Lecture 3 - Consumer Preferences and Utility

ECON 3070 - Intermediate Microeconomic Theory

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Overview

In this lecture, we will consider the following:

1. How we can represent and analyze consumer preferences.
2. Basic assumptions about consumer preferences.
3. Utility functions - Functions representing an individual's preferences for one or more goods.
4. Indifference curves as a graphical representation of consumer preferences.
5. What happens to consumers' desire for a good as they consume more of it?

Consumer Preferences

We all face trade-offs when it comes to our budget. Most people can't afford to buy everything they want.

- So how do we choose?
- Need a way of deciding between two or more 'baskets' of items.

Consumer Preferences

Consumer preferences tell us how an individual would rank any two baskets of goods (not considering their cost). We make a few assumptions about consumer preferences:

- Completeness: Consumer is able to rank any two baskets.
- Transitivity: If basket A is preferred to basket B, and B is preferred to C, then A is preferred to C.
- More is Better: Having more of a good is better for the consumer.

Consumer Preferences

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Cardinal rankings give us information about the intensity of consumer preferences. *I like this one 10, I like this one 25, etc.*

- What do these numbers mean? Units?

Utility Functions

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Utility functions measure the level of satisfaction that a consumer receives from any basket of goods. Examples:

- $U(X,Y) = 3X + 4Y$
- $U(X,Y) = X^{\frac{1}{2}}Y^{\frac{1}{2}}$
- $U(X,Y) = \log(XY)$

Why are we using functions?

Utility cannot actually be measured, so the exact specification is arbitrary.

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Utility explains how a person think about trade-offs

Utility cannot be compared *between* individuals, only *within* an individual.

Trade-offs and 'Marginal Utility'

Marginal utility refers to the rate at which total utility changes as the level of consumption of one good rises (holding constant the level of consumption of all other goods), and is given by:

$$MU_x = \frac{\Delta U}{\Delta x}$$

We will see that marginal utility is very helpful since it lets us compare *on the margin* how people compare more of different goods.

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- This is also the formula for the slope of the utility function. (U on the y-axis, good x on the x-axis)
- It is also the partial derivative with respect to x: $\frac{\delta U(x)}{\delta x}$.

Diminishing Returns

Why do you stop eating pizza when you do?

Marginal Utility

Diminishing marginal utility means that as an individual consumes more of a good, their additional (or marginal) utility from that good decreases.

- Think about the additional satisfaction you get from eating more pizza...
- Or watching additional episodes of a tv show on Netflix.

Marginal Utility

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- Think about the additional satisfaction you get from eating more pizza...
- Or watching additional episodes of a tv show on Netflix.
- At some point, we might have negative marginal utility, but for amounts that the consumer is likely to purchase, this probably is true.

Try It Yourself

Suppose that a consumer's preference can be represented by the utility function $U(X,Y) = 4X^{\frac{1}{2}}Y^{\frac{1}{2}}$.

What is the consumer's marginal utility function for good X?

Tradeoffs and Indifference Curves

In the case where a consumer is choosing quantities of two goods, the consumer's utility function can be represented graphically using indifference curves.

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- An **indifference curve** represents bundles of two goods which provide a consumer with the same level of utility.
- Consider the following illustration of a utility function with two goods:

