输入层　　　　　隐藏层1　　　　　输出层

x_1 $\underset{w_1=0.15}{\xrightarrow{\hspace{2cm}}}$ z_{h1} — a_{h1} $\underset{w_5=0.40}{\xrightarrow{\hspace{2cm}}}$ z_{o1} — a_{o1}
0.05

$w_2=0.2$

$w_3=0.25$　　　　　　　　　$w_6=0.45$　　　　　　　　　　　　　　　E_{o1}

x_2 $\underset{w_4=0.30}{\xrightarrow{\hspace{2cm}}}$ z_{h2} — a_{h2} $w_7=0.50$ z_{o2} — a_{o2}　$\Rightarrow$ $\begin{bmatrix} y_1 \\ 0.01 \\ y_2 \end{bmatrix}$ $-$ $\rangle E_t$ 反向传
0.10　　　　　　　　　　　　　　　　　　　　$w_8=0.55$　　　　　　　　　　　　　0.99　　　E_{o2}

$b_1=0.35$ $\uparrow$　　　　　　　$b_2=0.60$ $\uparrow$

正向传播

$$\begin{cases} z_{h1} = w_1 x_1 + w_2 x_2 + b_1 = 0.3775 \\ z_{h2} = w_3 x_1 + w_4 x_2 + b_1 = 0.3925 \end{cases}$$

激活函数 sigmoid: $S(x) = \dfrac{1}{1+e^{-x}}$　　$a_{h1} = 0.5933$

$$S'(x) = S(x) \cdot [1-S(x)]$$　　$a_{h2} = 0.5969$

$$\begin{cases} z_{o1} = w_5 a_{h1} + w_6 a_{h2} + b_2 = 1.1059 \\ z_{o2} = w_7 a_{h1} + w_8 a_{h2} + b_2 = 1.2249 \end{cases}$$

$$a_{o1} = \frac{1}{1+e^{-1.1059}} = 0.7514$$

$$a_{o2} = \frac{1}{1+e^{-1.2249}} = 0.7729$$

$$\begin{cases} E_{o1} = \frac{1}{2}(a_{o1} - y_1)^2 = 0.2748 \\ E_{o2} = \frac{1}{2}(a_{o2} - y_2)^2 = 0.0236 \end{cases}$$

$$E_t = \sum E_{oi} = 0.2984$$

MSE: $\quad E = \frac{1}{2} \sum (a_{oi} - y_i)^2$

反向传播1: $w_5 \sim w_8$ 更新

梯度 $\dfrac{\partial E_t}{\partial w_5} = \dfrac{\partial E_t}{\partial E_{o1}} \cdot \dfrac{\partial E_{o1}}{\partial a_{o1}} \cdot \dfrac{\partial a_{o1}}{\partial z_{o1}} \cdot \dfrac{\partial z_{o1}}{\partial w_5}$

$$= \frac{\partial \left[\frac{1}{2} \sum (a_{oi} - y_i)^2 \right]}{\partial a_{o1}} \cdot \frac{\partial \left[\frac{1}{1+e^{-z_{o1}}} \right]}{\partial z_{o1}} \cdot \frac{\partial \left[w_5 a_{h1} + w_6 a_{h2} + b_2 \right]}{\partial w_5}$$

$$= (a_{o1} - y_1) \cdot \frac{+e^{-z_{o1}}}{(1+e^{-z_{o1}})^2} \cdot a_{h1}$$

$$= (0.7514 - 0.01) \cdot \frac{+e^{-1.1059}}{(1+e^{-1.1059})^2} \cdot 0.5933 = 0.0822$$

$$\frac{\partial E_t}{\partial w_6} = \frac{\partial E_t}{\partial a_{o1}} \cdot \frac{\partial a_{o1}}{\partial z_{o1}} \cdot \frac{\partial z_{o1}}{\partial w_6}$$

