# APS 1022

# Financial Engineering II

# Course Project Part 1

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# Table of Contents

# Introduction

The project is required to use Mean-Variance Optimization (MVO) strategies to optimize a portfolio that consists of 30 different assets which consists of Dow Jones Industrial Average. The report will discuss the differences of portfolio construction under two circumstances: shortselling is allowed and shortselling is not allowed.

The historical data implemented in the model are daily adjusted closing prices of each assets between Jan 2015 and Dec 2017. The number of trading days is 755 days.

# Formulation

This project employs the following formulae to calculate the arithmetic return, geometric return and covariance. I used geometric returns for MVO, the arithmetic returns are need for the covariance computations.

Return of each asset at specific time i (using adjusted close):

$$r_i = \frac{(price_{i+1} - price_i)}{price_i}$$

Arithmetic average of asset i (T =  in this case):

$$\overline{r_i} = \frac{1}{T}\sum_{t=1}^{T} r_{it}$$

Geometric expected return of asset I (T = 754 in this case):

$$\mu_i = \left(\prod_{t=1}^{T}(1 + r_{it})\right)^{\frac{1}{T}} - 1$$

Covariance between two asset i and j (T = 754 in this case):

$$\sigma_{ij} = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \overline{r_i})(r_{jt} - \overline{r_j})$$

After knowing covariance and geometric expected return, MATLAB is used to find the goal R and optimal weights $x_i$, $x_j$, using the formula below:

$$\min_x \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} x_i x_j$$
$$s.t. \sum_{i=1}^{n} \mu_i x_i \geq R$$
$$\sum_{i=1}^{n} x_i = 1$$
$$(x \geq 0)$$

Correspondingly, the following optimization formula is typed in MATLAB and quadprog is used to solve it. Specifically, in this case Q is the covariance of two assets.

$$\min_x \frac{1}{2} x^T Q x$$
$$s.t. \mu^T x \geq R$$
$$e^T x = 1$$
$$(x \geq 0)$$

# Results

There are six portfolios which lie in two categories. The details of each portfolio are summarized below.

- **Shorting not allowed**

**Portfolio 1**: Shorting not allowed, with target return $R =$ median of $\mu_i's$
**Portfolio 2**: Change original $\mu_i's$ to a uniformly random number in the interval [0.95$\mu_i$, 1.05$\mu_i$], no shorting allowed, with target return $R =$ median of original $\mu_i's$
**Portfolio 3**: Average portfolio of portfolio 1, 2 and additional three simulations using strategy for portfolio 2

- **Shorting allowed**

**Portfolio 4**: Shorting allowed, with target return $R =$ median of $\mu_i's$
**Portfolio 5**: Change original $\mu_i's$ to a uniformly random number in the interval [0.95$\mu_i$, 1.05$\mu_i$], with shorting allowed, with target return $R =$ median of original $\mu_i's$
**Portfolio 6**: Average portfolio of portfolio 4, 5 and additional three simulations using strategy for portfolio 5

