

# Simulation of boson dynamics: A coherent-states-based method

Michal Horanský

University of Leeds

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# Introduction

## Curse of dimensionality

- ▶ A system governed by a Hamiltonian  $\hat{H}$  which preserves the total particle number  $S$  across all  $M$  modes.
- ▶ We wish to find the time evolution of some quantum state  $|\Psi\rangle$ .
- ▶ The Hilbert space is spanned by an occupancy number basis of dimension

$$\dim(\mathcal{H}) = \binom{S + M - 1}{M - 1}$$

which grows exponentially with  $M$ .

# Introduction

## Coherent states: construction

- ▶ Since  $H$  preserves  $S$ , its second quantization depends only on the dynamical operators  $\hat{a}_i^\dagger \hat{a}_j$ , which form the Lie algebra for the dynamical group  $G = SU(M) \otimes U(1)$ .
- ▶  $\mathcal{H}$  is spanned by the elements of  $G$  acting on some reference state  $|\Psi_0\rangle$ , here chosen to be  $|0, \dots, 0, S\rangle$ —hence  $\hat{g} |\Psi_0\rangle$  forms a basis to  $\mathcal{H}$ .
- ▶ There exists a stability subgroup  $S \subset G$  which leaves  $|\Psi_0\rangle$  invariant up to a constant factor.
- ▶ Hence a suitable (over-)complete basis is given by the unnormalized states  $|z\rangle = \exp [iz^i \hat{q}_i] |\Psi_0\rangle$ , where  $\hat{q}_i$  is an element of the Lie algebra of the quotient group  $G/S$ .
- ▶ This is one way of constructing  $SU(M)$  coherent states.

# Introduction

## Coherent states: utility

- ▶ Coherent states evolve in a way that captures the dynamics of the Hamiltonian.
- ▶ For this reason, a small sample of coherent states should form a suitable basis for the propagation of  $|\Psi\rangle$  (especially if the initial value of  $|\Psi\rangle$  is highly localized in  $|z\rangle$ ).
- ▶ Due to their structure, matrix elements of second-quantized Hamiltonians are easy to calculate in a coherent state basis.
- ▶ I chose to construct the states unnormalized, so that  $|z\rangle$  has no explicit dependence on  $\frac{\partial}{\partial z_i^*}$ , allowing the use of Wirtinger calculus for the equations of motion.
- ▶ Even if the basis states are unnormalized,  $|\Psi(t)\rangle$  remains normalized if we set the initial decomposition coefficients so that  $\langle\Psi(t=0)|\Psi(t=0)\rangle = 1$

# Results so far

## Two applications

- ▶ I tried to reproduce two papers studying  $SU(M)$  Hamiltonians:
  - ▶ Qiao and Grossmann's paper on the Bose-Hubbard model
  - ▶ Green and Shalashilin's paper on bosons in a displaced harmonic trap
- ▶ For each of these Hamiltonians, I have simulated the time-evolution of an initially pure coherent state  $|z_0\rangle$  in two ways:
  1. Uncoupled basis: Each basis state is propagated separately ( $\dim = M - 1$ ), then the decomposition coefficients are propagated on top of the evolved basis ( $\dim = N$ ).
  2. Variational method: The basis states and decomposition coefficients are fully coupled and propagated simultaneously ( $\dim = N \cdot M$ ).