Limits & Derivatives Practice Problems

Day 1 Topics

1. Evaluate the following limits
   (a) $\lim_{x\to 0} \frac{(3+x)^2-9}{x}$
   (b) $\lim_{x\to 4} 5x^2 - 2x + 3$
   (c) $\lim_{x\to -3} \frac{x^2-9}{2x^2+7x+3}$
2. Find $f'(x)$
   (a) $f(x) = x^5 + e^x$
   (b) $f(x) = 7x^2 + \ln(x)$
   (c) $f(x) = 2x^{-4/5}$
3. Find $f'(x)$
   (a) $f(x) = \sqrt{x}\ln(x)$
   (b) $f(x) = \ln(x^2) + e^x \ln(x)$
4. $f'(x)$
   (a) $f(x) = \frac{e^x}{1+x}$
   (b) $f(x) = \frac{x^2+x-2}{x^3+6}$
5. Find $f'(x)$
   (a) $f(x) = \ln(x)^2$
   (b) $f(x) = e^{(x^2+\ln(x))}$
6. Find $f''(x)$
   (a) $f(x) = xe^x$
   (b) $f(x) = (1+x)^6$

Day 2 Topics

7. Consider $f(x) = -x^3 + 2x^2 - x - 6$ is increasing/decreasing.
   (a) Examine where $f(x)$ is increasing/decreasing.
   (b) Examine where $f(x)$ is concave/convex.
8. Find y' by implicit differentiation of $x^2 + xy - y^3 = 0$.
9. Find $\frac{\partial f(x_1,x_2)}{\partial x_1}$, $\frac{\partial f(x_1,x_2)}{\partial x_2}$, and $\frac{\partial^2 f(x_1,x_2)}{\partial x_1 \partial x_2}$ for the following.
   (a) $f(x_1, x_2) = x_1^2 x_2 + \ln(x_1) x_2^3$
   (b) $f(x_1, x_2) = e^{(\sqrt{x_1})} \ln(x_2) + \frac{x_1}{x_2}$
   (c) $f(x_1, x_2) = A x_1^{\alpha} x_2^{\beta}$ with $0 < \alpha < 1$, $0 < \beta < 1$, & $A > 0$
10. If f is differentiable at x, find the expression for the derivative of $x^2 f(x) + [f(x)]^3$.
11. Find the Taylor Series Expansion of order $k = 4$ for $f(x) = e^{-6x}$ about $x = -4$