Figure: Indifference Curves in 2D

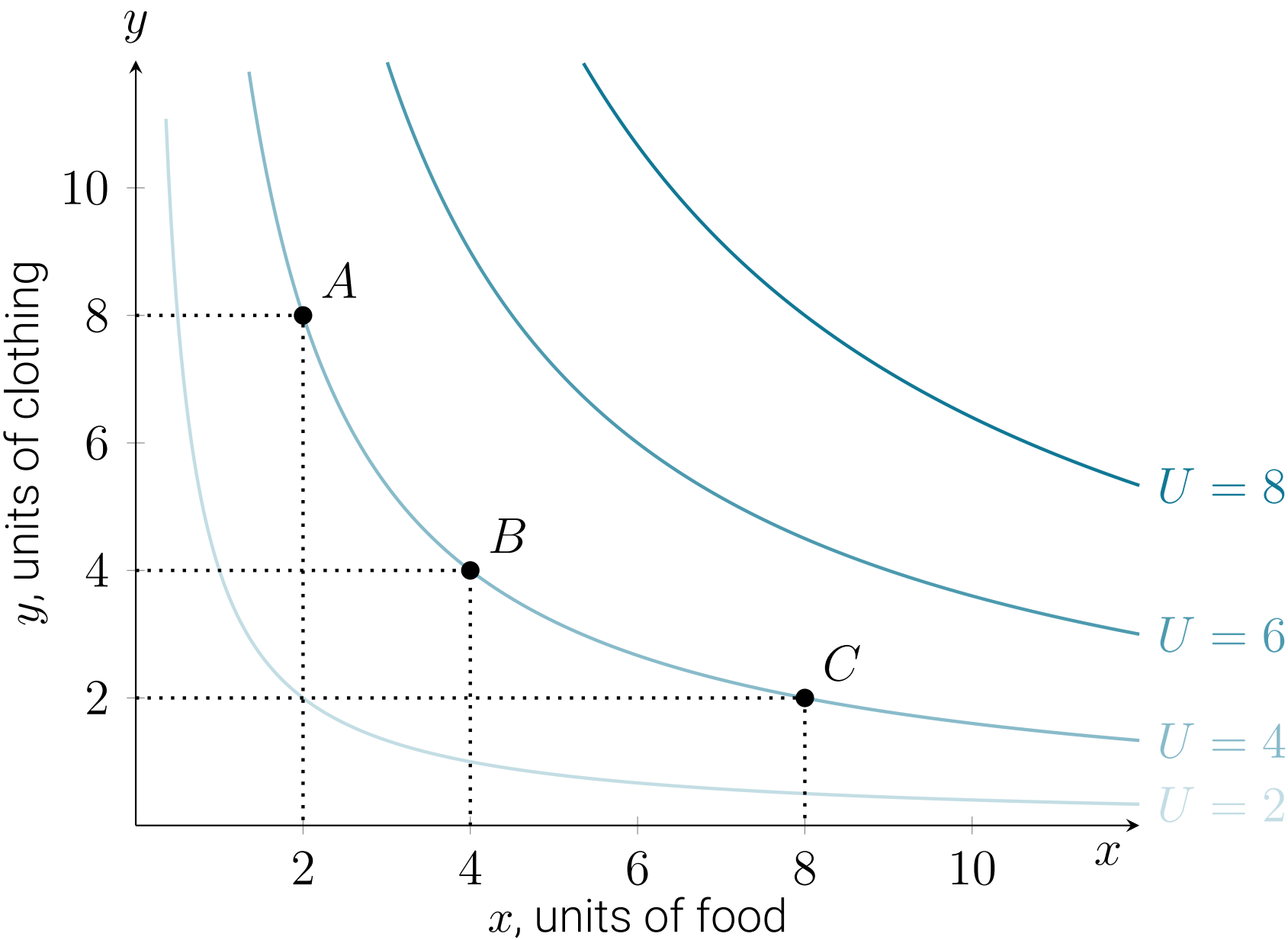
