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Supply, demand, and risk premiums in electricity markets[☆]

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ABSTRACT

We model the impact of supply and demand on risk premiums in electricity futures, using daily data for 2003–2014. The model provides a satisfactory fit and allows for unspanned economic risk not embedded in futures prices. Model-implied spot risk premiums and forward biases are large, negative, highly time-varying, and exhibit plausible seasonal patterns. They differ from existing models, especially in periods of market turmoil, have not decreased in size over time, and help predict future returns. Both demand and supply have an economically significant impact on risk premiums. The risk premium associated with supply is characterized by large positive outliers.

1. Introduction

The modeling of electricity prices and risk premiums in electricity markets is a long-standing research question, but the existing literature is relatively limited. One strand of the literature (Pirrong and Jermakyan, 2008; Cartea and Villaplana, 2008) provides empirical evidence that risk premia depend on demand and supply variables. Another approach, commencing with Lucia and Schwartz (2002), applies no-arbitrage techniques in models with latent variables to price electricity futures. This approach explicitly distinguishes between the physical and risk-neutral model dynamics, and therefore allows for the estimation of risk premia. Heretofore, these literatures have developed independently.

This paper contributes to the understanding of the pricing of electricity derivatives, and hence of electricity risk premia, by integrating these two approaches. Specifically, we estimate a no-arbitrage model that provides a good fit to electricity futures prices, while also quantifying the impact of supply, captured by the natural gas price, and demand, captured by temperature, on these prices. Separately modeling demand and supply may be helpful for all commodity markets, but it is especially relevant for electricity markets because electricity is nonstorable. Temporary imbalances can therefore not be resolved by drawing from existing inventory (see for instance Bessembinder and Lemmon, 2002). The model also allows for unspanned economic risk, which is risk captured by supply and demand variables but not identified by the futures prices. We use this model to estimate risk premiums embedded in electricity futures and study the characteristics and implications of these risk premiums. The model allows a decomposition of risk premiums into several components, including the components due to supply and demand variables. Our empirical analysis reveals several new findings.

We first document that the economic variables (temperature and the natural gas price) contain useful information about the risk premiums in electricity futures. After controlling for the information in the electricity futures curve, the economic variables have incremental forecasting power for future spot prices and returns on electricity futures. Second, while the supply and demand variables contain additional information on electricity risk premiums, the principal components of the futures curve summarize the majority of the information on futures prices. Specifically, we show that a model based on the first three principal components of the futures curve provides a good fit of the entire electricity futures curve.

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Third, the estimated spot risk premium in the unspanned model is negative and very large. It is on average −1.67% per day, but it is highly time-varying and exhibits very large negative and positive outliers. For instance, during the 2014 polar vortex the spot risk premium for the unspanned model fluctuates between −40% and 100% per day.

Fourth, the spot risk premium implied by the model displays strong seasonal patterns. It is much larger (more negative) in the peak demand seasons of winter and summer. This finding is consistent with the insight of Pirrong and Jermakyan (2008) that electricity futures prices incorporate a premium to compensate for the risk of price spikes that are more likely during peak demand periods, or when costs spike due to fuel price shocks.

Fifth, we confirm the findings of Pirrong and Jermakyan (2008) and Cartea and Villaplana (2008) that risk premia depend on both demand and supply variables. However, the estimated spot risk premium in the unspanned model is very different from risk premiums implied by a spanned model or other models with demand and supply variables. Our model provides better forecasts of future spot prices and returns on electricity futures compared to the risk premium of models that ignore this unspanned risk. We also find that unspanned economic risk associated with supply is the most important component of the spot risk premium on electricity futures.

Sixth, the forward bias is also negative on average, implying that forward prices exceed expected spot prices. The average forward bias ranges from -$3 for the one-month maturity to -$7 for the twelve-month maturity, and is highly time-varying regardless of the maturity of the contract, but with larger fluctuations and outliers for shorter-maturity contracts. For instance, the day-ahead forward bias reaches a maximum of $380 and a minimum of -$480 during the polar vortex period. For longer maturities, the forward bias is much larger than its sample average for an extended period between 2006 and mid-2008.

What is the economic meaning of the large risk premia we find in these markets? The most likely explanation is that the risk premia are caused by barriers to the entry of risk bearing capital into these markets (Hirshleifer, 1988; Bessembinder and Lemmon, 2002). This may suggest that electricity markets are not fully integrated with the broader financial markets.

This paper is related to several strands of literature. An important literature uses reduced-form no-arbitrage models with latent variables to price electricity futures (see, for example, Lucia and Schwartz, 2002; Cartea and Figueroa, 2005; Deng and Oren, 2006; Geman and Roncoroni, 2006; Benth et al., 2008; Geman, 2009). Our proposed model nests this class of models but augments them with economic supply and demand variables. We find that the economic variables are important in explaining the risk premium associated with the electricity futures.

Another literature uses a more structural approach to price electricity futures. These papers use a bottom-up approach by first specifying the dynamics for supply and demand variables and then derive the spot price as a function of those variables. This approach is more intuitively appealing because it exploits the information contained in supply and demand variables suggested by economic theory (see, for example, Pirrong and Jermakyan, 2008; Cartea and Villaplana, 2008; Pirrong, 2011). We show that while this approach is economically appealing and while the economic variables are important for explaining the risk premium, latent factors significantly improve model fit. We also demonstrate that it is critical to model the supply and demand variables as unspanned.

Because our model contains supply and demand variables, it is also related to the literature which develops equilibrium models to study the determinants of the risk premium of electricity futures (Bessembinder and Lemmon, 2002; Longstaff and Wang, 2004; Dong and Liu, 2007; Douglas and Popova, 2008; Bunn and Chen, 2013). Finally, several related papers emphasize the importance of economic variables for modeling the risk premium of commodity futures (see, for example, Khan et al., 2016; Heath, 2019).

The remainder of the paper proceeds as follows. Section 2 describes the data and provides a discussion of the economics of electricity markets. Section 3 outlines model specification and estimation. Section 4 discusses the estimation results and Section 5 discusses the model's implications for risk premiums. Section 6 discusses several robustness exercises. Section 7 concludes.

2. Electricity markets

We estimate the model using electricity data for the PJM (Pennsylvania-New Jersey-Maryland) Western Hub market. We now discuss the institutional features of the PJM market, the electricity futures prices and returns we use in the empirical analysis, and the economic demand and supply variables used to explain these prices.

PJM is a "Regional Transmission Organization" that operates centralized day-ahead and real time markets for electricity. Operators of generation assets submit offers to the RTO that indicate the amount of power they are willing to generate as a function of price the day prior to the operating day. Consumers of electricity ("load") submit bids to purchase electricity, where bids can vary by time of day. The RTO aggregates the generation offers to construct a supply curve, and uses the bids to construct a demand curve. For each hour of the operating day, the RTO sets the *day-ahead forward price* equal to that which clears the market, i.e., sets quantity supplied equal to quantity demanded.

In reality, things are somewhat more complicated due to the fact that production and consumption of electricity are spatially dispersed, and there can be rather complex constraints on transmitting power over distance to move from generators to load. Based on the generation offers and load bids, the PJM RTO solves a constrained optimization program that maximizes the sum of consumer and producer surpluses, subject to the transmission constraints. The RTO sets the day-ahead forward prices for each transmission constraint location in the network equal to the shadow prices associated with that constraint produced by the solution to this optimization problem.

In real time, electricity demand can vary randomly, and differ from the amount forecast the day before, which is represented by the bids. Operation in real time requires exact balancing of generation and load, and must respect transmission constraints. As load varies over time and across the PJM region, the RTO dispatches generation to ensure the system remains in balance. To optimize dispatch, the RTO solves the surplus maximization constrained optimization problem, and sets the market clearing *spot prices* equal to the relevant shadow prices in this optimization problem.

In addition to the day-ahead and real-time markets for physical energy, there are derivatives markets on PJM electricity. In particular, there are cash-settled *futures contracts* on PJM electricity. One such contract is the Peak PJM Western Hub Real Time contract. This contract has a payoff based on the arithmetic average of the PJM Western Hub market clearing real time price for each peak hour (8AM-11PM) of the contract calendar month. The notional quantity in this contract is 2.5 megawatts (MW). This contract is traded on the CME.

In our empirical analysis, our main results are obtained using daily data.[1] We use real-time peak hour spot and day-ahead peak hour prices in the PJM Western Hub market, and the prices of PJM

[1] To analyze the impact of economic variables on electricity markets, the daily frequency is a good compromise. An investigation of the (substantial) intraday high-frequency variation in prices would be an interesting extension of our approach.

Table 1

Descriptive Statistics. Electricity Prices and Economic Variables We report descriptive statistics for electricity prices and supply and demand variables. Panel A reports the number of observations, the sample mean, median, standard deviation, skewness, kurtosis, the minimum, maximum, and autocorrelation coefficient of order 1 for the spot and futures prices. We report on PJM Western Hub real-time spot, day-ahead futures (F^{DA}), and PJM Western Hub real-time peak calendar-month 2.5 MW futures with maturities between 1 month and 12 months (F^1 to F^{12}). The unit of the price is Dollar per MWh. Panel B reports log returns in daily percentage returns on futures. Panel C reports the descriptive statistics for the raw natural gas prices (PX) and temperature (T). The sample period is from May 2003 to May 2014.

	Nobs	Mean	Median	Std	Skew	Kurt	Min	Max	AC1
Panel A: Prices									
Spot	2767	58.1649	50.0676	32.4250	6.5112	101.9091	21.2089	757.4217	0.6136
F^{DA}	2767	58.0342	50.4025	29.8561	6.8500	105.4550	20.6838	683.1615	0.7566
F^1	2767	58.0873	53.9700	18.6850	1.7388	7.3168	31.3500	164.7500	0.9866
F^2	2767	58.9385	53.5800	19.3147	1.7136	6.9351	33.0600	154.3300	0.9932
F^3	2767	59.5083	54.0000	19.1678	1.5195	5.5145	35.1700	140.0000	0.9937
F^4	2767	59.8302	54.2100	18.9369	1.3944	4.8339	34.1700	139.2300	0.9937
F^5	2767	59.9655	54.4300	18.2606	1.1540	3.8122	35.0400	128.2900	0.9932
F^6	2767	60.1071	54.4100	17.9742	1.0423	3.3712	34.1100	128.4200	0.9928
F^7	2767	60.4287	54.9400	18.1264	1.0697	3.7185	34.3200	128.4200	0.9932
F^8	2767	61.0319	55.6600	18.5374	1.0138	3.5116	35.3600	127.0800	0.9939
F^9	2767	61.3238	55.5200	18.5972	0.9314	3.0770	35.3200	116.2500	0.9943
F^{10}	2767	61.3462	55.2600	18.1073	0.8598	2.8350	35.0000	114.3300	0.9935
F^{11}	2767	61.2828	55.4500	17.7731	0.8668	2.8897	34.3000	119.5000	0.9930
F^{12}	2767	61.2449	56.2900	17.9752	1.0133	3.6591	34.2600	138.4900	0.9932
Panel B: Returns									
F^{DA}	2767	−0.0136	−0.0240	0.2739	0.1532	6.6377	−1.6341	1.8042	0.1273
F^1	2636	−0.0006	−0.0008	0.0288	0.1636	57.0526	−0.4564	0.4668	0.0086
F^2	2767	−0.0006	−0.0007	0.0205	0.0625	6.9904	−0.1314	0.1270	0.0445
F^3	2767	−0.0005	0.0000	0.0183	0.1165	6.2947	−0.0959	0.1199	0.0518
F^4	2767	−0.0004	0.0000	0.0170	0.0774	6.9048	−0.1219	0.1065	0.0453
F^5	2767	−0.0003	0.0000	0.0157	0.2453	6.1922	−0.0868	0.1059	0.0444
F^6	2767	−0.0002	0.0000	0.0151	0.4245	7.6511	−0.0780	0.1178	0.0581
F^7	2767	−0.0001	0.0000	0.0144	0.3852	7.0972	−0.0634	0.1121	0.0462
F^8	2767	−0.0002	0.0000	0.0137	−0.2018	7.7555	−0.1275	0.0823	0.0451
F^9	2767	0.0000	0.0000	0.0130	0.2239	5.4150	−0.0501	0.0794	0.0177
F^{10}	2767	−0.0001	0.0000	0.0129	0.1182	5.1589	−0.0654	0.0633	0.0044
F^{11}	2767	−0.0001	0.0000	0.0129	0.2080	6.0141	−0.0628	0.0871	0.0302
F^{12}	2767	0.0002	0.0000	0.0128	0.3376	6.3818	−0.0654	0.0763	0.0719
Panel C: Economic Variables									
PX	2767	6.3659	5.7500	4.1495	10.6412	211.3608	1.9900	99.6600	0.7445
T	2767	56.5667	57.5000	17.2710	−0.2596	2.0204	9.5000	90.5000	0.9167

Western Hub real-time peak calendar-month 2.5 MW futures. The hourly real-time and day-ahead prices are downloaded from the PJM website and averaged over the day.[2] We model the day-ahead price as a short-term futures contract which matures in one day. Data on the PJM Western Hub real-time peak calendar-month 2.5 MW futures contracts are obtained from the CME. We include futures contracts with maturities of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 months. Each day, the sample therefore consists of thirteen futures prices and the spot price. The sample period is from May 1, 2003 to May 30, 2014.

PJM futures markets are quite liquid. Total open interest for PJM futures contracts as of the end of our sample period was 8,358,662 contracts, on three different exchanges, of which 76% was held by long commercials and 83% by short commercials.

The challenges in modeling these markets are apparent from Fig. 1 and Table 1. Panel A of Fig. 1 plots the time series of the daily spot prices. Panel A of Table 1 indicates that the average spot price over the sample is $58.16, but the fluctuations around this mean are enormous, with a minimum price of $21.21 and a maximum price of $757.42 over the sample period. The first row of Panel B of Table 1 reports descriptive statistics for log returns. The standard deviation of the daily log return is 27%, close to the 34% reported by Bessembinder and Lemmon (2002) for percentage returns in the 1997–2000 sample period. For comparison, the standard deviation for daily log returns on the *S*&*P*500 over our sample is half a percent.

Panel A of Table 1 indicates that for futures with a maturity of more than one month, the futures price on average exceeds the realized spot price.[3] The price of the twelve-month futures contract is $61.24 on average, or on average $3.08 higher than the average realized spot price. The differences in higher moments are larger. Compare the time series of the daily twelve-month futures price in Panel C of Fig. 1 with the spot price in Panel A. The futures price also fluctuates considerably, but these fluctuations are much smaller, resulting in a smaller standard deviation. The differences are even larger for the third fourth moments. The much lower kurtosis of the twelve-month contract is clearly visible in Fig. 1. The spot price in Panel A is very jagged and the futures price in Panel C is much smoother. Some of the stylized facts in the data are similar to the 1997–2003 sample of Pirrong and Jermakyan (2008), who argue that right skewness in forward prices induces left skewness in the payoffs to short forward positions, which requires a large risk premium for selling forward contracts.

