E YOUNG MODULUS CONCEPT

MATH CALCULUS 116

ENGIN MATHS II/CALCULUS

PAST QUESTIONS & SOLUTIONS WITH EXPLANATORY NOTES

WITH SHORTCUT & FORMULARS IN CALCULUS

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PLUS MTH 116 2016 SESSION

Department of Mathematics
Rivers State University of Science and Technology Nkpolu-Oroworukwo, Port Harcourt.
Second Semester Examinations 2015/2016 session
MTH 116 Calculus

<u>Instruction</u>: (1) Answer any FOUR questions and show your work clearly.
(2) Use of mobile phone is not allowed in this examination

<u>Time</u>: 2 Hours

1a) Evaluate the following limits: (i) $\lim_{x\to -2} x^2 + 2x - 1$ (ii) $\lim_{x\to 0} \frac{\tan x \cos x}{x}$ (iii) $\lim_{x\to 3} \frac{x^2-5x+6}{x-3}$

(iv) $\lim_{x\to\infty} \frac{4x^2+x-7}{6x^2+2x-1}$

b) For what values of x is the function $f(x) = \frac{x^2-x-2}{x^2-4}$ continuous.

c) Find the first derivative of $y = 5x^2 + 2x$ from first principle.

2a) (i) Find the maximum and minimum of $y = 2x^3 + 3x^2 - 12x -$ [illegible]

(ii) Find $\frac{dy}{dx}$ if $x^2y + 2y^3 - 4x = 1$

b) A cylindrical can with volume $125cm^3$ is to be made by cutting its top and bottom out of metal squares and forming its curved surface by bending a rectangular sheet of metal to match its ends. What radius r and height h of the can will maximize the amount of material required.

3a) Obtain $\frac{dy}{dx}$ for the following functions (i) $y = 2x^5 + 5x^3 - 4x + 2$ (ii) $y = \frac{e^{2x}+3x}{x+2}$

(iii) $y = \cos x \ln x$ (iv) $y = (4x^3 - 5x)^6$

b) If $y = 2x\sin x$ show that $y'' - 2y' + y = 4\cos x(1 - x) - 4\sin x$

4a) Evaluate (i) $\int x^4 e^{x^5-3} dx$ (ii) $\int \frac{dx}{2x^2-3x-2}$ (iii) $\int_0^2 (4x^3 + x - 1)dx$

b) Find the area bounded by the curve $y = x - 3x^2 + 1$, $x = 1$ and $x = 2$.

5a) A body moves in a straight line so that the distance, S (in meters) travelled by the body after t seconds is given by $S = t^3 - 2t^2 + t + 5$. Find the positions of the body when it is momentarily at rest. What is the acceleration of the body at these times.

b) Solve the equation $x\frac{dy}{dx} = y(1 + x^2)$

MTH 116 - CALCULUS
2015/2016 SESSION
DR. YOUNG'S SOLUTIONS

(1a.)(i) $\lim_{x \to -2} (x^2 + 2x - 1)$

$= ((-2)^2 + (2(-2)-1) = 4 - 4 = 1$

Ans $= -1$

(ii) $\lim_{x \to 0} \frac{\tan x \cos x}{x} = \frac{\tan 0 \cos 0}{0}$

$= \frac{0 \times 1}{0} = \frac{0}{0}$

Since we obtain $\frac{0}{0}$, it implies it is an indeterminate function that requires the use of L' Hopital's Rule to evaluate it's Limit (i.e differentiation method.

$\lim_{x \to 0} \left[\frac{\sin x}{\cos x} \cdot \cos x\right] = \frac{\sin x}{x}$

$\lim_{x \to 0} \left[\frac{\sin x}{x}\right] = 1$

how? $\frac{(\sin x)'}{(x)'} = \frac{\cos x}{1} = \cos 0 = 1$

Ans: 1

(iii) $\lim_{x \to 3} \frac{(x^2 - 5x + 6)}{x-3} = \frac{3^2 - 3(3) + 6}{3-3}$

$= \frac{-6+6}{+3-3} = \frac{0}{0}$

Using L' Hopitals Rule we have;