| | Portfolio 1 | Portfolio 2 | Portfolio 3 | Portfolio 4 | Portfolio 5 | Portfolio 6 |
|---|---|---|---|---|---|---|
| *AAPL* | 0.66% | 0.63% | 0.65% | 3.36% | 3.27% | 3.35% |
| *AXP* | 6.08% | 6.08% | 6.10% | 9.05% | 9.08% | 9.08% |
| *BA* | 0.04% | 0.04% | 0.03% | -0.55% | -0.27% | -0.50% |
| *CAT* | 0.37% | 0.37% | 0.37% | 3.55% | 3.53% | 3.58% |
| *CSCO* | 0.00% | 0.00% | 0.00% | -3.06% | -3.19% | -3.12% |
| *CVX* | 0.01% | 0.01% | 0.01% | -3.62% | -3.56% | -3.63% |
| *DIS* | 4.23% | 4.17% | 4.22% | 6.64% | 6.58% | 6.63% |
| *DWDP* | 0.02% | 0.02% | 0.02% | -0.75% | -0.79% | -0.80% |
| *GE* | 0.00% | 0.00% | 0.00% | -4.30% | -4.36% | -4.26% |
| *GS* | 0.00% | 0.00% | 0.00% | -3.95% | -4.04% | -3.98% |
| *HD* | 1.33% | 1.71% | 1.46% | 2.89% | 3.25% | 3.02% |
| *IBM* | 0.04% | 0.03% | 0.03% | 1.01% | 0.96% | 1.00% |
| *INTC* | 0.01% | 0.01% | 0.01% | -1.24% | -1.18% | -1.26% |
| *JNJ* | 14.66% | 14.53% | 14.68% | 15.93% | 15.78% | 15.93% |
| *JPM* | 0.00% | 0.00% | 0.00% | -4.17% | -4.13% | -4.18% |
| *KO* | 25.18% | 25.00% | 25.12% | 23.95% | 23.75% | 23.87% |
| *MCD* | 12.27% | 12.53% | 12.27% | 14.39% | 14.59% | 14.39% |
| *MMM* | 3.06% | 3.20% | 3.12% | 4.49% | 4.59% | 4.54% |
| *MRK* | 0.00% | 0.00% | 0.00% | -5.40% | -5.37% | -5.37% |

| | | | | | | |
|---|---|---|---|---|---|---|
| *MSFT* | 0.00% | 0.00% | 0.00% | -5.32% | -5.04% | -5.20% |
| *NKE* | 0.14% | 0.12% | 0.13% | 0.45% | 0.46% | 0.47% |
| *PFE* | 5.75% | 5.76% | 5.77% | 9.32% | 9.36% | 9.32% |
| *PG* | 6.98% | 6.92% | 7.01% | 8.52% | 8.42% | 8.50% |
| *TRV* | 0.12% | 0.10% | 0.11% | 1.90% | 1.94% | 1.94% |
| *UNH* | 4.03% | 3.93% | 3.91% | 5.01% | 4.86% | 4.90% |
| *UTX* | 1.05% | 0.97% | 1.04% | 3.87% | 3.74% | 3.84% |
| *V* | 0.01% | 0.01% | 0.01% | -1.04% | -1.21% | -1.13% |
| *VZ* | 7.97% | 7.89% | 7.93% | 9.31% | 9.22% | 9.25% |
| *WMT* | 5.86% | 5.87% | 5.88% | 5.14% | 5.18% | 5.16% |
| *XOM* | 0.11% | 0.09% | 0.10% | 4.62% | 4.60% | 4.66% |

