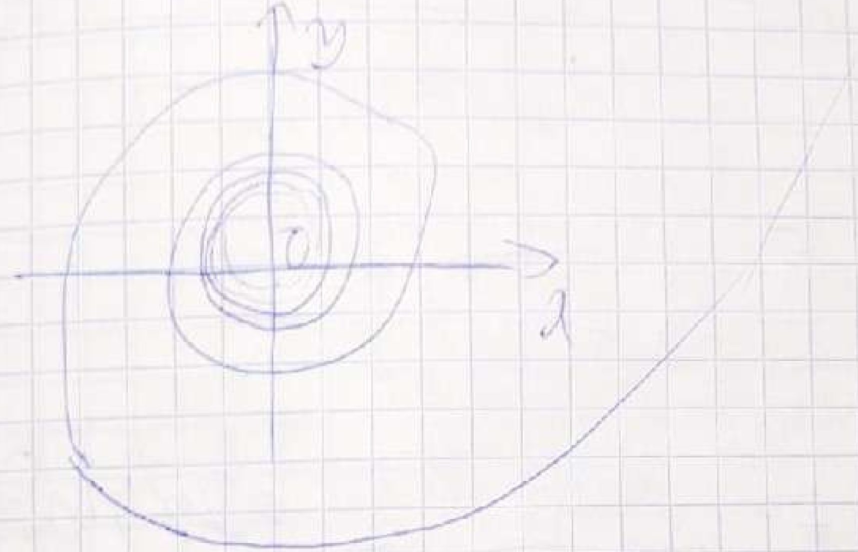
$$r = \sqrt[4]{\frac{-1}{4(t+C)}} \qquad t < -C$$

$$t \to -C \qquad r \to +\infty$$



№ 18

$$\dot{x}_1 = \lambda_1 x_1 + c_{20} x_1^2 + c_{11} x_1 x_2 + c_{02} x_2^2$$

$$\dot{x}_2 = \lambda_2 x_2 + d_{20} x_1^2 + d_{11} x_1 x_2 + d_{02} x_2^2$$

$$\lambda_2 = 2\lambda_1$$

$$\dot{x}_i = \lambda_i x_i + \sum_{k_1, k_2} g^i_{k_1 k_2} x_1^{k_1} x_2^{k_2}$$

$i = 1, 2,$ где $k_1 + k_2 = 2$

$\angle g^1_{..} = c_{..}$; $g^2_{..} = d_{..}$

$$p^i_{k_1k_2} = \frac{g^i_{k_1k_2}}{k_1\lambda_1 + k_2\lambda_2 - \lambda_i}$$

$$p^1_{20} = \frac{c_{20}}{-\lambda_1 + 2\lambda_1} = \frac{c_{20}}{\lambda_1}$$

$$p^2_{20} = \frac{d_{20}}{-\lambda_2 + 2\lambda_1}$$ — резонанс

$$p^1_{11} = \frac{c_{11}}{\lambda_2} = \frac{c_{11}}{2\lambda_1}$$

$$p^2_{11} = \frac{d_{11}}{\lambda_1}$$

$$p^1_{02} = \frac{c_{02}}{-\lambda_1 + 2\lambda_2} = \frac{c_{02}}{3\lambda_1}$$

$$p_{02}^2 = \frac{d_{02}}{\lambda_2} = \frac{d_{02}}{2\lambda_1}$$

$$x_1 = y_1 + \sum_{k_1+k_2=2} p^1_{k_1 k_2} y_1^{k_1} y_2^{k_2} =$$

$$= y_1 + \frac{c_{20}}{\lambda_1} y_1^2 + \frac{c_{11}}{2\lambda_1} y_1 y_2 + \frac{c_{02}}{3\lambda_1} y_2^2$$

$$x_2 = y_2 + \sum_{k_1+k_2=2} p^2_{k_1 k_2} y_1^{k_1} y_2^{k_2} =$$

$$= y_2 + \frac{d_{11}}{\lambda_1} y_1 y_2 + \frac{d_{02}}{2\lambda_1} y_2^2$$

$$(de_2 \quad p^2_{20})$$

$$\begin{cases} y_1 = \lambda_1 y_1 + \underline{O}\left(\sqrt{y_1^2+y_2^2}^{\,3}\right) \\ y_2 = \lambda_2 y_2 + d_{20} y_1^2 + \underline{O}\left(\sqrt{y_1^2+y_2^2}^{\,3}\right) \end{cases}$$

№ 79.

