

UCD Math Camp
Limits and Derivatives Problem Set

Day 1 Topics

1. Evaluate the following limits

(a) $\lim_{x \rightarrow 0} \frac{(3+x)^2 - 9}{x}$

(b) $\lim_{x \rightarrow 4} 5x^2 - 2x + 3$

(c) $\lim_{x \rightarrow -3} \frac{x^2 - 9}{2x^2 + 7x + 3}$

2. Find $f'(x)$

(a) $f(x) = x^5 + e^x$

(b) $f(x) = 7x^2 + \ln(x)$

(c) $f(x) = 2x^{-4/5}$

3. Find $f'(x)$

(a) $f(x) = \sqrt{x} \ln(x)$

(b) $f(x) = \ln(x^2) + e^x \ln(x)$

4. Find $f'(x)$

(a) $f(x) = \frac{e^x}{1+x}$

(b) $f(x) = \frac{x^2 + x - 2}{x^3 + 6}$

5. Find $f'(x)$

(a) $f(x) = \ln(x)^2$

(b) $f(x) = e^{(x^2 + \ln(x))}$

6. Find $f''(x)$

(a) $f(x) = xe^x$

(b) $f(x) = (1+x)^6$

Day 2 Topics

7. Consider $f(x) = -x^3 + 2x^2 - x - 6$ is increasing/decreasing.

- (a) Examine where $f(x)$ is increasing/decreasing.
- (b) Examine where $f(x)$ is concave/convex.

8. Find y' by implicit differentiation of $x^2 + xy - y^3 = 0$.

9. Find $\frac{\partial f(x_1, x_2)}{\partial x_1}$, $\frac{\partial f(x_1, x_2)}{\partial x_2}$, and $\frac{\partial^2 f(x_1, x_2)}{\partial x_1 \partial x_2}$ for the following.

- (a) $f(x_1, x_2) = x_1^2 x_2 + \ln(x_1) x_2^3$
- (b) $f(x_1, x_2) = e^{(\sqrt{x_1})} \ln(x_2) + \frac{x_1}{x_2}$
- (c) $f(x_1, x_2) = A x_1^\alpha x_2^\beta$ with $0 < \alpha < 1$, $0 < \beta < 1$, & $A > 0$

10. If f is differentiable at x , find the expression for the derivative of $x^2 f(x) + [f(x)]^3$.

11. Find the Taylor Series Expansion of order $k = 4$ for $f(x) = e^{-6x}$ about $x = -4$