CU Denver Math Camp - Limits & Derivatives

Additional Topics

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Additional Topics

- $\rightarrow$ Integral Calculus
- $\rightarrow$ Sets and Numbers

Integral Calculus

An antiderivative of a function $f(x)$ is a function $F(x)$ whose derivative is the original

$F : F' = f$

$\rightarrow$ The function F is also called the indefinite integral of f, $F(x) = \int f(x)dx$

Integral Rules

- $\int k\,dx = kx + C$ where k is a constant
- $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$
- $\int a \cdot f(x)\,dx = a \int f(x)\,dx$
- $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $\int \frac{1}{x}\,dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$
- Integration by parts
  - $\int u(x)v'(x)\,dv = u(x)v(x) - \int u'(x)v(x)\,du$

Indefinite Integral Example

Evaluate the following:

$$\int (3x^2 + 2x + 1)\, dx$$

Apply the addition rule:

$$\int 3x^2\, dx + \int 2x\, dx + \int 1\, dx$$

Apply the power rule:

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$$

Integrate each term:

$$3 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + x + C$$

$$x^3 + x^2 + x + C$$

Fundamental Theorem of Calculus

For numbers a and b, the definite integral of $f(x)$ from a to b is $F(b) - F(a)$, where $F(x)$ is an antiderivative of f.

$\rightarrow$ $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F' = f$

- The Fundamental Theorem of Calculus states that if we iterate each time dividing $[a, b]$ into smaller and smaller sub-intervals, in the limit we obtain the definite integral $\int_a^b f(x)\,dx$

  $\rightarrow$ $\lim_{\Delta \to 0} \sum_{i=1}^{N} f(x_i)\Delta = \int_a^b f(x)\,dx$

Sets

A set is any well-specified collection of elements

- $\rightarrow$ For any set A, we write $a \in A$ to indicate a is a member of set A, and $a \notin A$ to indicate that a is not in the set A
- $\rightarrow$ A set which contains no elements is called the empty set or null set and is denoted by $\varnothing$
- $\rightarrow$ Example of standard notation for sets: the set of all non-negative numbers is written as
  - $R_+ \equiv \{x \in R : x \geq 0\}$

Operations with Sets

- $A \cup B$, spoken "A union B," is the set of all elements that are either in A or in B (or in both)
  - $A \cup B \equiv \{x : x \in A$ or $x \in B\}$
- $A \cap B$, spoken "A intersect B," is the set of all elements that are common to both A and B
  - $A \cap B \equiv \{x : x \in A$ and $x \in B\}$
- $A - B$, or sometimes $A \setminus B$, spoken "A minus B," is the set of all elements of A that are not in B
  - $A - B \equiv \{x : x \in A$ and $x \notin B\}$

Number Sets

- **Natural Numbers** ($\mathbb{N}$):

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

  The set of positive whole numbers.

- **Integers** ($\mathbb{Z}$):

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

  The set of whole numbers including negative numbers and zero.

Number Sets

- **Rational Numbers** ($\mathbb{Q}$):

$$\mathbb{Q} = \left\{ \frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0 \right\}$$

  Numbers that can be expressed as a fraction of two integers.

- **Real Numbers** ($\mathbb{R}$):

$$\mathbb{R} = \text{All points on the number line, including rationals and irrationals}$$

  Includes rational and irrational numbers such as π, $\sqrt{2}$.