1-15

(1) Big O definition:

$T(n) = O(n^3)$ if $T(n) \leq c \cdot n^3$ for some $n \geq n_0$

$n^2 + 3n^3 \leq cn^3 \Rightarrow \frac{1}{n} + 3 \leq c$, Therefore, $n \geq n_0 = 1$
$c \geq 4 = (\frac{1}{1} + 3)$ □

(2) Omega definition:

$T(n) = O(n^3)$ if $T(n) \geq c \cdot n^3$ for some $n \leq n_0$

$n^2 + 3n^3 \geq cn^3 \Rightarrow \frac{1}{n} + 3 \geq c$, Therefore, $n \geq n_0 = 1$
$c \geq 4 = (\frac{1}{1} + 3)$ □

1-16

O: $f(n) = 6n^2 + 20n \leq 10n^3 \ \forall\ n > 1$, $(f(n) = O(g(n)))$, $g(n) = n^3$.

$\therefore f(n) \in O(n^3)$

Ω: $f(n) = 6n^2 + 20n \geq n^3$ but this is not true when n gets bigger

$\therefore f(n) \neq \Omega(n^3)$

Therefore $6n^2 + 20n = O(n^3)$ but $6n^2 + 20n \neq \Omega(n^3)$

1-17

$f(n) = 5n^5 + 4n^4 + 6n^3 + 2n^2 + n + 7$

for $n \geq 1$, $f(n) \leq 25n^2$
$\Rightarrow 5n^5 \leq f(n) \leq 25n^5$
for $n \geq 0$, $5n^5 \leq f(n)$

So $f(n) = 5n^5 + 4n^4 + 6n^3 + 2n^2 + n + 7 \in \Theta(n^5)$ $\forall n \geq 1$

1-18

$p(n) = a_k n^k + a_{k-1} n^{k-1} + \cdots + a_1 n + a_0$

Let $C = \sum_{i=0}^{k} a_i$, then $p(n) \le Cn^k$, so $p(n) = O(n^k)$

Let $C_2 = a_k$, then $p(n) \ge C_2 n^k$, so $p(n) = \Omega(n^k)$

$\because p(n) = O(n^k) = \Omega(n^k)$ $\therefore p(n) = \Theta(n^k)$ #

1-22 Group them by their complexity.

1. n^n & $n^n + \ln n$
2. $n!$ & $2^{n!}$
3. $10^n + n^{20}$
4. 4^n & e^n
5. $(\ln n)!$
6. $n^{\frac{5}{2}}$ x $5n^2 + 7n$ & $\sqrt{n}$
7. $5^{\ln n}$
8. $8n + 12$
9. $(\ln n)^2$
10. $n \ln n$ & $\ln(n!)$

Best case: $B(n) = 1$

analyze → $S[0] \;!= S[s.length() - 1]$

Worst case: $W(n) = n$

analyze → In the for loop in C++, when $n=4$, $n/2 = 2$
when $n=5$, $n/2 = 2$

$\therefore \lfloor n/2 \rfloor \Rightarrow W(n) = n$

Average case: $A(n) = n$.

analyze → $\sum_{i=1}^{\frac{n}{2}} k \times \frac{1}{n} = \frac{1}{n} \sum_{i=1}^{\frac{n}{2}} k = \frac{1}{n}\left(\frac{(\frac{n}{2}+1)\cdot(\frac{n}{2})}{2} \right) = \frac{n+2}{8}$

$\Rightarrow A(n) = n$

$T(n)$ D.N.E.

Since the input will affect the steps of the for loop.

so $T(n)$ D.N.E.