# Hopfield Neural Network Based Color Image Restoration

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***Abstract*—In this paper, Hopfield Neural Network (HNN) method for restoring color images is presented. Firstly, we describe the general restoration method of gray level image. Secondly, Hopfield Neural Network technique for restoring of monochromatic images is analyzed. Then, the color images are modeled as three spatially monochromatic images or channels, and the HNN based algorithm is introduced to restore color images. Finally, this algorithm is applied to the image blur model, and the result is analyzed and compared to Wiener Filter.**

## I. INTRODUCTION

Image restoration is an important problem in image processing. The restoration techniques are applied to remove degradations due to noise and blur. There are many methods [1], such as Inverse Filtering, Minimum Mean Square Error (Wiener) Filtering, Kalman Filtering, which are depending on some knowledge and the model of the degradation processing, and then applying the inverse processing on the degraded images. Due to the limitation of ill-posed equations of Inverse Filtering, quality images can be restored only at the condition that the Signal to Noise Ratio (SNR) is known. Wiener Filtering does not take the defects of Inverse Filtering, but it is on the assumption that the input image could be presented by a stationary stochastic process, and the correlation function is needed too, either of them are not common conditions for applications. The Kalman Filter could be applied on restoring of non-stationary images, but the calculation makes it diseconomy. Additionally, all the methods above take the positive constriction. Moreover, all the defects of the methods exist for color images. Zhou [2] has introduced an algorithm for gray level image based on Hopfield Neural Network, with the defect of huge calculations. Paik's improvement, which is called Modified Hopfield Neural Network (MHNN) [3], makes HNN a useful model for image restoration.

Compared to gray level images, each pixel of digital color images is a multi-dimension vector, which is defined by Color Models. In this paper, color images are divided into RGB distribution [4]. Then each sub-space can be regarded as a gray image space. 3-D matrix is used to represents a color image. for a color image $F$, the element $F(x,y)$ represents a pixel of the image and $F(x,y)=[r \quad g \quad b]^T$ defines the color of this pixel. HNN model and its algorithm are introduced to restore color images. Simulation results of the proposed method are presented in the last section.

## II. IMAGE DEGRADATION AND RESTORATION

As Fig. 1 shows, the degradation processing is modeled as a degradation function that, together with an additive noise term, operations on an input image $f(x,y)$ to produce a degraded image $g(x,y)$. Given $g(x,y)$, some knowledge about the degradation function H and the addition noise term $\eta(x,y)$, the objective of restoration is to obtain an estimate $\hat{f}(x,y)$ of the original image [1].

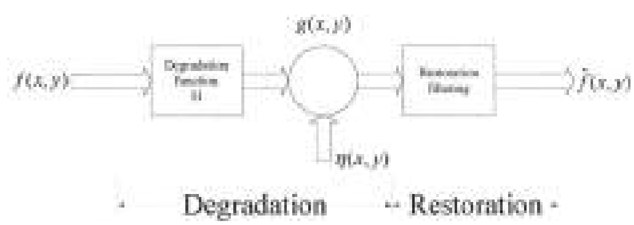


Figure 1. A model of the image degradation/restoration process

If H is a linear, position-invariant processing, then the degraded image is given in the spatial domain by

$$g(x,y)=h(x,y)\otimes f(x,y)+\eta(x,y) \tag{1}$$

Where $h(x,y)$ is the spatial representation of the degradation function or Point Spread Function (PSF) and the

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symbol $\otimes$ indicates spatial convolution, or written by matrix equation:

$$g = Hf + n \tag{2}$$

H is a block Toeplitz matrix, which can be approximated by a block circulant matrix. We assume the image is $M \times N$ in size resulting in H being an $MN \times MN$ matrix. So the restoration problem can be converted to a minimization problem as.

$$\|n\|^2 = \|g - H\hat{f}\|^2 = (g - H\hat{f})^T (g - H\hat{f}) \tag{3}$$

Zhou [2] has reformulate the restoration problem as one of minimizing an error function with constraints defined as

$$\begin{aligned} E &= \frac{1}{2}\|g - H\hat{f}\|^2 + \frac{1}{2}\lambda\|D\hat{f}\|^2 \\ &= \frac{1}{2} f^T (H^T H + \lambda D^T D) f - (H^T g)^T f \\ &\quad + \frac{1}{2} g^T g \end{aligned} \tag{4}$$

Where $\|\bullet\|$ is the $L_2$ norm. $\lambda$ is a constant. It is reasonable to solve the problem by Hopfield Neural Network (HNN).