*Table 1 Weights of Each Assets in Different Portfolios*

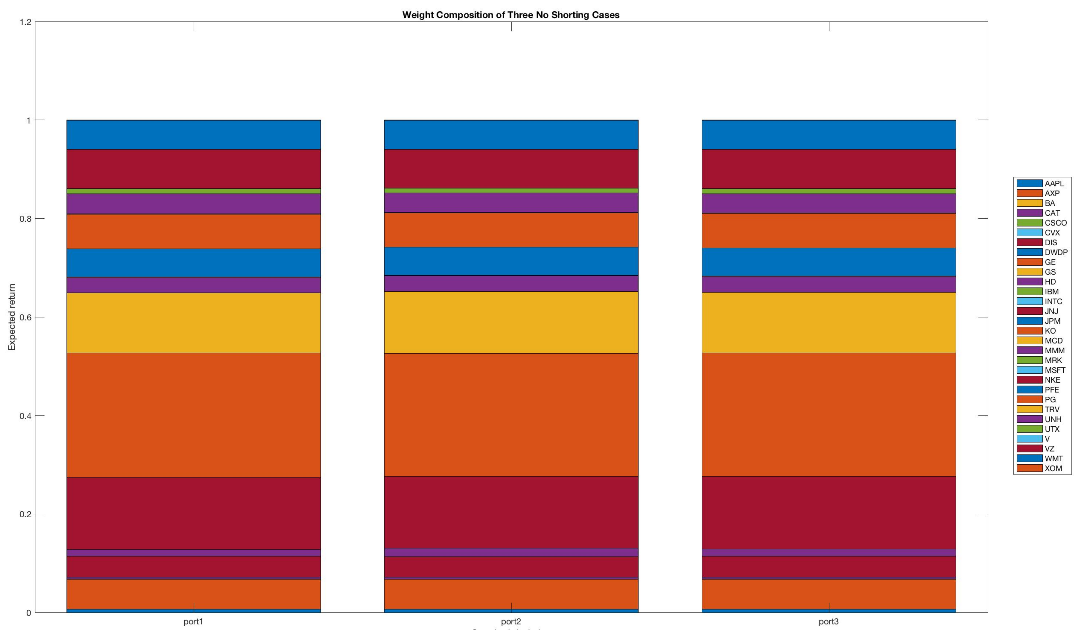


*Figure 1 Weight Composition of No Shorting Cases*

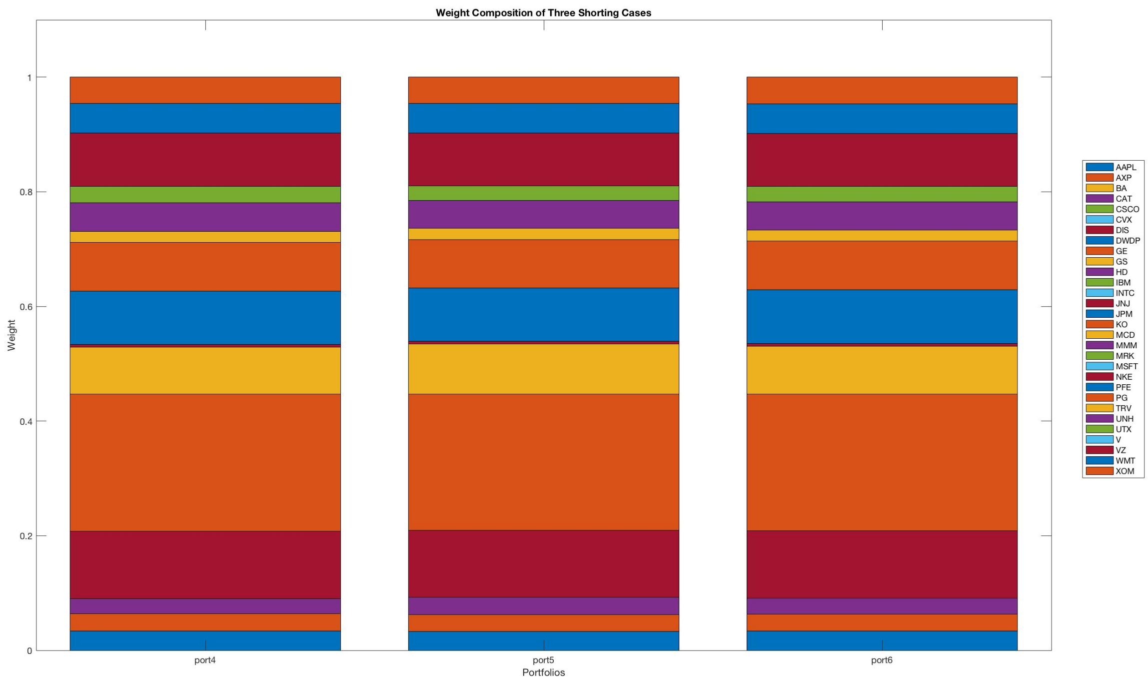


*Figure 2 Weight Composition of Shorting Cases*

| | Portfolio 1 | Portfolio 2 | Portfolio 3 | Portfolio 4 | Portfolio 5 | Portfolio 6 |
|---|---|---|---|---|---|---|
| *Expect Return* | 4.2742E-04 | 4.2742E-04 | 4.2742E-04 | 4.2815E-04 | 4.2809E-04 | 4.2822E-04 |
| *Variance* | 4.0205E-05 | 4.0239E-05 | 4.0202E-05 | 3.8048E-05 | 3.8109E-05 | 3.8050E-05 |
| *Sta Dev* | 6.3408E-03 | 6.3434E-03 | 6.3405E-03 | 6.1683E-03 | 6.1732E-03 | 6.1685E-03 |

