Algorithm Description for occfind2

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Assume that the Earth centered at the origin $[0, 0, 0] \in \mathbb{R}^3$. Let q_t denote the position of the Sun at time t. Let $a > 0$ denote the maximum radius of Earth, let $c > 0$ denote the maximum radius the Sun. The distance from the center of the Earth to the center of the Sun at time t can be computed using the Frobenius norm as follows.

$$d_t = \|\vec{q}_t\| = \sqrt{\vec{q}_t \cdot \vec{q}_t}$$

Note that the $\cdot$ operator above denotes the dot product. Let $\vec{n}_t = q_t/d_t$ denote the unit normal vector.

Define a truncated cone between the Earth and the Sun called the occultation zone as seen in Fig 1. An occultation event occurs when any part of an object is within this cone. For any distance $m \in \mathbb{R}$ such that $0 < m < d$, the radius of the cone with the plane cross-sectioned at $m\vec{n}_t$ can be computed via a linear interpolation as follows.

$$a + (m)(c - a)/d_t$$

Let $\vec{p}_t$ denote the position of the Moon at time t. Let $b > 0$ denote the maximum radius of Moon. The moon is said to enter the occultation zone if its center is within the widened cone as seen in Fig. 2. The nearest projection from p_t to the line generated by the vector n_t can be computed as $(\vec{q}_t \cdot \vec{n}_t)\vec{n}$. Some examples of this projection to the normal line can be seen in Fig. 3. When the dot product $\vec{q}_t \cdot \vec{n}_t < 0$, then the Moon is behind the Earth with respect to the Sun, and cannot be in the occultation zone. When the dot product $\vec{q}_t \cdot \vec{n}_t > 0$,

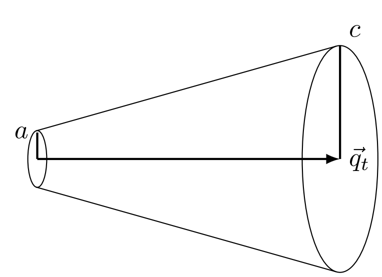

Figure 1: Illustration of truncated cone Occultation zone.

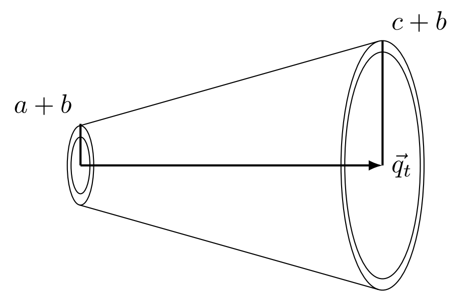

Figure 2: Occultation zone for object with maximum radius $b > 0$.

then the Moon is in the occultation zone if and only if the following inequality holds.

$$\|p_t - (\vec{p}_t \cdot \vec{n}_t)\vec{n}_t\| < a + b + (\vec{p}_t \cdot \vec{n}_t)(c - a)/d_t \tag{1}$$

An algorithm will be described for searching solar occultation events between time points $t_0, t_1 \in \mathbb{R}$ such that $t_0 < t_1$ will not be described. Let $\delta > 0$ denote the time step size.

1. Set current time to $t = t_0$
2. Obtain q_t from CSPICE as the state of the SUN in the J2000 frame with EARTH as the observer
3. Obtain p_t from CSPICE as the state of the MOON in the J2000 frame with EARTH as the observer
4. Compute distance $d_t = \|q_t\|$
5. Compute unit normal vector $\vec{n}_t = \vec{q}_t/d$
6. Compute dot product $m = \vec{p}_t \cdot \vec{n}_t$
7. If $m > 0$
    (a) Comptute $\|p_t - m\vec{n}_t\|$
    (b) Compute $a + b + m(c - a)/d_t$
    (c) If $\|p_t - m\vec{n}_t\| < a + b + m(c - a)/d_t$ report occultation event at time t.
8. update $t \mapsto t + \delta$
9. if $t > t_1$ then stop

When the dot product m computed in step 6 changes from being positive to negative, it will probably remain negative for a period of time that has a predictable infimum. If you have a lower bound for the amount of time that

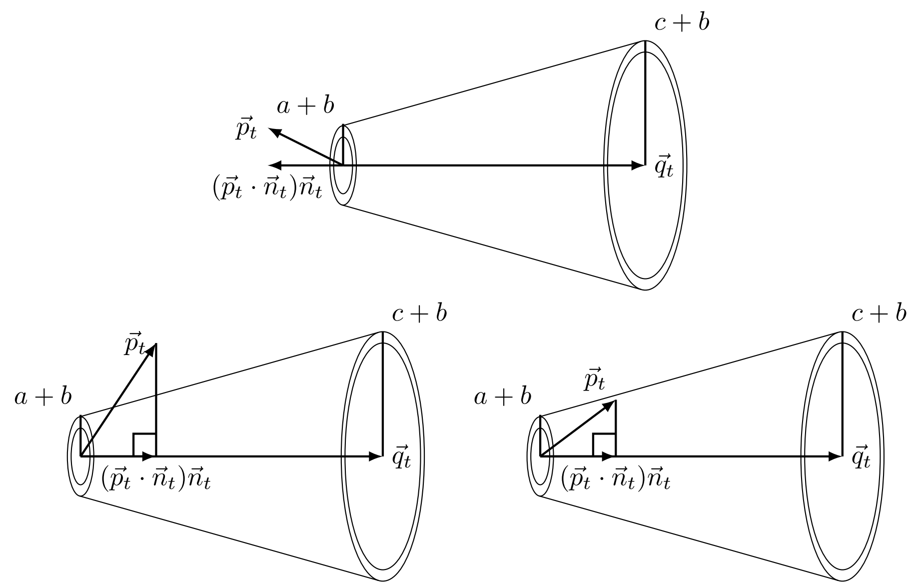

Figure 3: Top panel shows object with negative inner product. Bottom left shows object with positive inner product, but outside the occultation zone. Bottom right shows a point inside the occultation zone.

the Moon will remain behind the Earth with respect to the sun (for example 26/2.0), then you can speed up the search by skipping ahead in time by that amount. Another potential improvement to the algorithm may be obtained by using the velocity of the moon to compute roughly when and if it will intersect with the cone. This might be tricky to get just right, but it could speed up the algorithm considerably. Potentially, a coarser search could be made and interpolated between whenever a line segment intersects with the truncated cone.