chase strategy for a zero inventory plan, indicating a workforce with 107 initial workers in the first month:

A	B	C	D	E	F	G	H	I	J
Month	**Workdays**	**Demand**	**Adj. Demand** $(C-J)$	**Units / Worker** $(B \times K)$	**Worker Level** $(\lceil D/E \rceil)$	**Fire**	**Hire**	**Production** $(E \times F)$	**Inventory** $(I-D)$
1	26	850	850	8.008	107		107	856.856	6.856
2	24	1260	1253.144	7.392	170		63	1256.64	3.496
3	20	510	506.504	6.16	83	87		511.28	4.776
4	18	980	975.224	5.544	176		93	975.744	0.52
5	22	770	769.48	6.776	114	62		772.464	2.984
6	23	850	847.016	7.084	120		6	850.08	3.064
7	14	1050	1046.936	4.312	243		123	1047.816	0.88
8	21	1550	1549.12	6.468	240	3		1552.32	3.2
9	23	1350	1346.8	7.084	191	49		1353.044	6.244
10	24	1000	993.756	7.392	135	56		997.92	4.164
11	21	970	965.836	6.468	150		15	970.2	4.364
12	13	680	675.636	4.004	169		19	676.676	1.04
Total Hire / Fire:						**257**	**426**	**Total:**	**41.588**

Table 2: Zero Inventory Production Plan

We see that there is residual inventory build up every month (J), which we can use as leverage for the next month's demand. Thus, we introduce a new column (D) that accounts for this residual inventory build-up, by subtracting current demand with the previous inventory. To calculate the total cost of this plan, we need to calculate the cost of hiring and firing, as well as the residual inventory cost:

$$\begin{aligned}\text{Hiring Cost} &= C_H \times 426 \\ &= \$100 \times 426 \\ &= \$42600\end{aligned} \qquad \begin{aligned}\text{Firing Cost} &= C_F \times 257 \\ &= \$200 \times 257 \\ &= \$51400\end{aligned} \qquad \begin{aligned}\text{Inventory Cost} &= C_I \times 200 \\ &= \$0.1 \times 41.588 \\ &= \$4.1588\end{aligned}$$

The total cost is then the sum of the three, which gives us:

$$\begin{aligned}\text{Total Cost} &= 42600 + 51400 + 4.1588 \\ &= \$94004.1588 \\ &\approx \boxed{\$94004.16}\end{aligned}$$

(c)

Linear Program for $\mathbb{R}$ Variables

To develop a Linear Program (LP) to model this problem (variables treated as real numbers $\mathbb{R}$), we first need to define appropriate decision variables, constraints, and the objective function. We will define the following decision variables:

1. W_t: Number of workers employed in month t.
2. H_t: Number of workers hired at the beginning of month t.
3. F_t: Number of workers fired at the beginning of month t.
4. I_t: Inventory level at the end of month t.
5. P_t: Production Quantity in month t.

and the following constants and parameters:

1. D_t: Demand in month t
2. $c_I = \$0.10$: Holding cost per unit per month
3. $c_H = \$100$: Hiring cost per worker
4. $c_F = \$200$: Firing cost per worker
5. $K = 0.308$: Production rate per worker per day
6. n_t: Number of workdays in month t
7. $I_0 = 200$: Initial inventory level

Since our objective is to minimise total costs, we first define the objective function as:

$$\min \sum_{t=1}^{12} (c_I \cdot I_t + c_H \cdot H_t + c_F \cdot F_t) \tag{1}$$

subject to the following constraints:

$$\text{s.t. } W_t = W_{t-1} + H_t - F_t, \qquad \forall\, 1 \le t \le 12 \tag{2}$$
$$I_t = I_{t-1} + P_t - D_t, \qquad \forall\, 1 \le t \le 12 \tag{3}$$
$$P_t = K \cdot n_t \cdot W_t, \qquad \forall\, 1 \le t \le 12 \tag{4}$$
$$W_0 = 0$$
$$I_0 = 200$$
$$W_t,\, H_t,\ F_t,\ I_t \ge 0 \qquad \forall\, 1 \le t \le 12 \tag{5}$$

for a total of $T = 12$ months. Equation 2 represents the workforce conservation constraint, Equation 3 represents the Inventory Balance constraint, Equation 4 represents the Production Quantity constraint, and Equation 5 represents the non-negativity constraint.