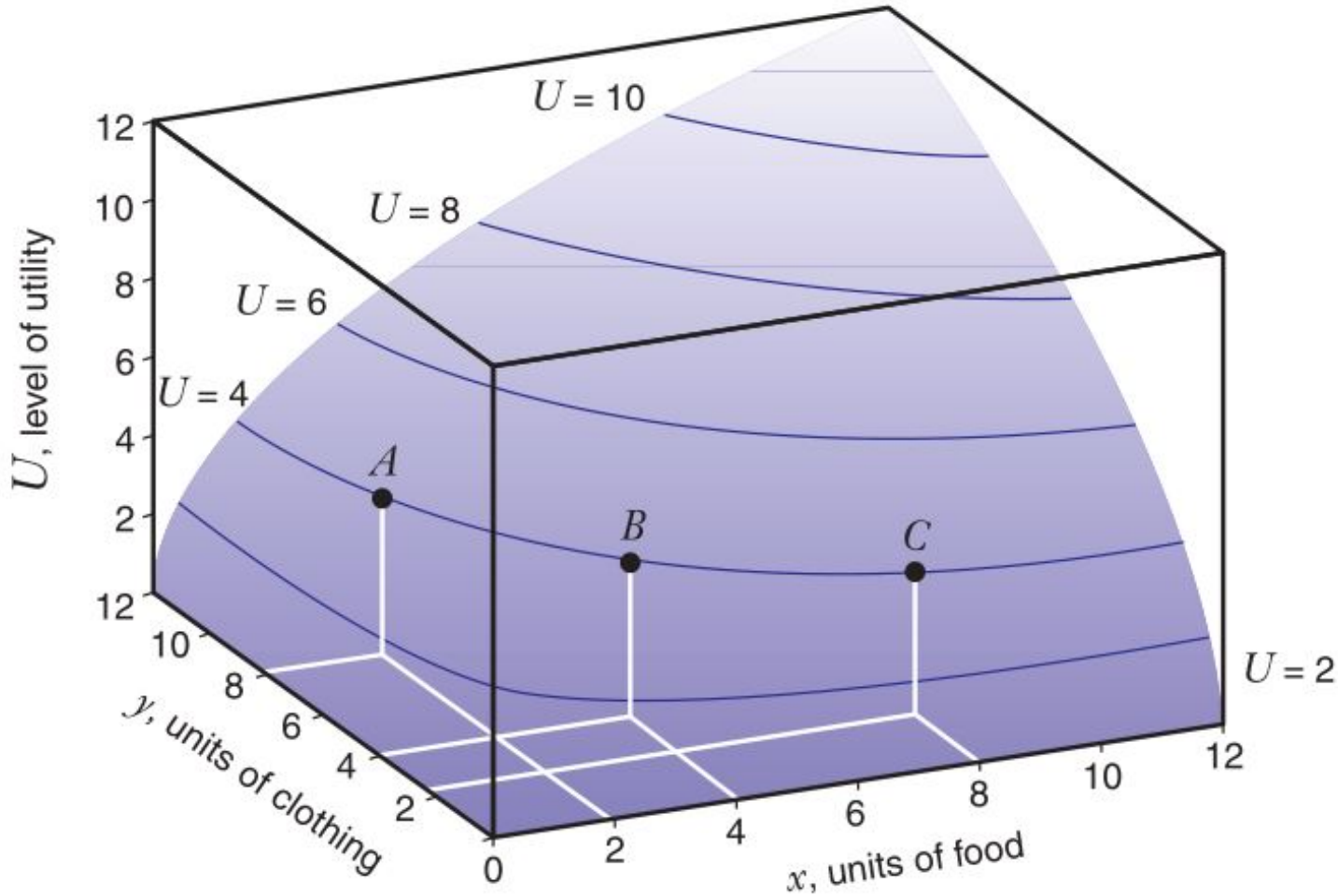