$$= (a_{o1} - y_1) \cdot \frac{e^{-1.1059}}{(1+e^{-1.1059})^2} \cdot a_{h2} = 0.0827$$

$$\frac{\partial E_t}{\partial w_7} = \frac{\partial E_t}{\partial a_{o2}} \cdot \frac{\partial a_{o2}}{\partial z_{o2}} \cdot \frac{\partial z_{o2}}{\partial w_7}$$

$$= (a_{o2} - y_2) \cdot \frac{e^{-1.2249}}{(1+e^{-1.2249})^2} \cdot a_{h1} = -0.0226$$

$$\frac{\partial E_t}{\partial w_8} = (a_{o2} - y_2) \cdot \frac{e^{-1.2249}}{(1+e^{-1.2249})^2} \cdot a_{h2} = -0.0227$$

①

权重更新 = $lr = 0.5$

$w_5{}^+ = w_5 - lr \cdot \dfrac{\partial E_t}{\partial w_5} = 0.40 - 0.5 \times 0.0822 = 0.3589$

$w_6{}^+ = 0.45 - 0.5 \times 0.0827 = 0.4087$

$w_7{}^+ = 0.50 - 0.5 \times (-0.0226) = 0.5113$

$w_8{}^+ = 0.55 - 0.5 \times (-0.0227) = 0.5614$

反向传播 2 : $w_1 \sim w_6$ 更新

$$\dfrac{\partial E_t}{\partial w_1} = \dfrac{\partial E_t}{\partial a_{o1}} \cdot \dfrac{\partial a_{o1}}{\partial z_{o1}} \cdot \dfrac{\partial z_{o1}}{\partial a_{h1}} \cdot \dfrac{\partial a_{h1}}{\partial z_{h1}} \cdot \dfrac{\partial z_{h1}}{\partial w_1}$$

$$+ \dfrac{\partial E_t}{\partial a_{o2}} \cdot \dfrac{\partial a_{o2}}{\partial z_{o2}} \cdot \dfrac{\partial z_{o2}}{\partial a_{h1}} \cdot \dfrac{\partial a_{h1}}{\partial z_{h1}} \cdot \dfrac{\partial z_{h1}}{\partial w_1}$$

$$= (a_{o1} - y_1) \cdot \dfrac{1}{1 + e^{-z_{o1}}} \cdot \left(1 - \dfrac{1}{1 + e^{-z_{o1}}}\right) \cdot w_5 \cdot \dfrac{e^{-z_{h1}}}{(1 + e^{-z_{h1}})^2} \cdot x_1$$

$$+ (a_{o2} - y_2) \cdot \dfrac{1}{1 + e^{-z_{o2}}} \cdot \left(1 - \dfrac{1}{1 + e^{-z_{o2}}}\right) \cdot w_7 \cdot \dfrac{e^{-z_{h1}}}{(1 + e^{-z_{h1}})^2} \cdot x_1$$

$$= (0.7514 - 0.01) \times \dfrac{e^{-1.1059}}{(1 + e^{-1.1059})^2} \times 0.4 \times \dfrac{e^{-0.3775}}{(1 + e^{-0.3775})^2} \times 0.05$$

$$+ (0.7729 - 0.99) \times \dfrac{e^{-1.2249}}{(1 + e^{-1.2249})^2} \times 0.5 \times \dfrac{e^{-0.3775}}{(1 + e^{-0.3775})^2} \times 0.05$$

$$= 0.000668 - \underset{0.000229}{\cancel{0.000453 = 0.000415}} = 0.000439$$

$$\dfrac{\partial E_t}{\partial w_2} = (a_{o1} - y_1) \times \dfrac{e^{-z_{o1}}}{(1 + e^{-z_{o1}})^2} \times w_5 \times \dfrac{e^{-z_{h1}}}{(1 + e^{-z_{h1}})^2} \times x_2$$