To solve this LP with the given data, the GLPK open-sourced solver was used to evaluate the results (code is attached in the Appendix). The result of the optimizer is shown in the table below:

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Month	**Workdays**	**Demand**	**Units / Worker** ($B \times K$)	**Optimal Workers**	**Optimal Fires**	**Optimal Hires**	**Production** ($E \times F$)	**Optimal Inventory**
1	26	850	8.008	155.87815		155.87815	1248.272	398.272
2	24	1260	7.392	155.87815			1152.251	290.524
3	20	510	6.16	155.87815			960.209	740.733
4	18	980	5.544	155.87815			864.188	624.921
5	22	770	6.776	155.87815			1056.230	911.152
6	23	850	6.776	155.87815			1104.240	1165.392
7	22	1050	6.776	155.87815			672.147	787.539
8	21	1550	6.468	155.87815			1008.220	245.759
9	23	1350	7.084	155.87815			1104.241	0.000
10	24	1000	7.392	155.87815			1152.251	152.251
11	21	970	6.468	155.87815			1008.220	190.471
12	13	680	4.004	155.87815			624.136	134.607
Total Fire / Hire:					**0**	**155.87815**	**Total:**	**5641.62**

Table 3: Optimal Solution obtained from Linear Program

From this, we obtain a minimum total cost value of $\boxed{\$16151.98}$.

The runtime of this code is $\boxed{5.168838}$ seconds (using an online Julia compiler, tends to be slower compared to running locally - 99.97% on compilation time with Replit).

Integer Program for $\mathbb{Z}$ Variables

Now, we require the worker level variables to be integer values. Since there is no change to the objective function, modifying some additional integrality constraints (10), we have:

$$\begin{aligned}
\text{s.t. } W_t &= W_{t-1} + H_t - F_t, && \forall\, 1 \le t \le 12 && (6)\\
I_t &= I_{t-1} + P_t - D_t, && \forall\, 1 \le t \le 12 && (7)\\
P_t &= K \cdot n_t \cdot W_t, && \forall\, 1 \le t \le 12 && (8)\\
W_0 &= 0 && &&\\
I_0 &= 200 && &&\\
W_t,\, H_t,\ F_t,\ I_t &\ge 0 && \forall\, 1 \le t \le 12 && (9)\\
W_t,\, H_t,\ F_t,\ I_t &\in \mathbb{Z} && && (10)
\end{aligned}$$

Similarly, solving this in Julia with GLPK gives us the following values:

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
Month	**Workdays**	**Demand**	**Units / Worker** $(B \times K)$	**Optimal Workers**	**Optimal Fires**	**Optimal Hires**	**Production** $(E \times F)$	**Optimal Inventory**
1	26	850	8.008	156		156	1248.272	399.248
2	24	1260	7.392	156			1152.251	292.400
3	20	510	6.16	156			960.209	743.360
4	18	980	5.544	156			864.188	628.223
5	22	770	6.776	156			1056.230	915.280
6	23	850	6.776	156			1104.240	1170.384
7	22	1050	6.776	156			672.147	793.056
8	21	1550	6.468	156			1008.220	252.064
9	23	1350	7.084	156			1104.241	7.168
10	24	1000	7.392	156			1152.251	160.320
11	21	970	6.468	156			1008.220	199.328
12	13	680	4.004	156			624.136	143.952
Total Fire / Hire:					**0**	**156**	**Total:**	**5704.784**

Table 4: Optimal Solution obtained from Linear Program

From this, we obtain a minimum total cost value of $\boxed{\$16170.48}$.

