

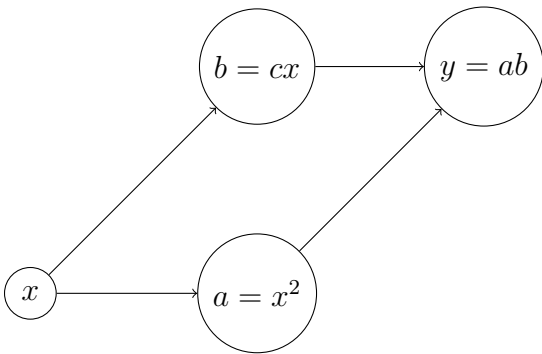
# Homework 3

Michael Pollack

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## Q1

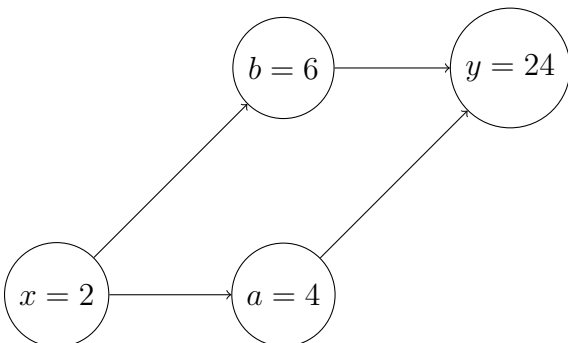
Draw a computation graph for this example:



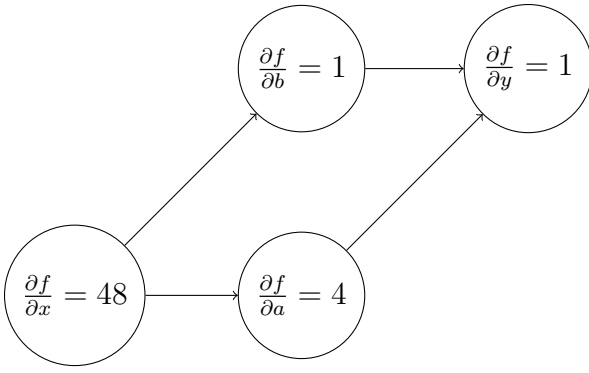
How many nodes are there (including input and output)?

4 (four) nodes

For  $x = 2$  and  $c = 3$ , Compute the value of each node in a forward pass:



For  $x = 2$  and  $c = 3$ , Compute  $\frac{df}{dn}$  for each node  $n$  using backpropagation:



Consider the node corresponding to  $x^2$  in the graph. For each of the following, write a symbolic expression and the numerical value (at  $x = 2, c = 3$ ) for

If we are using Binary Cross-Entropy, and therefore our loss function is:

$$L_{BCE}(y, \hat{y}) = -(y \log(\hat{y}) + (1 - y) \log(1 - \hat{y}))$$

Then we can assume that  $y = 1$  (for a positive result), then:

$$\frac{\partial L}{\partial \hat{y}} = -\left(\frac{1}{\hat{y}}\right)$$

If  $\hat{y}$  is the output of the function, then  $\hat{y} = a = 4$

$\therefore$

$$\frac{\partial L}{\partial \hat{y}} = \frac{\partial L}{\partial a} = -\left(\frac{1}{4}\right)$$

$\therefore$

- The upstream derivative:  $\frac{\partial L}{\partial a} = -\left(\frac{1}{4}\right)$

- The local derivative:  $\frac{\partial a}{\partial x} = 2x = 4$

- The downstream derivative:  $\frac{\partial L}{\partial x} = \frac{\partial L}{\partial a} \left(\frac{\partial a}{\partial x}\right) = -\left(\frac{1}{4}\right)(4) = -1$

### Q3

Epoch 0 average loss: 0.488037767606645

Epoch 1 average loss: 0.32633853972554966

Epoch 2 average loss: 0.28049176026340006

Epoch 3 average loss: 0.2572602522667666

Epoch 4 average loss: 0.24377345227392025

Epoch 5 average loss: 0.2353556996256047

Total training time: 1466.1704413890839