Question 1

a) Let $f(w) = a^\top (w \circ b)$
$= \Sigma_i a_i b_i (w \circ b)_i$
$df(w) / dw_i = a_i b_i$
$\nabla f(w) = (a_i b_i)^\top$
$\nabla f(w) = (a \circ b)$

b) $f(w) = tr(Bww^\top A) = \Sigma_{i,j} B_{ij} w_i w_j A_{ij}$
$df(w)/dw_i = \Sigma_j (B_{ij} w_j A_{ji} + B_{ji} w_i A_{ij})$
$= \Sigma_j B_{ij} (A^\top w)_i + (Aw)_i$
$\nabla f(w) = \Sigma_i (B(A^\top w)_i + (Aw)_i)$
$= BA(A^\top w) + B(Aw)$

c) $\nabla f(w) = d/dw(tr(w^\top ABw))$
$= (AB + (AB)^\top)w$
$H = d^2 f(w)/dw^2 = d/dw((AB + (AB)^\top)w)$
$= AB + (AB)^\top$

d) Code in other file.

e) $f(w) = \log(\sigma(w^\top x))$
$\sigma(a) = 1/(1 + e^{-a})$
$\sigma(w^\top x) = 1/(1 + e^{-w^\wedge Tx})$
$f(w) = \log(1/(1 + e^{-w^\wedge Tx})$
$\nabla f(w) = d/dw(\log(1/(1 + e^{-w^\wedge Tx})))$
$\nabla f(w) = (e^{-w^\wedge Tx} x)/ (1 + e^{-w^\wedge Tx})$