[2] See https://dataminer.pjm.com/dataminerui/pages/public/lmp.jsf.

[3] For the shorter-maturity contracts, the average price is less than the average spot price, but the median price exceeds the median spot price.

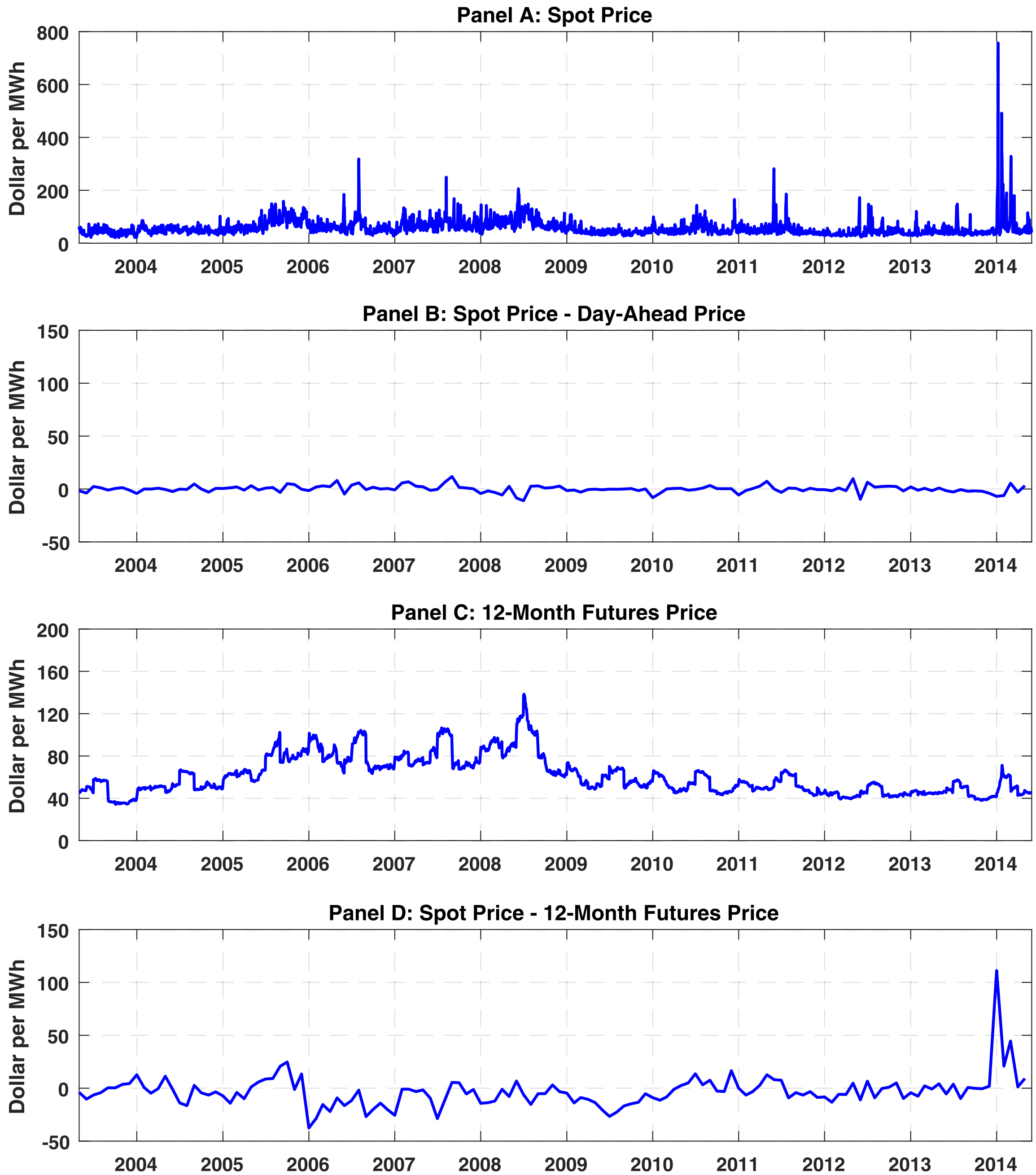

Fig. 1. Electricity Spot Prices and Futures Prices. We plot the daily spot price (Panel A), the difference between the spot price and the day-ahead price (Panel B), the price of the 12-month futures contract (Panel C), and the difference between them (Panel D) over the entire sample period. The difference is calculated as the average of daily differences in each month. The sample period is from May 2003 to May 2014.

The spot price in Panel A of Fig. 1 does not exhibit a pronounced trend in our sample and usually evolves in a fairly narrow range, but it is characterized by very sharp positive spikes, with a maximum of \$757.42 during the time of the polar vortex in 2014, and other large spikes in 2005, 2006, and 2008. This results in a large positive skewness of spot prices, consistent with the evidence in Bessembinder and Lemmon (2002) and Pirrong and Jermakyan (2008). The maximum value of the twelve-month futures contract is \$138.49, which occurs in 2008. The maximums for the spot and twelve-month futures prices therefore occur at different times. Panel D of Fig. 1 plots the difference between the spot price and the twelve-month futures price. We report monthly averages in Panel D, because for daily differences the extreme observations completely dominate the figure, and as a result it is not informative. As indicated in Table 1, the difference is negative on average, but Panel D indicates that it is also often positive. Similar observations apply to the other futures contracts. Finally, Panel B of Fig. 1 plots the difference between the spot price and the day-ahead price. This difference frequently changes sign, which makes the one-day contract very risky.

Panel A of Table 1 presents descriptive statistics on prices, but when we report on risk premiums we are effectively using (log) returns. Panel B of Table 1 therefore reports descriptive statistics for the relevant log returns. Not surprisingly, log returns have very different statistical properties. In our sample, the electricity spot price at the end of the sample is lower than at the start of the sample, which gives a negative average log spot return. The most important observation is that the spot price as well as the day-

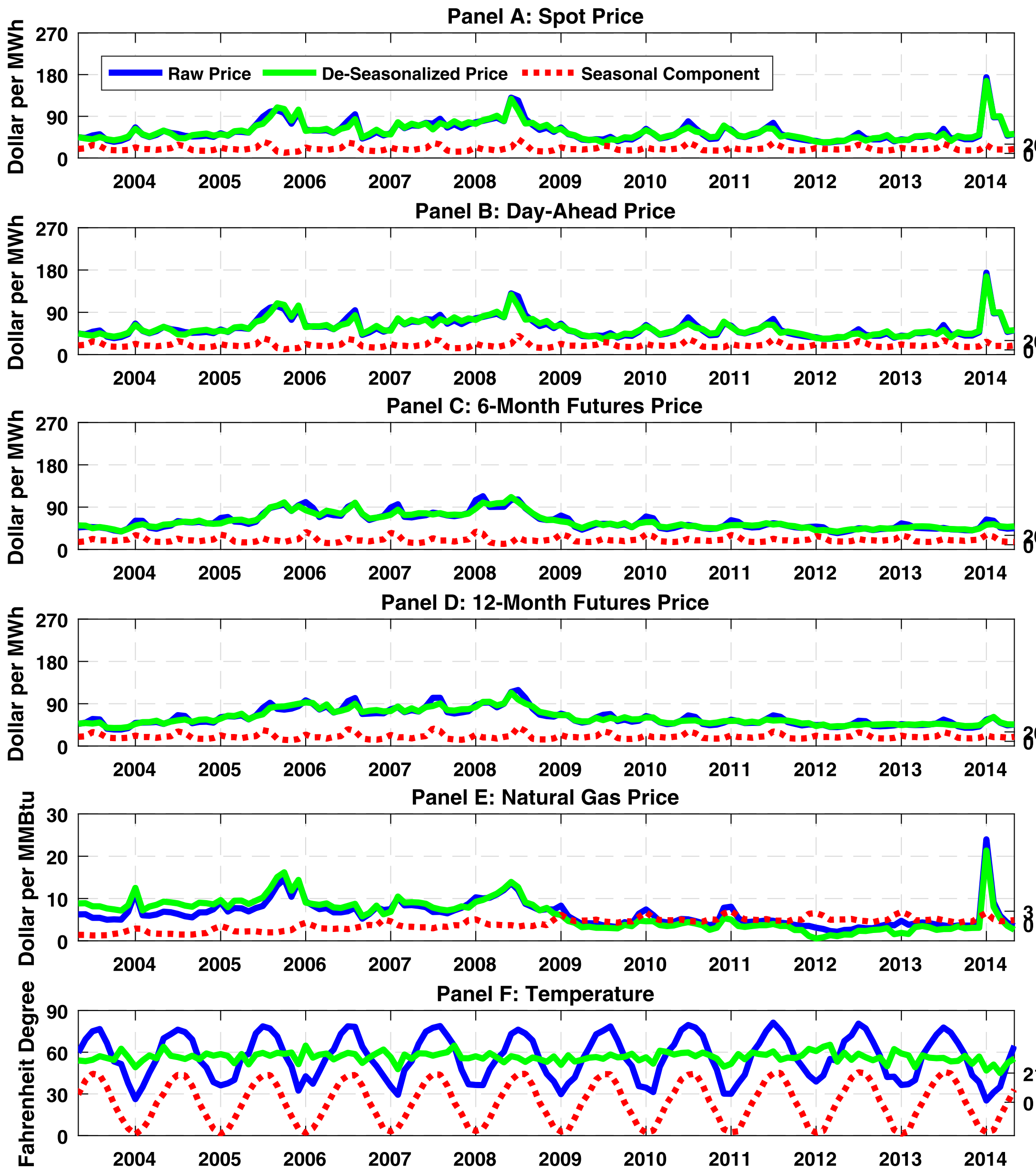

Fig. 2. Electricity Prices and Economic Variables. We plot the average spot price (Panel A), the day-ahead price (Panel B), the price of the 6-month futures contract (Panel C), the price of the 12-month futures contract (Panel D), the average natural gas price (Panel E), and the temperature (Panel F) in each month of the sample period. In each panel, we plot the raw price (the blue solid line), the seasonal component (the dotted line), and the de-seasonalized price (the green solid line). The sample period is from May 2003 to May 2014. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

ahead price are characterized by large positive kurtosis, but for the log returns the kurtosis is much smaller. Also, instead of the large positive skewness in prices, the skewness in returns is small. These results are partly due to the difference between returns and prices, and partly due to the use of log returns, because the logarithms effectively reduce the impact of outliers. The average return on most futures contracts is also negative.

Seasonalities are extremely important in electricity markets. We follow the existing literature and de-seasonalize the electricity prices as well as the economic variables. The de-seasonalization method is discussed in Section 3 below. Panels A-D of Fig. 2 plot the raw price, the seasonal component, and the de-seasonalized price for the spot price, the day-ahead contract and the 6-month and 12-month futures contracts. In order to better highlight the seasonalities we plot monthly averages rather than daily prices, which contain much high-frequency variation that is irrelevant for illustrating seasonalities. The seasonal patterns in the price data are readily evident from Fig. 2.

The economic data include demand and supply variables. Following Pirrong and Jermakyan (2008), we use the natural gas price (PX) as the supply variable. We utilize the price of natural gas as the supply variable because gas-fired generating units usually produce the marginal megawatt, and hence the price of gas is a primary determinant of the marginal cost of production, given that capacity is fixed in the short run. We obtain daily spot natural gas middle prices for Columbia Gas and Texas Eastern Pipeline zone M-3 from Bloomberg. The demand variable is the temperature. The temperature data is from the National Climatic Data Center (NCDC). We use the daily average temperature for Washington, D.C. and use it as a temperature proxy for the PJM Western Hub market. Panel C of Table 1 reports summary statistics for the supply and demand variables. Panel E-F of Fig. 2 plots the time series of the natural gas price and temperature. Again the seasonal patterns in the economic data are readily evident from Fig. 2.

It is instructive to compare the patterns in the spot and futures data with those in the economic variables in Fig. 2. For the supply variable in Panel E of Fig. 2, the natural gas price, it can be clearly seen that the spike in the natural gas price at the start of 2014 is accompanied by a large spike in the spot and day-ahead prices, but a much smaller increase in the twelve-month futures price. On the other hand, increases in the natural gas price in 2005 and 2008 are accompanied by increases in spot as well as futures prices in Fig. 2. It is less obvious to detect relations between the demand variable, temperature, and the price data in Fig. 2, perhaps partly because the raw data contain such strong seasonalities.

3. Models for electricity futures

This section presents three different models of electricity futures prices. We first outline a general affine framework which nests these three models. We then discuss the unspanned model, the spanned model with latent variables, and the spanned model with economic variables.

3.1. A Class of affine models

We outline a class of affine models, which nests the models that we investigate in our empirical work. Suppose that there are N state variables that fully determine the state of the electricity market. These variables can be latent variables or economic (demand and supply) variables. Generally denote this vector of state variables by X. We assume X follows a Gaussian VAR under the P measure, where the P-dynamic of X is denoted as follows:

$$X_{t+1} = \text{Seas}_{X,t+1} + K_0^P + K_1^P \times X_t + \Sigma^P \times \epsilon_{t+1}^P \tag{1}$$

where X_t is the state vector at time t, $\text{Seas}_{X,t}$ is an N by 1 vector denoting the seasonal component of the state variables, K_0^P is an N by 1 vector, K_1^P is an N by N matrix, Σ^P is an N by N upper triangular matrix, and ϵ_t^P is an N by 1 vector of independent standard normal innovations.[4]

The stochastic discount factor (SDF) is assumed to be of the following form:

$$\text{SDF}_{t+1} = e^{(\Lambda_0 + \Lambda_1 \times X_t)' \times \epsilon_{t+1}} \tag{2}$$

where Λ_0 is an N by 1 vector and Λ_1 is an N by N matrix.

Given these assumptions, we have the following dynamic of the state variables under the risk-neutral measure Q:

$$X_{t+1} = \text{Seas}_{X,t+1} + K_0^Q + K_1^Q \times X_t + \Sigma^Q \times \epsilon_{t+1}^Q \tag{3}$$

where K_0^Q is an N by 1 vector, K_1^Q is an N by N matrix, Σ^Q equals to Σ^P, and ϵ_t^Q is an N by 1 vector of independent standard normal innovations.