$\frac{(x^2 - 5x + 6)'}{(x-3)'} = \frac{2x-5}{1}$

$\lim_{x \to 3} \left[\frac{2x-5}{1}\right] = 2(3) - 5$

$= 6 - 5 = 1$

(IV) For limit as $x \to$ Infinity we divide all terms with x, with the highest power:

$$\frac{\frac{4x^2}{x^2} + \frac{x}{x^2} - 7/x^2}{\frac{6x^2}{x^2} + \frac{2x}{x^2} - 1/x^2}$$

$$= \frac{4 + 1/x - 7/x^2}{6 + 2/x - 1/x^2}$$

$$= \frac{4 + 1/\infty - 7/\infty^2}{6 + \frac{2}{\infty} - 1/\infty^2} = \frac{4}{6} = 2/3$$

(1B.) $F(x)$ is discontinuing whenever the denominator is zero. $x^2 - 4 = 0$, $(x-2)(x+2) = 0$

$x = 2$ or -2 hence point of discontinuity at $\langle 2, -2 \rangle$

1c.

From the first derivative we use the fact that:

$$\frac{dy}{dx} = \lim_{Dx \to 0} \frac{[F(x + \Delta x) - F(x)]}{Dx}$$

$$= \lim_{Dx \to 0} \frac{[5(x + \Delta x]^2 + 2(x + Dx) - (5x^2 + 2}{Dx}$$

$$\lim_{Dx\to 0} \frac{5(x^2+2xDx+Dx^2)+2x+2Dx - (5x^2+2x)}{Dx}$$

$$= \lim_{Dx\to 0}\left[\frac{5x^2+10xDx+5Dx^2+2x+2Dx-5x^2-2x}{Dx}\right]$$

$$= \lim_{Dx\to 0}\frac{[10xDx+5Dx^2+2Dx]}{Dx}$$

$$= \lim_{Dx\to 0}[10x+5Dx+2]$$

$= 10x+5(0)+2 = 10x+2$

(2a.) Given: $Y = 2x^3+3x^2-12x-7$

At Max and Min. point

$dy/dx = 0$

$dy/dx = 6x^2+6x-12 = 0$

$x^2+x-2 = 0$ [Solving]

$x = 1$ or -2

To determine the nature of the graph or point, we use the second derivative test for extrenum.

$\frac{d^2y}{dx^2} = 12x+6$ when $x=1$

$\frac{d^2y}{dx^2} = 12(1)+6 = 18 > 0$

which implies minimum point

$Y = 2(1)^3+3(1)^2-12(1)-7$

$= 2+3-12-7 = 5-19 = -14$

$(1,-14)$ minimum point.

Again, when $x=-2$

$\frac{d^2y}{dx^2} = 12(-2)+6 = -24+6 = -18$

which implies Maximum point he[re]

$Y = 2(-2)^3+3(-2)^2-12(-2)-7$

$= 2(-8)+3(4)+24-7 = -16+12+1[7]$

$= 13$

$(-2,13) \rightarrow$ Maximum points.

(2a.)

(ii) $x^2y+2y^3-4x=1$ is an implici[t] Function, whenever you differentia[te] y, we include dy/dx

$2xy + x^2\frac{dy}{dx} + 6y^2\frac{dy}{dx} - 4 = 0$

$dy/dx\,(x^2+6y^2) = 4-2xy$

$dy/dx = \frac{4-2xy}{x^2+6y^2}$

(3a.)

(i) $Y = 25x^5+5x^3-4x+2$

$\frac{dy}{dx} = 125x^4+15x^2-4$

(ii) $Y = \frac{e^{2x}+3x}{x+2}$

using Quotient rule

$Y' = \underline{Vu'-uv'}$

where : $u = e^{2x} + 3x$, $u' = 2e^{2x} + 3$

$v = x + 2$, $v' = 1$

$$y' = \frac{dy}{dx} = \frac{(x+2)(2e^{2x}+3) - (e^{2x}+3x)\times 1}{(x+2)^2}$$

$$= \frac{2xe^{2x} + 3x + 4e^{2x} + 6 - e^{2x} - 3x}{(x+2)^2}$$

$$= \frac{2xe^{2x} + 3e^{2x} + 6}{(x+2)^2}$$

$$= \frac{e^{2x}[2x+3] + 6}{(x+2)^2}$$

(iii) $y = \cos x \ln x$, using product rule:

$y' = uv' + vu'$ where: $u = \cos x$

$u' = -\sin x$; $v = \ln x$, $v' = 1/x$

$$y' = \frac{dy}{dx} = \cos x\left[\frac{1}{x}\right] + \ln x[-\sin x]$$

$$= \frac{\cos x}{x} - \sin x \ln x$$

(iv) $y = (4x^3 - 5x)^6$

using chain Rule:

$y = u^n$, $dy/dx = n.u^{n-1}.u'$

Let $u = 4x^3 - 5x$, $u' = 12x^2 - 5$

$$\frac{dy}{dx} = 6(4x^3 - 5x)^{6-1}(12x^2 - 5)$$

$$= (72x^2 - 30)(4x^3 - 5x)^5$$

3B

Given: $y = 2x\sin x$

using product rule we find:

$y' = 2x\cos x + 2\sin x$

$y'' = -2x\sin x + 2\cos x + 2\cos$

$y'' = -2x\sin x + 4\cos x$

Lets evaluate the LHS

$y'' - 2y' + y = -2x\sin x + 4\cos x - 2(2x\cos x + 2\sin x) + 2x\sin x$

$\Rightarrow -2x\sin x + 4\cos x - 4x\cos x - 4\sin x + 2x\sin x$

$\rightarrow 4\cos x(1-x) - 4\sin x$

which gives the RHS as required.

4a.

(i) $\int x^4 e^{x^5-3}\, dx$

This is integration by substitution. Let $u = x^5 - 3$

$$\frac{du}{dx} = 5x^4, \quad dx = \frac{du}{5x^4}$$

$\int x^4 . e^u . \frac{du}{5x^4}$; $\frac{1}{5}\int e^u du$

$$= \frac{e^{x^5-3}}{5} + c$$

(ii) $\int \frac{dx}{2x^2 - 3x - 2}$

This is an integration by path Fraction Method. hence into part.

into partial fraction before carrying integration.

$$\frac{1}{2x^2-3x-2} = \frac{A}{(x-2)} + \frac{B}{(2x+1)}$$

$$1 = A(2x+1) + B(x-2)$$

$$1 = 2Ax + A + Bx - 2B$$

$$-2B + A = 1 \quad \text{---(1)}$$

$$2A + B = 0 \quad \text{---(2)}$$

Hence: $A - 2B = 1$ ---(1)

$2A + B = 0$ ---(2)

From eqn (2) $B = -2A$

$A - 2(-2A) = 1$: $A + 4A = 1$

$5A = 1$; $A = \frac{1}{5}$; $B = -\frac{2}{5}$

$$\int \frac{dx}{(x-2)(2x+1)} = \int \frac{\frac{1}{5}\,dx}{(x-2)} - \int \frac{2/5}{2x+1}dx$$

$$= \frac{1}{5}\ln(x-2) - \frac{2}{5} \times \frac{1}{2}\ln(2x+1) + c$$

$$= \frac{1}{5}\ln(x-2) - \frac{1}{5}\ln(2x+1) + c$$

$$y = \ln\left[\frac{x-2}{2x+1}\right]^{1/5} + c$$

(iii) $\int_0^2 (4x^3 + x - 1)\,dx$

$$= \frac{4x^4}{4} + \frac{x^2}{2} - x \Big|_0^2$$

$$= \left[\frac{4(2)^4}{4} + \frac{2^2}{2} - 2\right] - [0]$$

$$= 2^4 + \frac{4}{2} - 2 - 0$$

$$= 16 + 2 - 2 = 16$$

4B

$$\int_1^2 (x - 3x^2 + 1)\,dx$$

$$A = \frac{x^2}{2} - \frac{3x^3}{3} + x \Big|_1^2$$

$$A = \frac{2^2}{2} - \frac{3(2)^3}{3} + 2 - \left[\frac{1}{2} - 1 + 1\right]$$

$$A = (2-8+2) - \frac{1}{2} = -4 - \frac{1}{2}$$

$$= -\frac{9}{2}$$

$$|A| = \left|-\frac{9}{2}\right| = \frac{9}{2}\ \text{unit}^2$$

$$= 4.5\ \text{unit}^2$$

5A

$$S = t^3 - 2t^2 + t + 5$$

When Momentairly at rest initial velocity becomes zero.