The runtime of this code is $\boxed{9.001999}$ seconds (using an online Julia compiler, tends to be slower compared to running locally - 99.95% on compilation time with Replit).

Question 3

(a)

In the context of "Chewy" cereals, we shall assume that the demand for the product is a known constant of $\lambda = 280$ kg/year. The ordering setup cost is $K = \$45$, and the purchase price is $c = \$2.40$/kg. Since the holding costs are assessed at a 20% interest rate, the annual holding cost per unit is calculated as

$$\begin{aligned} h &= \$2.40 \times 0.2 \\ &= \$0.48 \end{aligned}$$

Using the Economic Order Quantity (EOQ) model to find the optimal order quantity (defined as Q), we have that:

$$\text{Ordering Cost per unit time} = \frac{K + cQ}{T} \qquad \text{Average Inventory Holding Cost} = \frac{hQ}{2}$$

We thus want to minimise the overall cost function $G(Q)$:

$$\begin{aligned} G(Q) &= \frac{K + cQ}{T} + \frac{hQ}{2} \\ &= \frac{K + cQ}{Q/\lambda} + \frac{hQ}{2} \\ &= \frac{K\lambda}{2} + \lambda c + \frac{hQ}{2} \end{aligned}$$

Since this is a convex function, it is trivial to obtain the minimum value Q^* by setting the first derivative to be 0, and the expression is given by:

$$\begin{aligned} Q^* &= \sqrt{\frac{2K\lambda}{h}} \\ &= \sqrt{\frac{2 \times 45 \times 280}{0.48}} \\ &= 50\sqrt{21} \\ &\approx \boxed{230 \text{ kg}} \end{aligned}$$

This is also the minimum as proven in class due to the second derivative being positive. Thus, the optimal order quantity of muesli ingredients is approximately 230 kgs.

(b)

The time T between order placements can be obtained by:

$$\begin{aligned} T &= \frac{Q}{\lambda} \\ &= \frac{50\sqrt{21}}{280} \\ &\approx \boxed{0.818 \text{ years}} \end{aligned}$$

(c)

The total annual setup cost can be calculated by:

$$\begin{aligned}\text{Annual setup cost} &= \frac{K}{T} \\ &= \frac{45}{0.818} \\ &= 54.9909 \\ &\approx \boxed{\$54.99/\text{year}}\end{aligned}$$

The holding cost per cycle is given by:

$$\text{Inventory Holding Cost} = \frac{hQ}{2}$$

Thus, the annual holding cost can be obtained by:

$$\begin{aligned}\text{Annual Holding Cost} &= \frac{hQ}{2T} \\ &= \frac{0.48 \times 50\sqrt{21}}{2 \times 0.818} \\ &= \boxed{\$67.20/\text{year}}\end{aligned}$$

(d)

Since there are 52 weeks in a year, the 3 week order replenishment lead time is equivalent to $\tau = \frac{3}{52} = 0.0577$ years $< T = 0.818$ years. We can find the reorder point R by:

$$\begin{aligned}R &= \lambda\tau \\ &= 280 \times \frac{3 \times 7 \text{ days}}{365 \text{ days}} \\ &= \frac{1176}{73} \\ &\approx \boxed{16.1 \text{ kg}}\end{aligned}$$

Thus, Chewy's employees should place the replenishment order when the inventory level falls at or below 16.1 kg.

