

$$\begin{aligned}
\left[ \hat{a}_m^\dagger \hat{a}_m, \hat{a}_\alpha^\dagger \hat{a}_\beta^\dagger \hat{a}_\gamma \hat{a}_\delta \right] &= -\hat{a}_\alpha^\dagger \hat{a}_\beta^\dagger \hat{a}_\delta \hat{a}_m \delta_{m,\gamma} - \hat{a}_\alpha^\dagger \hat{a}_\beta^\dagger \hat{a}_\gamma \hat{a}_m \delta_{m,\delta} \\
&\quad + \hat{a}_m^\dagger \hat{a}_\beta^\dagger \hat{a}_\gamma \hat{a}_\delta \delta_{m,\alpha} + \hat{a}_m^\dagger \hat{a}_\alpha^\dagger \hat{a}_\gamma \hat{a}_\delta \delta_{m,\beta}
\end{aligned}$$