## III. HOPFIELD NEURAL NETWORKS BASED ON IMAGE RESTORATION

John Hopfield of California Institute of Technology proposed the Hopfield Neural Network during the early 1980s. The publication of his work in 1982 significantly contributed to the renewed interest in research in artificial neural networks. As a classic neural model, HNN has been applied to a lot of fields such as content-addressable memory, optimal computation and pattern recognition [5].

### A. General Description of Hopfield Neural Network

Fig. 2 is a Hopfield model with six units, where each node is connected to every other node in the network.

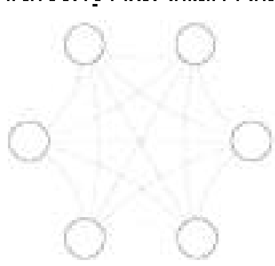

Figure 2. Hopfield model

An N-order Discrete Hopfield Network (DHN) is a six-element set composed with N neurons: [6]

$$net_{DHN}^{N} = \langle V_N, A_N, IF, WA, OA \rangle \tag{5}$$

$V_N = \{v_i, i = 1, 2, \dots, N\}$ is a set of the nodes.

$A_N = W = \{w_{ij}\}_{N \times N}$ is the connection matrix.

$$A_N = W = \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & \cdots & V_N \end{array} \\ \left[\begin{array}{cccc} w_{11} & w_{12} & \cdots & w_{1N} \\ w_{21} & w_{22} & \cdots & w_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ w_{N1} & w_{N2} & \cdots & w_{NN} \end{array}\right] \end{array} \begin{array}{c} \\ V_1 \\ V_2 \\ \vdots \\ V_N \end{array}$$

Where $w_{ij}$ is the linking intensity of $V_i \to V_j$.

The input domain $IF \subseteq V_N$

The output domain $OF \subseteq V_N$

Working Algorithm: $WA: IF \to OF$

$DHN\_W: (o(0), o(t))$

Step1. Initialization:

Set $t = 0$ and determine Working Subset $V_N^{(0)}$.

Step2. Integration:

$$u_j(t) = \sum_{i=1}^{N} w_{ij} o_i(t) \quad \left(V_j \in V_N^{(t)}\right) \tag{6}$$

Step3. Excitation:

$$o_j(t+1) = \begin{cases} o_j(t) & V_j \notin V_N^{(t)} \\ -1 & V_j \in V_N^{(t)},\ u_j(t) < \theta_j \\ +1 & V_j \in V_N^{(t)},\ u_j(t) \ge \theta_j \end{cases} \tag{7}$$

Step4. Conditional stop:

If $net_{DHN}^{N}$ is stable, stop.

Step5. Unconditional jump:

Set $t = t + 1$, determine a new working subset $V_N^{(t+1)}$ and jump to Step2.

Organize algorithm:

$$OA: \{W^t\} \to \{W^*\} \tag{8}$$

Where $W^{(t+1)} = W^{(t)} + \sigma o(t) o^T(t+1)$, and $\sigma (> 0)$ is the learning rate

In fact, a Hopfield NN can be identified by Connection Matrix $W$ and Threshold Vector $b$.

$$new_{\langle W,b\rangle}^{(N)} = \langle W, b \rangle \tag{9}$$

The Lyapunov function (Energy Function) is:

$$\begin{aligned} E &= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} u_i(t) u_j(t) - \sum_{i=1}^{N} b_i u_i(t) \\ &= -\frac{1}{2}\mathbf{u}^T(\mathbf{t})\mathbf{W}\mathbf{u}(\mathbf{t}) - \mathbf{b}^T\mathbf{u}(\mathbf{t}) \end{aligned} \tag{10}$$

Hopfield [6] has proved that a N ordered DHN network $new_{\langle W,b\rangle}^{N}$ is stable at the condition that:

- $new_{\langle W,b\rangle}^{(N)}$ is working on the sequential model
- $W$ is symmetric matrix of which the diagonal elements are nonnegative

Alternatively

- $new_{\langle W,b\rangle}^{(N)}$ is working on the parallel model.
- $W$ is nonnegative defined matrix, that is $W \geq 0$

We know that a stable neural network always converges to a stable state $\Omega_i \in \Sigma_{M \times N}$ with any starting state. And this property can be used to solve optimization problems.

*B. The Design of Hopfield Neural Network for Color Image Restoration*

Consider a digital color image $\mathbf{F}_{M \times N} = \left[f_{ij}\right]$, of which each element is a 3-D vector $f_{ij} = \left[f_{ij}^1 \quad f_{ij}^2 \quad f_{ij}^3\right]^T$, degraded by a linear, position-invariant system $H = \left[H_1 \quad H_2 \quad H_3\right]$, adding the independent identical distribution (i.i.d) random noise $\eta = \left[\eta_1 \quad \eta_2 \quad \eta_3\right]^T$, the degraded imaged can be written in matrix form:

$$(G) = \begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix}_i = (H_i)(F^i)_i + \eta_i \tag{11}$$

That is to say, each channel of a color image can be treated independently, and the PSF of each channel is the same.

