Homework 1

Question 1

A. Convert the following numbers to their decimal representation. Show your work.

1. $10011011_2 = 1 \times 2^0 + 1 \times 2^1 + 1 \times 2^3 + 1 \times 2^4 + 1 \times 2^7 = 1 + 2 + 8 + 16 + 128 = 155_{10}$

2. $456_7 = 6 \times 7^0 + 5 \times 7^1 + 4 \times 7^2 = 6 + 35 + 196 = 237_{10}$

3. $38A_{16} = 10 \times 16^0 + 8 \times 16^1 + 3 \times 16^2 = 10 + 128 + 768 = 906_{10}$

4. $2214_5 = 4 \times 5^0 + 1 \times 5^1 + 2 \times 5^2 + 2 \times 5^3 = 4 + 5 + 50 + 250 = 309_{10}$

B. Convert the following numbers to their binary representation.

1. $69_{10} = 64 + 4 + 1 = 1 \times 2^6 + 1 \times 2^2 + 1 \times 2^0 = 1000101_2$

2. $485_{10} = 256 + 128 + 64 + 32 + 4 + 1 = 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^2 + 1 \times 2^0 = 111100101_2$

3. $6D1A_{16} = (0110)(1101)(0001)(1010) = 110110100011010_2$

Hex Digit	4 bit binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
a	1010
b	1011
c	1100
d	1101
e	1110
f	1111

C. Convert the following numbers to their hexadecimal representation.

1. $1101011_2 = (0110)(1011) = 6B_{16}$

2. $895_{10} = 3 \times 256 + 7 \times 16 + 15 = 3 \times 16^2 + 7 \times 16^1 + 15 \times 16^0 = 37F_{16}$

Question 2

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = 14303_8$

1 1 1 1

$$\begin{array}{r} 7\,5\,6\,6_8 \\ +\,4\,5\,1\,5_8 \\ \hline 1\,4\,3\,0\,3_8 \end{array}$$

2. $10110011_2 + 1101_2 = 11000000_2$

1 1 1 1 1 1

$$\begin{array}{r} 1\,0\,1\,1\,0\,0\,1\,1_2 \\ +\quad 1\,1\,0\,1_2 \\ \hline 1\,1\,0\,0\,0\,0\,0\,0_2 \end{array}$$

3. $7A66_{16} + 45C5_{16} = C02B_{16}$

1 1

$$\begin{array}{r} 7\,A\,6\,6_{16} \\ +\,4\,5\,C\,5_{16} \\ \hline C\,0\,2\,B_{16} \end{array}$$

4. $3022_5 - 2433_5 = 34_5$

$$\begin{array}{r} \dot{3}\,\dot{0}\,\dot{2}\,2_5 \\ -\;2\,4\,3\,3_5 \\ \hline 0\;\;0\,3\,4_5 \end{array}$$

Question 3

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = 64 + 32 + 16 + 8 + 4 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 = 1111100_2 = 01111100_{8\ bit\ 2's\ comp}$

2. $-124_{10} = 10000100_{8\ bit\ 2's\ comp}$

$$
\begin{array}{r}
{\scriptstyle 1\,1\,1\,1\,1\,1} \\
0\,1\,1\,1\,1\,1\,0\,0_2 \\
\underline{+\,1\,0\,0\,0\,0\,1\,0\,0_2} \\
1\,0\,0\,0\,0\,0\,0\,0\,0_2
\end{array}
$$

3. $109_{10} = 64 + 32 + 8 + 4 + 1 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0 = 1101101_2 = 01101101_{8\ bit\ 2's\ comp}$

4. $-79_{10} = -(64 + 8 + 4 + 2 + 1) = -(1 \times 2^6 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0) = -1001111_2 = 10110001_{8\ bit\ 2's\ comp}$

$$
\begin{array}{r}
{\scriptstyle 1\,1\,1\,1\,1\,1\,1\,1} \\
0\,1\,0\,0\,1\,1\,1\,1_2 \\
\underline{+\,1\,0\,1\,1\,0\,0\,0\,1_2} \\
1\,0\,0\,0\,0\,0\,0\,0\,0_2
\end{array}
$$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\ bit\ 2's\ comp} = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 = 30_{10}$

2. $11100110_{8\ bit\ 2's\ comp} = -11010_2 = -(1 \times 2^4 + 1 \times 2^3 + 1 \times 2^1)_{10} = -26_{10}$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\ \ 1\,1\,1\ \ } \\ 1\,1\,1\,0\,0\,1\,1\,0_2 \\ +\,0\,0\,0\,1\,1\,0\,1\,0_2 \\ \hline 1\,0\,0\,0\,0\,0\,0\,0\,0_2 \end{array}$$