As in the log price model in Lucia and Schwartz (2002), we assume that the natural logarithm of the electricity spot price is a linear function of the state variables. Denoting the natural logarithm of the spot price S_t at time t as s_t, this gives:

$$s_t = \text{Seas}_{s,t} + \rho_0 + \rho_1 \times X_t \tag{4}$$

where ρ_0 is a scalar, ρ_1 is an 1 by N matrix, and $\text{Seas}_{s,t}$ is a scalar denoting the seasonal component of the log spot rate. This seasonal component is a scaled version of the seasonal component of the state vector.

Based on Eq. (4), futures prices can be derived recursively. Denoting the log price of the futures contract with maturity j at time t as f_t^j, we can show that f_t^j is given by

$$f_t^j = \text{Seas}_{f,t+j} + A_j + B_j \times X_t \tag{5}$$

where $\text{Seas}_{f,t+j}$ denotes the seasonal component of the forward contract with maturity $t+j$ and

$$A_j = A_{j-1} + B_{j-1} K_0^Q + \frac{1}{2} B_{j-1} \Sigma^Q \Sigma'^Q B'_{j-1} \tag{6}$$

$$B_j = B_{j-1} K_1^Q \tag{7}$$

$$A_0 = \rho_0 \quad \text{and} \quad B_0 = \rho_1 \tag{8}$$

3.2. The unspanned model

We now assume that there are N^S state variables that fully determine the price of the electricity futures. Denote the vector of those state variables as X^S. The unspanned model assumes that the information in the futures price can only span part of the information in the economy. Denote the part that cannot be explained, or the unspanned part, by US_t, and rewrite X_t as follows.

$$X_t = \begin{bmatrix} X_t^S \\ \text{US}_t \end{bmatrix} \tag{9}$$

where $X_t^S \cup \text{US}_t = X_t$ and $X_t^S \cap \text{US}_t = 0$. Substituting Eq. (9) into Eq. (1), we get the P-dynamic of the unspanned model.

$$\begin{bmatrix} X_{t+1}^S \\ \text{US}_{t+1} \end{bmatrix} = \text{Seas}_{X,t+1} + K_0^P + K_1^P \times \begin{bmatrix} X_t^S \\ \text{US}_t \end{bmatrix} + \Sigma^P \times \epsilon_{t+1}^P \tag{10}$$

In these models, the variables can be rotated, which means that we can re-define an equivalent model that is written in terms of different variables. In our empirical work, we rotate the unspanned part of economic variables to the economic variables EC_t themselves in order to provide a more intuitive interpretation of the estimated coefficients. We therefore estimate the following version of the unspanned model:

$$X_t = \begin{bmatrix} X_t^S \\ \text{EC}_t \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} X_{t+1}^S \\ \text{EC}_{t+1} \end{bmatrix} = \text{Seas}_{X,t+1} + K_0^P + K_1^P \times \begin{bmatrix} X_t^S \\ \text{EC}_t \end{bmatrix} + \Sigma^P \times \epsilon_{t+1}^P \tag{12}$$

$$X_{t+1}^S = \text{Seas}_{X^S,t+1} + K_0^Q + K_1^Q \times X_t^S + \Sigma^Q \times \epsilon_{t+1}^Q \tag{13}$$

$$\text{SDF}_{t+1} = e^{(\Lambda_0 + \Lambda_1 \times X_t)' \times \epsilon_{t+1}} \tag{14}$$

$$s_t = \text{Seas}_{s,t} + \rho_0 + \rho_1 \times X_t^S \tag{15}$$

Joslin et al. (2014) model the term structure of interest rates using this model and Heath (2019) uses a similar model to study oil futures. Joslin et al. (2014) show that under certain assumptions, one can use principal components (PCs) of the futures data to estimate the unspanned model. Moreover, they show that it is possible to obtain consistent estimates of the P- and Q-parameters by breaking up the estimation problem in two parts. We follow Joslin et al. (2014) and use the PCs of the electricity futures prices as the state variables under the risk neutral measure Q. We augment the PCs with economic variables to get the state vector under the physical measure P. Because the PCs and the economic variables are both observed, we can use a vector autoregressive approach to estimate the physical dynamic given in Eq. (12). Subsequently, we estimate the Q parameters in Eq. (13) by minimizing the root mean squared error based on the difference between observed futures prices and model prices.

[4] Given the sharp spikes in the data, a natural alternative to this specification would be to use jump processes. This approach has been pursued in no-arbitrage models of electricity markets by, among others, Johnson and Barz (1999) and Geman and Roncoroni (2006). Note that it is possible to model jump intensities as functions of economic variables using methods that are similar to the ones we consider here. We keep this extension for future research.

3.3. The spanned model with latent factors

To highlight the importance of the unspanned relation between the demand and supply variables and the latent variables, we consider alternative models which remove the unspanned economic component. The first alternative model removes the unspanned economic component by dropping the economic variables. We refer to this model as the spanned model with latent factors. In this model with latent factors, the state variables under both the P- and Q-dynamics are equal to X^S. The dynamics for this model are:

$$X^S_{t+1} = \text{Seas}_{X^S,t+1} + K^P_0 + K^P_1 \times X^S_t + \Sigma^P \times \epsilon^P_{t+1} \quad (16)$$

$$X^S_{t+1} = \text{Seas}_{X^S,t+1} + K^Q_0 + K^Q_1 \times X^S_t + \Sigma^Q \times \epsilon^Q_{t+1} \quad (17)$$

$$\text{SDF}_{t+1} = e^{(\Lambda_0 + \Lambda_1 \times X^S_t)' \times \epsilon_{t+1}} \quad (18)$$

$$s_t = \text{Seas}_{s,t} + \rho_0 + \rho_1 \times X^S_t \quad (19)$$

This model belongs to a class of reduced-form models that only use latent factors to price futures. In this class of models, Lucia and Schwartz (2002) propose two models that are based on the log power price. Both those models are closely related to the model in this section.

We estimate the spanned model with latent factors using a method very similar to the one used for the unspanned model. First, use the PCs of the futures curve to estimate the P-parameters in Eq. (16) using a vector autoregressive approach. Subsequently the Q parameters in Eq. (17) are estimated by minimizing the root mean squared error.

3.4. The spanned model with economic variables

Another special case of the unspanned model is a model without latent variables. In this case, the state variables under the physical measure only consist of economic variables.[5] The same economic variables are also the state variables under the risk-neutral measure Q and thus fully determine the prices of electricity futures. The futures prices fully span the economy, and conversely the economic variables are fully spanned by the electricity futures. We refer to this model as the spanned model with economic variables. The resulting model is related to the framework of Pirrong and Jermakyan (2008) and Pirrong (2011), who exclusively use demand and supply variables to price electricity futures. In summary, this model is given by:

$$\text{EC}_{t+1} = \text{Seas}_{EC,t+1} + K^P_0 + K^P_1 \times \text{EC}_t + \Sigma^P \times \epsilon^P_{t+1} \quad (20)$$

$$\text{EC}_{t+1} = \text{Seas}_{EC,t+1} + K^Q_0 + K^Q_1 \times \text{EC}_t + \Sigma^Q \times \epsilon^Q_{t+1} \quad (21)$$

$$\text{SDF}_{t+1} = e^{(\Lambda_0 + \Lambda_1 \times \text{EC}_t)' \times \epsilon_{t+1}} \quad (22)$$

$$s_t = \text{Seas}_{s,t} + \rho_0 + \rho_1 \times \text{EC}_t \quad (23)$$

The economic variables are observed and thus we can estimate the P-dynamic in Eq. (20) using a vector autoregressive approach. Then, we use the economic variables as the state variables under Q and we estimate the Q-dynamic in Eq. (21) by minimizing the dollar root mean squared errors.

[5] Strictly speaking, it is incorrect to refer to this model as being nested by the unspanned model. In the unspanned model, the state variables under P consist of the unspanned part of the economic variables, whereas in a spanned model with economic variables, the state variables are the economic variables themselves. We can refer to the unspanned part of the economic variables as the economic variables due to the rotation.

3.5. Modeling the seasonal component

We specify the seasonal component of the log of the electricity spot price following Lucia and Schwartz (2002). For the log spot rate:

$$\text{Seas}_{s,t} = \beta_1 \times M_1(t) + \beta_2 \times M_2(t) + \cdots + \beta_{12} \times M_{12}(t) \quad (24)$$

where $M_i(t), i = 1, 2, \ldots, 12$ are monthly dummies. For example, $M_1(t)$ is defined as follows.

$$M_1(t) = \begin{cases} 1, & \text{if t is in January} \\ 0, & \text{otherwise} \end{cases} \quad (25)$$

The other $M_i(t), i = 2, 3, \ldots, 12$ are defined similarly.

Following Lucia and Schwartz (2002), we first use OLS to estimate the following regression to get the seasonal component of the log spot price.

$$s_t = \beta_1 \times M_1(t) + \beta_2 \times M_2(t) + \beta_3 \times M_3(t) + \cdots + \beta_{12} \times M_{12}(t) + \epsilon_t \quad (26)$$

Then we de-seasonalize the log spot price and obtain the corresponding de-seasonalized series.

$$s_{t,DS} = s_t - (\hat{\beta}_1 \times M_1(t) + \hat{\beta}_2 \times M_2(t) + \cdots + \hat{\beta}_{12} \times M_{12}(t)) \quad (27)$$

The de-seasonalized log futures prices are obtained by adjusting the raw log futures prices with the value of the seasonal component of the spot rate in the month when the futures mature. The definition of de-seasonalized futures price is thus as follows.

$$f^j_{t,DS} = f^j_t - (\hat{\beta}_1 \times M_1(t+j) + \hat{\beta}_2 \times M_2(t+j) + \cdots + \hat{\beta}_{12} \times M_{12}(t+j)) \quad (28)$$

We use the de-seasonalized series in Eqs. (27) and (28) to estimate the model parameters.

This de-seasonalization approach deserves some comment. Theory suggests that the risk premiums in futures prices, and hence futures prices themselves, depend on the likelihood and magnitude of price spikes (Bessembinder and Lemmon, 2002; Pirrong and Jermakyan, 2008). Furthermore, the likelihood of price spikes is seasonal because spikes are more likely to occur when capacity utilization is high, which is most likely during seasonal demand peaks that occur in the summer and winter in the United States. Thus, risk premia are likely to be seasonal.

Deseasonalizing futures prices themselves using standard techniques would make it impossible to detect any such seasonality in risk premia. The approach we implement quantifies the seasonality in the expectation of the spot price under the physical measure, and by removing this seasonal component we can identify seasonalities in the risk premium.

For the economic variables, the de-seasonalized series is defined as follows:

$$\text{De-Seasonalized EC}_t = \text{EC}_t - \text{Expected EC}_t \quad (29)$$

where EC_t is the level of economic variable in time t and Expected EC_t is the expected level of economic variable in time t. We use four different methods to estimate the Expected EC_t to ensure our results are robust:

$$\begin{array}{ll} \text{Method I:} & \text{Expected EC}_t = \text{Average EC}_t \text{ in past K years} \\ \text{Method II:} & \text{Expected EC}_t = \text{Average EC}_t \text{ estimated by a} \\ & \text{non-parametric regression with Gaussian kernel} \\ \text{Method III:} & \text{Expected EC}_t = \text{Average EC}_t \text{ estimated by a} \\ & \text{non-parametric regression with Epanechnikov kernel} \\ \text{Method IV:} & \text{Expected EC}_t = \text{Average EC}_t \text{ estimated by a} \\ & \text{parametric regression with a trigonometric function} \end{array} \quad (30)$$

We use Method I with K = 10 as our benchmark implementation. For example, for June 15th 2018, the expected temperature

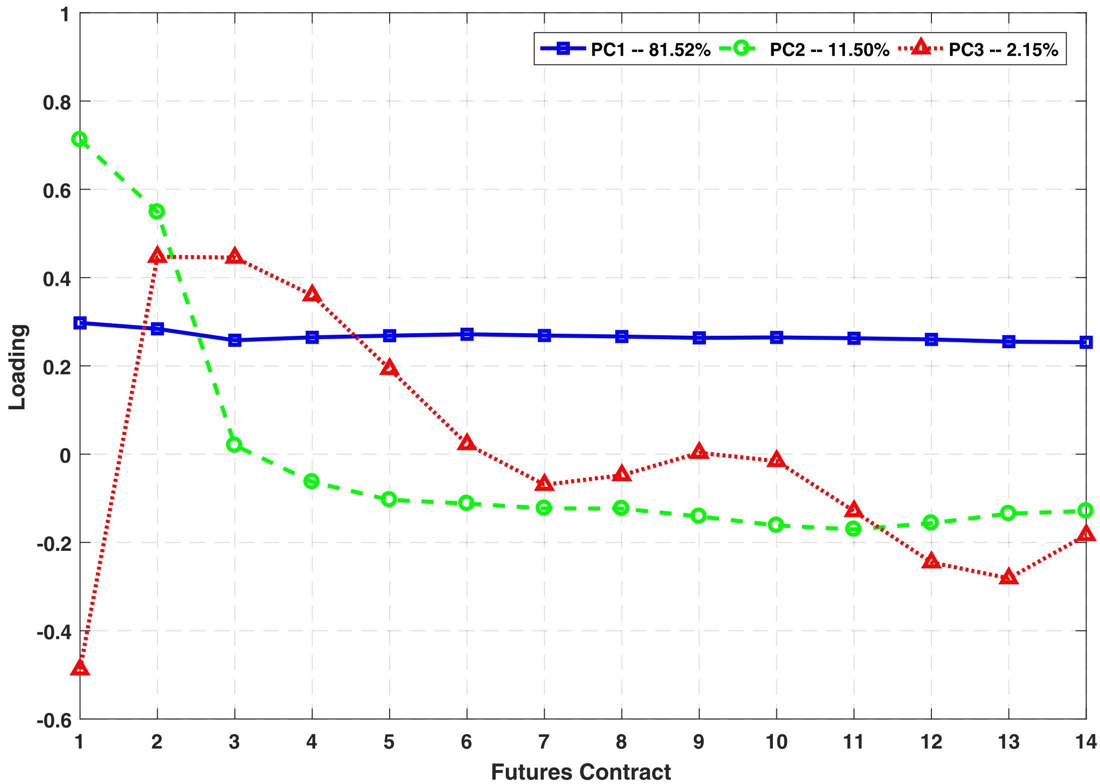

Fig. 3. Principal Components of Electricity Prices. We plot the loadings of the first three principal components (PCs) of log electricity prices. The first PC is represented by the line with squares, the second PC is represented by the line with circles, and the third PC is represented by the line with triangles. The legend displays the fraction of the total variance explained by each of the principal components. The sample period is from May 2003 to May 2014.

is the average temperature on June 15th for the years 1998 to 2017. We use the other three methods in our robustness analysis. The details of these three alternative methods are provided in Appendix A.2.

