Question 1:

a) p(Water = warm|Play = yes), p(Water = warm|Play = no)
   p(Water = warm|Play = yes) = 2/3
   p(Water = warm|Play = no) = 1

b) p(Play = yes|Water = warm), p(Play = no|Water = warm)
   p(Play = yes|Water = warm) = 2/3
   p(Play = no|Water = warm) = 1/3

c) p(Play = yes|Forecast = same), p(Play = yes|Forecast = change)
   p(Play = yes|Forecast = same) = 1
   p(Play = yes|Forecast = change) = 1/2

d) p(Water = warm|Play = yes), p(Water = warm|Play = no) with Laplace smoothing
   p(Water = warm|Play = yes) = 3/5
   p(Water = warm|Play = no) = 2/3

Question 2:

a) Let's consider a counterexample where k_1 (2, 2) = x^2 and $k_2(x, 2)$ = 22. If we choose a_1=1 and a_2=1, the function $k(x,z)=k_1(x,z) – k_2(x,z)=x^2 - z^2$ is not positive semi-definite.

   Therefore, the function $k(x, z) = a_1k_1(x, z) - a_2k_2(x, z)$ may not always be a valid kernel function.

b) Given A = k(x, z) = e x⊤z σ2, B = k(x, z) = e − ||x−z||2 2 2σ2.
   B would be a valid kernel due to the product of two Gaussian functions is also a Gaussian function.

c) Mercer's theorem disqualifies this kernel due to non-guaranteed symmetry and positive semi-definiteness.