$\ddot{\alpha} + \sin\alpha = 0 \qquad w(\alpha) - ?$

$$\sin\alpha = \alpha - \frac{\alpha^3}{6} + \underline{o}(\alpha^3)$$

$u := \dot{\alpha}$

$$\begin{cases} \dot{\alpha} = u \\ \dot{u} = -\alpha + \dfrac{\alpha^3}{6} + \underline{o}(\alpha^3) \end{cases}$$

$$\begin{pmatrix} \dot{\alpha} \\ \dot{u} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}}_{\mathcal{A}} \begin{pmatrix} \alpha \\ u \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{\alpha^3}{6} \end{pmatrix} + \underline{o}(\alpha^3)$$

$$\det(\mathcal{A} - \lambda E) = \cancel{0}\ \lambda^2 + 1 = 0$$

$\lambda = \pm i \qquad \lambda_1 = i,\ \lambda_2 = -i$

$$\lambda_1: \begin{pmatrix} -i & 1 \\ -1 & -i \end{pmatrix} \overline{h_1} = 0 \qquad \overline{h_1} = \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$\lambda_2: \begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix} h_2 = 0$

$h_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$ Замена:

$$\begin{cases} \hat{x} = x + iy = h_1^T \begin{pmatrix} x \\ y \end{pmatrix} \\ \hat{u} = x - iy = h_2^T \begin{pmatrix} x \\ y \end{pmatrix} \end{cases} \Longleftrightarrow \begin{cases} x = \frac{\hat{x} + \hat{u}}{2} \\ y = -i \cdot \frac{\hat{x} - \hat{u}}{2} \end{cases}$$

$$\dot{\hat{x}} = y + i\left(-x + \frac{x^3}{6}\right) + O(x^5) =$$

$$= -i\frac{(\hat{x} - \hat{u})}{2} + i\left(-\frac{\hat{x} + \hat{u}}{2} + \left(\frac{\hat{x} + \hat{u}}{2}\right)^3 \cdot \frac{1}{6}\right) =$$

$$= -i\hat{x} + \frac{i}{48}\left(\hat{x}^3 + 3\hat{x}^2\hat{u} + 3\hat{x}\hat{u}^2 + \hat{u}^3\right)$$

$$\dot{\hat{u}} = y - i\left(-x + \frac{x^3}{6} + \ldots\right) =$$

$$= i\hat{u} - \frac{i}{48}\left(\hat{x}^3 + 3\hat{x}^2\hat{u} + 3\hat{x}\hat{u}^2 + \hat{u}^3\right)$$

Проверим на резонанс:

$$\lambda_i \overset{(?)}{=} k_1\lambda_1 + k_2\lambda_2, \quad k_1+k_2=3$$

$\lambda_1 = i, \ \lambda_2 = -i$

$\tilde{a}^3: \quad k_1 = 3; \quad k_2 = 0$

$$\pm i \overset{(?)}{=} 3 \cdot i \text{ — нет}$$

$\tilde{a}^2 \tilde{u}:$

$$\pm i \overset{(?)}{=} 2i - i = i \Rightarrow$$

$\Rightarrow$ резонанс для $\tilde{a}$

$\tilde{a}\tilde{u}^2: \quad \pm i \overset{(?)}{=} i - 2i = -i \Rightarrow$

$\Rightarrow$ резонанс для $\tilde{u}$

$\tilde{u}^3: \quad \pm i \overset{(?)}{=} -3i$ — нет $\Rightarrow$

$$\Rightarrow \quad \dot{\tilde{a}} = -i\tilde{a} + \frac{i}{16}\tilde{a}^2\tilde{u}$$

$$\dot{\hat{u}} = i\hat{u} - \frac{i}{16}\hat{u}^2\hat{x}$$

$$\dot{\hat{x}}\hat{u} = -i\hat{x}\hat{u} + \frac{i}{16}\hat{x}^2\hat{u}^2$$

$$\hat{x}\dot{\hat{u}} = i\hat{u}\hat{x} - \frac{i}{16}\hat{x}^2\hat{u}^2$$

$$\Rightarrow \dot{\hat{x}}\hat{u} + \dot{\hat{u}}\hat{x} = 0 = \frac{d(\hat{x}\hat{u})}{dt} \Rightarrow$$