4. Model estimates

We first establish that electricity futures prices can be adequately summarized by their first three principal components (PCs). Then we show that the demand and supply variables contain additional information beyond the PCs, suggesting that they are unspanned by the electricity futures. We then estimate the model, discuss the fit and economic implications of the unspanned model, and compare it with other models. We also discuss the implications of the spanning assumption.

4.1. The information in the principal components of electricity futures and the economic variables

We investigate if the supply and demand variables are spanned by the electricity futures. To this end we first need to parsimoniously represent the information in the electricity futures. We use principal component analysis to analyze the electricity futures curve. Figure 3 shows the loadings of the first three principal components (PCs) and the fraction of total variance explained by each PC. The first three PCs explain more than 95% of the total variation of the price of electricity futures. We therefore conclude that most information in the electricity futures curve can be adequately summarized by the first three PCs.

The interpretation of these three PCs is similar to that of yield curve PCs. The loadings on the first PC are virtually identical for all maturities from one day to 12 months as well as the spot, meaning that this first component affects the prices of all maturities similarly, and therefore causes parallel shifts in the forward curve; this is a level effect. The time series of PC^1 is therefore very similar to Panel C of Fig. 1. The loadings of PC^2 are large and positive for short maturities, and negative and relatively small (in absolute value) for longer maturities. This PC corresponds to the slope of the futures curve. The time series of PC^2 is highly correlated with Panel D of Fig. 1, but note that Panel D of Fig. 1 reports monthly averages. The time series of PC^2 therefore contains much more short-term variation. The loadings of PC^3 are large and negative for both short and long maturities. This PC corresponds to the curvature of the forward curve.

We next verify if the economic supply and demand variables (EC) contain unspanned information that affect the risk premium in electricity futures. We use the supply and demand variables to forecast returns on electricity futures and changes in the first three PCs. The regressions are specified as follows:

$$\begin{aligned} \text{Ret}_{t\to t+1}^{DA-12} &= \text{Const.} + \beta_{pc}\text{PC}_t^{1-5} + \beta_{EC}\text{EC}_t + \epsilon_t \\ \Delta\text{PC}_{t\to t+1}^{1-3} &= \text{Const.} + \beta_{pc}\text{PC}_t^{1-5} + \beta_{EC}\text{EC}_t + \epsilon_t \end{aligned} \tag{31}$$

If the ECs are unspanned by the electricity futures, the loading on EC in this forecasting regression should be statistically significant and the adjusted R^2 should increase when adding ECs to the set of regressors.[6]

The results in Table 2 indicate that this is indeed the case. Regardless of whether we use returns on electricity futures (Panel A) or changes in PCs (Panel B), the loading on the natural gas price (PX) and the temperature (T) are significant and positive

[6] Alternatively one can implement these regressions in two stages. See for example Heath (2019). In a first stage one finds the unexplained part of the economic variables by regressing the economic variables on the PCs. In the second stage the unexplained components of the economic variables are used in the forecasting regressions. We verified that this approach gives similar results.

Table 2

Spanning Regressions. We forecast returns on different futures and changes in the first three principal components using the unspanned component of demand and supply variables, controlling for lagged values of the PCs. The regression is specified as follows:

$$\begin{aligned} \mathrm{Ret}^{DA-12}_{t\to t+1} &= \mathrm{Const.} + \beta_{pc}\mathrm{PC}^{1-5}_t + \beta_{EC}\mathrm{EC}_t + \epsilon_t \\ \Delta\mathrm{PC}^{1-3}_{t\to t+1} &= \mathrm{Const.} + \beta_{pc}\mathrm{PC}^{1-5}_t + \beta_{EC}\mathrm{EC}_t + \epsilon_t \end{aligned}$$

For all regressions, standard errors are adjusted by Newey-West (1987) method with 8 lags. The sample period is from May 2003 to May 2014.

Panel A: Returns								
	PX_t	SE	T_t	SE	Adj. R^2 (PCs only)	Adj. R^2 (PCs + EC)	DM Stat	Nobs
$\mathrm{Ret}^{DA}_{t\to t+1}$	1.3903	0.2772	0.2192	0.0698	0.0585	0.0909	2.1476	2766
$\mathrm{Ret}^{1}_{t\to t+1}$	0.0369	0.0259	0.0179	0.0062	0.0120	0.0154	1.3391	2635
$\mathrm{Ret}^{2}_{t\to t+1}$	0.0251	0.0106	0.0171	0.0049	0.0080	0.0131	1.7179	2766
$\mathrm{Ret}^{3}_{t\to t+1}$	0.0089	0.0076	0.0155	0.0044	0.0042	0.0082	1.7665	2766
$\mathrm{Ret}^{4}_{t\to t+1}$	0.0118	0.0068	0.0137	0.0042	0.0029	0.0066	1.7418	2766
$\mathrm{Ret}^{5}_{t\to t+1}$	0.0081	0.0071	0.0131	0.0039	0.0017	0.0056	1.7434	2766
$\mathrm{Ret}^{6}_{t\to t+1}$	0.0107	0.0063	0.0123	0.0038	0.0023	0.0061	1.7409	2766
$\mathrm{Ret}^{7}_{t\to t+1}$	0.0090	0.0056	0.0089	0.0037	0.0028	0.0048	1.3400	2766
$\mathrm{Ret}^{8}_{t\to t+1}$	0.0056	0.0057	0.0101	0.0034	0.0053	0.0082	1.4667	2766
$\mathrm{Ret}^{9}_{t\to t+1}$	0.0106	0.0051	0.0102	0.0034	0.0047	0.0084	1.6538	2766
$\mathrm{Ret}^{10}_{t\to t+1}$	0.0063	0.0048	0.0083	0.0033	0.0051	0.0072	1.2797	2766
$\mathrm{Ret}^{11}_{t\to t+1}$	0.0118	0.0075	0.0092	0.0032	0.0030	0.0062	1.6793	2766
$\mathrm{Ret}^{12}_{t\to t+1}$	0.0236	0.0123	0.0083	0.0032	0.0045	0.0100	3.3371	2766
Panel B: Change of PCs								
$\Delta\mathrm{PC}^{1}_{t\to t+1}$	0.0079	0.0010	0.0011	0.0003	0.1292	0.1721	1.5320	2766
$\Delta\mathrm{PC}^{2}_{t\to t+1}$	−0.0152	0.0020	−0.0012	0.0006	0.1721	0.2130	1.4595	2766
$\Delta\mathrm{PC}^{3}_{t\to t+1}$	0.0005	0.0002	0.0001	0.0001	0.0450	0.0455	0.7501	2766

in most cases, suggesting that economic variables impact the future realized changes in the futures prices and PCs. Moreover, after including the economic variables, the adjusted R^2 substantially increases. The increment is statistically significant based on the Diebold and Mariano (1995) test. Higher adjusted R^2s mean that the demand and supply variables contain useful information about future changes in electricity prices, which provides strong support for the hypothesis that the demand and supply variables are unspanned by the electricity futures.[7] This finding is perhaps not surprising. The importance of demand and supply in electricity markets has been documented by Pirrong and Jermakyan (2008) and Cartea and Villaplana (2008). Heath (2019) shows the unspanned nature of supply and demand in crude oil markets. Because of the non-storability of electricity, we would expect such a result to hold a fortiori in electricity markets.

There are two economic explanations for a factor that is material to prices and yet unspanned: Either risk premiums and the effect on the price forecast are exactly offsetting, or market participants ignore or overreact to the factor. We now provide evidence that supports the latter explanation.[8] The full-spanning condition in a model with macro factors requires not only that the macro factors cannot contribute forecasting power for returns after conditioning on current prices, but also that the macro factors cannot contribute forecasting power for the macro factors themselves after conditioning on current prices. In other words, when putting future values of PX and T on the left hand side of the regressions, the current values of PX and T should not load. Table A.1 in the Appendix presents the results of this regression. Current economic variables help to predict future value of economic variables, even after controlling for the PCs of electricity prices. This provides additional evidence against full spanning and supports the interpretation that market participants ignore or overreact to economic variables.

4.2. The dynamics of the unspanned model

Now that we have established that the demand and supply variables are unspanned by the electricity futures, we proceed to estimate the physical (P) and risk neutral (Q) dynamics of the unspanned model.

Panel A of Table 3 reports the risk-neutral model estimates. The upper left entry of the K_1^Q matrix is very close to one and highly statistically significant. This parameter captures the persistence of the model implied spot price under the risk neutral measure. The deseasonalized spot price is close to a unit root process under this measure. This of course reflects not only the dynamic of the spot price under the physical measure, but also the risk premium.

The loading of PC^1 on PC^2 is negative and statistically significant. A larger PC^2 indicates a flatter slope, thus the negative sign indicates that the level of the electricity price will decrease when the slope flattens. The loading of PC^1 on PC^3 suggests that PC^3 also impacts the level of the electricity price.

The loadings on the second row of K_1^Q are all significant, indicating that the slope of the futures curve is affected by the level, itself, and the curvature of the futures curve. Finally, the bottom row of K_1^Q indicates that the curvature is related to both the level and slope of the futures curve.

Panel B of Table 3 reports the estimated P dynamic for the unspanned model. Not surprisingly, the first PC, which captures the level of the futures prices, is much more persistent than the second PC, which captures the slope, and the third PC, which captures the curvature. Nevertheless, the loading of PC^1_{t+1} on PC^1_t is 0.96, which is not very high given that the data are daily and we are investigating the pricing implications of this factor one month or one year ahead. The supply and demand variables are stationary: shocks to the supply and demand variables do not persist, and

[7] Longstaff and Wang (2004) find that electricity risk premiums are related to the conditional volatilities of demand and the spot price. We therefore repeated the forecasting regressions with these volatilities as additional regressors. The additional explanatory power of the economic variables in excess of the control variables is very similar.

[8] We would like to thank an anonymous referee for this suggestion.

Table 3
Estimates of the Unspanned Model. We report the estimated Q- and P-dynamics of the unspanned model. The Q- and P-dynamics are specified as follows:

$$\text{States in } Q_{t+1} = K_0^Q + K_1^Q \times \text{States in } Q_t + \Sigma^Q \times \epsilon_{t+1}^Q$$
$$\text{States in } P_{t+1} = K_0^P + K_1^P \times \text{States in } P_t + \Sigma^P \times \epsilon_{t+1}^P$$

where ϵ_t^Q and ϵ_t^P are standard Brownian motions, and Σ^Q is the left upper 3 by 3 sub-matrix of Σ^P. The state variables in the Q-dynamic are PC1, PC2, and PC3. The state variables in the P-dynamic are PC1, PC2, PC3, natural gas price, and temperature. Standard errors are reported in parentheses. The sample period is from May 2003 to May 2014.

Panel A: Q-Dynamic

	K_0^Q	K_1^Q		
		$PC1_t$	$PC2_t$	$PC3_t$
$PC1_{t+1}$	−0.0667	0.9906	−0.1484	0.3106
	(0.0023)	(0.0004)	(0.0024)	(0.0551)
$PC2_{t+1}$	0.0476	−0.0197	0.6925	0.6948
	(0.0011)	(0.0009)	(0.0057)	(0.1216)
$PC3_{t+1}$	0.0264	0.0043	0.0549	0.8402
	(0.0050)	(0.0009)	(0.0110)	(0.0058)

Panel B: P-Dynamic

	K_0^P	K_1^P				
		$PC1_t$	$PC2_t$	$PC3_t$	PX_t	T_t
$PC1_{t+1}$	−0.0080	0.9656	−0.1445	0.0602	0.0083	0.0009
	(0.0024)	(0.0031)	(0.0067)	(0.0155)	(0.0007)	(0.0003)
$PC2_{t+1}$	−0.0157	−0.0678	0.6660	0.2288	0.0166	0.0009
	(0.0047)	(0.0062)	(0.0133)	(0.0310)	(0.0013)	(0.0006)
$PC3_{t+1}$	−0.0010	0.0037	0.1019	0.6189	0.0009	0.0003
	(0.0023)	(0.0029)	(0.0063)	(0.0148)	(0.0006)	(0.0003)
PX_{t+1}	0.2966	0.9252	−0.2415	2.4581	0.6589	0.0085
	(0.0549)	(0.0718)	(0.1541)	(0.3601)	(0.0153)	(0.0066)
T_{t+1}	0.5319	0.6913	−0.2388	−0.7105	−0.2396	0.5676
	(0.1286)	(0.1679)	(0.3606)	(0.8428)	(0.0359)	(0.0155)

Panel C: Estimates of $\Sigma^P(\Sigma^P)'$

	$PC1_t$	$PC2_t$	$PC3_t$	PX_t	T_t
$PC1_t$	0.0144	0.0255	−0.0058	0.0351	−0.0200
$PC2_t$		0.0572	−0.0142	0.0567	−0.0344
$PC3_t$			0.0130	0.0152	−0.0241
PX_t				7.7173	−0.1109
T_t					42.2709

have a bigger impact in the short term than over longer horizons. Therefore, shocks to the supply and demand variables may be informative about short-term movements in prices, but have little information about longer term movements. This comports with basic economic considerations, as discussed in Pirrong (2011).

The estimates in Panel B of Table 3 also capture the interaction between the electricity prices and the demand and supply variables under the physical measure. The estimates reflect that natural gas is the marginal fuel for electricity production, and consequently the natural gas price is closely tied to electricity prices. The loading of the level of the electricity price PC^1 on PX_t is positive and highly statistically significant. The positive sign reflects the economic relation between production cost and electricity price.

The natural gas price not only affects the spot price but also the slope of the electricity futures curve, as the loading of PC^2_{t+1} on PX_t is also positive and significant. A smaller PC^2 indicates a steeper futures slope, so a higher natural gas price predicts a flatter slope. This reflects that the natural gas price mainly affects the short-term electricity price.

The model also indicates that the electricity price affects the natural gas price. The loadings of PX_{t+1} on PC^1_t are positive and significant. These positive signs indicate that higher electricity spot prices lead to higher natural gas prices. The high electricity price might result from high demand for electricity, which in turn leads to a higher usage of natural gas and thus higher prices.

Temperature is a proxy for electricity demand. The results in Table 3 are therefore consistent with economic intuition. First, temperature positively affects the PC^1. This reflects the fact that higher temperature generally leads to higher electricity demand. Second, because a higher PC^2_{t+1} implies a flatter futures curve, the positive impact of temperature on PC^2_{t+1} implies that temperature negatively affects the slope of the futures curve.

