Historical emissions, growth, and who should abate

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1 Introduction

The world has 1,150 billions of tons of CO_2 left before reaching 2°C of warming (MCC Berlin, 2021). In 2023 alone, the world emitted 37.8 billion tons (Ritchie & Roser, 2020). To prevent the worst harms of climate change, the world would need to reach net zero within the next couple of decades.

However, who should bear the responsibility of abatement is highly contentious. The world's richest countries also tend to have the highest cumulative emissions, while the countries with the most to gain from growth (and hence pollution) are usually those who have emitted least.

Historically, countries have developed by emitting greenhouse gases. For instance, the United Kingdom has long been a major emitter. Its cumulative CO_2 emissions of 78.8 billion tons dwarf those of a developing country like Pakistan (5.7 billion tons), despite Pakistan having almost four times the UK's population in 2023 and many more people living in extreme poverty (Hasell et al., 2022; Ritchie & Roser, 2020; UN Population Division, 2024).

Given these inequalities in wealth and cumulative emissions, how should we allocate the remaining carbon budget? Which countries should grow, and which must abate?

I modify the Solow model to investigate these questions. I assume that a country's capital stock represents both its historical emissions and its wealth. Given each country's level of capital, I solve a simple two-period model for a Pareto-efficient allocation of the remaining carbon budget. I show that in the efficient allocation, countries with more initial capital (greater historical emissions) must choose lower savings rates and cleaner production technologies than countries with less capital.

This short paper introduces the model in Section 2. Section 3 presents the first-order conditions for the model, comparative statics, and solves a numerical example. Section 4 concludes.

2 Model

2.1 Previous literature

Chichilnisky, 1993 introduces a model with many different countries, all with potentially different income levels. She maximizes the sum of utilities, and shows that the income each country spends on abatement is proportion to that country's income level. The approach in this paper is inspired by Chichilnisky, 1993, but extends the model to consider historical emissions and growth.

Brock and Taylor, 2010 introduce a 'Green Solow Model'. I take a similar approach, but I represent emissions and abatement differently, and reduce the Solow model to two periods for simplicity.

2.2 Model setup

2.2.1 Unequal wealth, emissions, and levels of capital

I assume that countries are primarily differentiated by their level of capital. Wealthy countries have high levels of capital and poorer countries have very little. I assume further that countries with high cumulative emissions have large capital stocks, and those with large capital stocks have high cumulative emissions. To have high cumulative emissions, a country must have been wealthy enough to pollute significantly in the past (thus having high levels of capital already). To have a high capital stock, a country must have produced the capital in the past and polluted in the process.

Countries can vary by productivity and labour in my model. Productivity and labour influence a country's wealth, but this paper will not consider either in depth. Of course, cumulative emissions, present wealth, and levels of capital are not perfectly correlated with each other, but simply allowing capital to vary in the model is sufficient to illustrate the effects of inequality.

2.2.2 What countries choose

Given an initial distribution of capital among countries, I will solve the social planner's problem and derive a Pareto optimal allocation of the carbon budget.

There are two periods in the model. Period t can be interpreted as the current generation, and $t+1$ as the next. Countries invest in their capital stock and grow the economy in the first period for the next generation in the second period.

Each country i chooses a savings rate s_i for time t and a carbon intensity ϕ_i for both periods, and cares about output from both periods. The social planner solves for s_i and ϕ_i for every country to maximize the sum of utilities of consumption while staying within the remaining carbon budget.

2.3 Production

Each country i has the same continuous, concave, constant returns to scale aggregate production function $F(K_i, A_iL_i)$. I assume no population change and no change in A.

Then in period $t+1$, we have capital stock $K_{i,t+1}$. The capital stock reaches $K_{i,t+1}$ after $F(K_{i,t}, A_iL_i)$ is produced in period t, proportion $s_i \in [0,1]$ is saved (with total investment $s_iF(K_{i,t}, A_iL_i)$), and proportion $\delta \in [0,1]$ depreciates. Depreciation δ is common between countries.

$$K_{i,t+1} = K_{i,t} + s_iF(K_{i,t}, A_iL_i) - \delta K_{it}$$

Out of this, $s_iF(K_{i,t}, A_iL_i)$ is invested and $(1-s_i)F(K_{i,t}, A_iL_i)$ is available for consumption or abatement.

Production in period $t+1$ is given by

$$F(K_{i,t+1}, A_iL_i) = F\big((K_{i,t} + s_iF_i(K_{i,t}, A_iL_i) - \delta K_{i,t}), A_iL_i\big)$$

All of the output in $t+1$ is consumed.