*Table 2 Key Measures of Each Portfolio*

## Comparison and Discussion

The target return R is set to be 4.2740E-04 (median of $\mu_i{}'s$). For shorting not allowed cases, the expected returns of portfolio 1,2,3 remain the same while the variance varies in a subtle range. Since portfolio 1 is a point on the efficient frontier, portfolio 2 is under and portfolio 3 is above the efficient frontier. The change of variance is because of the change of $\mu_i's$. In portfolio 2, majority of the uniformly random generated $\mu_i{}'s$ appear within the range of [$0.95\mu_i$, $\mu_i$], which are less than the original $\mu_i{}'s$. Thus, in order to achieve the same expect return as portfolio 1, more weights will be allocated into more risky assets, resulting larger variance. Similarly, if the uniformly random generated $\mu_i{}'s$ are larger than the original $\mu_i{}'s$, the variance of the portfolio will be lower. The weight allocations of first three portfolios follow the same pattern. Weights concentrate on The Coca-Cola Company, Johnson & Johnson, and McDonald's Corporation. The change of weight allocation is subtle between each other.

Regarding shorting allowed cases, the expected returns are higher than the shorting not allowed cases. Meanwhile, the variances are lower than the other case because shorting diversifies the asset choices. The changes of expected return and variance among portfolio 4,5,6 are still owing to the changes of uniformly random generated $\mu_i{}'s$. Weights still concentrate on The Coca-Cola Company, Johnson & Johnson, and McDonald's Corporation. However, it is noticeable that for those assets with 0% weight in no shorting allowed case become negative (shorting) in this case.

## Appendix A – A List of Dow 30 Companies

The following are the companies included in the index, as of January 4, 2018:

— 3M (MMM)

— American Express (AXP)

— Apple (AAPL)

— Boeing (BA)

— Caterpillar (CAT)

— Chevron (CVX)

— Cisco (CSCO)

— Coca-Cola (KO)

— Disney (DIS)

— DowDuPont Inc (DWDP)

— Exxon Mobil (XOM)

— General Electric (GE)

— Goldman Sachs (GS)

— Home Depot (HD)

— IBM (IBM)

— Intel (INTC)

— Johnson & Johnson (JNJ)

— JPMorgan Chase (JPM)

— McDonald's (MCD)

— Merck (MRK)

— Microsoft (MSFT)

— Nike (NKE)

— Pfizer (PFE)

— Procter & Gamble (PG)

— Travelers Companies Inc (TRV)

— United Technologies (UTX)

— United Health (UNH)

— Verizon (VZ)

— Visa (V)

— Walmart (WMT)

