APS 1022

Financial Engineering II

Course Project Part 2

Submission on: May 14th, 2018

Tong Li 999502328

Jun Zhang 999857631

Mechanical & Industrial Engineering
UNIVERSITY OF TORONTO

Table of Contents

Introduction **...** ***1***

Methodology **...** ***2***

Result **...** ***4***

Discussion **...** ***6***

Conclusion **...** ***7***

Reference **..** *Error! Bookmark not defined.*

Appendix **...** ***8***

Introduction

Mean-variance optimization (MVO) is a well-developed strategy for figuring out the optimal allocation of investments. It is straightforward, because it only utilizes few variables including mean, variance, and covariance of asset returns. However, the limitations of MVO are noticeable as well. The first drawback is that it only takes use of the first two moments, which are the mean and variance of the return distribution. In addition, MVO is a static approach. The period of our optimization procedure has to be determined before the calculation. In this case, the algorithm loses the adaptability to the growing data. Moreover, it is difficult to forecast accurate asset returns using historical data. The errors of estimation have a significant impact on asset allocation, especially the errors occur in the estimated expected returns. Therefore, MVO leads to a highly concentrated portfolio if short selling is not allowed, which is quite normal in practice.

In order to overcome the shortcomings of MVO, Black-Litterman model (BL model) was developed by Fisher Black and Robert Litterman at Goldman Sachs in the 1990s. BL Model is an asset allocation model that allows portfolio managers to incorporate views into CAPM (Capital Asset Pricing Model) equilibrium returns and to create more diversified portfolios than those generated by traditional MVO. Overall, BL model tries to overcome high-concentration, input sensitivity, and estimation error maximization problems that are inherent in the MVO model.

Another improvement to the MVO model is called Risk Parity Portfolio or equal risk contribution (ERC) portfolio. Similar to equal weighted portfolio, ERC portfolio minimize the risk contribution of each asset to be the same despite the weight contribution. ERC portfolio is less risky than the Equal Weighted Portfolio while more diversified than minimum variance portfolio.

This assignment followed the steps introduced in paper "A step-by-step guide to the Black-Litterman Model" by Thomas M. Idzorek to reimplement the Black-Litterman Model using eight assets in different categories – US Bonds, International Bonds, US Large Growth, US Large Value, US Small Growth, US Small Value, International Developed Equity, and International Emerging Equity. The assets we chose in this report are: Fidelity US Bond Index Premium (FSITX), Templeton Global Bond A (TPINX), Vanguard Dividend Appreciation ETF (VIG), iShares Russell 1000 Value ETF (IWD), T. Rowe Price New Horizons (PRNHX), Vanguard Small Cap Value Index Inv (VISVX), Fidelity International Index Investor (FSIIX) and iShares MSCI Emerging Markets ETF (EEM). Additionally, an ERC portfolio was also constructed and compareed with BL portfolio.

Methodology

The following equations were employed in the implementation of Black Litterman Model and equal risk contribution portfolio construction. N is the number of assets and K is the number of views.

$$\boldsymbol{\Pi} = \boldsymbol{\lambda} \mathbf{Q} \boldsymbol{w}_{\mathbf{mkt}} \quad (\mathbf{1})$$

Where:

Π: $Implied\ Equilibrium\ Excess\ Return\ Vector\ (Nx1\ column\ vector)$

$\lambda: Risk\ aversion\ coefficient$

$Q: The\ covariance\ matrix\ of\ the\ excess\ returns\ (NxN\ matrix)$

$w_{mkt} = the\ market\ capitalization\ weight\ (Nx1\ column\ vector)\ of\ the\ chosen\ assets$

$$\boldsymbol{w} = (\boldsymbol{\lambda}\mathbf{Q})^{-\mathbf{1}} \boldsymbol{\mu}\ (\mathbf{2})$$

Where:

$\mu: Excess\ Return\ vector$

$w: Assets'weightings$

$$\boldsymbol{\lambda} = \frac{\boldsymbol{E}(\boldsymbol{r}) - \boldsymbol{r_f}}{\boldsymbol{\sigma}^2} \quad (\mathbf{3})$$

Where:

$E(r): The\ expected\ market\ portfolio\ return$

$r_f: The\ risk\ free\ rate$

$\sigma^2:\ w_{mkt}^T Q w_{mkt}\ is\ the\ variance\ of\ the\ market\ excess\ returns$

$$\boldsymbol{\mu} = \boldsymbol{E}[\boldsymbol{R}] = [(\boldsymbol{\tau}\boldsymbol{Q}_{\boldsymbol{new}})^{-\mathbf{1}} + \boldsymbol{P}^{\boldsymbol{T}}\boldsymbol{\Omega}\boldsymbol{P}]^{-\mathbf{1}}\ [(\boldsymbol{\tau}\boldsymbol{Q}_{\boldsymbol{new}})^{-\mathbf{1}}\boldsymbol{\Pi} + \boldsymbol{P}^{\boldsymbol{T}}\boldsymbol{\Omega}\boldsymbol{q}] \quad (\mathbf{4})$$