Figure: Indifference Curves



How do you draw an indifference curve?

Say the utility function is $U(x, y) = x + 2y$. To draw an indifference curve:

1. Pick a value of utility $\bar{U}$. Solve for y.
   e.g. $\bar{U} = 2 = x + 2y \implies y = 1 - \frac{1}{2x}$
2. Pick points and 'trace out' indifference curve shape.
   In this case, we have a line, so drawing it should be easy.

Tip: Pick a $\bar{U}$ so the math works well.

Try It Yourself

Draw three indifference curves for the utility function

$U(x, y) = 2x + 2y$

Properties of Indifference Curves

Indifference curves have four properties:

- If both products are 'goods', curves slope downward.
- Indifference curves **cannot** intersect.
- Every consumption basket lies on only one indifference curve.
- Indifference curves are not "thick".

Marginal Rate of Substitution

Note that for a consumer to stay on a given indifference curve (keeping utility constant), if they receive more of one good, they must give up a certain amount of the other. Why is that?

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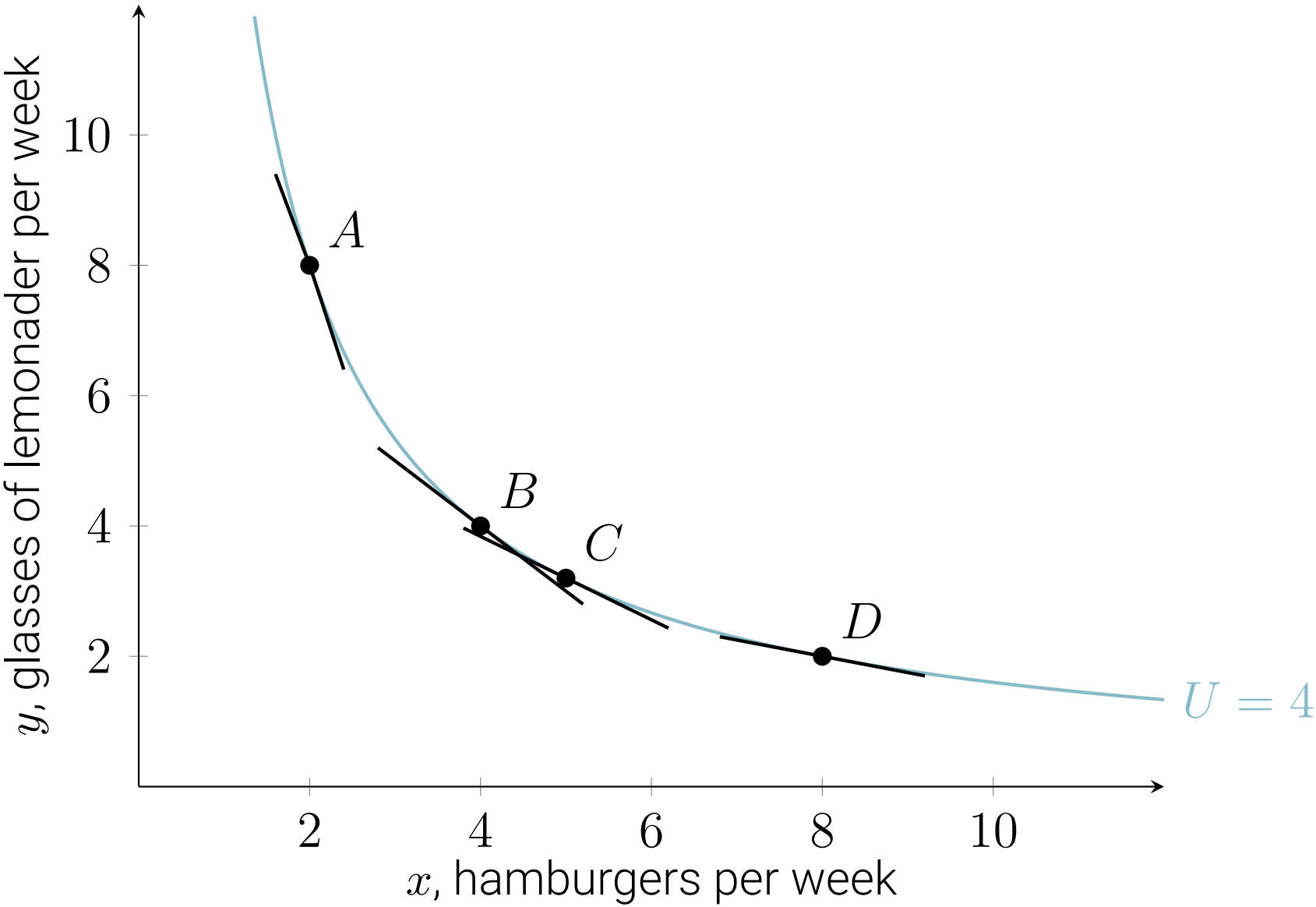
But how much of the other good do we have to give up to stay on the indifference curve?

