$$\left[\hat{a}_\gamma\hat{a}_\delta, \hat{a}_\alpha^\dagger\hat{a}_\beta^\dagger\right] = \hat{a}_\beta^\dagger\hat{a}_\delta\delta_{\gamma,\alpha} + \hat{a}_\alpha^\dagger\hat{a}_\delta\delta_{\gamma,\beta} + \hat{a}_\beta^\dagger\hat{a}_\gamma\delta_{\delta,\alpha} + \delta_{\gamma,\beta}\delta_{\delta,\alpha} + \hat{a}_\alpha^\dagger\hat{a}_\gamma\delta_{\delta,\beta} + \delta_{\gamma,\alpha}\delta_{\delta,\beta} \quad (1)$$