Where:

$\tau: A\ scalar\ number\ indicating\ the\ level\ of\ confidence\ of\ the\ distribution$

$P: A\ matrix\ with\ investors\ view\ (KxN\ vector)$

$q: Expected\ returns\ difference\ of\ the\ portfolios\ given\ the\ veiws$

$\Omega = Diagonal\ covariance\ matrix\ of\ view\ uncertainties$

$$\mathbf{Q_{new}} = [(\boldsymbol{\tau\Sigma})^{-\mathbf{1}} + \mathbf{P^T \Omega P}]^{-\mathbf{1}} + \mathbf{Q}\ \ (\mathbf{5})$$

Where:

$Q_{new}: New\ covariance\ matrix$

The following views were applied to the model:

View 1: International Developed Equity will have an absolute excess return of 0.5%
View 2: International Bonds will outperform US Bonds by 5 basis points
View 3: US Large Growth and US Small Growth will outperform US Large Value and US Small Value by 0.6%

$$P_{ij} = \begin{cases} 1 \quad if\ view\ i\ is\ postive\ about\ asset\ j \\ 0 \quad if\ view\ i\ is\ irrelevant\ to\ asset\ j \\ \dfrac{weight_j}{weight_j + weight_k}\ if\ asset\ j\ outperforms\ asset\ k \\ -\dfrac{weight_j}{weight_j + weight_k}\ \ if\ asset\ j\ overperforms\ asset\ k \end{cases}$$

The variance of the error terms that form the diagonal elements of the diagonal covariance matrix of uncertainties (Ω) were found using the following equation:

$$\mathbf{\Omega_{nn} = P_n Q P_n^T} \quad \mathbf{(6)}$$

where,

$$\Omega_{nn} = diagonal\ elememetns\ of\ variance\ of\ error\ terms$$

$$\mathbf{\min \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{x_i (Qx)_i}{\sqrt{x^T Q x}} - \frac{x_j (Qx)_j}{\sqrt{x^T Q x}}\right)^2} \quad \mathbf{(7)}$$

where,

$$x_i = weight\ of\ asset\ i$$
$$Q = covariance\ matrix$$

The following procedures were applied in the MATLAB code as:

1. Expected returns were converted to monthly excess returns and the historical expected excess returns and covariance were calculated based on this.
2. Eqn. (3) was used to calculate lambda.
3. Eqn. (1) was used to calculate the implied equilibrium excess returns.
4. Eqn. (6) was used to construct the errors in our views
5. Matrix P that identifies the assets involved in the views and the view vector Q were constructed based on the views
6. Eqn. (4) was used to calculate the Black-Litterman expected excess returns.
7. Eqn. (7), ERC portfolio was solved by using MATLAB function ‘fmincon’.

Result

The information of assets that used to implement BL model are listed in Table 1.

Category	Name	Symbol	Net Asset
US Bonds	Fidelity US Bond Index Premium	FSITX	\$36.15B
International Bonds	Templeton Global Bond A	TPINX	\$38.28B
US large growth	Vanguard Dividend Appreciation ETF	VIG	\$33.99B
US large value	iShares Russell 1000 Value ETF	IWD	\$35.92B
US small growth	T. Rowe Price New Horizons	PRNHX	\$22.85B
US small value	Vanguard Small Cap Value Index Inv	VISVX	\$29.34B
International Equity	Fidelity International Index Investor	FSIIX	\$22.87B
Emerging markets	iShares MSCI Emerging Markets ETF	EEM	\$42.17B

Table 1 Assets Information

The portfolio expected return and weightings for Black-Litterman and market is shown as:

Asset Class	New Combined Return Vector E[R]	Implied Equilibrium Return Vector Π	Difference E[R] - Π	New Weight $\hat{w}$	New Weight (normalized) $\hat{w}_{norm}$	Market Capitalization Weight w_{mkt}	Difference $\hat{w} - w_{mkt}$
US Bonds: FSITX	0.060%	0.017%	0.044%	23.711%	23.454%	13.820%	9.891%
Int'l Bonds: TPINX	0.130%	0.134%	-0.005%	4.050%	4.006%	14.635%	-10.585%
US Large Growth: VIG	0.264%	0.271%	-0.007%	108.421%	107.244%	12.995%	95.426%
US Large Value: IWD	0.150%	0.269%	-0.119%	-74.728%	-73.917%	13.732%	-88.460%
US Small Growth: DTSGX	0.797%	0.388%	0.409%	72.887%	72.096%	8.736%	64.151%
US Small Value: VISVX	0.181%	0.322%	-0.141%	-61.039%	-60.376%	11.217%	-72.256%
Int'l Dev. Equity: FISSX	0.485%	0.335%	0.150%	12.067%	11.936%	8.743%	3.323%
Int'l Emerg. Equity: EEM	0.514%	0.418%	0.097%	15.729%	15.558%	16.122%	-0.393%
Sum				101.10%	100.00%	100.00%	1.097%

Table 2 Return Vectors and Resulting Portfolio Weights

The comparison of portfolio statistics between BL and market:

Monthly Stats	Market Capitalization-Weighted Portfolio w_{mkt}	BL Portfolio $\hat{w}$
Excess Return	0.261%	0.795%
Variance	5.661e-04	1.708e-03
Standard Deviation	2.379e-02	4.133e-02
Beta	1.000	1.057
Residual Return		4.811e-03
Residual Risk		3.280e-02
Active Return		4.980e-03
Active Risk		3.283e-02
Sharpe Ratio	0.1096	0.1924
Information Ratio		0.1517

Table 3 Portfolio Statistic

Assets' weighting for portfolio BL and ERC:

Asset	BL weight	ERC weight	Difference
US Bonds: FSITX	23.454%	63.052%	-39.598%
Int'l Bonds: TPINX	4.006%	21.748%	-17.742%
US Large Growth: VIG	107.244%	10.818%	96.426%
US Large Value: IWD	-73.917%	11.166%	-85.083%
US Small Growth: DTSGX	72.096%	6.164%	65.932%
US Small Value: VISVX	-60.376%	7.556%	-67.932%
Int'l Dev. Equity: FISSX	11.936%	13.032%	-1.096%
Int'l Emerg. Equity: EEM	15.558%	-33.536%	49.094%

Table 4 BL and ERC weightings

The comparison of expected return, portfolio variance between BL and ERC:

	BL portfolio	ERC portfolio	Difference
Excess Return	0.795%	0.194%	0.601%
Variance	1.708e-03	1.376e-04	-1.571e-03
Standard Deviation	4.133e-02	1.173e-02	2.960e-02

Table 5 ERC,BL statistics comparison

Discussion

Given in Table 1, our chosen assets have fairly close market values. This was chosen in order to mitigate the potential variability generated due to the large differences among the asset values.

As shown in Table 2, the implied equilibrium return vector gives a general idea about how the market performs, in terms of excess return. Except for US and International bonds, the performance of the rest bonds does not deviate too much, based on the implied equilibrium excess return. However, in the BL excess return, FSITX grew to 0.06%, which reflected our personal view number 2. Similarly, DTSGX grew to 0.797% and both IWD and VISVX dropped dramatically. This also indicated our personal view number 3, which is about the performance between US Growth Bonds and US Value Bonds, in the result. FISSX increased accordingly as well. However, the difference between the implied equilibrium return and BL return is not exactly the same as our views, which means it incorporated the errors during the calculation.

The market weightings in Table 2 gives a relatively sparse portfolio, since most of the assets have weights around 10%. However, BL gives a portfolio that is relatively extreme. The two bonds, US Large and Small Values, were decided to be shorted the most. This large difference in assets' weights indicates how our confidence level worked in the formula. The more confident you are about the views, the stronger the views are going to affect the final BL excess return.

Table 3 gives a summary of the portfolio statistics of BL portfolio and ERC portfolio. BL has a much larger expected excess monthly portfolio return, compared to the one of the market portfolio. As a trade-off, BL portfolio also a much larger portfolio risk, compared to the market portfolio. This difference in variance is caused by the uncertainty in our views. However, the monthly residual return of BL is really small, which indicates that the BL portfolio does not outperform our benchmark that much on a monthly base. Similarly, the monthly active return is really small as well. But luckily, the monthly residual risk and active risk are acceptably small, even though they are larger than the returns. Besides, the sharpe ratio of BL is almost twice as it of market portfolio, which indicates a good return per unit risk in BL portfolio, compared to the market. The information ratio is also fairly good, compared to the sharpe ratio value.

