

Simulation of boson dynamics: A coherent-states-based method

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Introduction

Curse of dimensionality

- ▶ A system governed by a Hamiltonian \hat{H} which preserves the total particle number S across all M modes.
- ▶ We wish to find the time evolution of some quantum state $|\Psi\rangle$.
- ▶ The Hilbert space is spanned by an occupancy number basis of dimension

$$\dim(\mathcal{H}) = \binom{S + M - 1}{M - 1}$$

which grows exponentially with M .

Introduction

Coherent states: construction

- ▶ Since H preserves S , its second quantization depends only on the dynamical operators $\hat{a}_i^\dagger \hat{a}_j$, which form the Lie algebra for the dynamical group $G = SU(M) \otimes U(1)$.
- ▶ \mathcal{H} is spanned by the elements of G acting on some reference state $|\Psi_0\rangle$, here chosen to be $|0, \dots, 0, S\rangle$ —hence $\hat{g} |\Psi_0\rangle$ forms a basis to \mathcal{H} .
- ▶ There exists a stability subgroup $S \subset G$ which leaves $|\Psi_0\rangle$ invariant up to a constant factor.
- ▶ Hence a suitable (over-)complete basis is given by the unnormalized states $|z\rangle = \exp [iz^i \hat{q}_i] |\Psi_0\rangle$, where \hat{q}_i is an element of the Lie algebra of the quotient group G/S .
- ▶ This is one way of constructing $SU(M)$ coherent states.

Introduction

Coherent states: utility

- ▶ Coherent states evolve in a way that captures the dynamics of the Hamiltonian.
- ▶ For this reason, a small sample of coherent states should form a suitable basis for the propagation of $|\Psi\rangle$ (especially if the initial value of $|\Psi\rangle$ is highly localized in $|z\rangle$).
- ▶ Due to their structure, matrix elements of second-quantized Hamiltonians are easy to calculate in a coherent state basis.
- ▶ I chose to construct the states unnormalized, so that $|z\rangle$ has no explicit dependence on $\frac{\partial}{\partial z_i^*}$, allowing the use of Wirtinger calculus for the equations of motion.
- ▶ Even if the basis states are unnormalized, $|\Psi(t)\rangle$ remains normalized if we set the initial decomposition coefficients so that $\langle\Psi(t=0)|\Psi(t=0)\rangle = 1$

Results so far

Two applications

- ▶ I tried to reproduce two papers studying $SU(M)$ Hamiltonians:
 - ▶ Qiao and Grossmann's paper on the Bose-Hubbard model
 - ▶ Green and Shalashilin's paper on bosons in a displaced harmonic trap
- ▶ For each of these Hamiltonians, I have simulated the time-evolution of an initially pure coherent state $|z_0\rangle$ in two ways:
 1. Uncoupled basis: Each basis state is propagated separately ($\dim = M - 1$), then the decomposition coefficients are propagated on top of the evolved basis ($\dim = N$).
 2. Variational method: The basis states and decomposition coefficients are fully coupled and propagated simultaneously ($\dim = N \cdot M$).

Results so far

Bose-Hubbard model - uncoupled basis propagation

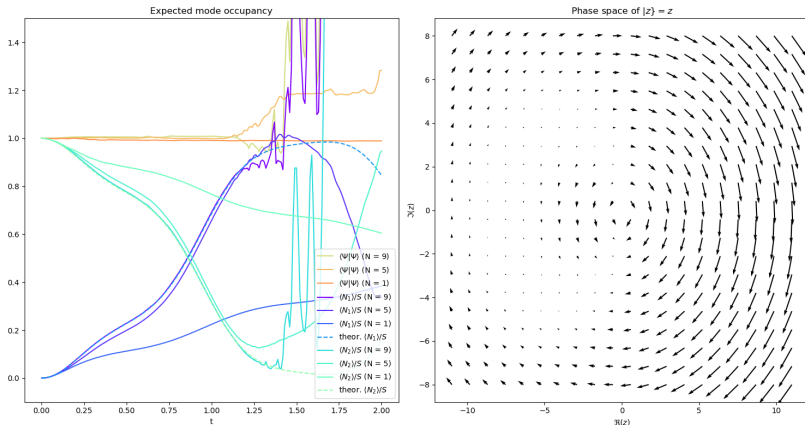


Figure: $M = 2, S = 10, |\Psi(t=0)\rangle = n|0.0 + 0.0i\rangle$.
 $J(t) = 1 + \frac{1}{2} \cos(2\pi t), U = 0.1, K = 0$