2.4 Emissions and abatement

Let $\phi_i \geq 0$ be the carbon emissions per unit output for country i. Assume that intensity ϕ_i is the same for both periods. Total pollution by country i over both periods is then

$$e_i = \phi_i[F(K_{i,t+1}, A_iL_i) + F(K_{i,t}, A_iL_i)]$$

The higher ϕ_i is, the dirtier the production. Switching to cleaner technology has a fixed cost $C(\phi_i)$, where C is continuous, decreasing, and convex.

Assume that countries pay for abatement using their production in t, so that after saving and abating, $(1-s_i)F(K_{i,t}, A_iL_i) - C(\phi_i)$ is available for consumption by country i [1].

All countries face the same frontier for clean technology represented by C. Changing ϕ_i substitutes between clean and dirty technologies, but does not change the production capacity. Importantly, paying $C(\phi)$ sets the *intensity* of emissions, and not the level. For the same cost of $C(\phi_i)$, countries who produce more will have intensity ϕ_i apply to more units of output.

[1] Implicitly, transfers of money between countries might be efficient in this world, if developing countries cannot afford to both save and abate in period t, but wealthy countries could pay them to abate at low cost $C(\phi_i)$.

2.5 Preferences and the social planner's problem

Assume countries have Cobb-Douglas (log) utilities of consumption. Consumption today and tomorrow are treated as separate goods that cannot be freely substituted for each other. Utility is concave in consumption, so the first units of consumption (corresponding to basic needs) are valued extremely highly.

Period $t+1$ has discount factor $1-\rho$. The discount factor is the same across all countries.

$$U_i = \rho \log[F(K_{i,t+1}, A_i L_i)] + \log[(1-s_i)F_i(K_{i,t}, A_i L_i) - C(\phi_i)]$$

To simplify notation, call $\eta_{i,t}$ the consumption of country i at time t (as a mnemonic, countries eat η).

$$\begin{aligned} \eta_{i,t} &= (1-s_i)F_i(K_{i,t}, A_i L_i) - C(\phi_i) \quad &\text{at } t \\ \eta_{i,t+1} &= F(K_{i,t+1}, A_i L_i) \quad &\text{at } t+1 \end{aligned}$$

With countries indexed by i, WNO (World Nations Official, or Wise Neutral Observer) solves

$$\begin{aligned} \max_{s_i,\phi_i} \quad & \sum_i U_i = \sum_i \big[\, \rho \log(\eta_{i,t+1}) + \log(\eta_{i,t}) \,\big] \\ \text{subject to} \quad & \sum_i \phi_i [F(K_{i,t+1}, A_i L_i) + F(K_{i,t}, A_i L_i)] \leq E \\ & 0 \leq s_i \leq 1 \quad \forall i \\ & 0 \leq \phi_i \quad \forall i \end{aligned}$$

In other words, the social planner maximizes the sum of utilities while staying within the global carbon budget E. This arrives at a Pareto-efficient allocation [2].

2.6 Model summary

Symbol	*Independent variable*
$K_{i,t}$	Capital stock of country i at time t
A_i	Productivity of labour in country i (assumed constant over periods)
L_i	Labour in country i (assumed constant over periods)
	Choice variable
s_i	Savings rate of country i
ϕ_i	Carbon intensity of output in country i (assumed constant across periods)
	Other parameter, function, or quantity
$\eta_{i,t} = (1-s_i)F_i(K_{i,t}, A_i L_i) - C(\phi_i)$	Consumption of country i in time t ...
$\eta_{i,t+1} = F(K_{i,t+1}, A_i L_i)$	... and in time $t+1$
$F(K_{i,t}, A_i L_i)$	Production function of country i at time t
$K_{i,t+1} = K_{i,t} + s_i F_i(K_{i,t}) - \delta K_{it}$	Capital of country i at time $t+1$
δ	Rate of capital depreciation (constant over countries and over time)
e_i	Total emissions of country i across both periods
$C(\phi_i)$	Cost of country i achieving carbon intensity ϕ_i
$U_i(s_i, \phi_i)$	Utility of country i across both periods
ρ	Discount factor of future consumption (common across countries)
E	Remaining global carbon budget for all countries and all present/future periods

Table 1: Summary of variables, parameters, and functions

[2] Maximizing any weighted sum of utilities is sufficient for Pareto optimality. Here countries are weighted equally, but weighting the countries by population would have slightly different results.

3 Results

3.1 The Lagrangian

We maximize the Lagrangian.