## Appendix B – MATLAB Subroutine

```
clc;
clear all;
format long

%% a)
% Input 30 dow jones stock files
AAPL= 'AAPL.csv';
AXP = 'AXP.csv';
BA = 'BA.csv';
CAT = 'CAT.csv';
CSCO = 'CSCO.csv';
CVX = 'CVX.csv';
DIS = 'DIS.csv';
DWDP = 'DWDP.csv';
GE = 'GE.csv';
GS = 'GS.csv';
HD = 'HD.csv';
IBM = 'IBM.csv';
INTC = 'INTC.csv';
JNJ = 'JNJ.csv';
JPM = 'JPM.csv';
KO = 'KO.csv';
MCD = 'MCD.csv';
MMM = 'MMM.csv';
MRK = 'MRK.csv';
MSFT = 'MSFT.csv';
NKE = 'NKE.csv';
PFE = 'PFE.csv';
PG = 'PG.csv';
TRV = 'TRV.csv';
UNH = 'UNH.csv';
UTX = 'UTX.csv';
V = 'V.csv';
VZ = 'VZ.csv';
WMT = 'WMT.csv';
XOM = 'XOM.csv';


file_name = {AAPL AXP BA CAT CSCO CVX DIS DWDP GE GS HD IBM INTC JNJ JPM
KO...
    MCD MMM MRK MSFT NKE PFE PG TRV UNH UTX V VZ WMT XOM};


N = length(file_name);

% Read daily prices
for i = 1:N
% Read daily prices
    if(exist(file_name{1,i},'file'))
      fprintf('\nReading Monthly prices datafile - %s\n', file_name{1,i});
      fid = fopen(file_name{1,i});
      vheader{i} = textscan(fid, '%[^,]%*[^\n]');
      dates{i} = vheader{1}(1:end); % separate d into different cells
      fclose(fid);
```

```
      data_prices{i} = dlmread(file_name{1,i}, ',',[1,5,755,5]);
    else
      error('Daily prices datafile does not exist')
    end
    consolidated_price(:,i) = data_prices{1,i};
end

% 36 months in total, and 35 rates of return
T = length(dates{1,1}{1,1}) - 2;

for i = 1:N
    geo_return = 1;
     for j = 1:T
        returns(j,i) = data_prices{1,i}(j+1) / data_prices{1,i}(j) - 1;
        geo_return = geo_return * (1+returns(j,i));
     end
     arithmetic_average_return(i) = mean(returns(:,i));
     geometic_expected_return(i) = geo_return^(1/T) - 1;
     sta_dev(i) = std(returns(:,i));
end

Q = cov(returns)*(T-1)/T;
mu_i = geometic_expected_return;
R = median(geometic_expected_return);

%% b) MVO -- portfolio 1

weight_wo_1 = zeros(N,1);
weight_1 = zeros(N,1);

c = zeros(1,N); %[0 0 0]
A = - mu_i;
b = - R;
Aeq = ones(1,N); %[1 1 1]
beq = 1;
ub = ones(N,1) * inf; %[inf; inf; inf];
lb_without = zeros(N,1); % [0; 0; 0]% without short selling
lb = -ones(N,1) * inf;

[x_wo, fval_wo, exitflag_wo] = quadprog(2*Q, c, A, b, Aeq, beq, lb_without,
ub);
if exitflag_wo == 1
    weight_wo_1(:,1) = x_wo;
    value_wo_1 = fval_wo;
else
    weight_wo_1(:,1) = zeros(N,1);
    value_wo_1 = 0;
end

[x, fval, exitflag] = quadprog(2*Q, c, A, b, Aeq, beq, lb, ub);
if exitflag == 1
    weight_1(:,1) = x;
    value_1= fval;
else
    weight_1(:,1) = zeros(N,1);
    value_1 = 0;
end
```

```
mu_wo_1 = mu_i * x_wo;
var_wo_1 = x_wo' * Q * x_wo;
sta_wo_1 = sqrt(var_wo_1);

mu_1 = mu_i * x;
var_1 = x' * Q * x;
sta_1 = sqrt(var_1);
%% c)

% generate ramdom value
a = 0.95;
b = 1.05;
rdm = (b-a).*rand(30,1) + a;

rdm_mu_i = mu_i .* rdm';

weight_wo_2 = zeros(N,1);

c = zeros(1,N); %[0 0 0]
A = - rdm_mu_i;
b = - R;
Aeq = ones(1,N); %[1 1 1]
beq = 1;
ub = ones(N,1) * inf; %[inf; inf; inf];
lb_without = zeros(N,1); % [0; 0; 0]% without short selling

[x_wo, fval_wo, exitflag_wo] = quadprog(2*Q, c, A, b, Aeq, beq, lb_without,
ub);