For each channel, we have the error function:

$$\begin{aligned} E_i &= \frac{1}{2}F_i^T(H_i^T H_i + \lambda D_i^T D_i)F_i \\ &- (H_i^T G_i)^T F_i - \frac{1}{2}G_i^T G_i \end{aligned} \tag{12}$$

Discard the constant term $\frac{1}{2}G_i^T G_i$:

$$\begin{aligned} E_i &= \frac{1}{2}F_i^T(H_i^T H_i + \lambda D_i^T D_i)F_i \\ &- (H_i^T G_i)^T F_i \end{aligned} \tag{13}$$

We can get the $\hat{f}$ by train the HNN which has energy function as the same form of (13), that is:

$$E_{nn} = -\frac{1}{2}\mathbf{v}^T\mathbf{T}\mathbf{v} - \mathbf{b}^T\mathbf{v} \tag{14}$$

Where $v = F_i$, $T = -H_i^T H_i - \lambda D_i^T D$ and $b = H_i^T G_i$.

The states equation is:

$$v_i(t+1) = G(\sum_j w_{ij} v_j(t) - b_i) \tag{15}$$

Where

$$G(u) = \begin{cases} 1, u \geq 0 \\ 0, u < 0 \end{cases} \tag{16}$$

In image restoration, $v \geq 0$ $T < 0$ the energy decline is not guaranteed. Zhou [2] use a time checking method, which cost more calculation. Another defect is that Zhou use a number of neurons to represent a pixel value, which is ranging form 0 to 255, through simple summation, that costs much more neurons and increase the calculation too.

We chose Paik's MHN network [5] [7] [8] [9], of which each neuron represents a pixel vector component value from 0 to 255, the Updating Rule (U R) is:

$$x_i(t+1) = g(x_i(t) + \Delta x_i), \quad i = 1, \ldots, n \tag{17}$$

Where

$$g(v) = \begin{cases} 0, & v < 0 \\ v, & 0 \leq v \leq 255 \\ 255, & v > 255 \end{cases} \tag{18}$$

$$\Delta x_i = f(u_i) = \begin{cases} -1, & u_i < -\theta_i \\ 0, & |u_i| \leq \theta_i \\ 1, & u_i > \theta_i \end{cases} \tag{19}$$

$$u_i = b_i - \sum_j t_{ij} x_j(t) = e_i^T (b - Tx(t)) \quad (20)$$

And $e_i$ represents the $i$th unit vector. Below is the block diagram of the modified Hopfield network model:

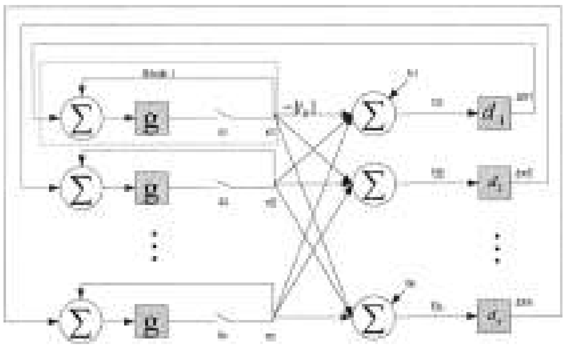


Figure 3. Diagram of Modified Hopfield Network Model

As a color image has several channels, which is added by independent noise, we choose RGB color model because the CCD / CMOS imaging mechanism, which obtain a color image in RGB model. In this color model, we can restore each channel of the color image:

$$G = \begin{bmatrix} G_r \\ G_g \\ G_b \end{bmatrix} = \begin{bmatrix} HF_r \\ HF_g \\ HF_b \end{bmatrix} + \begin{bmatrix} \eta_r \\ \eta_g \\ \eta_b \end{bmatrix} \quad (21)$$

That is to say, we can use three independent MHNN to restore the degraded color image by processing each channel separately. Fig. 4 is the system structure:

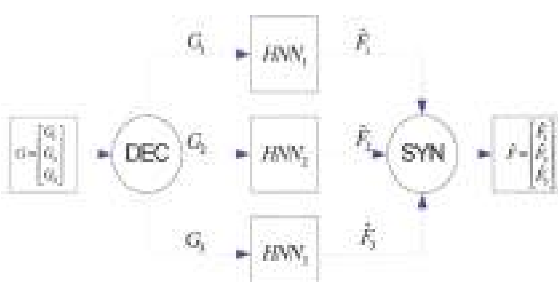


Figure 4. The System Structure of Color Image Restoration

Each $HNN_i$ in the system takes the same algorithm:

Step0. $x(0) = D, z$, $t = 0$ and $i = 1$.

