INTRO TO DATA SCIENCE

LINEAR ALGEBRA REVIEW

Linear algebra is the branch of mathematics concerning **vector spaces** and **linear mappings** between such spaces.

https://en.wikipedia.org/wiki/Linear_algebra

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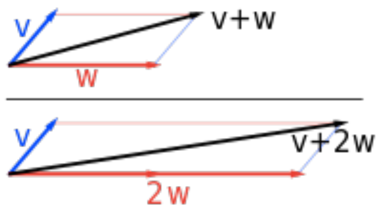
A **vector space** (also called a linear space) is a collection of objects called vectors, which may be added together and multiplied ("scaled") by numbers, called scalars in this context.



https://en.wikipedia.org/wiki/Vector_space

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A **linear mapping** is a mapping $V \rightarrow W$ between spaces that preserves the operations of addition and scalar multiplication.

https://en.wikipedia.org/wiki/Linear_map

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SCALAR MULTIPLICATION:
takes any scalar *a* and any vector **v** and outputs a new vector *a***v**

$$\mathbf{z} = a\mathbf{v}$$

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Associativity of addition	u + (v + w) = (u + v) + w

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Compatibility of multiplication	$a(bv) = (ab)v$
Identity element of scalar multiplication	$1v = v$

u, v and w are vectors in V, and a and b are scalars in F.

A linear transformation T between two vector spaces V and W is compatible with scalar multiplication and vector addition:

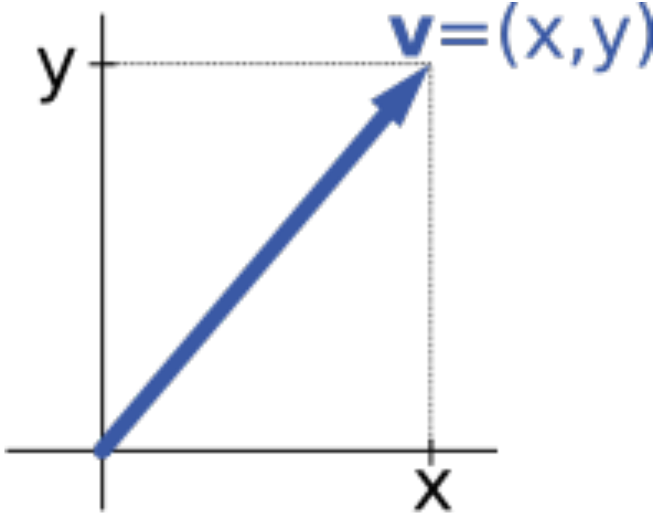
T: V —>W

satisfies:

$$T(a\,\mathbf{u} + b\,\mathbf{v}) = a\,T(\mathbf{u}) + b\,T(\mathbf{v})$$

Vectors can be represented by a list of numbers, their coordinates:

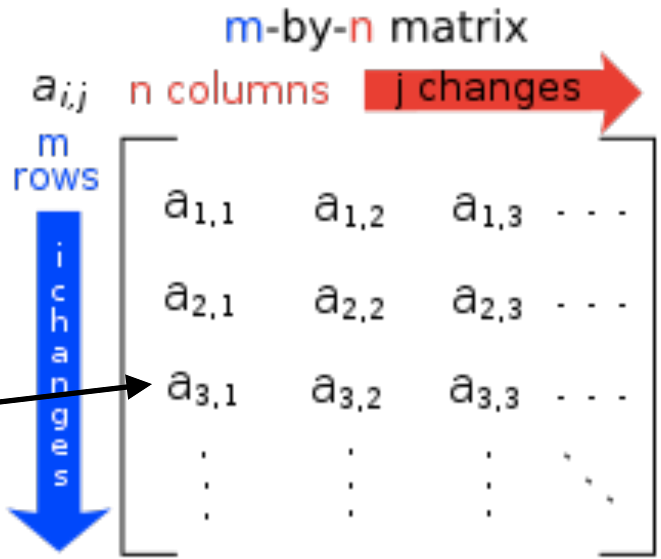
(give me some examples of vector quantities)



Linear mappings can be represented by **matrices**

Matrices are an array of real numbers with m rows and n columns

Each value in a matrix is called an entry.



The size of a matrix is defined by the number of rows and columns.

Examples:

Name	Size	Example
Row vector	$1 \times n$	$\begin{bmatrix} 3 & 7 & 2 \end{bmatrix}$
Column vector	$n \times 1$	$\begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$
Square matrix	$n \times n$	$\begin{bmatrix} 9 & 13 & 5 \\ 1 & 11 & 7 \\ 2 & 6 & 3 \end{bmatrix}$

https://en.wikipedia.org/wiki/Matrix_(mathematics)

Rule 1!

Matrices can be added together only when they are the same size. If they are not the same size, their sum is **undefined**.

$$[1\ 3\ 9\ 2]+[2\ 5\ 9\ 4] = [3\ 8\ 18\ 6]$$

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$$[1\ 3\ 9\ 2]+[2\ 5\ 9\ 4] = [3\ 8\ 18\ 6]$$

$$[8\ 72\ 3\ 1]+[17\ 55\ 3\ 10] = ?$$

Rule 2!