$$V_0 = \frac{ds}{dt} = 3t^2 - 4t + 1 = 0$$

$$3t^2 - 4t + 1 = 0$$

When solving the above;

$t = 1$ or $\frac{1}{3}$ at $t = 1$

Position: $S(1) = 1^3 - 2(1)^2 + 1 + 5$

$= 1 - 2 + 1 + 5 = 5m$

which is the same if we let $t=0$ from rest.

$S(0) = 0 - 0 + 0 + 5 = 5m$

at $t = \frac{1}{3}$

Position: $S(\frac{1}{3}) = (\frac{1}{3})^3 - 2(\frac{1}{3})^2 + \frac{1}{3} + 5$

which is not too far from the Former position, hence the posi- -tions are : 5m and 5.15M

$a = \frac{dv}{dt} = 6t - 4$ at $t = 1sec.$

$a = 6(1) - 4 = 2m/s^2$

At $t = 1/3$ $a = \frac{6}{3} - 4 = 2 - 4$

$= -2m/s^2$

i.e it accelerate and de- -celerate at equal amount as it comes to rest.

5B

$x \frac{dy}{dx} = y(1+x^2)$

This is an ODE, We use seperation of variable tec- -hnique. $\frac{dy}{y} = \frac{(1+x^2)dx}{x}$

$\int \frac{dy}{y} - \int (\frac{1}{x} + x) dx = \int 0$

$\ln y - \ln x - \frac{x^2}{2} = c$

$\ln(xy) - \frac{x^2}{2} = c$

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RIVERS STATE UNIVERSITY OF SCIENCE AND TECHNOLOGY, PORT HARCOURT

DEPARTMENT MATHEMATICS AND COMPUTER SCIENCE

SECOND SEMESTER EXAMINATION 2014/2015 SESSION

Course: MTH 116 (Calculus)

Options: Med Lab Sci, Applied and Environmental Biology

Time: 2hrs

Instruction: Answer any Four (4) Questions

1a. Evaluate $\lim_{x \to 2} (x^2 + 2x + 1)$

b. Find the value of $\lim_{x \to \infty} \frac{4x - 5}{2x + 3}$

c. Given that $f(x) = x^2 + x$, find the value of

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \; h \neq 0$$

2(a) Determine the gradient of the tangent line to the curve

$y = 3x^2 - 5x + 2$ at the point $(2, 1)$

b. Show that the function

$y = Ae^{2x} + Be^{3x}$ satisfies the equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

where A, B are constants.

c. Using appropriate substitution, find the derivative of the function

$$y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$$

3(a) Use the second derivative test to determine the maximum and minimum values of the function

$y = 2x^3 - 9x^2 + 12x$

b. The radius of a cylinder is decreasing at the rate of 4cm/sec; while the height is increasing at the rate of 2cm/sec. Find the rate of change of the volume when the radius is 2cm and the height is 6cm.

Evaluate the integrals

(a) $\int (4x^3 + 3x^2 + 2)\,dx$

(b) $\int 8x\sqrt{2x^2 + 1}\,dx$

(c) Calculate the volume of the solid generated when the area bounded by the curve $y = \sqrt{2x + 5}$, the axis of x and the lines x = 1 and x = 3 is rotated about the x-axis through four right angles.

5(a) Find the anti-derivatives of

(i) $\frac{x^3 + 4}{x^2 + 2x}$ (ii) $\cos^3 x \sin^2 x$

b. A particle moves in a straight line such that its acceleration a m/sec^2 after passing through a fixed point O is given by $a = 8\cos 2t - 4\sin 2t$.

Find the velocity of the particle

6(a) Calculate the value of $\int_1^2 (x^2 + 3)\,dx$

(b) Shade the region bounded by the curves

$y = x + 3$ and $y = x^2 + 1$ in the xy-place.