Question 4

(a)

Now we have a finite production rate $P = 1120$ kg / year, with all other variables the same as before. We note that $P > \lambda = 280$ for feasibility. **Assuming that there are no shortages**, the costs will also remain the same as before. To account for a finite production rate, we must now use the Production Order Quantity (POQ) model, accounting for the fact that production and consumption occur simultaneously. We define H to be the maximal inventory level, where $H \neq Q$. We have that:

$$\frac{H}{T_1} = P - \lambda \qquad \text{(Surplus Produced)} \tag{11}$$

and per cycle,

$$T_1 = \frac{Q}{P} \tag{12}$$

From the above equations, we obtain an expression for H:

$$H = Q\left(1 - \frac{\lambda}{P}\right) \tag{13}$$

Now, our annual cost function must be modified:

$$\begin{aligned} G(Q) &= \frac{K}{T} + \frac{hH}{2} \\ &= K\frac{\lambda}{Q} + \frac{hQ}{2}\left(1 - \frac{\lambda}{P}\right) \end{aligned}$$

With this, to find the optimal Q^* that must be ordered, we take the first derivative and set to 0 to obtain:

$$\begin{aligned} G'(Q) &= -\frac{K\lambda}{Q^2} + \frac{h}{2}\left(1 - \frac{\lambda}{P}\right) \\ \frac{K\lambda}{Q^2} &= \frac{h}{2}\left(1 - \frac{\lambda}{P}\right) \end{aligned}$$

$$\implies Q^* = \sqrt{\frac{2K\lambda}{h\left(1 - \frac{\lambda}{P}\right)}} \tag{14}$$

Using equation 14, we get the optimal order quantity to be:

$$\begin{aligned} Q^* &= \sqrt{\frac{2 \times 45 \times 280}{0.48 \times \left(1 - \frac{280}{1120}\right)}} \\ &= 100\sqrt{7} \\ &\approx \boxed{264.58 \text{ kg}} \end{aligned}$$

This is also a minimum point as proved in class with the second derivative being positive.

(b)

The cycle length is given by T, where $Q = \lambda T$. The time between order placements can be obtained with:

$$\begin{aligned} T &= \frac{Q}{\lambda} \\ &= \frac{100\sqrt{7}}{280} \\ &\approx \boxed{0.945 \text{ years}} \end{aligned}$$

(c)

The maximal inventory level is defined in equation 13:

$$\begin{aligned} H &= Q\left(1 - \frac{\lambda}{P}\right) \\ &= 100\sqrt{7}\left(1 - \frac{280}{1120}\right) \\ &\approx \boxed{198.4 \text{ kg}} \end{aligned}$$

Question 5

We are given that the demand is $\lambda = 140$ units/year, the setup costs $K = \$30$, and the inventory holding costs have an annual interest rate of $h = 18\%$. Since this is an **incremental quantity discount schedule**, we first define the ordering cost $C(Q)$:

$$C(Q) = \begin{cases} 350Q, & 0 \leq Q < 26 \\ 350(25) + 315(Q - 25), & 26 \leq Q < 51 \\ 350(25) + 315(25) + 285(Q - 50), & 51 \leq Q \end{cases}$$

and thus the ordering cost per unit is

$$\frac{C(Q)}{Q} = \begin{cases} 350, & 0 \leq Q < 26 \\ 315 + \frac{875}{Q}, & 26 \leq Q < 51 \\ 285 + \frac{2375}{Q}, & 51 \leq Q \end{cases} \tag{15}$$

Now, the average annual cost function is given by:

$$G(Q) = \frac{\lambda C(Q)}{Q} + \frac{K\lambda}{Q} + I\left[\frac{C(Q)}{Q}\right]\frac{Q}{2} \tag{16}$$

which we will solve for the optimal Q^* that minimises $G(Q)$ for each of the functions of $\frac{C(Q)}{Q}$. Here, we have 3 different representations of $G(Q)$ depending on which interval Q falls into. Since $C(Q)$ is continuous, $G(Q)$ must also be continuous. The optimal solution will occur at one of the minimum of the 3 average annual cost curves $G(Q)$, which we define as $G_0(Q)$, $G_1(Q)$, and $G_2(Q)$:

$$\begin{aligned} G_0(Q) &= 140 \times 350 + \frac{30 \times 140}{Q} + 0.18\left[\frac{350Q}{2}\right] \\ G_0'(Q) &= -\frac{4200}{Q^2} + 31.5Q \end{aligned}$$