3. $00101101_{8\ bit\ 2's\ comp} = 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0 = 45_{10}$

4. $10011110_{8\ bit\ 2's\ comp} = -1100010_2 = -(1 \times 2^6 + 1 \times 2^5 + 1 \times 2^1)_{10} = -98_{10}$

$$\begin{array}{r} {\scriptstyle 1\,1\,1\,1\ \ 1\,1\,1\ \ } \\ 1\,0\,0\,1\,1\,1\,1\,0_2 \\ +\,0\,1\,1\,0\,0\,0\,1\,0_2 \\ \hline 1\,0\,0\,0\,0\,0\,0\,0\,0_2 \end{array}$$

Question 4

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

(b) Truth table for $\neg(p \vee q)$

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(c) Truth table for $r \vee (p \wedge \neg q)$

p	q	r	¬q	$p \wedge \neg q$	$r \vee (p \wedge \neg q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. Exercise 1.3.4, sections b, d

(b) Truth table for (p→q) →(q→p)

p	q	p→q	q→p	(p→q) →(q→p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d) Truth table for $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	¬q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7, sections b, c

(b) $(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$

(c) $B \vee (D \wedge M)$

2. Exercise 1.3.7, sections b – e

(b) $y \rightarrow p$

(c) $p \rightarrow y$

(d) $p \leftrightarrow y$

(e) $p \rightarrow (s \vee y)$

3. Exercise 1.3.9, sections c, d

(c) $c \rightarrow p$

(d) $c \rightarrow p$

Question 6

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6, sections b – d

(b) If Joe is eligible for the honors program, then Joe maintains a B average.

(c) If Rajiv can go on the roller coaster, then Rajiv is at least four feet tall.

(d) If Rajiv is at least four feet tall, then Rajiv can go on the roller coaster.

2. Exercise 1.3.10, sections c – f

The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

(c) $(p \vee r) \leftrightarrow (q \wedge r)$

$p \vee r$	T
$q \wedge r$	F
$(p \vee r) \leftrightarrow (q \wedge r)$	F

(d) $(p \wedge r) \leftrightarrow (q \wedge r)$

$p \wedge r$	Unknown
$q \wedge r$	F
$(p \vee r) \leftrightarrow (q \wedge r)$	Unknown

(e) $p \longrightarrow (r \vee q)$

$r \vee q$	Unknown
$p \longrightarrow (r \vee q)$	Unknown

(f) $(p \wedge q) \longrightarrow r$

$p \wedge q$	F
$(p \wedge q) \longrightarrow r$	T

Question 7

Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

(b) The first sentence can be expressed as $\neg j \to (l \vee \neg r)$

j	l	r	¬j	¬r	$l \vee \neg r$	$\neg j \to (l \vee \neg r)$
T	T	T	F	F	T	T
T	T	F	F	T	T	T
T	F	T	F	F	F	T
T	F	F	F	T	T	T
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	F	F
F	F	F	T	T	T	T

The second sentence can be expressed as $(r \wedge \neg l) \to j$

j	l	r	¬l	$r \wedge \neg l$	$(r \wedge \neg l) \to j$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	T	F
F	F	F	T	F	T

Thus these two expressions are logically equivalent.

(c) The first sentence can be expressed as $j \to \neg l$, while the second sentence can be expressed as $\neg j \to l$.

j	l	¬j	¬l	$j \to \neg l$	$\neg j \to l$
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	F

Thus these two expressions are **NOT** logically equivalent.

(d) The first sentence can be expressed as $(r \vee \neg l) \rightarrow j$

j	l	r	¬l	$r \vee \neg l$	$(r \wedge \neg l) \rightarrow j$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	T
F	F	T	T	T	F
F	F	F	T	T	F

The second sentence can be expressed as $j \rightarrow (r \wedge \neg l)$

j	l	r	¬l	$r \wedge \neg l$	$j \rightarrow (r \wedge \neg l)$
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	T	T
F	F	F	T	F	T

Thus these two expressions are **<u>NOT</u>** logically equivalent.