Marginal Rate of Substitution

But how much of the other good do we have to give up to stay on the indifference curve? This is the **marginal rate of substitution (MRS)**

- The rate at which a consumer must substitute one good for another to stay on the same indifference curve is given by $\Delta y/\Delta x$.
- $MRS_{x,y}$ is therefore also the slope of the indifference curves $(\Delta y/\Delta x)$

Figure: Marginal Rate of Substitution



Try It Yourself

Suppose that Tom is indifferent between two meals: Meal A contains 3 cheeseburgers (C), and 1 large french fry (F). Meal B contains 2 cheeseburgers and 3 large french fries. What is Tom's $MRS_{C,F}$?

Remember: $MRS_{C,F} = |\frac{\Delta F}{\Delta C}|$

MRS and Marginal Utility

Can also express MRS as ratio of marginal utilities of two goods

- To stay on the same indifference curve, for any small change in quantity of good x, **the change in utility from good y must offset the change in utility from good x.**

  That is,

$$MU_x * \Delta x = -MU_y * \Delta y$$

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  That is,

$$MU_x * \Delta x = -MU_y * \Delta y$$

Rearranging the terms, gives us our formula we will use for the class

$$-\frac{\Delta y}{\Delta x} = \frac{MU_x}{MU_y} = MRS_{x,y}$$

How to interpret MRS

The way you interpret MRS is:

For a 1 unit decrease of x, you would have to get $MRS_{x,y}$ units of y (and vice-versa)

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$MRS_{x,y}$ means you **substitute** x **for** y

Try It Yourself

Calculate the marginal rate of substitution, $MRS_{x,y}$, for the following utility functions:

$$U(x,y) = 2x + y \quad \text{and} \quad U(x,y) = x^{\frac{1}{2}} y^{\frac{1}{2}};$$

Diminishing Marginal Rate of Substitution

Remember that for many goods, as consumption increases, the additional utility gained from consuming the good (MU) decreases.

$$MRS_{x,y} = \frac{MU_x}{MU_y}$$

- More of good $y \implies$ lower $MU_y \implies$ higher $MRS_{x,y}$.

Diminishing Marginal Rate of Substitution

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- More of good $y \implies$ lower $MU_y \implies$ higher $MRS_{x,y}$.
- Therefore to get one unit of good x, the more good y we would give up for .
- Said another way, we would be willing to give less of good x for each additional unit of good y we consume, holding x constant.

The three utility functions in this class

There are three kinds of utility functions we will give you in this course.

If you want to do well in this class, you should practice these three functions a bunch *(calculate MRS, draw indifference curves, and later on calculate demand curves for x and y)*

Preference #1: Perfect Substitutes

If two goods are **perfect substitutes** for a consumer, then the marginal rate of substitution is constant (would always trade them at a constant rate)