What is the area of your shaded region?

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MTH 116.2 2014/2015 SESSION [DEYOUNG SOLUTION]

1a. $\lim [x^2 + 2x + 1]$

$= 2^2 + 2(2) + 1 = 9$

1b) $\lim_{x\to\infty} \frac{4x-5}{2x+3} = \frac{\frac{4x}{x} - 5/x}{\frac{2x}{x} + 3/x}$

$= \frac{4 - 5/x}{2 + 3/x}$

$\lim \left[\frac{4 - 5/x}{2 + 3/x}\right] = \frac{4-0}{2+0} = 2$

(1c.) $\lim_{h\to 0} \frac{\left[(x+h)^2 + (x+h) - (x^2+x)\right]}{h}$

$\lim = \frac{(x^2 + 2xh + h^2) + x + h - x^2 - x}{h}$

$\lim_{h\to 0} \frac{[2xh + h^2 + h]}{h}$

$= \lim_{h\to 0} (2x + h + 1)$

$= 2x + 1$

(2a.) $y = 3x^2 - 5x + 2$

$\frac{dy}{dx} = M = 6x - 5 \quad (2,1)$

$M = 6(2) - 5 = 7$

using $y - y_1 = M(x - x_1)$ for tangent

$y - 1 = 7(x - 2)$

$y = 7x - 14 + 1$; $y = 7x - 13$ or

$-7x + y + 13 = 0$: $7x - y - 13 = 0$

(2B) Given $y = Ae^{2x} + Be^{3x}$

$y' = 2Ae^{2x} + 3Be^{3x}$

$y'' = 4Ae^{2x} + 9Be^{3x}$

Hence Lets substitute:

$y'' - 5y' + 6y = 0$

$4Ae^{2x} + 9Be^{3x} - 5(2Ae^{2x} + 3Be^{3x}) + 6(Ae^{2x} + Be^{3x})$

$\Rightarrow -6Ae^{2x} + 6Ae^{2x} = 0$

$-6Ae^{3x} + 6Be^{3x} = 0$; $0 + 0 = 0$

hence we can say;

$y = Ae^{2x} + Be^{3x}$ is the solution to $y'' - 5y' + 6 = 0$

2c

Given $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$

Let $u = \frac{1+x}{1-x}$

$y = \tan^{-1} u$

using chain rule we have;

$y = (\tan u)^{-1}$

$\frac{dy}{dx} = 1 - \sec^2 u \cdot u' \cdot (\tan u)^{-2}$

$$u' = \frac{(1-x)(1+x)' - (1+x)(+x)'}{(1-x)^2}$$

$$= \frac{(1-x)' - (1+x)(-1)}{(1-x)^2}$$

$$= \frac{1-x+1+x}{(1-x)^2} = \frac{2}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{-2}{(1-x)^2} \cdot \sec^2 u \, . (\tan u)^{-2}$$

$$= \frac{-2}{(1-x)^2} \cdot \frac{1}{\cos^2 u} \cdot \left[\frac{\sin u}{\cos u}\right]^{-2}$$

$$= \frac{-2}{(1-x)^2} \cdot \frac{1}{\cos^2 u} \cdot \frac{\sin^{-2} u}{\cos^{-2} u}$$

$$= \frac{-2}{(1-x)^2 \sin^2 u}$$

$$= \frac{-2}{(1-x)^2 \sin^2\left[\frac{1+x}{1-x}\right]} \;//$$

3a.

$y = 2x^3 - 9x^2 + 12x$

$\frac{dy}{dx} = 6x^2 + 18x + 12 = 0$

At stationary point

$x^2 - 3x + 2 = 0$

$x = 2$ or 1

To determine the maxima and Minima we use:

$\frac{d^2y}{dx^2} = 12x - 18$

when $x = 2$

$\frac{d^2y}{dx^2} = 12(2) - 18 = 24 - 18$
$= 6 > 0$

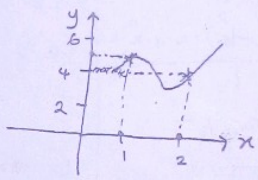
which implies minimum point, hence y value is :

$y = 2(2)^3 - 9(2)^2 + 12(2)$
$= 2(8) - 9(4) + 24$
$= 16 - 36 + 24 = 4 \quad (2,4)$

Also, to obtain the maximum point. $\frac{d^2y}{dx^2} = 12(1) - 18 = -6 < 0$

$y = 2(1)^3 - 9(1)^2 + 12(1)$
$= 2 - 9 + 12 = -7 + 12 = 5$

$(1,5) \rightarrow$ Maximum.