The comparison of weightings between BL and ERC portfolios is shown in Table 4 and 5. As we can see, BL generates a portfolio that is much more extreme than ERC. Only EEM is shorted 33.56% in ERC. In BL portfolio, IWD and VISVX are shorted 73.9% and 60.38% respectively, and VIG is brought to more than 100%. This extreme portfolio causes the BL a relative large portfolio variance, which is 1.78*10^-3, compared to ERC portfolio, which is only 1.376*10^-4. As a trade-off, the expected portfolio return of ERC is also much smaller, compared to BL portfolio.

Conclusion

To sum up, the report followed the steps that introduced in the paper "A step-by-step guide to the Black-Litterman Model" by Thomas M. Idzorek and reimplement the Black-Litterman Model using eight assets in different categories. As expected, the results of the portfolio using Black Litterman model achieves higher excess return as well as larger variance compared to market portfolio with similar views in the reference paper. Furthermore, Black Litterman portfolio also outperforms equal risk contribution portfolio in terms of excess return but with larger risks. Throughout the implementation, we gained a better understanding of the procedure as well as the advantages and drawbacks of Black Litterman model. Regarding future work, τ and view matrix q could be further tuned in order to examine the sensitivity of the model.

Appendix

A1: Historical expected excess returns:

Asset class	Historical return
US Bonds: FSITX	-0.042%
Int'l Bonds: TPINX	-0.052%
US Large Growth: VIG	0.703%
US Large Value: IWD	0.660%
US Small Growth: DTSGX	1.227%
US Small Value: VISVX	0.793%
Int'l Dev. Equity: FISSX	0.410%
Int'l Emerg. Equity: EEM	0.318%

A2: Covariance Matrix of eight assets

Asset class	FSITX	TPINX	VIG	IWD	DTSGX	VISVX	FISSX	EEM
FSITX	7.443E-05	1.509E-05	1.841E-05	-2.036E-05	6.552E-05	-1.117E-05	4.745E-05	9.553E-05
TPINX	1.509E-05	3.514E-04	2.324E-04	2.620E-04	3.652E-04	2.930E-04	3.730E-04	4.608E-04
VIG	1.841E-05	2.324E-04	8.180E-04	7.472E-04	6.909E-04	8.057E-04	7.046E-04	8.063E-04
IWD	-2.036E-05	2.620E-04	7.472E-04	8.168E-04	6.501E-04	9.138E-04	6.519E-04	7.642E-04
DTSGX	6.552E-05	3.652E-04	6.909E-04	6.501E-04	2.362E-03	1.015E-03	1.186E-03	1.091E-03
VISVX	-1.117E-05	2.930E-04	8.057E-04	9.138E-04	1.015E-03	1.265E-03	7.479E-04	8.170E-04
FISSX	4.745E-05	3.730E-04	7.046E-04	6.519E-04	1.186E-03	7.479E-04	1.164E-03	1.221E-03
EEM	9.553E-05	4.608E-04	8.063E-04	7.642E-04	1.091E-03	8.170E-04	1.221E-03	2.005E-03