4.3. Model fit

Table 4 reports the fit of the unspanned model and compares its performance with that of the spanned model with economic variables. We do not report on the fit of the spanned model, because by definition it is identical to the fit of the unspanned model. For each of these three models, Table 4 reports the root mean squared error (RMSE) and the relative root mean squared error (RRMSE) for the spot and each futures contract, as well as the overall RMSE and RRMSE.

Not surprisingly, the unspanned model has a much smaller RMSE and RRMSE. The overall RMSE (RRMSE) is 5.04 (0.0649) for the unspanned model, compared to 36.44 (0.5459) for the spanned model. The poor fit of the spanned model with economic variables is not surprising: the spanning assumption forces all the information in the economic variables to enter the futures prices, which results in a poor fit. The main objective of a model with economic variables only is not to provide the best possible fit, but rather to

Table 4
Model Fit. We report the fit of the unspanned model (Panel A) and the spanned model with economic variables (Panel B) for futures prices. The economic variables are the natural gas price and temperature. For each model, we report both the root mean squared error (RMSE) and relative root mean squared error (RRMSE), defined as follows:

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{F}^i - F^i)^2} \qquad \mathrm{RRMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\frac{(\hat{F}^i - F^i)^2}{(F^i)^2}}$$

where $\hat{F}^i$ is the fitted futures price and F^i is the realized futures price. Overall RMSE and RRMSE are calculated as simple averages of the RMSE and RRMSE over all futures contracts. The sample period is from May 2003 to May 2014.

	The Unspanned Model		The Spanned Model with Economic Variables	
	RMSE	RRMSE	RMSE	RRMSE
Spot	6.6906	0.0595	74.2019	0.7622
F^{DA}	11.6164	0.0893	72.8245	0.4302
F^1	6.0108	0.0877	58.7756	0.6685
F^2	3.7946	0.0579	44.0974	0.8183
F^3	4.2285	0.0592	34.2346	0.7124
F^4	4.3773	0.0582	30.1756	0.6165
F^5	3.9984	0.0558	27.0885	0.5158
F^6	3.9192	0.0578	28.1335	0.4210
F^7	4.1716	0.0627	25.6745	0.4309
F^8	4.6189	0.0669	22.6175	0.4603
F^9	4.5299	0.0651	22.2761	0.4876
F^{10}	3.9168	0.0559	22.1166	0.4737
F^{11}	4.0623	0.0600	22.7272	0.4419
F^{12}	4.6704	0.0722	25.2338	0.4036
Overall	5.0433	0.0649	36.4412	0.5459

provide the best possible economic intuition. Our proposed model captures such economic intuition, but in addition provides a good fit, similar to a spanned model.

5. Analyzing risk premiums

The main conclusion from Table 4 is that the unspanned model fits futures prices well. We emphasize that by construction this fit is identical to the fit of the spanned model. The difference between the two modeling approaches emerges when studying risk premiums. This highlights the fact that the demand and supply variables contain additional information that is relevant for the futures prices under the physical measure, and hence the risk premiums. It is important to isolate this information, and it is to this task that we now turn. We first discuss the estimates and properties of the spot premium. We then discuss the forward bias.

5.1. The spot premium in the unspanned model

We analyze the spot premium implied by the unspanned model and compare it with spot premiums implied by alternative models. The spot premium measures the compensation required by investors for investing in electricity futures. It is defined as the difference of the physical and risk-neutral expectations of the log day-ahead price.

$$\text{Spot Premium}_t = E_t^P[\mathrm{Log}(S_{t+1})] - E_t^Q[\mathrm{Log}(S_{t+1})] \tag{32}$$

Table 5 reports the average spot premium for the unspanned model. We report results for the entire sample period as well as by season. The average estimated spot premium is approximately −1.67% per day. Pirrong and Jermakyan (2008) also find a negative risk premium, using a different sample period and a model with economic variables only. Our estimate of the spot premium is negative on average in every season, but it is larger (more negative) in the winter and the summer. This is consistent with the intuition that there is a greater risk of power price spikes in the peak seasons (summer and winter). Those who are short power (i.e., distribution companies that must buy power at the market price to sell to customers at fixed rates) are at risk to these price spikes, which can impose large losses on them. Risk averse physical shorts can hedge these risks by purchasing futures, thereby creating hedging pressure on prices: this pressure tends to cause upward biased futures prices, which in the context of the model means a negative risk premium.[9] In the non-peak seasons, price spikes are less likely, and the need to hedge is commensurately less. The lower hedging pressure from power consumers reduces the upward bias in futures prices. Indeed, since there can be short hedging pressure from generation operators looking to hedge electricity price risk, prices can actually be biased downwards, especially in the low-demand "shoulder" months of the spring and fall.

Figure 4 highlights the fluctuations in the spot premium over time as well as the differences between the spot risk premiums for the different models. It plots the time series of the spot premium for the unspanned model (Panel A), the spanned model (Panel B), and the model with economic variables (Panel C). A first important observation is that regardless of the model, the average spot risk premium does not seem to decrease over time. Moreover, for all three models the latter part of the sample contains larger outliers, but the nature of these outliers differs across models.

Panel D of Fig. 4 compares the three models to illustrate these differences; here we report weekly averages because the three daily plots are too noisy in one panel. The properties of the spot premium for the unspanned model are quite different from the other two models. Figure 4 clearly indicates that the risk premium in the unspanned model is most variable, followed by the spanned model, and the model with economic variables. More importantly, Fig. 4 shows that the unspanned model is capable of generating occasional large positive spikes in the spot risk premium, most notably in 2014, at the time of the polar vortex. The two other models generate large negative risk premiums on that occasion. The unspanned model is able to capture this spike due to the spike in the natural gas price, evident from Fig. 2. While the model with economic variables of course also includes the natural gas price, it is constrained because it does not allow for pricing factors other than the economic variables, which restricts its flexibility to capture atypical patterns in risk premiums in the polar vortex period.

Panel E in Fig. 4 provides more perspective on these fluctuations and the differences with other models by plotting the spot premiums of the different models during the 2014 polar vortex period. The models with economic variables (the unspanned model and the spanned model with economic variables) exhibit dramatic changes in risk premiums during this period, whereas models without economic variables (the spanned model with latent variables) cannot. While the estimated spot risk premium in the unspanned model is on average −1.67% per day, during the polar vortex period it fluctuates between approximately −40% and 100% per day.

Finally, to further investigate the dynamics of the spot premium, we decompose the spot premium into five components plus a constant. The five components represent the component associated with the level of the electricity futures curve, the component associated with the slope of the electricity futures curve, the component associated with the curvature of the electricity futures curve, the component associated with the natural gas price, and

[9] See Keynes (1923), Hirshleifer (1988), or Hirshleifer (1990) for models of commodity markets in which hedging pressure is a determinant of price bias and risk premia. Upward bias is associated with a negative risk premium because a negative risk premium means that spot prices drift up more (down less) in the equivalent (pricing) measure than the physical measure.

Table 5
Estimated Spot Premiums. We report the daily average of the spot premiums for the unspanned model. The spot premium is defined as follows:

$$\text{Spot Premium}_t = E_t^P[\text{Log}(S_{t+1})] - E_t^Q[\text{Log}(S_{t+1})]$$

For each model, we report the average spot premium in each season as well as over the entire sample period. The definition of the seasons is as follows: Winter is defined as December, January, and February. Spring is defined as March, April, and May. Summer is defined as June, July, and August. Fall is defined as September, October, and November. The economic variables that are used to calculate the spot premium of the unspanned model are the natural gas prices and temperature. The reported numbers are raw daily log returns. The sample period is from May 2003 to May 2014.

	Nobs	Mean	Std	Skew	Kurt	Min	Max	AC1
Winter	657	−0.0237	0.0922	3.8619	43.4904	−0.3857	0.9872	0.4067
Spring	722	−0.0060	0.0628	−0.2487	2.8504	−0.2321	0.1538	0.7241
Summer	709	−0.0274	0.0685	−0.1549	3.0081	−0.2570	0.1583	0.7500
Fall	679	−0.0099	0.0547	0.3481	2.9310	−0.2024	0.1648	0.6792
All Seasons	2767	−0.0167	0.0711	1.8572	29.7300	−0.3857	0.9872	0.6101

the component associated with the temperature. The Appendix provides details on this decomposition.

Figure 5 depicts the time-series of these five components. Several conclusions obtain. First, the components associated with the electricity level (PC1) and the slope (PC2) are time-varying. The component associated with the slope is sometimes positive and sometimes negative, indicating that investors sometimes require compensation to bear this risk while at times paying to hedge this risk. Second, the risk premiums associated with the natural gas price contain very large outliers, which are much larger than the ones associated with the temperature. This suggests that episodes where economic variables have large impacts on risk premiums mainly originate on the supply side rather than the demand side, confirming the evidence in Fig. 2.

5.2. Predicting returns with the estimated spot premium

Our results indicate that the spot premiums from different models have different properties. While the spot premium from the unspanned model seems to have some plausible properties, strictly speaking this does not prove it is a superior measure of the risk preferences of investors in electricity markets. We therefore conduct an out-of-sample exercise in which we use the estimated spot premium to predict the future realized return on day-ahead electricity price. The regression specification is as follows:

$$s_t - f_{t-1}^{\text{DA}} = \text{Const.} + \beta \times \text{Spot Premium}_{t-1} + \epsilon_t \tag{33}$$

If the spot premium of the unspanned model is a better indicator of the risk premium people pay for electricity futures, then the prediction of the unspanned model should be statistically significant and more importantly, the adjusted R^2 should be higher compared to the other models.

Table 6 shows results for the regression (33) for the entire sample period and two sub-samples. We also control for lagged returns. Among the three models we consider, the unspanned model has the highest adjusted R^2. This results holds when we discuss the forecasting power of the estimated spot premiums separately as well as jointly and are statistically significant based on the Diebold and Mariano (1995) test. The same result holds for the sub-samples, and when including control variables. These findings suggest that the spot premium of the unspanned model better captures the risk preferences of investors in electricity markets.

5.3. The forward bias

There are several ways to characterize risk premiums in forward contracts. We focus on the forward bias, the difference between the expected average spot price under the P measure and the one under the Q measure. More precisely, it is defined as follows:

$$\text{Forward Bias}_t^i = \mathrm{E}_t^P[\bar{S}_{t+i}] - \mathrm{E}_t^Q[\bar{S}_{t+i}] \tag{34}$$

where $\mathrm{E}_t^P[\bar{S}_{t+i}]$ and $\mathrm{E}_t^P[\bar{S}_{t+i}]$ denote the time-t average expected spot price in the maturity month t+i, where the expectation is under the P and Q measure, respectively.

Figure 6 plots time series of the forward bias in the unspanned model. Panel A plots the forward bias for the day-ahead contract, Panel B for the 1-month futures contract, and Panel C for the 12-month futures contract. Note that the forward bias is expressed in dollar terms, whereas the spot premium in Fig. 4 is expressed in daily percentage returns. We report weekly averages in Fig. 6, because for daily differences the extreme observations completely dominate the figure in Panel A, and as a result it is not informative. Figure 6 indicates that the forward bias can be both negative and positive in our sample, but on average it is negative. This is also the case when computing the forward bias for the maturities that are not reported in Fig. 6.

To save space we do not report the forward biases for the spanned model, but they differ from the forward bias implied by the unspanned model for short maturities. For the day-ahead contract, the correlation between the forward bias in the two models is only 26%, while it is near 100% for the longest maturities. This finding is consistent with the intuition that the supply and demand variables are mean reverting. Shocks to the supply and demand variables do not persist, and have a bigger impact in the short term than over longer horizons. Put differently, shocks to the supply and demand variables mainly affect the risk premium in shorter maturity futures.

The forward bias in the unspanned model is positively correlated across maturities, but the correlation quickly decreases as the difference in maturity increases. For example, the correlation between the forward bias in Panels A and B is 44% but the correlation between the forward bias in Panels A and C is only 1%. Although the one-month forward bias in Panel B and the twelve-month forward bias in Panel C display a positive correlation (10%), there are some important differences. There is a large negative spike in the one-month forward bias at the time of the polar vortex in 2014, which is much less pronounced for the twelve-month contract. The twelve-month forward bias remains negative and large for an extended period of time during 2005–2009, and this is not the case for on one-month forward bias. More generally, for longer maturities, reversion to the mean of the forward bias is substantially slower. The forward bias in Panel C first becomes more negative between 2004 and 2009 and then decreases in absolute value, eventually turning positive. No such patterns are evident in Panels A and B. It is also worth pointing out that no such patterns are evident from the slope of the futures curve in Panel C of Fig. 1.