Question 8

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

(c) $(p \to q) \land (p \longrightarrow r) \equiv p \to (q \land r)$

$LHS \equiv^{11a} (\neg p \lor q) \land (\neg p \lor r) \equiv^{4a} \neg p \lor (q \land r) \equiv^{11a} p \to (q \land r) \equiv RHS$

(f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$LHS \equiv^{9a} \neg p \land \neg(\neg p \land q) \equiv^{9b} \neg p \land (\neg\neg p \lor \neg q) \equiv^{7} \neg p \land (p \lor \neg q) \equiv^{4b} (\neg p \land p) \lor (\neg p \land \neg q) \equiv^{5a} \neg p \land \neg q \equiv RHS$

(i) $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$LHS \equiv^{11a} \neg(p \land q) \lor r \equiv^{9b} (\neg p \lor \neg q) \lor r \equiv^{2b} \neg p \lor (\neg q \lor r)$

$RHS \equiv^{11a} \neg(p \land \neg r) \lor \neg q \equiv^{9b} (\neg p \lor \neg\neg r) \lor \neg q \equiv^{7} (\neg p \lor r) \lor \neg q \equiv^{2b} \neg p \lor (r \lor \neg q) \equiv^{3a} \neg p \lor (\neg q \lor r)$

Thus $LHS \equiv RHS$

Table 1.5.1: Laws of propositional logic.

1.	Idempotent laws:	$p \lor p \equiv p$	$p \land p \equiv p$
2.	Associative laws:	$(p \lor q) \lor r \equiv p \lor (q \lor r)$	$(p \land q) \land r \equiv p \land (q \land r)$
3.	Commutative laws:	$p \lor q \equiv q \lor p$	$p \land q \equiv q \land p$
4.	Distributive laws:	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
5.	Identity laws:	$p \lor F \equiv p$	$p \land T \equiv p$
6.	Domination laws:	$p \land F \equiv F$	$p \lor T \equiv T$
7.	Double negation law:	$\neg\neg p \equiv p$	
8.	Complement laws:	$p \land \neg p \equiv F$ $\neg T \equiv F$	$p \lor \neg p \equiv T$ $\neg F \equiv T$
9.	De Morgan's laws:	$\neg(p \lor q) \equiv \neg p \land \neg q$	$\neg(p \land q) \equiv \neg p \lor \neg q$
10.	Absorption laws:	$p \lor (p \land q) \equiv p$	$p \land (p \lor q) \equiv p$
11.	Conditional identities:	$p \to q \equiv \neg p \lor q$	$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

2. Exercise 1.5.3, sections c, d

(c) $\neg r \lor (\neg r \to p) \equiv \neg r \lor (r \lor p) \equiv (\neg r \lor r) \lor p \equiv p$

(d) $\neg(p \to q) \to \neg q \equiv \neg\neg(p \to q) \lor \neg q \equiv (p \to q) \lor \neg q \equiv (\neg p \lor q) \lor \neg q \equiv \neg p \lor (q \lor \neg q) \equiv \neg p$

Question 9

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3, sections c, d

(c) $\exists x, x = x^2$

(d) $\forall x, x \leq x^2$

2. Exercise 1.7.4, sections b – d

(b) $\forall x, \ \neg S(x) \wedge W(x)$

(c) $\forall x, \ S(x) \rightarrow \neg W(x)$

(d) $\exists x, S(x) \wedge W(x)$

Question 10

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c – i

(c) F. Because P(c) = F.

(d) T. Because Q(e) = R(e) = T.

(e) T. Because Q(a) = P(d) = T.

(f) T. Because this is true for all the cases, including x = a, c, d, e.

(g) F. Because P(c) = R(c) = F.

(h) T. Because this is true for all the cases, including x = a, b, c, d, e.

(i) T. Because Q(a) = T.

2. Exercise 1.9.2, sections b – i

(b) F.

(c) T. Because x=1 satisfies the statement.

(d) F.

(e) T. Because y=2 satisfies the statement.

(f) F.

(g) F.

(h) T.

(i) T. Because $\neg S(x, y)$ is always true.

Question 11

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c – g

(c) $\exists x\ \exists y, (x + y = xy)$

(d) $\forall x\ \forall y, \left((x > 0) \wedge (y > 0) \rightarrow \left(\frac{x}{y} > 0\right)\right)$

(e) $\forall x, \left((0 < x < 1) \rightarrow \left(\frac{1}{x} > 1\right)\right)$

(f) $\forall x\ \exists y, (y < x)$

(g) $\forall x\ \exists y, \left((x \neq 0) \rightarrow (xy = 1)\right)$

2. Exercise 1.10.7, sections c – f

(c) $\exists x, \left(N(x) \wedge D(x)\right)$

(d) $\forall x, \left(D(x) \rightarrow P(Sam, x)\right)$

(e) $\exists x\ \forall y, \left(N(x) \wedge P(x, y)\right)$

(f) $\exists x, \left(N(x) \wedge D(x) \wedge \forall y\ \left((x \neq y) \rightarrow \neg D(y)\right)\right)$