Suppose there are n countries. Then let $\vec{s} = (s_1, ..., s_n) \in \mathbb{R}^n_+$ and $\vec{\phi} = (\phi_1, ..., \phi_n) \in \mathbb{R}^n_+$.

$$\begin{aligned}\mathscr{L}(\vec{s}, \vec{\phi}) = \sum_i U_i &= \sum_i \big[\rho \log[F(K_{i,t+1}, A_iL_i)] + \log\big[(1-s_i)F(K_{i,t}, A_iL_i) - C(\phi_i)\big]\big] \\ &- \lambda \sum_i (\phi_i[F\big(K_{i,t+1}, A_iL_i\big) + F(K_{i,t}, A_iL_i)] - E) \\ &- \sum_i \lambda_{s_i,0}(-s_i) - \sum_i \lambda_{s_i,1}(s_i - 1) - \sum_i \lambda_{\phi_i}(-\phi_i)\end{aligned}$$

Using the Karush-Kuhn-Tucker conditions, we add shadow prices λ for the remaining carbon budget, $\lambda_{s_i,0}$ for the $s_i \geq 0$ constraint, $\lambda_{s_i,1}$ for $s_i \leq 1$, and λ_{ϕ_i} for $\phi_i \geq 0$. These shadow prices are 0 if the constraint does not bind, and positive if the constraint binds.

Shadow price	*Corresponding constraint*
λ	Total emissions from both periods must not exceed remaining global carbon budget
$\lambda_{s_i,0}$	$s_i \geq 0$ for country i
$\lambda_{s_i,1}$	$s_i \leq 1$ for country i
λ_{ϕ_i}	$\phi_i \geq 0$ for country i

Table 2: Shadow prices

The global carbon budget constraint always binds, since for any savings rate, it is cheaper to abate less. No country emits less than what they have to. Since utilities are Cobb-Douglas, countries have positive consumption in each period, so $s_i < 1$ always.

3.2 First-order conditions and comparative statics

Take first-order conditions for each country i. (Observe, however, that while utilities are concave, the feasible set is not necessarily convex, so our necessary conditions for optimality are not guaranteed to be sufficient).

$$0 = \frac{\partial \mathscr{L}}{\partial \phi_i} \Leftrightarrow \frac{-C'(\phi_i)}{\eta_{i,t}} + \lambda_{\phi_i} = \lambda[F(K_{i,t+1}, A_iL_i) + F(K_{i,t}, A_iL_i)] \tag{1}$$

$$0 = \frac{\partial \mathscr{L}}{\partial \lambda} \Leftrightarrow \sum_i \phi_i[F(K_{i,t+1}, A_iL_i) + F(K_{i,t}, A_iL_i)] = E \tag{2}$$

$$\begin{aligned}0 = \frac{\partial \mathscr{L}}{\partial s_i} \Leftrightarrow \rho\frac{F'(K_{i,t+1}, A_iL_i)F(K_{i,t}, A_iL_i)}{\eta_{i,t+1}} &+ \lambda_{s_i,0} \\ = \frac{F(K_{i,t}, A_iL_i)}{\eta_{i,t}} &+ \lambda\phi_i F'(K_{i,t+1}, A_iL_i)F(K_{i,t}, A_iL_i)\end{aligned} \tag{3}$$

Condition (1) equates the loss in marginal utility due to abatement with the shadow price of the constraint. Another way to say this is that the left-hand side is the marginal cost (in utility) of decreasing ϕ_i, and the right-hand side is the marginal benefit of decreasing ϕ_i in terms of emissions. (If $\phi_i \geq 0$ binds, then marginal cost $-C'(\phi_i)/\eta_{i,t}$ of decreasing ϕ_i is less than the marginal benefit of cleaner production by a wedge $\lambda_{\phi_i} > 0$).

What happens if $K_{i,t}$ is higher? On the left-hand side, the marginal cost of decreasing ϕ_i is lower, since $\eta_{i,t}$ is higher. On the right-hand side, $F(K_{i,t+1}, A_iL_i) + F(K_{i,t}, A_iL_i)$ increases, so the marginal benefit of reducing ϕ_i increases. Intuitively, richer countries choose lower ϕ_i, both because it is easier for them to afford the abatement cost, and also because each unit reduction of ϕ_i (the emissions *intensity*) reduces their emissions *level* by more.

Condition (2) says the total emissions from every country must not exceed the remaining carbon budget.

Condition (3) equates the marginal benefit with the marginal cost of saving.

The left-hand side is the marginal benefit from saving in t and consuming more in $t+1$. The denominator divides by consumption in $t+1$ since the marginal utility of consumption diminishing. The numerator is the benefit of raising the savings rate (increasing $s_i F(K_{i,t}, A_i L_i)$ and hence $F(K_{i,t+1}, A_i L_i)$).