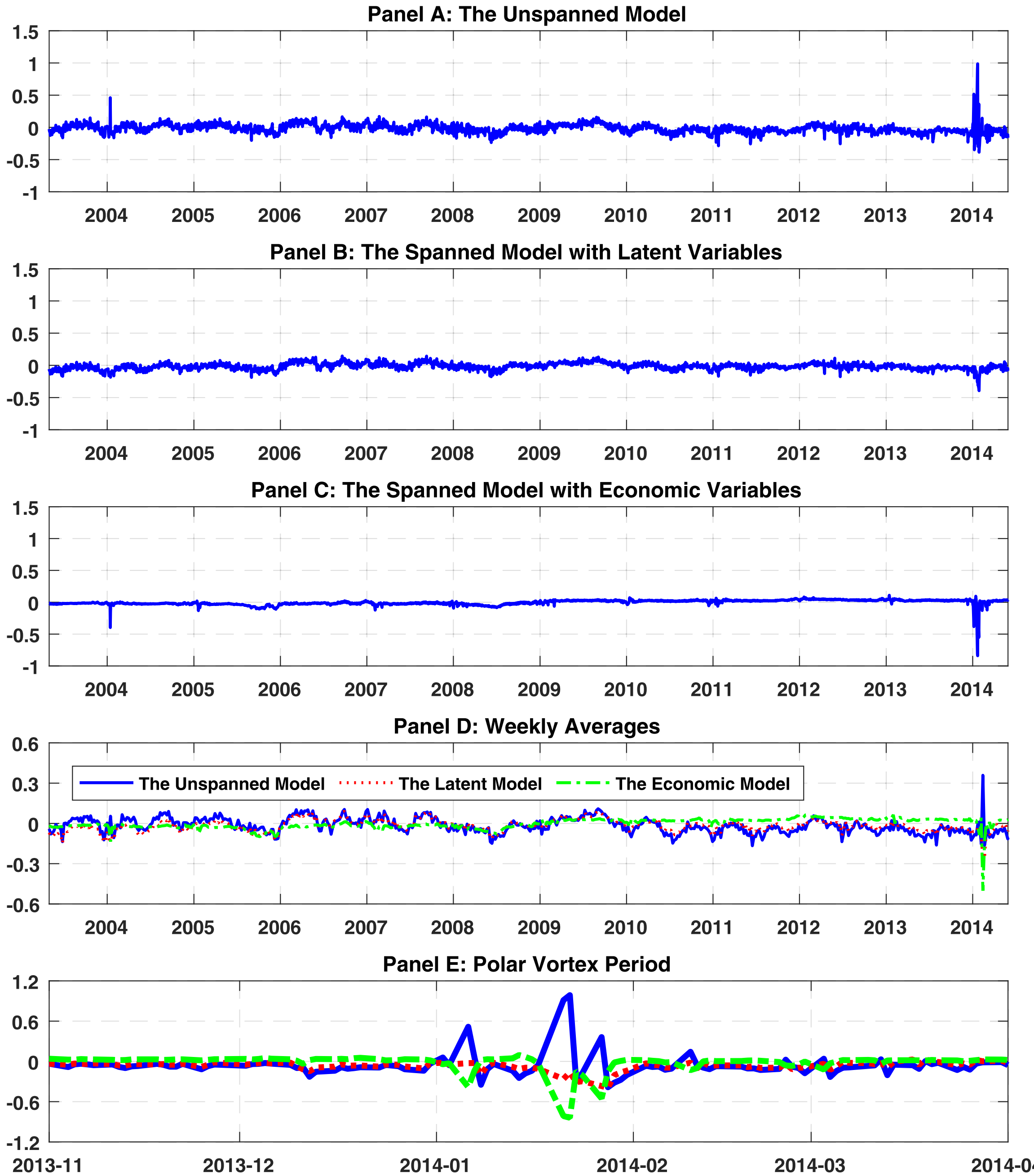

Fig. 4. Spot Premiums. Various Models. We plot the model-implied spot premium for the unspanned model (Panel A), the model with latent variables (Panel B), the model with economic variables (Panel C), their weekly averages (Panel D), and during the 2014 Polar Vortex period (Panel E). The economic variables that are used to calculate the spot premium are the natural gas price and temperature. The sample period is from May 2003 to May 2014.

The time series of the forward bias for contracts with maturities between one month and one year (not reported) show that as the maturity of the contract gets longer, the pattern in the time series of the forward bias increasingly resembles the pattern in Panel C.

Figure 7 plots the average difference between the expected average spot rate and the forward rate in each season (Panels A-D) and for the entire sample (Panel E). In each panel, we plot this difference as a function of the time to maturity of the forward contract, which ranges up to twelve months. Note that the horizontal axis refers to the contract number, with 1 denoting the spot rate and 13 the twelve-month forward. This analysis follows Pirrong and Jermakyan (2008), who conduct a similar exercise for a single day. Figure 7 indicates that this premium increases (becomes more negative) as a function of maturity. The forward bias is very large and economically significant. Recall from Table 1 that over our sample period the average spot and futures price is approximately \$60. Panel E of Fig. 7 indicates that on average the forward bias is -\$3 for the one-month maturity and -\$7 for the twelve-month maturity, which in both cases represents a significant percentage of the price. However, Fig. 6 shows that the forward bias dramatically fluctuates around the sample average, with larger fluctuations and outliers for shorter-maturity contracts. For instance, the day-ahead forward bias contains a positive outlier of \$380 and a negative outlier of -\$481 during the polar vortex period. The one-month forward bias has negative outliers of approximately -\$93 in 2014 and -\$37 in 2005 and 2008. Note that the magnitudes in Fig. 6 are different because these are weekly averages.

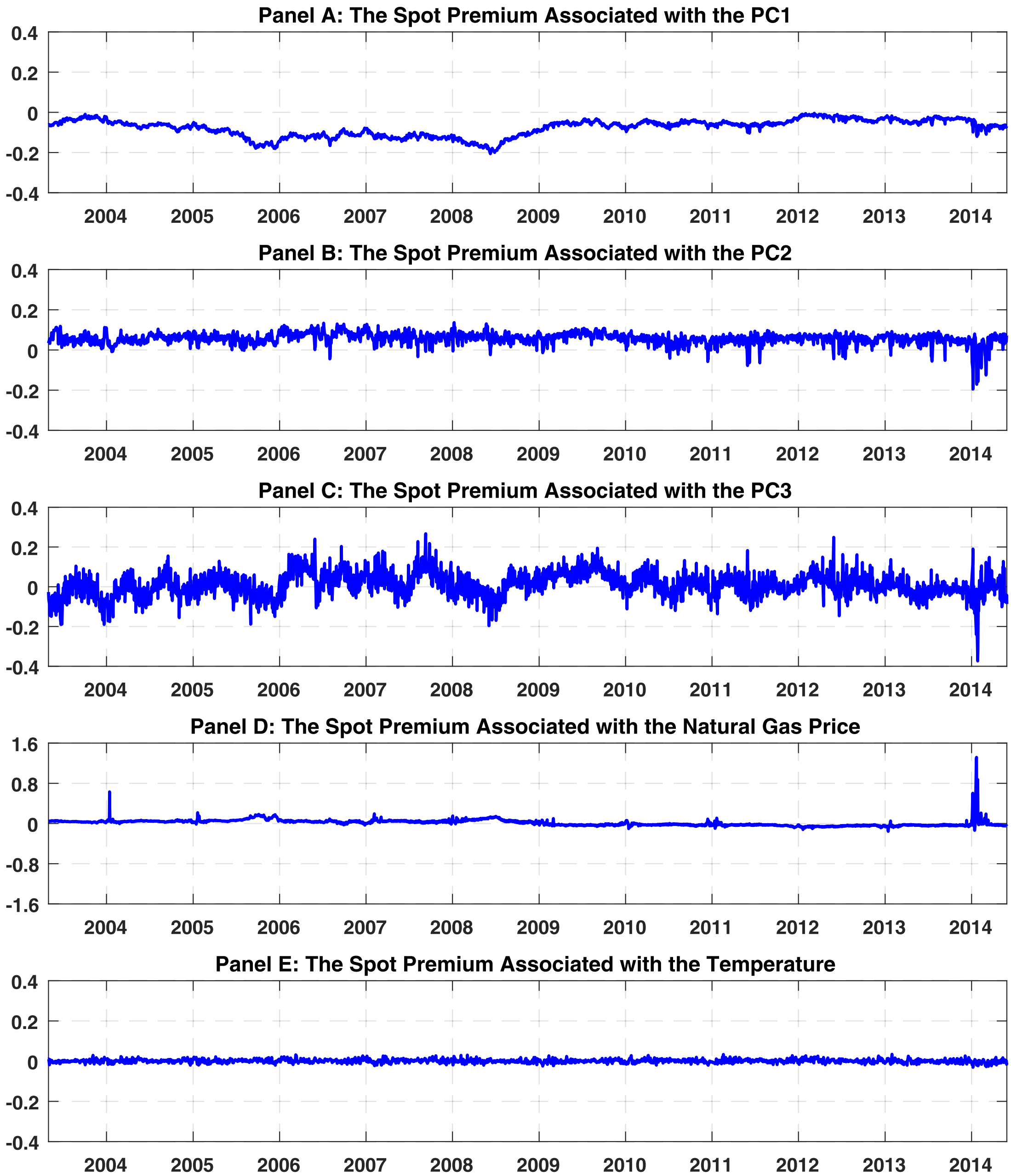

Fig. 5. Decomposing the Spot Premium of the Unspanned Model. We decompose the spot premium of the unspanned model. Panel A plots the spot premium associated with PC^1, Panel B plots the spot premium associated with PC^2, Panel C plots the spot premium associated with PC^3, Panel D plots the spot premium associated with the natural gas price, and Panel E plots the spot premium associated with the temperature. The details of the decomposition are given in the Appendix. The sample period is from May 2003 to May 2014.

6. Robustness analysis

The above results are based on daily data. We re-estimate the model using monthly data as a robustness check. Following Fama and French (1987), we use the futures price on the first day of each month to construct a monthly futures price sample. There are two major advantages to use monthly prices. First, the maturity of individual futures contract remains constant over the whole sample period. This makes return on the same futures at different time periods directly comparable. Second, return on daily futures might have abnormal behavior when the futures approaches to its maturity, thus using a monthly sample with the price at the first day of each month will solve this issue. After constructing the futures sample, we match the futures price in each month with the average economic variables in the last month. This is to avoid introducing a look-ahead bias.

These results are reported in the Appendix. Table A.2 confirms that with monthly data, supply and demand variables still contain unspanned information on the risk premium of electricity futures. Table A.3 shows the average risk premium with monthly data is negative and displays a seasonal pattern. Finally, Table A.4 shows that the estimated risk premium of the unspanned model helps predict the monthly return on electricity futures.

We also apply alternative de-seasonalization methods on economic variables. Table A.5 reports the results of spanning regression using three alternative de-seasonalization methods specified in Eq. (30). With all three alternative de-seasonalization methods, the regression results are consistent with the ones in Table 2, sug-

Table 6
Predicting the Log Day-Ahead Return with the Estimated Spot Premium. We compare the predictive power of the spot premium implied by different models for the log day-ahead return. The predictive regression is specified as follows:

$$s_t - f_{t-1}^{\mathrm{DA}} = \mathrm{Const.} + \beta \times \text{Spot Premium}_{t-1} + \epsilon_t$$

where s_t is the log of the spot price at time t and f_{t-1}^{DA} is the log of the day-ahead futures price at time t - 1. The economic variables that are used in the unspanned model and the spanned model with economic variables are the natural gas price and temperature. Standard errors are reported in parentheses. The sample period is from May 2003 to May 2014.

Panel A: Baseline Regression

Unspan	1.2317			1.3841
	(0.1210)			(0.2314)
Span with Latents		1.3574		−0.1410
		(0.1241)		(0.3031)
Span with Econs			−0.2659	0.4556
			(0.1981)	(0.2227)
Const.	0.0066	0.0087	−0.0142	0.0075
	(0.0055)	(0.0049)	(0.0048)	(0.0053)
Adj. R^2	0.1018	0.0627	0.0014	0.1043
N.Obs.	2766	2766	2766	2766
DM Stat		−2.6389	−4.6890	0.6591

Panel B: Sub-Samples

	2003 - 2008				2009 - 2014			
Unspan	1.3136			1.1937	1.2297			2.1744
	(0.1378)			(0.3356)	(0.1982)			(0.3756)
Span with Latents		1.3225		0.2207		1.4053		−1.3035
		(0.1310)		(0.3204)		(0.2292)		(0.5236)
Span with Econs			0.6482	−0.3782			−0.5089	1.6544
			(0.2610)	(0.5415)			(0.4150)	(0.6405)
Const.	−0.0034	0.0171	0.0118	−0.0099	0.0172	0.0002	−0.0100	−0.0139
	(0.0062)	(0.0063)	(0.0094)	(0.0167)	(0.0107)	(0.0074)	(0.0147)	(0.0170)
Adj. R^2	0.1005	0.0728	0.0028	0.1002	0.1037	0.0541	0.0050	0.1129
N.Obs.	1410	1410	1410	1410	1356	1356	1356	1356
DM Stat		−2.3192	−4.4281	0.4351		−2.0628	−3.0044	0.8152

Panel C: Controlling for Lagged Returns

Unspan	1.1919				1.4125
	(0.1232)				(0.2342)
Span with Latents		1.2719			−0.2613
		(0.1349)			(0.3210)
Span with Econs			−0.2113		0.5291
			(0.1741)		(0.2206)
Lagged Returns	0.0341	0.0458	0.1257	0.1280	0.0392
	(0.0218)	(0.0258)	(0.0223)	(0.0229)	(0.0233)
Const.	0.0064	0.0079	−0.0124	−0.0121	0.0066
	(0.0053)	(0.0047)	(0.0043)	(0.0043)	(0.0051)
Adj. R^2	0.1026	0.0643	0.0168	0.0160	0.1053
N.Obs.	2765	2765	2765	2765	2765
DM Stat		−2.7420	−4.6233	−4.3694	0.7800

gesting our results are robust to different de-seasonalization methods.

Next we repeat the spanning regressions on subsamples. Electricity markets are very volatile, and it is important to show robustness of the results across subsamples in order to avoid spurious findings and conclusions based on a particular sample period. One major outlier in our sample period is the 2014 Polar Vortex episode, and this may affect model estimates and our qualitative and quantitative conclusions. We therefore consider subsamples that omit the latter part of the sample. Specifically, we use the intial 50%, 60%, 70% and 80% of the original sample. All subsamples yield very similar conclusions. Table A.6 presents results on the spanning regressions for the 70% subsample to illustrate. The results are consistent with our benchmark results, i.e. economic variables help to predict the return on electricity prices and change in PCs.

Lucia and Schwartz (2002) and Geman and Roncoroni (2006) argue that constant volatility affine models need to be augmented with jumps in order to capture the dynamics of electicity markets, due to the large day-to-day movements in prices. This raises the question whether our finding that unspanned economic risk is significant is due to model misspecification due to the absence of jumps. The ideal way to investigate this question is to augment the model with jumps. However, while jumps can be easily accommodated in models with latent state variables, it is not straightforward to implement an extension of our model with jump because the state variables are observed. We therefore provide indirect evidence: We estimate the two-factor model based on the log spot price in Lucia and Schwartz (2002), which we refer to as the Lucia and Schwartz model, and the corresponding model with two factors as well as a jump in Villaplana (2003), which we refer to as the Villaplana model. To directly compare these models with the unspanned model, we generalize the Lucia and Schwartz model to include seasonal risk premium parameters and the Villaplana model to include a seasonal risk premium, as well as a seasonal jump intensity and jump size parameters.