if exitflag_wo == 1
    weight_wo_2(:,1) = x_wo;
    value_wo_2 = fval_wo;
else
    weight_wo_2(:,1) = zeros(N,1);
    value_wo_2 = 0;
end

[x, fval, exitflag] = quadprog(2*Q, c, A, b, Aeq, beq, lb, ub);
if exitflag == 1
    weight_2(:,1) = x;
    value_2 = fval;
else
    weight_2(:,1) = zeros(N,1);
    value_2 = 0;
end

mu_wo_2 = rdm_mu_i * x_wo;
var_wo_2 = x_wo' * Q * x_wo;
sta_wo_2 = sqrt(var_wo_2);

mu_2 = rdm_mu_i * x;
var_2 = x' * Q * x;
sta_2 = sqrt(var_2);
%% d)

weight_wo_3 = zeros(N,3);
```

```
mu_wo_3 = zeros(3,1);
var_wo_3 = zeros(3,1);
sta_wo_3 = zeros(3,1);

weight_3 = zeros(N,3);
mu_3 = zeros(3,1);
var_3 = zeros(3,1);
sta_3 = zeros(3,1);

for i = 1: 3

    a = 0.95;
    b = 1.05;
    rdm = (b-a).*rand(30,1) + a;
    rdm_mu_i = mu_i .* rdm';

    c = zeros(1,N); %[0 0 0]
    A = - rdm_mu_i;
    b = - R;
    Aeq = ones(1,N); %[1 1 1]
    beq = 1;
    ub = ones(N,1) * inf; %[inf; inf; inf];
    lb_without = zeros(N,1); % [0; 0; 0]% without short selling

    [x_wo, fval_wo, exitflag_wo] = quadprog(2*Q, c, A, b, Aeq, beq,
lb_without, ub);
    if exitflag_wo == 1
        weight_wo_3(:,i) = x_wo;
        value_wo_3 (i) = fval_wo;
    else
        weight_wo_3(:,i) = zeros(N,1);
        value_wo_3 (i) = 0;
    end

    [x, fval, exitflag] = quadprog(2*Q, c, A, b, Aeq, beq, lb, ub);
    if exitflag == 1
        weight_3(:,i) = x;
        value_3(i)= fval;
    else
        weight_3(:,i) = zeros(N,1);
        value_3(i) = 0;
    end

    mu_wo_3(i) = rdm_mu_i * x_wo;
    var_wo_3(i) = x_wo' * Q * x_wo;
    sta_wo_3(i) = sqrt(var_wo_3(i));

    mu_3(i) = rdm_mu_i * x;
    var_3(i) = x' * Q * x;
    sta_3(i) = sqrt(var_3(i));
end

% construct the average portfolio
avg_weight_wo = (weight_wo_1 + weight_wo_2 + sum(weight_wo_3,2))/5;
avg_mu_wo = (mu_wo_1 + mu_wo_2 + sum(mu_wo_3))/5;
avg_var_wo = avg_weight_wo' * Q * avg_weight_wo;
```

```
avg_sta_wo = sqrt(avg_var_wo);

avg_weight = (weight_1 + weight_2 + sum(weight_3,2))/5;
avg_mu = (mu_1 + mu_2 + sum(mu_3))/5;
avg_var = avg_weight' * Q * avg_weight;
avg_sta = sqrt(avg_var);
%% e) this part has been encoded in the above parts

%% plotting
figure(1);
set(gcf, 'color', 'white');
weight = [weight_wo_1';weight_wo_2';avg_weight_wo'];
c = categorical({'port1','port2','port3'});
bar(c,weight,'stacked')
title('Weight Composition of Three No Shorting Cases');
xlabel('Portfolios');
ylabel('Weight');
ylim([0 1.1])
legend({'AAPL' 'AXP' 'BA' 'CAT' 'CSCO' 'CVX' 'DIS' 'DWDP' 'GE'...
    'GS' 'HD' 'IBM' 'INTC' 'JNJ' 'JPM' 'KO' 'MCD' 'MMM' 'MRK' 'MSFT' ...
    'NKE' 'PFE' 'PG' 'TRV' 'UNH' 'UTX' 'V' 'VZ' 'WMT' 'XOM'});

figure(2);
set(gcf, 'color', 'white');
weight = [weight_1';weight_2';avg_weight'];
c = categorical({'port4','port5','port6'});
bar(c,weight,'stacked')
title('Weight Composition of Three Shorting Cases');
xlabel('Portfolios');
ylabel('Weight');
ylim([0 1.1])
legend({'AAPL' 'AXP' 'BA' 'CAT' 'CSCO' 'CVX' 'DIS' 'DWDP' 'GE'...
    'GS' 'HD' 'IBM' 'INTC' 'JNJ' 'JPM' 'KO' 'MCD' 'MMM' 'MRK' 'MSFT' ...
    'NKE' 'PFE' 'PG' 'TRV' 'UNH' 'UTX' 'V' 'VZ' 'WMT' 'XOM'});
```