Matrices can be multiplied by a scalar (single entity) value.

Each value in the matrix is multiplied by the scalar value.

$$[1\ 3\ 9\ 2]*3=[3\ 9\ 27\ 6]$$

$$[8\ 72\ 3\ 1]*2=?$$

Rule 3!

Matrices and vectors can be multiplied together given that the matrix columns are as wide as the vector is long.

What shape will the result take?

$$\begin{bmatrix} 1 & 3 & 9 & 2 \\ 2 & 4 & 6 & 8 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 6 \\ 5 \end{bmatrix} = \quad ?$$

2x4 4x1

Rule 3!

Matrices and vectors can be multiplied together given that the matrix columns are as wide as the vector is long.
The result will always be a vector.

$$\begin{bmatrix} 1 & 3 & 9 & 2 \\ 2 & 4 & 6 & 8 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 6 \\ 5 \end{bmatrix} = \begin{matrix} (2+\ \ 9+54+10) \\ (4+12+36+40) \end{matrix} = \begin{bmatrix} 75 \\ 92 \end{bmatrix}$$

$$\underset{2x4}{} \qquad \underset{4x1}{} \qquad \underset{2x1}{}$$

Rule 4!

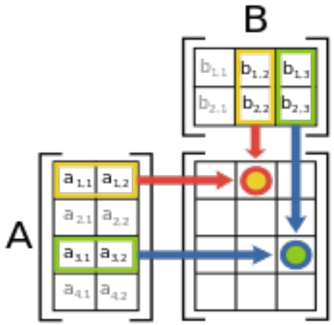
Matrices can be multiplied together using the same rules that we have from matrix-vector multiplication.
What shape will the result take?

$$\begin{bmatrix} 1 & 3 & 9 & 2 \\ 2 & 4 & 6 & 8 \end{bmatrix} * \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 6 & 0 \\ 5 & 4 \end{bmatrix} = \quad ?$$

Rule 4!

Matrices can be multiplied together using the same rules that we have from matrix-vector multiplication.
The result will always be a matrix.

$$\begin{bmatrix} 1 & 3 & 9 & 2 \\ 2 & 4 & 6 & 8 \end{bmatrix} * \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 6 & 0 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} \begin{matrix}(2+9+54+10) \\ =75\end{matrix} & \begin{matrix}(1+6+0+8) \\ =15\end{matrix} \\ \begin{matrix}(4+12+36+40) \\ =92\end{matrix} & \begin{matrix}(2+8+0+32) \\ =42\end{matrix} \end{bmatrix}$$



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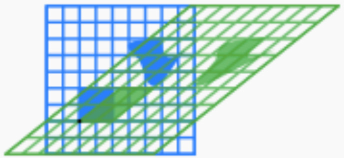
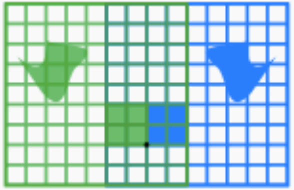
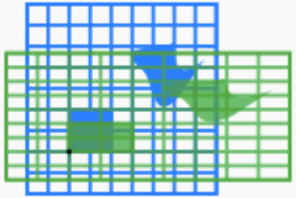
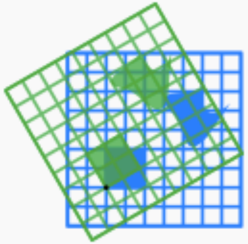
IMPORTANT NOTE:

Matrix multiplication is NOT COMMUTATIVE. The order of matrix multiplication DOES matter. The number of columns of the first matrix must match the number of rows of the second matrix.

https://en.wikipedia.org/wiki/Matrix_(mathematics)#Matrix_multiplication

Here are some examples of operations in a 2D vector space with the corresponding matrix.

Each point in this space is represented by the vector of its coordinates P = (x, y)

Horizontal shear with m=1.25.	Horizontal flip	Squeeze mapping with r=3/2	Scaling by a factor of 3/2	*Rotation by $\pi/6^R = 30°$*
$\begin{bmatrix}1 & 1.25\\0 & 1\end{bmatrix}$	$\begin{bmatrix}-1 & 0\\0 & 1\end{bmatrix}$	$\begin{bmatrix}3/2 & 0\\0 & 2/3\end{bmatrix}$	$\begin{bmatrix}3/2 & 0\\0 & 3/2\end{bmatrix}$	$\begin{bmatrix}\cos(\pi/6^R) & -\sin(\pi/6^R)\\\sin(\pi/6^R) & \cos(\pi/6^R)\end{bmatrix}$
				

https://en.wikipedia.org/wiki/Matrix_(mathematics)

LINKS

https://en.wikipedia.org/wiki/Matrix_(mathematics)
http://mathworld.wolfram.com/Matrix.html
http://ed.ted.com/lessons/how-to-organize-add-and-multiply-matrices-bill-shillito