Table A.7 reports the model fit of the unspanned model, the Lucia and Schwartz model, and the Villaplana model. The fit of the unspanned model is from Table 4. The fit of the Lucia and Schwartz model and the Villaplana model are a bit worse than that of the unspanned model because they impose more restrictions on the

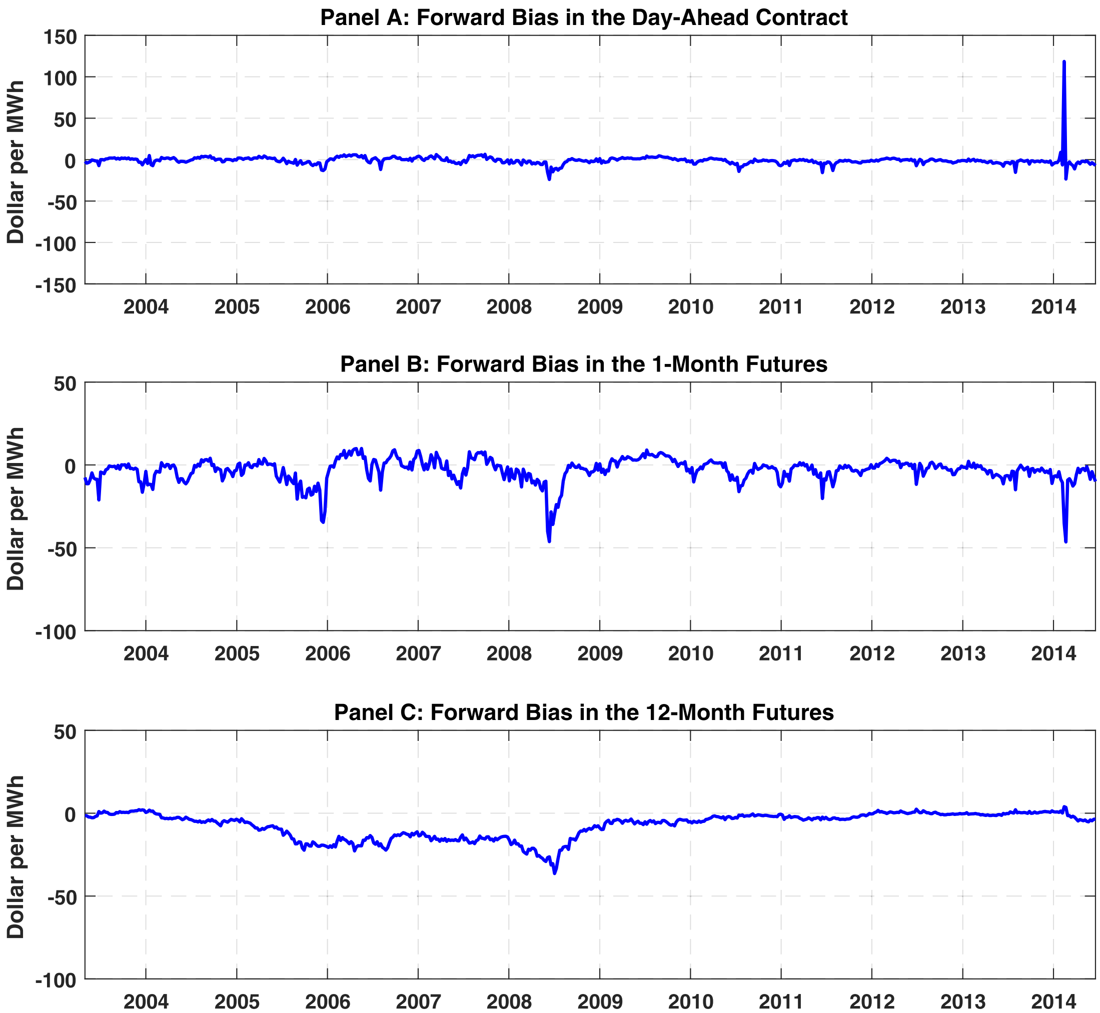

Fig. 6. The Forward Bias. Various Contracts. We plot the forward bias in the unspanned model. For a futures contract maturing in month i, the forward bias is defined as follows: Forward Bias$_t^i = \mathrm{E}_t^P[\bar{S}_{t+i}] - \mathrm{E}_t^Q[\bar{S}_{t+i}]$ where $\mathrm{E}_t^P[\bar{S}_{t+i}]$ and $\mathrm{E}_t^Q[\bar{S}_{t+i}]$ denote the time-t average expected spot price in the maturity month (t+i), where the expectation is under the P and the Q measure, respectively. Panels A to C plot the weekly averages of the forward bias for contracts with maturities equal to 1 day, 1 month, and 12 months respectively. The economic variables that are used to calculate the expected spot price under P are the natural gas price and temperature. The sample period is from May 2003 to May 2014.

state variable dynamics. The Villaplana model fits the spot price somewhat better than the Lucia and Schwartz model because it captures the jump component in electricity spot prices, but it underperforms in fitting the day-ahead price. However, overall the fit of the three models is not substantially different.

Using these estimates, we study the effect of the jump component on risk premiums. We check its effect on both the spot premium and the log forward bias. We focus on the log forward bias because it is easier to decompose into different components. Table A.8 reports the results. The economic variables have a positive risk premium while the jump component has a negative risk premium. A negative jump premium suggests that investors who are long futures pay a premium to hedge out jump risk in the electricity market. A positive unspanned economic risk premium suggests that long investors absorb risk from net sellers, thus earning positive returns. Note that the risk premium is the expected price under the P measure minus the expected price under the Q measure. Because economic variables do not impact the expected price under the Q measure in the unspanned model, a positive risk premium associated with economic variables suggests a positive relation between the level of the economic variables and the level of expected electricity prices under the P measure. This is consistent with Table 2, which shows that economic variables help to predict returns on electricity prices in addition to the PC of electricity prices. It is also consistent with the bottom two plots in Fig. 5, which indicate that the contribution of economic variables to the spot premium are generally positive. Overall these findings suggest that the unspanned economic component and jump component affect the risk premium of electricity prices through different channels.

7. Concluding remarks

We model the impact of supply and demand on the price of electricity futures in a no-arbitrage model, using data for 2003–2014. By design, the model fits electricity futures as well as a fully latent model. Additionally, the model allows for unspanned economic risk which is captured by the supply and demand variables but not identified by the futures prices.

We find that demand and supply variables contain valuable information about the spot risk premiums embedded in electricity futures prices. The spot risk premiums implied by the unspanned model are on average negative, very different from the risk premiums implied by the spanned model, display strong seasonal patterns, and better predict future changes in spot prices. We decompose the spot risk premium into components associated with de-

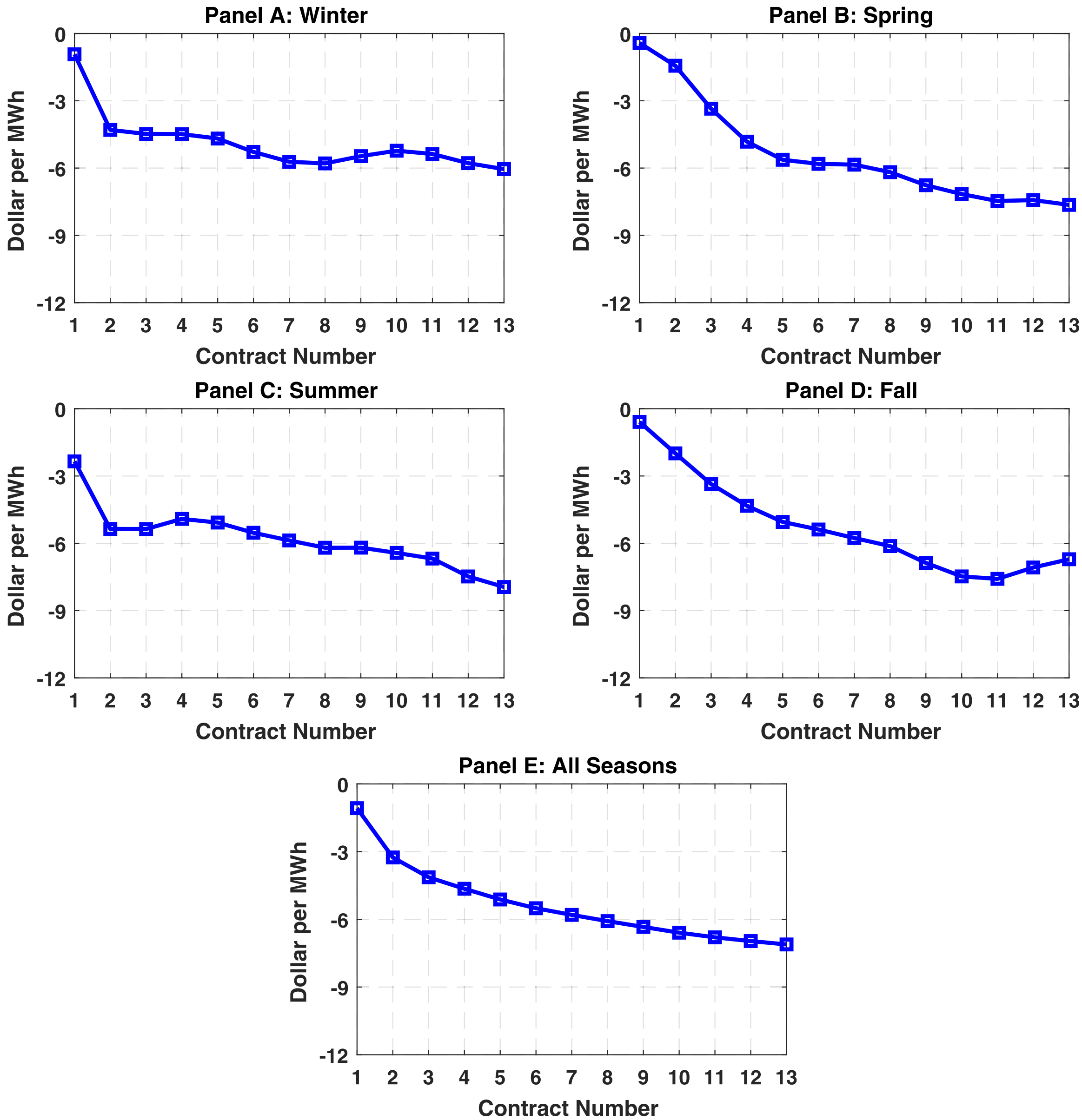

Fig. 7. The Forward Bias by Season. We plot the average forward bias in winter (Panel A), spring (Panel B), summer (Panel C), and fall (Panel D). Panel E plots the average forward bias over the entire sample period. In each panel, the maturity ranges from 1 month (contract number = 2) up to 12 months (contract number = 13). For contract number 1, the maturity equals 1 day. The economic variables that are used to calculate the expected spot price under P are the natural gas price and temperature. The sample period is from May 2003 to May 2014.

mand and supply. In periods of market turmoil, the unspanned risk premium associated with supply contains large outliers which are a very important component of the total spot risk premium embedded in electricity futures.

The forward bias implied by the model is also on average negative and large. It is highly time-varying and increases as a function of maturity. These findings suggest that while the use of latent variables to model electricity prices provides a good fit to the data, including unspanned economic variables generates more plausible economic implications, especially in periods of market turmoil. Several extensions of our approach may prove interesting. We show that the estimated spot premium predicts the future realized return on the day-ahead price, but this analysis uses estimates obtained using the entire sample. It may prove interesting to repeat this exercise using a recursive estimation. Furthermore, to capture the sharp spikes in the data, an alternative approach could model use jump processes with intensities that are functions of the demand and supply variables. Another possible extension is the estimation of a quadratic model to investigate the robustness of our results on spanning and the measurement of risk premiums to the assumption that the log spot price is linear in the state variables.

Appendix

A1. Decomposing the spot premium in the unspanned model

We define the spot risk premium as the expected log return of holding a day-ahead contract:

$$\text{Spot Premium}_t = E_t^P[\text{Log}(S_{t+1}) - \text{Log}(F_t^{DA})] \tag{A.1.1}$$

where S_{t+1} denotes the electricity spot price at time t+1 and F_t^{DA} denotes the day-ahead price at time t. We can show that the spot premium is equal to the difference between the log expected spot

Table A.1
Spanning Regressions for the Economic Variables. We forecast future values of the economic variables using the current demand and supply variables, controlling for the PCs of the electricity prices. The regression is specified as follows:

$$\mathrm{EC}_{t+1} = \mathrm{Const.} + \beta_{pc}\mathrm{PC}_t^{1-5} + \beta_{EC}\mathrm{EC}_t + \epsilon_t$$
$$\Delta\mathrm{EC}_{t\to t+1} = \mathrm{Const.} + \beta_{pc}\mathrm{PC}_t^{1-5} + \beta_{EC}\mathrm{EC}_t + \epsilon_t$$

For all regressions, standard errors are adjusted by Newey-West (1987) method with 8 lags. The sample period is from May 2003 to May 2014.

Panel A: Values of ECs								
	PX_t	SE	T_t	SE	Adj. R^2 (PCs only)	Adj. R^2 (PCs + EC)	DM Stat	Nobs
PX_{t+1}	0.6581	0.0727	0.0089	0.0064	0.4011	0.6431	2.7732	2766
T_{t+1}	−0.2381	0.0496	0.5666	0.0166	0.0176	0.3527	9.0814	2766
Panel B: Changes of ECs								
$\Delta PX_{t\to t+1}$	−0.3418	0.0727	0.0089	0.0064	0.0172	0.1711	1.2520	2766
$\Delta T_{t\to t+1}$	−0.1901	0.0369	−0.2891	0.0165	−0.0013	0.1336	7.0204	2766

Table A.2
Spanning Regressions. Monthly Data. We forecast returns on different futures and changes in the first three principal components using the unspanned component of demand and supply variables, controlling for lagged values of the PCs. The regression is specified as follows:

$$\mathrm{Ret}_{t\to t+1}^{1-12} = \mathrm{Const.} + \beta_{pc}\mathrm{PC}_t^{1-5} + \beta_{EC}\mathrm{EC}_t + \epsilon_t$$
$$\Delta\mathrm{PC}_{t\to t+1}^{1-3} = \mathrm{Const.} + \beta_{pc}\mathrm{PC}_t^{1-5} + \beta_{EC}\mathrm{EC}_t + \epsilon_t$$

For all regressions, standard errors are adjusted by Newey-West (1987) method with 8 lags. The sample period is from May 2003 to May 2014.