$$+ (a_{o2} - y_2) \times \dfrac{e^{-z_{o2}}}{(1 + e^{-z_{o2}})^2} \times w_7 \times \dfrac{e^{-z_{h1}}}{(1 + e^{-z_{h1}})^2} \times x_2$$

$$= (0.7514 - 0.01) \times \dfrac{e^{-1.1059}}{(1 + e^{-1.1059})^2} \times 0.4 \times \dfrac{e^{-0.3775}}{(1 + e^{-0.3775})^2} \times 0.10$$

$$+ (0.7729 - 0.99) \times \dfrac{e^{-1.2249}}{(1 + e^{-1.2249})^2} \times 0.5 \times \dfrac{e^{-0.3775}}{(1 + e^{-0.3775})^2} \times 0.10$$

$$= 0.001337 - 0.000460 = 0.000877$$

$$\dfrac{\partial E_t}{\partial w_3} = (0.7514 - 0.01) \times \dfrac{e^{-1.1059}}{(1 + e^{-1.1059})^2} \times 0.45 \times \dfrac{e^{-0.3925}}{(1 + e^{-0.3925})^2} \times 0.05$$

$$+ (0.7729 - 0.99) \times \dfrac{e^{-1.2249}}{(1 + e^{-1.2249})^2} \times 0.55 \times \dfrac{e^{-0.3925}}{(1 + e^{-0.3925})^2} \times 0.05$$

$$= 0.000750 - 0.000252 = 0.000498$$

$$\dfrac{\partial E_t}{\partial w_4} = (0.7514 - 0.01) \times \dfrac{e^{-1.1059}}{(1 + e^{-1.1059})^2} \times 0.45 \times \dfrac{e^{-0.3925}}{(1 + e^{-0.3925})^2} \times 0.10$$

$$(0.7729 - 0.99) \times \dfrac{e^{-1.2249}}{(1 + e^{-1.2249})^2} \times 0.55 \times \dfrac{e^{-0.3925}}{(1 + e^{-0.3925})^2} \times 0.10$$

$$= 0.001500 - 0.000504 = 0.000996$$

权重更新 $lr = 0.5$

$w_1^+ = 0.15 - 0.5 \times 0.000439 = 0.14978$

$w_2^+ = 0.20 - 0.5 \times 0.000877 = 0.19956$

$w_3^+ = 0.25 - 0.5 \times 0.000698 = 0.24975$

$w_4^+ = 0.30 - 0.5 \times 0.000996 = 0.29950$

所有权重更新完毕
第一轮完成

迭代正向：

$z_{h_1}^+ = w_1^+ x_1 + w_2^+ x_2 + b_1 = 0.37745$

$z_{h_2}^+ = w_3^+ x_1 + w_4^+ x_2 + b_1 = 0.39244$

$a_{h_1}^+ = \dfrac{1}{1 + e^{-0.37745}} = 0.59326$

$a_{h_2}^+ = \dfrac{1}{1 + e^{-0.39244}} = 0.59687$

$z_{o_1}^+ = w_5^+ a_{h_1} + w_6^+ a_{h_2} + b_2 = 0.3589 \times 0.59326 + 0.4087 \times 0.59687 + 0.6 = 1.05686$

$z_{o_2}^+ = 0.5113 \times 0.59326 + 0.5614 \times 0.59687 + 0.60 = 1.23842$

$a_{o_1}^+ = \dfrac{1}{1 + e^{-1.0569}} = 0.7421$

$a_{o_2}^+ = \dfrac{1}{1 + e^{-1.2384}} = 0.7753$

$\therefore E_t^+ = \dfrac{1}{2}\left[(0.7421 - 0.01)^2 + (0.7753 - 0.99)^2\right] = 0.2910$

反向传播 $w_5 \sim w_8$ 更新 $\Rightarrow$ $w_1 \sim w_4$ 更新 $\Rightarrow$ 求新 E_t 循环迭代

③