Step1. Check termination.

Step2. Choose $l(t) = \{i\}$.

Step3. $temp = g(x(t) + \Delta x_i e_i)$ Where $\Delta x_i$ is given by (18).

Step4. If $temp \neq x(t)$ then $x(t+1) = temp$ and $t = t+1$.

Step5. $i = i + 1$ (if $i > n$, $i = i - n$) and go to step (1).

## IV. Simulation Results

### A. System Design

As shown in Fig. 4, the system could be threaded into 3 independent threads, which is an efficient way for running program on multi-core CPU.

In each thread, we choose the sequential algorithm of MHNN which is discussed in Section III.

We chose Lena, $180 \times 180$ pixels in size, as the test image, and tried to add 20dB noise to the original image, then compared the results between experiments with and without noise. We also figured out the change of energy at each iteration during the restoring of noised blur image. And for a subjective comparison, the result of classic Wiener filter was shown.

### B. Results

- Original Image

Figure 5. Original Image

- Degraded image without noise

Figure 6. Degraded image without noise

Figure 7. Restored image

- Degraded image with noise ($SNR = 20dB$)

Figure 8. Degraded image with noise

Figure 9. Restored image (Iteration number: 100)

Compared to the result of Wiener filter:

Figure 10. Restored image by Wiener Filter

Energy change by iterations:

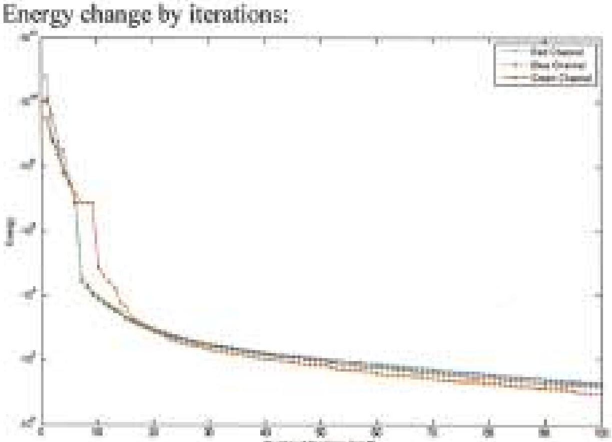


Figure 11. Changes of separated channels in energy at each interation

## V. CONCLUSION

In this paper, we have proposed Hopfield Neural Network model for color image restoration.

In our simulation on the degradation model of image blur, we find that our method restored the blur image without noise to quality image (Fig. 7). For noised blur image, the result (Fig. 9) was subjectively better than Wiener Filter (Fig. 10). The energy of HNN declined quickly during the first 20 iterations, then change slowly (Fig. 11). That is to say the algorithm has a quick convergence, though the calculation is some of huge. Our future work is to find better method to improve the SNR and reduce the time complexity.


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## REFERENCES


[1] R.C. Gonzalez, Digital image processing, second edition, Beijing: [illegible], 2003.

[2] Y.T. Zhou, R. Chellappa, A. Vaid, and B.K. Jenkins, "Image restoration using a neural network", IEEE Trans. On Acoustic Speech and Signal Proc. Vol.36, pp.[illegible], Jul. 1988.

[3] J.K. Paik, and A.K. Katsaggelos, "Image restoration using a modified Hopfield network", IEEE Trans. on Image Proc. Vol.1, pp.49-63, Jan. 1992.

[4] A.K. Katsaggelos, J.K. Paik, "Iterative color image restoration algorithms", in International Conf. on Acoustics, Speech and Signal Processing, pp.1028-1031, Vol.2, Apr. 1988.

[5] M. [illegible] Hagan, Neural network design, Beijing: CMP, 2002.

[6] [illegible] Kuan, Neural Computing, Beijing: [illegible] 2000.

[7] [illegible] Wang, and X.H. He, "Improved Hopfield network for image restoration", Computer Engineering, Vol.31, pp.[illegible], Sept. 2005.

[8] [illegible], "Image restoration based on modified Paik Hopfield Neural Network", Journal of applied sciences, Vol.[illegible], No.2, pp.[illegible], Mar. 2005.

[9] [illegible], and [illegible], "Application of Neural Network in image processing", Information and control, Vol.[illegible], No.[illegible], pp.[illegible], Aug. [illegible].