which is minimised at

$$\begin{aligned} 0 &= -\frac{4200}{Q^2} + 31.5Q \\ \frac{4200}{Q^2} &= 31.5 \\ Q^{(0)} &= \sqrt{\frac{4200}{31.5}} \\ &= \lceil 11.547 \rceil \\ &= \boxed{12} \end{aligned}$$

to prove that it is indeed a minimum, consider

$$G_0''(Q) = \frac{8400}{Q^3} > 0, \quad \forall\, Q > 0$$

Similarly, we have

$$\begin{aligned} G_1(Q) &= \lambda\left(315+\frac{875}{Q}\right)+\frac{K\lambda}{Q}+0.18\left(315+\frac{875}{Q}\right)\times\frac{Q}{2} \\ G_1'(Q) &= -\frac{875\lambda+K\lambda}{Q^2}+\frac{0.18\times 315}{2} \end{aligned}$$

which is minimised at

$$\begin{aligned} 0 &= -\frac{875\lambda+K\lambda}{Q^2}+\frac{0.18\times 315}{2} \\ \frac{875\lambda+K\lambda}{Q^2} &= \frac{0.18\times 315}{2} \\ Q^{(1)} &= \sqrt{\frac{875\times 140+30\times 140}{(0.18\times 315)/2}} \\ &= \lceil 66.8515\rceil \\ &= \boxed{67} \end{aligned}$$

to prove that it is indeed a minimum, consider:

$$G_1''(Q)=\frac{2(875\lambda+K\lambda)}{Q^3}>0, \quad \forall\, Q,\ \lambda,\ K>0$$

Lastly, we have

$$\begin{aligned} G_2(Q) &= \lambda\left(285+\frac{2375}{Q}\right)+\frac{K\lambda}{Q}+0.18\left(285+\frac{2375}{Q}\right)\times\frac{Q}{2} \\ G_2'(Q) &= -\frac{2375\lambda+K\lambda}{Q^2}+\frac{0.18\times 285}{2} \end{aligned}$$

which is minimised at

$$\begin{aligned} 0 &= -\frac{2375\lambda+K\lambda}{Q^2}+\frac{0.18\times 285}{2} \\ \frac{2375\lambda+K\lambda}{Q^2} &= \frac{0.18\times 285}{2} \\ Q^{(2)} &= \sqrt{\frac{2375\times 140+30\times 140}{(0.18\times 285)/2}} \\ &= \lceil 114.5718\rceil \\ &= \boxed{115} \end{aligned}$$

to prove that it is indeed a minimum, consider:

$$G_2''(Q)=\frac{2(2375\lambda+K\lambda)}{Q^3}>0, \quad \forall\, Q,\ \lambda,\ K>0$$

With this, we have 3 possible values for the minimum, namely

$$
\begin{aligned}
Q^{(0)} &= 12 \\
Q^{(1)} &= 67 \\
Q^{(2)} &= 115
\end{aligned}
$$

These values were rounded up via the ceiling function, to account for the fact that microcontroller units are discrete in nature. From the given range of values of Q in Equation 15, we note that $Q^{(1)}$ is out of the range $26 \leq Q < 51$, thus it is not realizable. The optimal solution is then obtained by comparing $G_0(Q^{(0)})$ and $G_2(Q^{(2)})$. The two values are given by

$$
\begin{aligned}
G_0\left(Q^{(0)}\right) &= 140 \times 350 + \frac{30 \times 140}{12} + 0.18\left(\frac{350 \times 12}{2}\right) \\
&= \boxed{\$49728} \\
G_2\left(Q^{(2)}\right) &= 140\left(285 + \frac{2375}{115}\right) + \frac{30 \times 140}{115} + 0.18\left(285 + \frac{2375}{Q}\right) \times \frac{115}{2} \\
&= \boxed{\$45991.326}
\end{aligned}
$$

Clearly, the value of $G_2(Q^{(2)})$ is smaller, hence the most economic order size for Axis systems is 115 units of microcontrollers.

Appendix