Results so far

Bose-Hubbard model - fully variational method

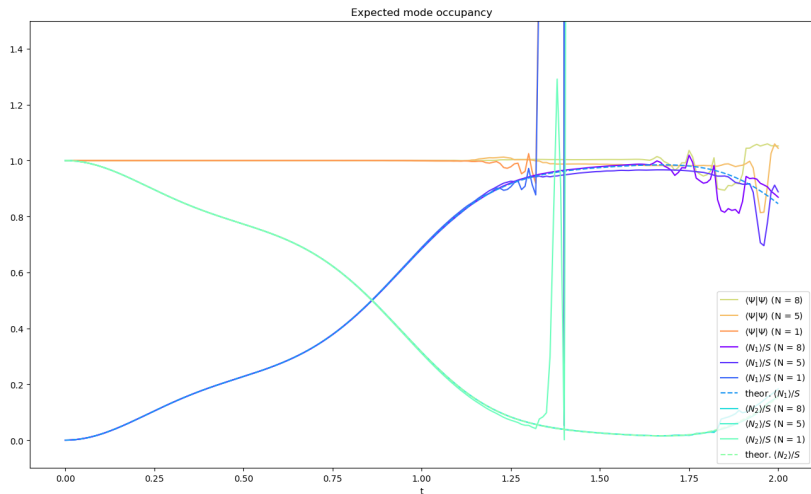


Figure: $M = 2, S = 10, |\Psi(t=0)\rangle = n|0.0 + 0.0i\rangle$.
 $J(t) = 1 + \frac{1}{2} \cos(2\pi t), U = 0.1, K = 0$

Results so far

Bose-Hubbard model - uncoupled basis propagation

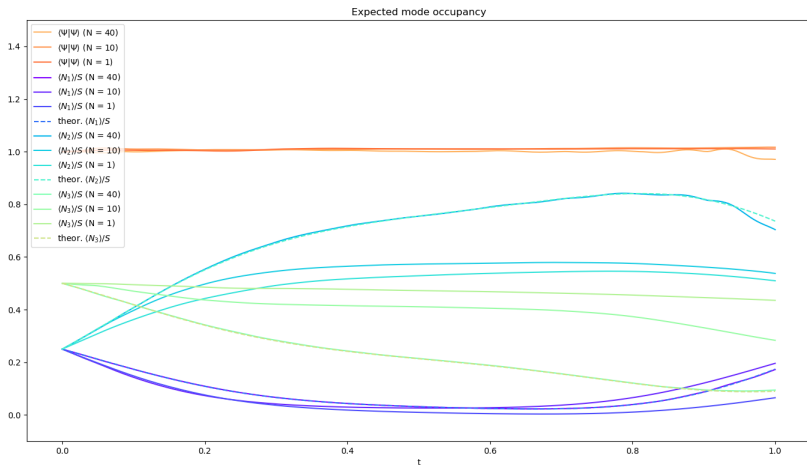


Figure: $M = 3, S = 10, |\Psi(t=0)\rangle = n|0.0 + 0.0i\rangle$.
 $J(t) = 1 + \frac{1}{2} \cos(2\pi t), U = 0.1, K = 0$

Results so far

Bose-Hubbard model - fully variational method

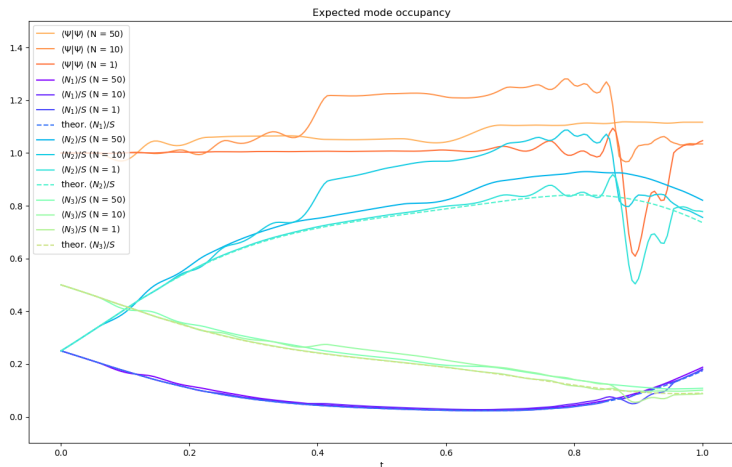


Figure: $M = 3, S = 10, |\Psi(t=0)\rangle = n|0.5 + 0.5i, 0.5 - 0.5i\rangle$.
 $J(t) = 1 + \frac{1}{2} \cos(2\pi t), U = 0.1, K = 0$

