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CS222 Assignment 1

Quest 3

Limits: i. $\lim_{n\to\infty} \frac{\ln(n)+4}{5n^4+7n^2+6}$

$= \lim_{n\to\infty} \frac{d/dx\,(\ln(n)+4)}{d/dx\,(5n^4+7n^2+6)}$

$= \lim_{n\to\infty} \frac{1}{n(20n^3+14n)}$

$= \lim_{n\to\infty} \frac{1}{20n^4+14n^2}$

Applying constant multiple rule: $\lim_{n\to\infty} cf(n) = c\lim_{n\to\infty} f(n)$

$C = 1/2$

$f(n) = \frac{1}{n^2(10n^2+7)}$

$\lim_{n\to\infty} \frac{1}{20n^4+14n} = \left(\frac{1}{2}\lim_{n\to\infty} \frac{1}{n^2(10n^2+7)}\right)$

$\frac{\frac{1}{2}\lim_{n\to\infty} 1}{\lim_{n\to\infty} n^2(10n^2+7)}$

$= \frac{1}{2\lim_{n\to\infty} n^2(10n^2+7)} = 0/2$

$\therefore \lim_{n\to\infty} \frac{\ln(n)+4}{5n^4+7n^2+6} = 0$

ii. $\lim_{n\to\infty} \frac{2^n}{\log_2 n}$

$= \lim_{n\to\infty} \frac{d/dx(2^n)}{d/dx(\log_2 n)}$

$= \lim_{n\to\infty} \frac{2^n \ln(2)}{1/n \ln(2)}$

$= \lim_{n\to\infty} \ln^2(2) \cdot 2^n n$

Applying constant multiple rule

$\ln^2(2) \lim_{n\to\infty} 2^n \cdot n$

$= \ln^2(2) \lim_{n\to\infty} (2^n) \cdot \lim_{n\to\infty} (n)$

$= \ln^2(2) \cdot \infty \cdot \infty$

$= \infty$

Quick approximation

i. $\sum_{k=0}^{30} k^2$

$= n^3/3$

$= \frac{30^3}{3}$

$= 9000$

ii. $\sum_{k=0}^{100} k^3$

$= n^4/4$

$= \frac{100^4}{4}$

$= 25000000$

Quest 4

i. $T(n) = 7T(n/2) + n^2$

$a = 7$

$n/b = n/2$

$f(n) = n^2$

$\log_b a = \log_2 7 = 2.81 > 2$

$n^{\log_b a} = n^{\log_2 7}$

$f(n) = n^2 = O(n^2)$

$\therefore T(n) = O(n^{\log_2 7}) = O(n^{2.81})$

ii. $T(n) = 5T(n/3) + O(n)$

$a = 5$, $n/b = n/3$, $f(n) = n$

$\log_b a = \log_3 5 = 1.46 > 1$

$T(n) = O(n^{\log_b a}) = O(n^{1.46})$

iii. $T(n) = 3T(n/2) + 3/4 n + 1$

$a = 3$

$n/2 = n/b$

$b = 2$

$f(n) = 3/4 n + 1$

$\log_b a = \log_2 3 = 1.58$

$T(n) = O(n^{\log_b a}) = O(n^{1.58})$

Induction

i. $\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$

when $k=1$

$$\sum_{k=1}^{n} k\binom{n}{k} = n\sum_{k=1}^{n}\binom{n-1}{k-1}$$

$$= n\sum_{k=0}^{n-1}\binom{n-1}{k} = n2^{n-1}$$

ii. $2\sum_{k=0}^{n} 3^k = 3^{n+1} - 1$

When $n=1$

$2(3^0 + 3^1 + 3^n) = 3^{n+1} - 1$

$2(3^0 + 3^1) = 3^{1+1} - 1$

$2(4) = 8$

$8 = 8$

when $n=k$

$2(3^0 + \ldots + 3^k) = 3^{k+1} - 1$

when $n = k+1$

$2(3^0 + \ldots + 3^k + 3^{k+1}) = 3^{(k+1)+1} - 1$

$2(3^0 + \ldots + 3^k) + 2(3^{k+1}) = 3^{k+2} - 1$

$3^{k+1} - 1 + 2(3^{k+1}) = 3^{k+2} - 1$

$3^{k+1}(1+2) - 1 = 3^{k+2} - 1$

$3^{k+1} \cdot 3 - 1 = 3^{k+2} - 1$

$3^{k+2} - 1 = 3^{k+2} - 1$