Notice that on the left-hand side, the marginal utility in $t+1$ of saving in t diminishes with $K_{i,t}$. Since $\frac{\partial^2 F}{\partial K^2} < 0$, $F'(K_{i,t+1}, A_i L_i)$ decreases in $K_{i,t+1}$ and hence in $K_{i,t}$. That is, the marginal product of capital is lower for richer countries (who have higher $K_{i,t}$). When $K_{i,t}$ increases, $\eta_{i,t+1}$ also increases (since $K_{i,t+1}$ increases) and hence the left-hand side decreases again. That is, the marginal utility of consumption diminishes, since wealthier countries already have high consumption.

The right hand side totals the marginal costs of saving. The first term is the effect of reducing consumption now to invest in the future. The second term accounts for the shadow price of the constraint. It penalizes the extra emissions due to saving and growing.

If $s_i \geq 0$ binds, then the marginal cost of saving is greater than the marginal benefit by wedge $\lambda_{s_i,0} > 0$.

There are also two further conditions which apply only if s_i and/or ϕ_i have a corner solution.

$$0 = \frac{\partial \mathscr{L}}{\partial \lambda_{\phi_i}} \Leftrightarrow \phi_i = 0 \tag{4}$$

$$0 = \frac{\partial \mathscr{L}}{\partial \lambda_{s_i,0}} \Leftrightarrow s_i = 0 \tag{5}$$

In general, it is not possible to derive a closed-form solution for s_i and ϕ_i. In each equation, the choice for ϕ_i affects the choice for s_i and vice versa, and conditions (1) and (3) and complicated and nonlinear [3]. We cannot isolate s_i or ϕ_i. The best we can do is to apply a numerical root-finding algorithm to solve the above Karush-Kuhn-Tucker conditions.

3.3 Numerical example

Consider a world with only Country 1 and Country 2, and a remaining global carbon budget of $E = 6000$ (the numbers and units will be more or less arbitrary in this example).

They share a Cobb-Douglas aggregate production function $F(K, AL) = AL^{0.5}K^{0.5}$, where $A_1 L_1^{0.5} = A_2 L_2^{0.5} = 20$. Both countries face the same cost of abatement $C(\phi) = e^{-0.2\phi}$. I choose this functional form instead of a simpler one (such as $C(\phi) = \phi^{-1}$) to allow for the case that $\phi = 0$, representing net zero. Both countries have their capital depreciate at $\delta = 0.1$ and discount factor $\rho = 1$.

I hold the total stock of capital constant at $K_{1,t} + K_{2,t} = 100$ and solve the model for different scenarios where $K_{1,t}$ ranges between 10 and 51. For this case, it is easier to directly solve the constrained optimization problem numerically in Python than to solve the KKT conditions [4].

Figure 1 plots both countries' choices of s_i and ϕ_i. The larger the initial capital stock for each country, the lower the savings rate and carbon intensity. Since country 2 (almost always) begins wealthier than country 1, it also (almost always) grows less and has a lower carbon intensity than country 2.

Figure 2 plots levels of consumption, growth and emissions. Both countries grow, but the wealthier country 2 grows noticeably less when the world is more unequal. Country 2 chooses emissions intensities which are so low that despite its larger economic output, it actually emits less than country 1 over both periods.

4 Discussion

Two major assumptions may affect the qualitative results of this model. First, every country can choose the same emissions *intensity* ϕ_i at cost $C(\phi_i)$. Scaling the cost by the size of each economy would make richer countries abate less. Second, this analysis has mostly ignored $A_i L_i$, but since $A_i L_i$ influences the marginal product of capital, different $A_i L_i$ across countries might significantly change each country's level of savings.

[3] For instance, in (3), s_i appears on both the left-hand side as part of $K_{i,t+1}$ and the right-hand-side as part of $\eta_{i,t}$ and $K_{i,t+1}$. Furthermore, both sides of the equation are nonlinear in s_i and the right-hand-side also depends on the choice of ϕ_i.

[4] Code is available at the author's GitHub (link).

With those caveats, this paper derives a Pareto-efficient allocation of the global carbon budget between highly unequal countries. Future work could calibrate the model on actual emissions and macroeconomic data and derive the efficient allocation for multiple countries (for instance, the UK, Pakistan, and the rest of the world).