Results so far

Displaced harmonic trap - uncoupled basis propagation

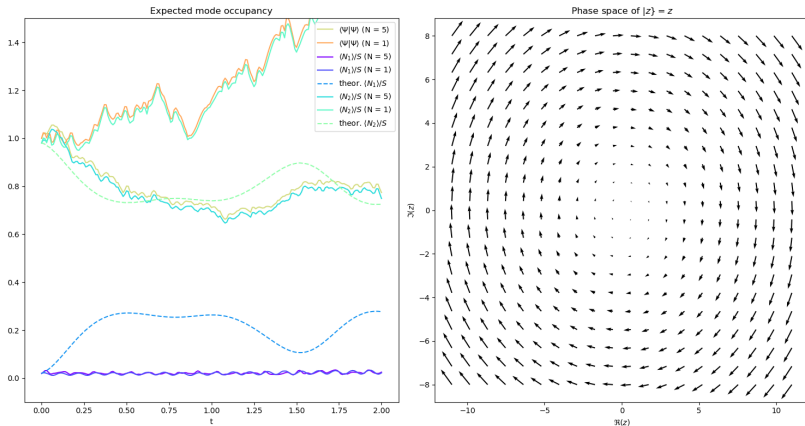
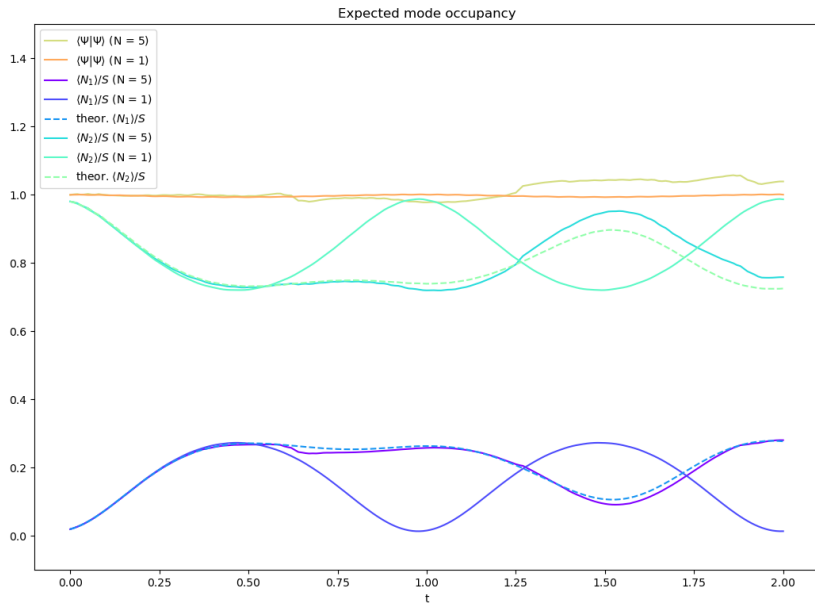


Figure: $M = 2, S = 10, |\Psi(t = 0)\rangle = n|0.1 + 0.1i\rangle$. $\xi = 2.1, \lambda_0 = 0.01$

Results so far

Displaced harmonic trap - fully variational method



Results so far

Displaced harmonic trap - fully variational method

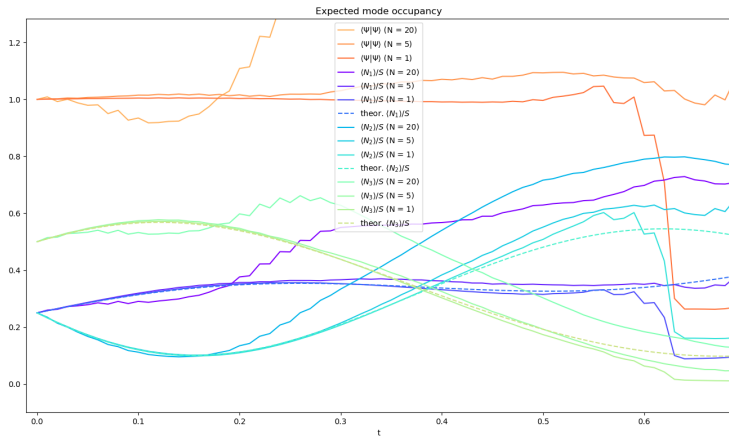


Figure: $M = 3$, $S = 10$, $|\Psi(t = 0)\rangle = n |0.5 + 0.5i, 0.5 - 0.5i\rangle$.
 $\xi = 2.1$, $\lambda_0 = 0.01$

Outlook

- ▶ Much faster (DHT with $M = 3$: 1.5 minutes for an $N = 20$ basis vs. 25 minutes for the occupancy basis solution)
- ▶ Numerically unstable (big M , big S , conditioning, regularisation...)
- ▶ Systems with different dynamical groups?

Thank you for your attention