$$\Rightarrow \hat{x}\hat{u} = c = const$$

$$\hat{u} = \frac{c}{\hat{x}}$$

$$\dot{\hat{x}} = -i\hat{x} + \frac{i}{16}\hat{x}c$$

$$\dot{\hat{x}} + \left(i - \frac{ic}{16}\right)\hat{x} = 0 \Rightarrow$$

$$\Rightarrow \hat{x} = \hat{x}_0 e^{\frac{i}{16}(c-16)t}$$

$$\hat{u} = \hat{u}_0 e^{-\frac{i}{10}(c-16)t}$$

$$\hat{x}_0\hat{u}_0 = c$$

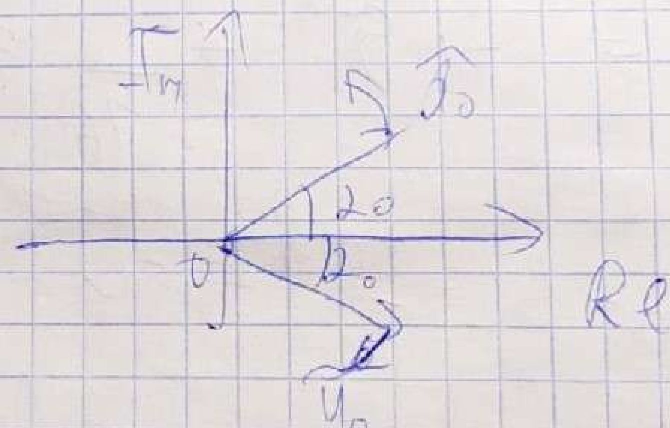
$$x = \frac{\tilde{x} + \tilde{y}}{2} = \frac{Re(\tilde{x}) + Re(\tilde{u})}{2} =$$

$$= \cancel{\ldots\cos\omega t} \quad \tilde{y}_0 \quad \tilde{x}_0 + \frac{c}{\tilde{x}_0} \in R$$

$$\frac{\tilde{x} + \tilde{u}}{2} \in R \quad \frac{\tilde{x}_0 + \tilde{u}_0}{2} \in R$$

$$= |\tilde{x}_0| \cos(\omega_0 t + \alpha_1) + |\tilde{u}_0| \cos(\omega_0 t + \alpha_2) =$$

$$\omega_0 = \left| \frac{c}{x_0} - 1 \right|$$



$$x = x_0 \cos(\omega_0 t + \alpha)$$

$y_0 = (-i) \cdot \frac{\tilde{a} - \tilde{u}}{2}$

$\cancel{x_0 = \frac{\tilde{a}_0 + \tilde{u}_0}{2}; \quad y_0 = (-i)\frac{\tilde{a}_0 - \tilde{u}_0}{2}}$

$\cancel{\tilde{a}_0 \tilde{u}_0 = c} \qquad \cancel{x_0 = a}$

$\cancel{y_0 = a\omega_0}$

$x_0 = a; \quad y_0 = a\omega_0$

$|\tilde{a}|^2 = x^2 + [illegible]\, y^2 = x_0^2 + [illegible]\, y^2 =$

$= [illegible] \quad \tilde{a}_0 \tilde{a}_0^* = |\tilde{u}|^2 \neq = \tilde{a}_0 \tilde{u}_0^* =$

$= \frac{c}{\tilde{a}_0} \frac{c}{\tilde{a}_0^*}$

$|\tilde{a}_0| = |\tilde{u}_0| \; \cancel{= \sqrt{c}}\; ; c \in \mathbb{R}$

$x = 2|\tilde{a}_0| \cos(\omega_0 t + \alpha_1)$

$|\tilde{a}_0|^2 = c$

$x_0^2 + y_0^2 = a^2 + a^2\omega_0 = c$

$$a^2\left(1+\left|\frac{c}{16}-1\right|\right)=c$$

$$\cancel{a^2(1+\omega)=\omega}$$

1) $\frac{c}{16}<1$

$$a^2(1+\omega)=(1-\omega)\cdot 16$$

$$\omega=\frac{16-a^2}{16+a^2}$$

2) $\frac{c}{16}>1$

$$a^2(1+\omega)=(1+\omega)\cdot 16$$

$a^2=16$ - лож

Ответ: $\omega=\frac{16-a^2}{16+a^2}$