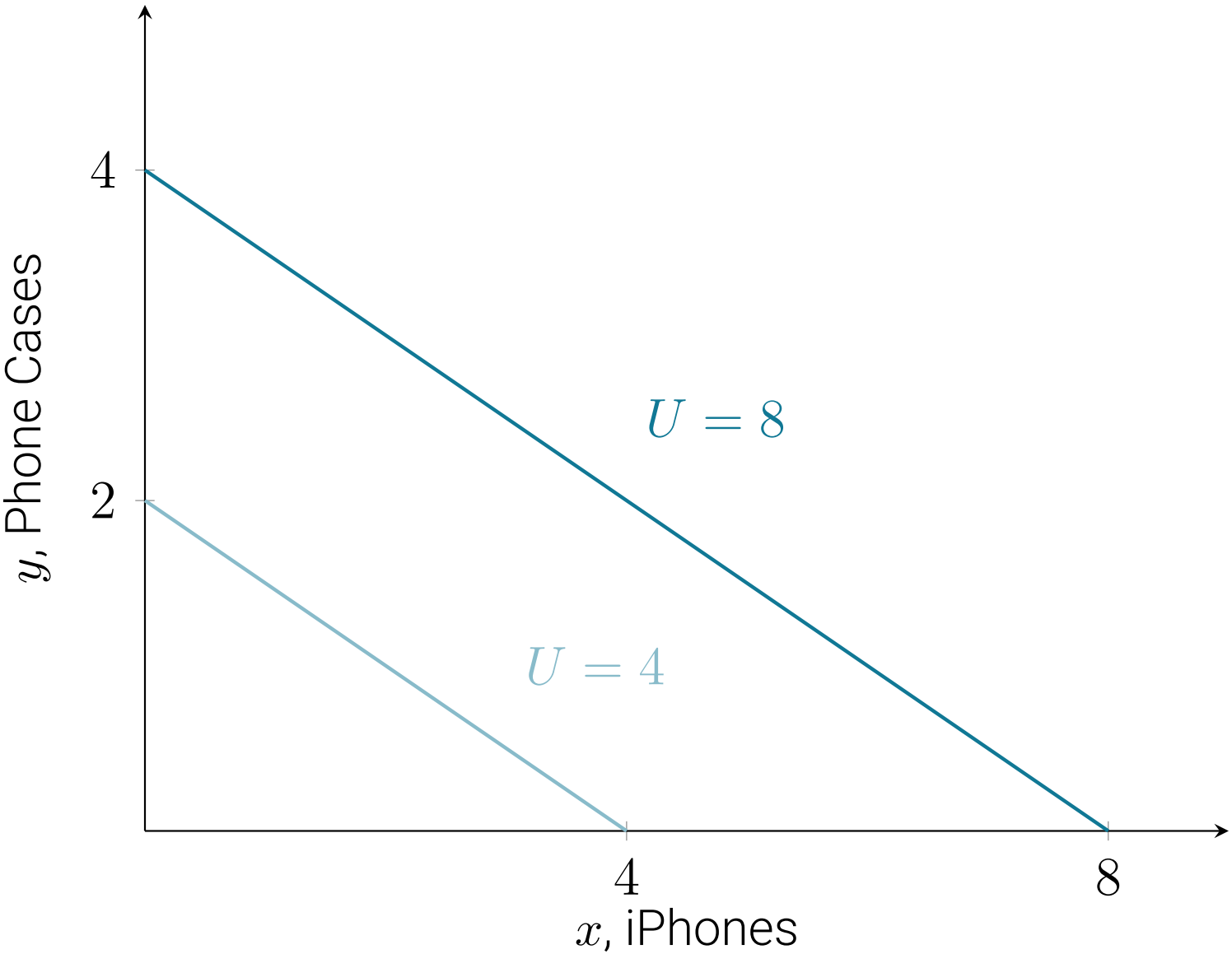
These are given by $U(x, y) = ax + by$ where a and b are two numbers that describe the rate you would trade them.

Preference #1: Perfect Substitutes

Try It Yourself

Calculate the $MRS_{x,y}$ for $U(x,y) = ax + by$. Then draw two indifference curves

Figure: Perfect Substitutes



Preferences #2: Cobb-Douglas

Utility functions of the form $U = x^{\alpha}y^{\beta}$ are known as **Cobb-Douglas Utility Functions**. α and β are numbers (typically between 0 and 1)

