Bosonic and fermionic particle-preserving coherent states, their dynamics and applications

Transfer Report

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I love coherent states

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get that adiabatic shit outta here lil bro

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he bossin

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me when i quantise the modes

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ground state would be nice aha

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Here we write that sweet history of coherent states

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Truly coherent, mm

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we exponentiating in this mofo

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me when i group

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surprisingly easy to generalise this shiss

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de aguiar is my man

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He do be overlappin doe

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i work hard yes

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transitioning into a better person rn

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quote the two body and one body integrals matey

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we want that nobel ngl

Bibliography

[1] Shalashilin, D. V. (2011), Multiconfigurational Ehrenfest approach to quantum coherent dynamics in large molecular systems. *Faraday Discussions*, **153**, pp. 105–116

[2] Schrodinger, E. (1926), Der stetige übergang von der Mikro-zur Makromechanik. *Naturwiss.* 14, pp. 664-666. Translated into English in Schrodinger, E. (1928), *Collected Papers in Wave Mechanics.* 1st edn. London: Blackie & Son, pp. 41–44

[3] Glauber, R. J. (1963), Coherent and Incoherent States of the Radiation Field. *Phys. Rev.*, **131**, 6, pp. 2766–2788

[4] Klauder, J. R., Skagerstam, B. S. (1985), *Coherent States: Applications in Physics and Mathematical Physics.* 1st eng. edn. Singapore: World Scientific.

[5] Perelomov, A. M. (1972), Coherent states for arbitrary Lie group. *Commun. Math. Phys.*, **26**, pp. 222–236

[6] Gilmore, R. (1972), Geometry of symmetrized states. *Ann. Phys.*, **74**, 2, pp. 391–463

[7] Zhang, W. M., Feng, D. H., Gilmore, R. (1990), Coherent states: Theory and some applications. *Rev. Mod. Phys.*, **62**, pp. 867–927

[8] Viscondi, T. F., Grigolo, A., de Aguiar, M. A. M. (2015), Semiclassical Propagator in the Generalized Coherent-State Representation. arXiv:1510.05952 [**quant-ph**]

[9] Block, I., Dalibard, J., Zwerger, W. (2008), Many-body physics with ultracold gases. *Rev. Mod. Phys.*, **80**, pp. 885–964

[10] Qiao, Y., Grossmann, F. (2021), Exact variational dynamics of the multimode Bose-Hubbard model based on $SU(M)$ coherent states. *Phys. Rev. A*, **103**, 042209

[11] Green, J. A., Shalashilin, D. V. (2019), Simulation of the quantum dynamics of indistinguishable bosons with the method of coupled coherent states. *Phys. Rev. A*, **100**, 013607

[12] Bertlmann, R. A., Krammer, P. (2008), Bloch vectors for qudits. arXiv:0806.1174 **[quant-ph]**

[13] Buonsante, P., Penna, V.(2008), Some remarks on the coherent-state variational approach to nonlinear boson models. *J. Phys. A: Math. Theor.*, **41**, 175301

[14] Grigolo, A., Viscondi, T. F., de Aguiar, M. A. M. (2016), Multiconfigurational quantum propagation with trajectory-guided generalized coherent states. *J. Chem. Phys.*, **144**, 094106

Appendix A

Auxilary theorems

This appendix presents mathematical results which were used to derive the mathematical properties of bosonic and fermionic coherent states, but which were chosen to be omitted from the main body of text due to their abstract mathematical nature.

The author does not claim originality of these theorems or their proofs, but chooses to include them, as he was unable to find them in existing literature. The work presented in this appendix is the author's, except for mathematical identities which are stated explicitly.

A.1 counting minors