MATLAB Code

```
clc;
clear all;
format long
%% Part A
% Data used for each asset class
file_name = {'FSITX.csv' 'TPINX.csv' 'VIG.csv' 'IWD.csv' ...
    'PRNHX.csv' 'VISVX.csv' 'FSIIX.csv' 'EEM.csv'};

N = length(file_name);

% Read monthly prices
for i = 1:N
    if(exist(file_name{1,i},'file'))
      fprintf('\nReading Monthly prices datafile - %s\n', file_name{1,i});
      fid = fopen(file_name{1,i});
      vheader{i} = textscan(fid, '%[^,]%*[^\n]');
      dates{i} = vheader{1}(1:end); % separate d into different cells
      fclose(fid);
      data_prices{i} = dlmread(file_name{1,i}, ',',[1,5,61,5]);
    else
      error('Daily prices datafile does not exist')
    end
    consolidated_price(:,i) = data_prices{1,i};
end

% US 10-years T bill is considered as risk free rate
% since our data is from 2013 to 2018, we take the average number
rf=(1.91+2.58+2.43+2.09+1.88+2.86)/6;

% Calculate monthly return excess for each assets
returns = consolidated_price(2:61,:) ./ consolidated_price(1:61-1,:) - 1;
returns = returns - rf/1200;

% Calculate the expected monthly excess returns for each asset
mu_i = mean(returns);


% Calculate the covariance matrix of the excess return for 8 assets
Q = cov(returns);

% Net asset values for all eight assets. From left to right, they are:
% 'FSITX' 'TPINX' 'VIG' 'IWD' 'PRNHX' 'VISVX' 'FSIIX' 'EEM'
% the values are measured in Billion
net_asset=[36.15 38.28 33.99 35.92 22.85 29.34 22.87 42.17];

% Calculate the market weights for eight assets we chose above
w_mkt=(net_asset/sum(net_asset))';

% Calculate the parameter lambda
lambda=((w_mkt'*mu_i')-(rf/1200))/(w_mkt'*Q*w_mkt);

% Implied Equilibrium excess return vector
pi=lambda*Q*w_mkt;
```

```
% Market capitalization weighted portfolio expected excess return
r_mkt=w_mkt'*pi;

% Market capitalization weighted portfolio variance
var_mkt = w_mkt' * Q * w_mkt;

% Beta
beta = [1.04,-0.24,0.85,0.96,1.01,0.98,0.91,1.15];

% the scaling factor for voariability of the new expected return
tau=0.025;

% View 1: FSIIX will have an absolute excess monthly return of 0.5%
% View 2: TPINX will outperform FSITX by 5 basis points
% View 3: VIG and PRNHX will outperform IWD and VISVX by 0.6%

p33 = net_asset(3)/(net_asset(3)+net_asset(5));
p34 = - net_asset(4)/(net_asset(4)+net_asset(6));
p35 = net_asset(5)/(net_asset(3)+net_asset(5));
p36 = - net_asset(6)/(net_asset(4)+net_asset(6));


P=[0,0,0,0,0,0,1,0;-1,1,0,0,0,0,0,0;0,0,p33,p34,p35,p36,0,0];

% Set the view as our expectation
q=[0.5/100;0.05/100;0.6/100];

% o1,o2,o3: our confidence level to view 1,2 and 3
o1=tau*P(1,:)*Q*P(1,:)';
o2=tau*P(2,:)*Q*P(2,:)';
o3=tau*P(3,:)*Q*P(3,:)';

% Construct the Omega matrix using o1,o2,o3
Ome=diag([o1,o2,o3]);

% Variance of the View Portfolios
v_vp1=P(1,:)*Q*P(1,:)';
v_vp2=P(2,:)*Q*P(2,:)';
v_vp3=P(3,:)*Q*P(3,:)';

% Calculate the expected return of black-litterman
exp_r=inv(inv(tau*Q)+P'*inv(Ome)*P)*(inv(tau*Q)*pi+P'*inv(Ome)*q);

% New covariance of the new combined distribution
cov_BL=inv(inv(tau*Q)+P'*inv(Ome)*P)+Q;


% Black-litterman asset weights
w_bl=inv(lambda * cov_BL)*exp_r;
w_bl=w_bl/sum(w_bl);

% Black_litterman portfolio excess return
r_bl=w_bl'*exp_r;
```

```
% Black_litterman portfolio excess return variance
var_bl=w_bl'*cov_BL*w_bl;

% Difference from black_litterman mu and pi
mu_diff=exp_r-pi;

% Difference from black_litterman portfolio and market portfolio
w_diff=w_bl-w_mkt;

% Market beta normalizer
beta_norm=beta*w_mkt;

% Black-litterman beta
beta_bl=beta*w_bl/beta_norm;

% Black-litterman Residual Return
rret_bl=r_bl-beta_bl*w_mkt'*exp_r;

% Black-litterman Residual Risk
rrsk_bl=sqrt(var_bl-(beta_bl^2)*var_mkt);

% Active Portfolio Beta
beta_pa=beta_bl-1;

% Black-litterman Active Return
aret_bl=r_bl-w_mkt'*exp_r;

% Black-litterman Active Risk
arsk_bl=sqrt((rrsk_bl^2)+(beta_pa^2)*var_mkt);

% Market portfolio sharpe ratio
sr_mkt=r_mkt/sqrt(var_mkt);

% Black-litterman sharpe ratio
sr_bl=r_bl/sqrt(var_bl);

% Black-litterman information ratio:
ir_bl=aret_bl/arsk_bl;

%%%%%%%%%%%%%%%%%%%% ERC portfolio
n=8;
X=repmat(1.0/n, n, 1);
Sigma=Q;

f = @(X) var(X.*(Sigma*X))*10^13;

w_erc = fmincon(f,X,[],[],ones(1,length(X)),1);
for i=1:8
    l=Q*w_erc;
    var{i}=w_erc(i)*l(i);
end

% ERC profolio expected excess return
r_erc=mu_i*w_erc;
```

```
% Return difference between ERC and black-litterman portfolio
r_ercbl=r_erc-r_bl;

% ERC portfolio variance
var_erc=w_erc'*Q*w_erc;

% Variance difference between ERC and black-litterman portfolio
var_ercbl=var_erc-var_bl;

% Weights difference between ERC and black-litterman portfolio
w_ercbl=w_erc-w_bl;
```