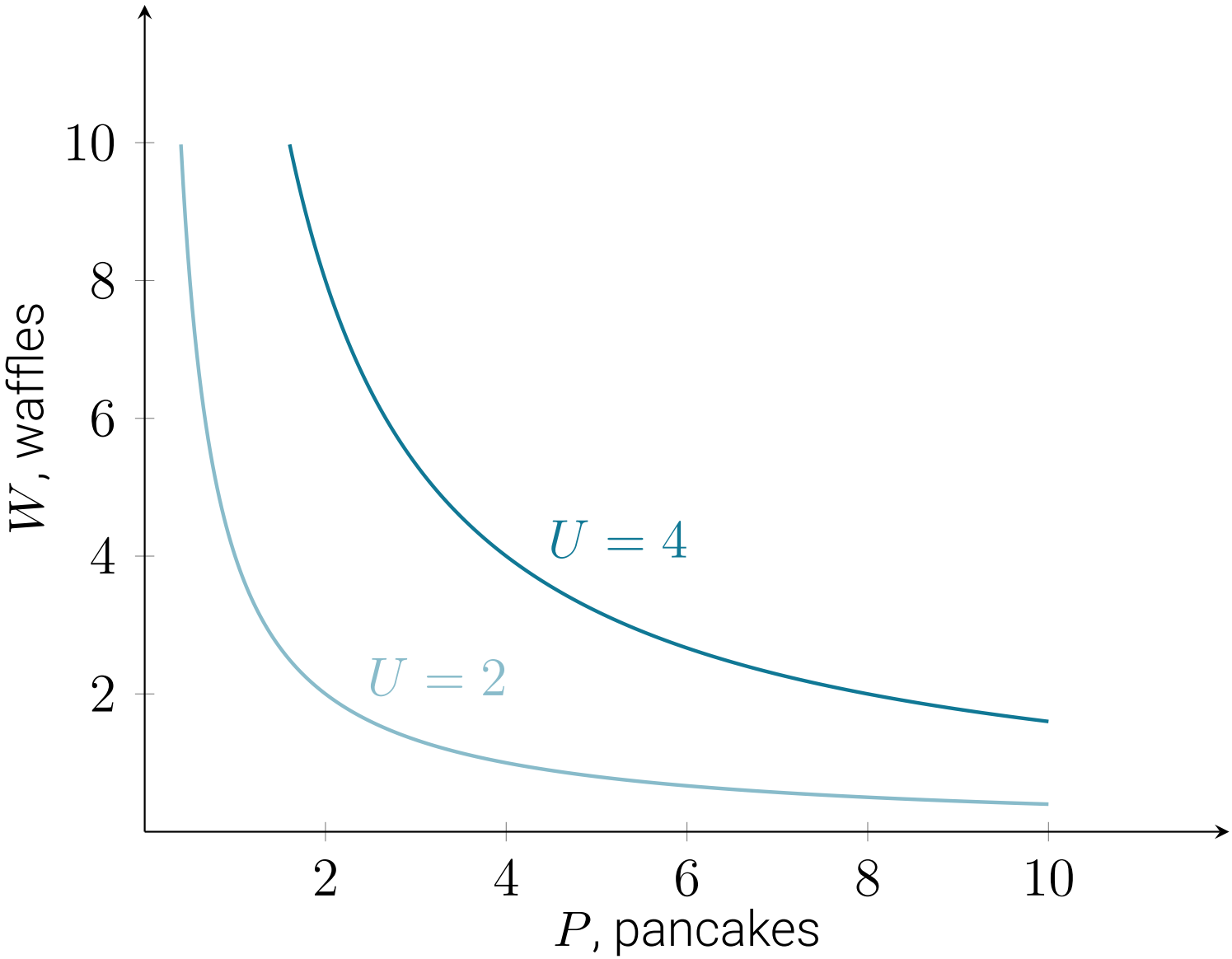
Note this utitlity function displays diminishing marginal rates of substitutions which is the most common case in the real world.

Preference #2: Cobb-Douglas

Try It Yourself

Calculate the $MRS_{x,y}$ for $U(x,y) = x^{\frac{1}{3}}y^{\frac{2}{3}}$. Then draw two indifference curves

Figure: Cobb-Douglas



Preference #3: Perfect Complements

If two goods are **perfect complements** for a consumer, then the consumer only consumes the good in a constant proportion.

They are given by $U(x, y) = \min\left(ax, by\right)$. You will always consumer such that $ax = by$. Why?