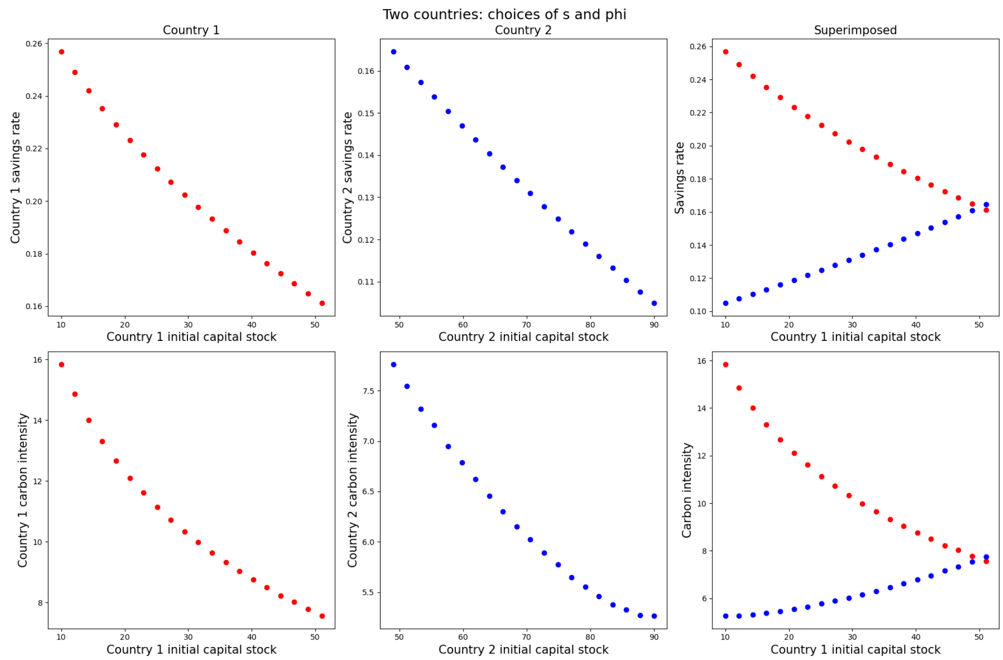

Figure 1: Different scenarios where $K_{1,t} + K_{2,t} = 100$ and $K_{1,t}$ ranges between 10 and 51. In the rightmost column, every point on the x-axis is a different scenario.

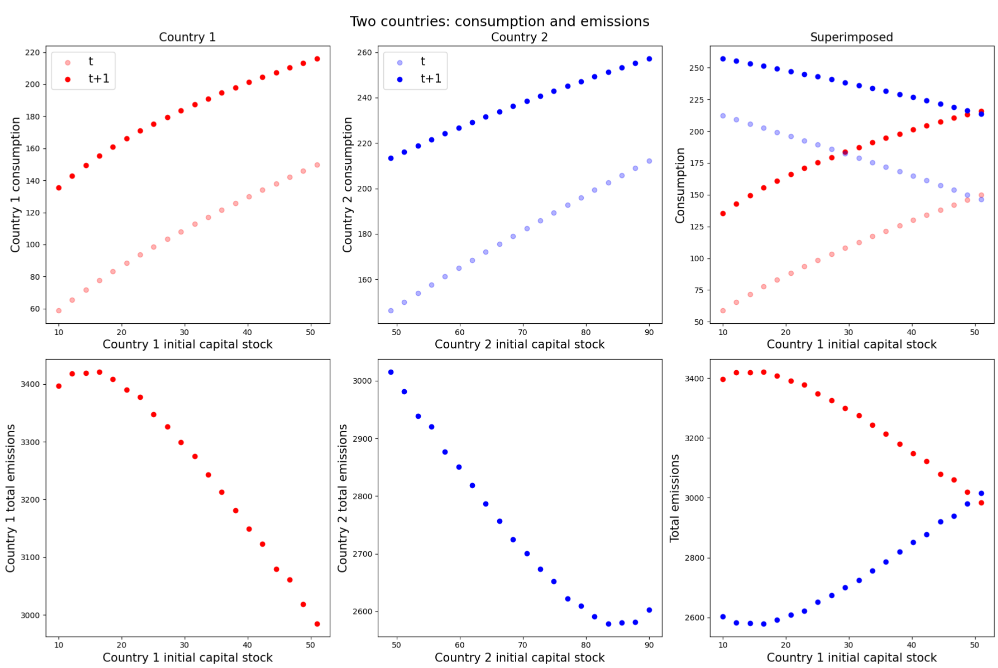

Figure 2: The top row gives $(1 - s_i)F(K_{i,t}, A_iL_i)$ at time t (faint) and $F(K_{i,t+1}, A_iL_i)$ at $t + 1$ (solid). The bottom row gives each country's total emissions for both periods.

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