3. Exercise 1.10.10, sections c – f

(c) $\forall x, \left(T(x, Math\ 101) \wedge \left(\exists y (y \neq Math\ 101) \wedge T(x, y)\right)\right)$

(d) $\exists x\ \forall y, T(x, y)$

(e) $\forall x\ \exists y\ \exists z, (T(x, y) \wedge T(x, z) \wedge (y \neq z))$

(f) $\exists x\ \exists y, \left(T(Sam, x) \wedge T(Sam, y) \wedge (x \neq y) \wedge \left(\forall z (z \neq x) \wedge (z \neq y) \rightarrow \neg T(Sam, z)\right)\right)$

Question 12

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2, sections b – e

(b) Every patient was given the medication or the placebo or both.

- $\forall x,\ D(x) \vee P(x)$
- Negation: $\neg\forall x,\ D(x) \vee P(x)$
- Applying De Morgan's Law: $\exists x, \neg D(x) \wedge \neg P(x)$
- English: Some patients was given neither the medication nor the placebo.

(c) There is a patient who took the medication and had migraines.

- $\exists x,\ D(x) \wedge M(x)$
- Negation: $\neg\exists x,\ D(x) \wedge M(x)$
- Applying De Morgan's Law: $\forall x, \neg D(x) \vee \neg M(x)$
- English: All the patients, either did not take medication, or do not have migraines, or both.

(d) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, p → q ≡ ¬p ∨ q.)

- $\forall x,\ P(x) \rightarrow M(x)$
- Negation: $\neg\forall x,\ P(x) \rightarrow M(x)$
- Applying De Morgan's Law: $\exists x, \neg(P(x) \rightarrow M(x)) \equiv \exists x, \neg(\neg P(x) \vee M(x)) \equiv \exists x, P(x) \wedge \neg M(x))$
- English: There is a patient who took the placebo and did not have migraines.

(e) There is a patient who had migraines and was given the placebo.

- $\exists x,\ M(x) \wedge P(x)$
- Negation: $\neg\exists x,\ M(x) \wedge P(x)$
- Applying De Morgan's Law: $\forall x, \neg M(x) \vee \neg P(x)$
- English: All the patients, either did not have migraines, or was not given the placebo, or both.

2. Exercise 1.9.4, sections c – e

(c) $\neg\left(\exists x \forall y \big(P(x,y) \rightarrow Q(x,y)\big)\right) \equiv \forall x\ \exists y, \neg\big(P(x,y) \rightarrow Q(x,y)\big) \equiv \forall x\ \exists y, \neg\big(\neg P(x,y) \vee Q(x,y)\big) \equiv \forall x\ \exists y, \big(P(x,y) \wedge \neg Q(x,y)\big)$

(d) $\neg\left(\exists x\ \forall y, \big(P(x,y) \leftrightarrow P(y,x)\big)\right) \equiv \forall x\ \exists y, \neg\big(P(x,y) \leftrightarrow P(y,x)\big)$

Let P(x,y) = Q, P(y,x) = R, then

$$\neg\big(P(x,y) \leftrightarrow P(y,x)\big) \equiv \neg(Q \leftrightarrow R) \equiv \neg\big((Q \rightarrow R) \wedge (R \rightarrow Q)\big) \equiv \neg(Q \rightarrow R) \vee \neg(R \rightarrow Q)$$
$$\equiv \neg(\neg Q \vee R) \vee \neg(\neg R \vee Q) \equiv (Q \wedge \neg R) \vee (R \wedge \neg Q)$$

Thus $\neg\left(\exists x\ \forall y, \big(P(x,y) \leftrightarrow P(y,x)\big)\right) \equiv \forall x\ \exists y, \big(P(x,y) \wedge \neg P(y,x)\big) \vee \big(P(y,x) \wedge \neg P(x,y)\big)$

(e) $\neg\big(\exists x\ \exists y, P(x,y) \wedge \forall x\ \forall y\ Q(x,y)\big) \equiv \forall x\ \forall y, \neg(P(x,y) \wedge \forall x\ \forall y\ Q(x,y)) \equiv \forall x\ \forall y, \neg P(x,y) \vee \neg\big(\forall x\ \forall y\ Q(x,y)\big) \equiv \forall x\ \forall y, \neg P(x,y) \vee \big(\exists x\ \exists y\ \neg Q(x,y)\big)$