Question 1:

1. p(Water = warm|Play = yes), p(Water = warm|Play = no)

p(Water = warm|Play = yes) = 2/3

p(Water = warm|Play = no) = 1

1. p(Play = yes|Water = warm), p(Play = no|Water = warm)

p(Play = yes|Water = warm) = 2/3

p(Play = no|Water = warm) = 1/3

1. p(Play = yes|Forecast = same), p(Play = yes|Forecast = change)

p(Play = yes|Forecast = same) = 1

p(Play = yes|Forecast = change) = 1/2

1. p(Water = warm|Play = yes), p(Water = warm|Play = no) with Laplace smoothing

p(Water = warm|Play = yes) = 3/5

p(Water = warm|Play = no) = 2/3

Question 2:

1. Let's consider a counterexample where k₁ (2, 2) = x² and k₂(x, 2) = 22. If we choose a1=1 and a2=1, the function k(x,z)=k1(x,z) – k2(x,z)=x2 - z2 is not positive semi-definite.

Therefore, the function k(x, z) = a1k1(x, z) - a2k2(x, z) may not always be a valid kernel function.

1. Given A = k(x, z) = e x⊤z σ2, B = k(x, z) = e − ||x−z||2 2 2σ2.

B would be a valid kernel due to the product of two Gaussian functions is also a Gaussian function.

1. Mercer's theorem disqualifies this kernel due to non-guaranteed symmetry and positive semi-definiteness.