(4a.)

$\int (4x^3 + 3x^2 + 2)\, dx$

$= \frac{4x^4}{4} + \frac{3x^3}{3} + 2x + c$

$y = x^4 + x^3 + 2x + c$

4b

$\int (8x \sqrt{2x^2+1}\,)\, dx$

Let $u = 2x^2 + 1$

$\frac{du}{dx} = 4x, \; dx = \frac{du}{4x}$

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$$8x \int u^{1/2} \frac{du}{4x}$$

$$2\int u^{1/2}\,du = 2\left[\frac{u^{3/2}}{3/2}\right] + c$$

$$= \frac{4}{3}u^{3/2} + c$$

$$= 4/3\,(2x^2+1)^{3/2} + c$$

5a

(i) $\int \left(\frac{x^3+4}{x^2+2x}\right) dx$

Using integrating by partial Fraction, we have to use Long division since the fraction is improper.

$$\begin{array}{r|l} & x-2 \\ \hline x^2+2x & x^3+4 \\ & x^3+2x^2 \\ \hline & -2x^2+4 \\ & -2x^2-4x \\ \hline & 4x+4 \end{array}$$

$$\int\left(\frac{4x+4}{x^2+2x} + x-2\right)dx$$

$$= \int\left(\frac{4x+4}{x^2+2x}\right)dx + \int(x-2)\,dx$$

We use partial fraction method: $\frac{4x+4}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$$4x+4 = A(x+2) + Bx$$
$$= Ax + Bx + 2A$$
$$= x(A+B) + 2A$$

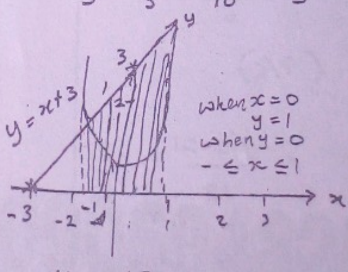
By Comparing;

$A+B = 4$; $2A = 4$, $A = 2$

$B = 2$

6A

$$\int_1^2 (x^2+3)\,dx$$
$$= \frac{x^3}{3} + 3x \Big|_1^2$$
$$= \left(\frac{2^3}{3} + 3(2)\right) - \left(\frac{1}{3} + 3(1)\right)$$
$$= \left(\frac{8}{3}+6\right) - \left(\frac{1}{3}+3\right]$$
$$= \left[\frac{8+18}{3}\right] - \left[\frac{1+9}{3}\right]$$
$$= \frac{26}{3} - \frac{10}{3} = \frac{16}{10} = \frac{8}{5}$$



$y = x+3$

when $x=0$, $y=3$

$y=0$ $x=3$

for $y = x^2+1$

when $x=0$ $y=1$

When $y=0$

$- \le x \le 1$ (Latus Rectum)

10

The value of x is obtain

at $x+3 = x^2+1$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1$ or 2

$\int_{-1}^{2} (x^2 - x - 2)\,dx$

$\frac{x^3}{3} - \frac{x^2}{2} - 2x \Big|_{-1}^{2}$

$\left[\frac{2^3}{3} - \frac{2^2}{2} - 2(2)\right] - \left[\frac{-1}{3} - \frac{1}{2} + 2\right]$

$\left[8/3 - 2 - 4\right] - \left[\frac{-2-3+12}{6}\right]$

$\left[\frac{8-6-12}{3}\right] - \left[\frac{-5+12}{6}\right]$

$\left(\frac{8-18}{3}\right) - (7/6)$

$-\frac{10}{3} - \frac{7}{6} = -\frac{60-21}{18} = -\frac{81}{18}$

Area $= \left|\frac{-81}{18}\right| = 9/2$

$= 4.5$ unit^2