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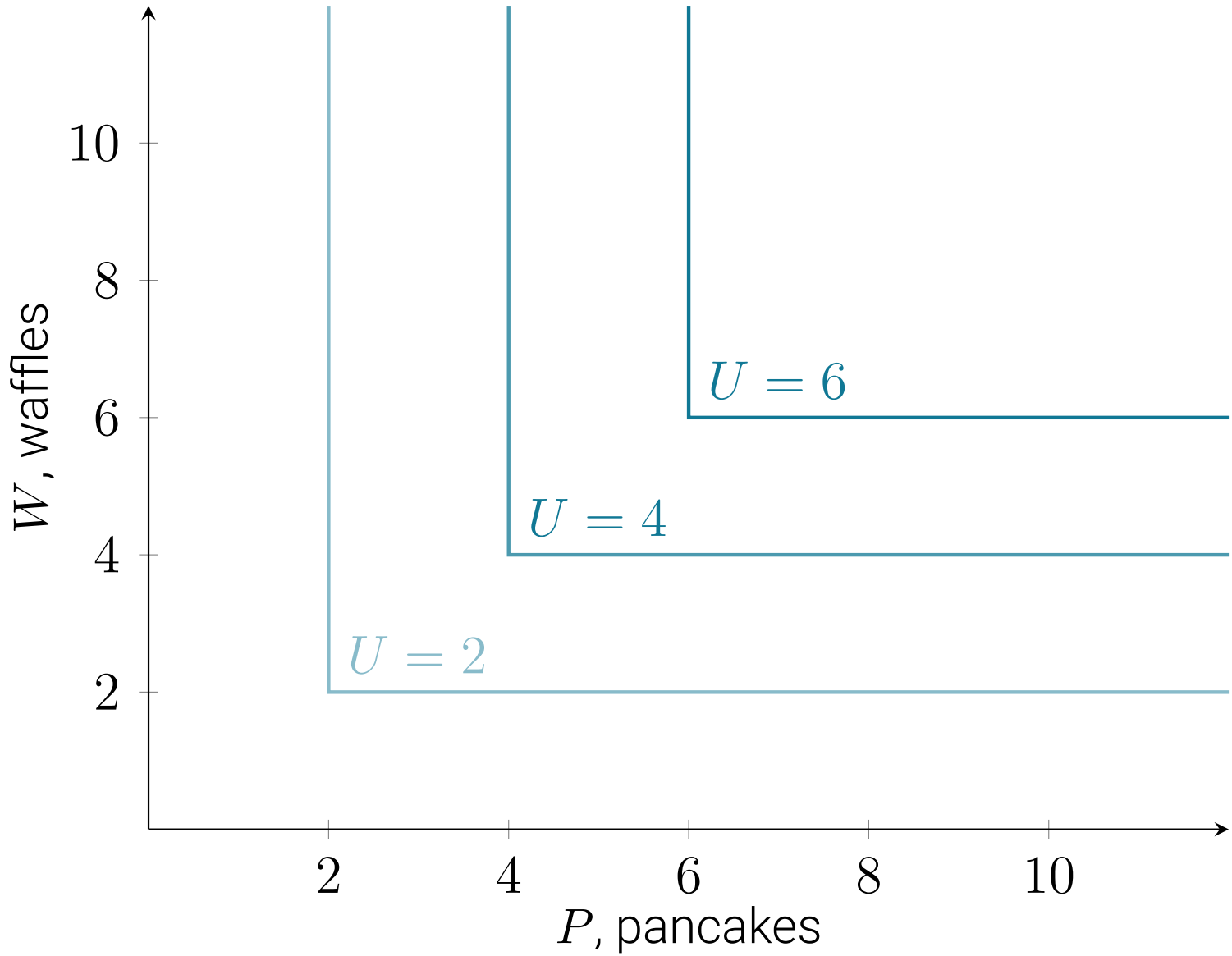
Holding y fixed:

- if $ax < by$, then you are spending too much money on y since $\min\left(ax, by\right) = ax$.
- if $ax > by$, then you are spending too much money on x since $\min\left(ax, by\right) = by$.
- Thus, you consume $ax = by$.

Figure: Perfect Complements



Perfect Complements and Marginal Rate of Substitution

Note you can't calculate MRS since you can't take the derivative of the $\min\left(ax, by\right)$.