Panel A: Returns								
	PX_t	SE	T_t	SE	Adj. R^2 (PCs only)	Adj. R^2 (PCs + EC)	DM Stat	Nobs
$\mathrm{Ret}^1_{t\to t+1}$	0.0117	0.0082	0.0056	0.0047	0.1740	0.1754	0.9739	132
$\mathrm{Ret}^2_{t\to t+1}$	0.9417	0.3835	2.4554	0.4710	0.2151	0.2492	1.6960	132
$\mathrm{Ret}^3_{t\to t+1}$	0.8074	0.3427	2.3558	0.3823	0.1658	0.1977	2.0042	132
$\mathrm{Ret}^4_{t\to t+1}$	0.9394	0.3496	2.6871	0.4498	0.1574	0.2111	2.1114	132
$\mathrm{Ret}^5_{t\to t+1}$	0.9659	0.3582	2.6961	0.3745	0.1542	0.2133	1.9498	132
$\mathrm{Ret}^6_{t\to t+1}$	0.9568	0.3419	2.7986	0.3322	0.1459	0.2033	1.9022	132
$\mathrm{Ret}^7_{t\to t+1}$	0.9058	0.3002	3.0172	0.3327	0.1573	0.2236	2.0492	132
$\mathrm{Ret}^8_{t\to t+1}$	0.8417	0.3103	2.7125	0.3191	0.1419	0.2027	1.8318	132
$\mathrm{Ret}^9_{t\to t+1}$	0.7159	0.2955	2.4230	0.2998	0.1469	0.1989	1.5290	132
$\mathrm{Ret}^{10}_{t\to t+1}$	0.6178	0.2699	2.2886	0.2953	0.1722	0.2125	1.5078	132
$\mathrm{Ret}^{11}_{t\to t+1}$	0.7998	0.2575	3.1060	0.1978	0.1804	0.2385	1.9245	132
$\mathrm{Ret}^{12}_{t\to t+1}$	1.0652	0.2526	4.2173	0.1748	0.2844	0.3708	2.4453	132
Panel B: Change of PCs								
$\Delta\mathrm{PC}^1_{t\to t+1}$	0.0290	0.0109	2.6483	0.0129	0.0879	0.1420	1.9385	132
$\Delta\mathrm{PC}^2_{t\to t+1}$	0.0006	0.0048	0.1326	−0.0015	0.8389	0.8367	0.3191	132
$\Delta\mathrm{PC}^3_{t\to t+1}$	−0.0003	0.0019	0.0019	0.0015	0.9318	0.9314	0.5386	132

Table A.3
Estimated Spot Premiums. Monthly Data. We report the daily average of the spot premiums of the unspanned model. The spot premium is defined as follows:

$$\text{Spot Premium}_t = E_t^P[\mathrm{Log}(S_{t+1}) - \mathrm{Log}(F_t^{DA})]$$

For each model, we report the average spot premium in each season as well as over the entire sample period. The definition of the seasons is as follows: Winter is defined as December, January, and February. Spring is defined as March, April, and May. Summer is defined as June, July, and August. Fall is defined as September, October, and November. The economic variables that are used to calculate the spot premium of the unspanned model are the natural gas prices and temperature. The reported numbers are monthly log returns. The sample period is from May 2003 to May 2014.

	Nobs	Mean	Std	Skew	Kurt	Min	Max	AC1
Winter	33	−0.0238	0.1729	2.4334	12.2815	−0.3020	0.7314	0.1818
Spring	33	0.0463	0.0984	0.3170	3.5879	−0.1515	0.3139	0.1403
Summer	34	−0.0798	0.1323	−0.5313	2.5609	−0.3911	0.1330	0.2950
Fall	33	0.0038	0.1280	0.1569	3.4206	−0.3085	0.3082	0.2548
All Seasons	133	−0.0139	0.1417	0.8414	8.0886	−0.3911	0.7314	0.5020

Table A.4
Predicting the Log F1 Return with the Estimated Spot Premium. Monthly. We compare the predictive power of the spot premium implied by different models for the log day-ahead return. The predictive regression is specified as follows:

$$s_t - f^1_{t-1} = \text{Const.} + \beta \times \text{Spot Premium}_{t-1} + \epsilon_t$$

where s_t is the log of the spot price at time t and f^1_{t-1} is the log of the 1-month futures price at time t - 1. The economic variables that are used in the unspanned model and the spanned model with economic variables are the natural gas price and temperature. Standard errors are reported in parentheses. The sample period is from May 2003 to May 2014.

Panel A: Baseline Regression

Unspan	0.9699			1.1095
	(0.0958)			(0.4348)
Span with Latents		0.9627		−0.1475
		(0.1022)		(0.4668)
Span with Econs			0.0524	0.0331
			(0.2674)	(0.2261)
Const.	0.0001	0.0000	−0.0042	0.0052
	(0.0183)	(0.0190)	(0.0421)	(0.0456)
Adj. R^2	0.3117	0.2905	−0.0073	0.3015
N.Obs.	132	132	132	132
DM Stat		−1.8754	−2.7828	0.2127
DM P-Value		0.0304	0.0027	0.5842

Panel B: Sub-Samples

	2003 - 2008				2009 - 2014			
Unspan	1.0397			1.2132	0.9215			0.6535
	(0.1445)			(0.7603)	(0.1359)			(1.0069)
Span with Latents		1.0688		−0.1840		1.0088		0.3378
		(0.1364)		(0.7725)		(0.1848)		(1.2858)
Span with Econs			0.4536	−0.0136			−0.3547	0.1144
			(0.3266)	(0.2494)			(0.3233)	(0.3522)
Const.	0.0112	0.0295	0.0407	0.0053	−0.0066	−0.0258	−0.0278	0.0009
	(0.0216)	(0.0231)	(0.0660)	(0.0499)	(0.0299)	(0.0293)	(0.0492)	(0.0581)
Adj. R^2	0.4083	0.3889	0.0085	0.3902	0.2072	0.2002	0.0027	0.1822
N.Obs.	68	68	68	68	64	64	64	64
DM Stat		−0.9572	−2.7223	0.1319		−0.3684	−1.8211	0.1761
DM P-Value		0.1692	0.0032	0.5525		0.3563	0.0343	0.5699

Panel C: Controlling for Lagged Returns

Unspan	0.9266				1.1255
	(0.1392)				(0.4294)
Span with Latents		0.9095			−0.2262
		(0.1539)			(0.4756)
Span with Econs			0.3045		0.0511
			(0.1971)		(0.2463)
Lagged Returns	0.0309	0.0383	0.3843	0.3528	0.0453
	(0.0963)	(0.0993)	(0.0840)	(0.0665)	(0.1068)
Const.	0.0010	0.0008	0.0428	−0.0052	0.0090
	(0.0180)	(0.0185)	(0.0407)	(0.0198)	(0.0481)
Adj. R^2	0.2989	0.2769	0.1248	0.1194	0.2890
N.Obs.	131	131	131	131	131
DM Stat		−2.1735	−2.7604	−2.9168	0.2774
DM P-Value		0.0149	0.0029	0.0018	0.6092

under the P measure and the log expected spot under the Q measure plus a constant:

$$\begin{aligned}
\text{Spot Premium}_t &= E^P_t[\text{Log}(\text{S}_{t+1}) - \text{Log}(\text{F}^{\text{DA}}_t)] \\
&= E^P_t[\text{s}_{t+1} - f^{\text{DA}}_t] \\
&= E^P_t[\text{s}_{t+1}] - f^{\text{DA}}_t \\
&= E^P_t[\text{s}_{t+1}] - (E^Q_t[\text{s}_{t+1}] + \tfrac{1}{2}\sigma_s^2) \\
&= E^P_t[\text{s}_{t+1}] - E^Q_t[\text{s}_{t+1}] - \tfrac{1}{2}\sigma_s^2
\end{aligned} \tag{A.1.2}$$

where s_{t+1} denotes the log of the spot price at time t+1, f^{DA}_t denotes the log of the day-ahead price at time t, and σ_s denotes the volatility of the log spot price.

Suppose that we use [PC1, PC2, PC3, PX, T] as the state variables for the unspanned model, where PC1 denotes the first principal component of the log electricity price (log spot price and log futures price), PC2 denotes the second principal component of the log electricity price, PC3 denotes the third principal component of the log electricity price, PX denotes the natural gas price, and T denotes the temperature. Based on Eq. (12), the expected log spot price under P can be expressed as:

$$\begin{aligned}
E^P_t[\text{s}_{t+1}] &= \text{Seas}_{s,t+1} + \rho_0 \\
&\quad + \sum_{i=1}^{5} \rho_1(i) \times (K^P_0(i) + K^P_1(i,\text{PC1}) \times \text{PC1}_t + K^P_1(i,\text{PC2}) \\
&\quad \times \text{PC2}_t + K^P_1(i,\text{PC3}) \times \text{PC3}_t + K^P_1(i,\text{PX}) \times \text{PX}_t + K^P_1(i,\text{T}) \times \text{T}_t)
\end{aligned} \tag{A.1.3}$$

where $\rho_1(i)$ denotes the ith element in ρ_1, $K^P_0(i)$ denotes the ith element of the column vector K^P_0, $K^P_1(i,\text{PC1})$ is the element in row i and column one of the matrix K^P_1, $K^P_1(i,\text{PC2})$ is the element in row i and column two of the matrix K^P_1, $K^P_1(i,\text{PC3})$ is the element in row i and column three of the matrix K^P_1, $K^P_1(i,\text{PX})$ is the element in row i and column four of the matrix K^P_1, and $K^P_1(i,\text{T})$ is the element in row i and column five of the matrix K^P_1.

Similarly, Eq. (13) implies that the expected price under Q can be expressed as:

$$\begin{aligned}
E^Q_t[\text{s}_{t+1}] &= \text{Seas}_{s,t+1} + \rho_0 \\
&\quad + \sum_{i=1}^{3} \rho_1(i) \times (K^Q_0(i) + K^Q_1(i,\text{PC1}) \times \text{PC1}_t + K^Q_1(i,\text{PC2}) \times \text{PC2}_t \\
&\quad + K^Q_1(i,\text{PC3}) \times \text{PC3}_t)
\end{aligned} \tag{A.1.4}$$

Table A.5

Spanning Regressions. Alternative De-Seasonalization Methods. We use three alternative methods to de-seasonalize economic variables and use the de-seasonalized series to forecast returns on different futures and changes in the first three principal components. The alternative de-seasonalization methods are:

Method 1: T - average T estimated by nonparametric regression with Gaussian kernel
Method 2: T - average T estimated by nonparametric regression with Epanechnikov kernel
Method 3: T - average T estimated by a trigonometric function

The regression is specified as follows:

$$\text{Ret}^{DA-12}_{t\to t+1} = \text{Const.} + \beta_{pc}\text{PC}^{1-5}_t + \beta_{EC}\text{EC}_t + \epsilon_t$$
$$\Delta\text{PC}^{1-3}_{t\to t+1} = \text{Const.} + \beta_{pc}\text{PC}^{1-5}_t + \beta_{EC}\text{EC}_t + \epsilon_t$$

The sample period is from May 2003 to May 2014.

Panel A: Returns

	Gaussian Kernel			Epanechnikov Kernel			Trigonometric Function		
	Adj. R^2 (PCs only)	Adj. R^2 (PCs + EC)	DM Stat	Adj. R^2 (PCs only)	Adj. R^2 (PCs + EC)	DM Stat	Adj. R^2 (PCs only)	Adj. R^2 (PCs + EC)	DM Stat
$\text{Ret}^{DA}_{t\to t+1}$	0.0622	0.0956	2.1330	0.0622	0.0954	2.1297	0.0622	0.0956	2.1137
$\text{Ret}^{1}_{t\to t+1}$	0.0114	0.0148	1.3193	0.0114	0.0149	1.3457	0.0114	0.0145	1.2104
$\text{Ret}^{2}_{t\to t+1}$	0.0077	0.0129	1.7286	0.0077	0.0129	1.7355	0.0077	0.0126	1.6326
$\text{Ret}^{3}_{t\to t+1}$	0.0040	0.0077	1.7667	0.0040	0.0078	1.7664	0.0040	0.0073	1.6833
$\text{Ret}^{4}_{t\to t+1}$	0.0027	0.0064	1.7803	0.0027	0.0065	1.7682	0.0027	0.0061	1.7005
$\text{Ret}^{5}_{t\to t+1}$	0.0016	0.0054	1.7691	0.0016	0.0054	1.7449	0.0016	0.0052	1.7011
$\text{Ret}^{6}_{t\to t+1}$	0.0022	0.0057	1.7642	0.0022	0.0058	1.7534	0.0022	0.0056	1.7097
$\text{Ret}^{7}_{t\to t+1}$	0.0027	0.0044	1.3139	0.0027	0.0044	1.2983	0.0027	0.0042	1.2623
$\text{Ret}^{8}_{t\to t+1}$	0.0051	0.0080	1.5165	0.0051	0.0080	1.5023	0.0051	0.0078	1.4754
$\text{Ret}^{9}_{t\to t+1}$	0.0046	0.0082	1.6928	0.0046	0.0082	1.6682	0.0046	0.0080	1.6554
$\text{Ret}^{10}_{t\to t+1}$	0.0050	0.0069	1.2864	0.0050	0.0069	1.2580	0.0050	0.0068	1.2635
$\text{Ret}^{11}_{t\to t+1}$	0.0029	0.0058	1.6943	0.0029	0.0058	1.6726	0.0029	0.0059	1.6601
$\text{Ret}^{12}_{t\to t+1}$	0.0044	0.0096	3.3004	0.0044	0.0097	3.4257	0.0044	0.0096	2.7007
Panel B: Change of PCs									
$\Delta\text{PC}^{1}_{t\to t+1}$	0.1318	0.1768	1.5587	0.1318	0.1767	1.5570	0.1318	0.1767	1.5530
$\Delta\text{PC}^{2}_{t\to t+1}$	0.1758	0.2193	1.5027	0.1758	0.2192	1.5017	0.1758	0.2192	1.5010
$\Delta\text{PC}^{3}_{t\to t+1}$	0.0453	0.0457	0.7119	0.0453	0.0457	0.7122	0.0453	0.0457	0.7393

Table A.6

Spanning Regressions in Subsamples. We use the first 70% of the sample period and run the spanning regression for this subsample. The regression is specified as follows:

$$\text{Ret}^{DA-12}_{t\to t+1} = \text{Const.} + \beta_{pc}\text{PC}^{1-5}_t + \beta_{EC}\text{EC}_t + \epsilon_t$$
$$\Delta\text{PC}^{1-3}_{t\to t+1} = \text{Const.} + \beta_{pc}\text{PC}^{1-5}_t + \beta_{EC}\text{EC}_t + \epsilon_t$$

For all regressions, standard errors are adjusted by Newey-West (1987) method with 7 lags. The sample period is from May 2003 to February 2011.

Panel A: Returns

	PX_t	SE	T_t	SE	Adj. R^2 (PCs only)	Adj. R^2 (PCs + EC)	DM Stat	Nobs
$\text{Ret}^{DA}_{t\to t+1}$	1.4844	0.4322	0.2993	0.0827	0.0425	0.0634	2.5959	1935
$\text{Ret}^{1}_{t\to t+1}$	−0.0090	0.0291	0.0175	0.0078	0.0193	0.0213	1.1612	1843
$\text{Ret}^{2}_{t\to t+1}$	0.0016	0.0265	0.0196	0.0067	0.0084	0.0120	1.4369	1935
$\text{Ret}^{3}_{t\to t+1}$	−0.0044	0.0214	0.0184	0.0063	0.0037	0.0077	1.4305	1935
$\text{Ret}^{4}_{t\to t+1}$	0.0108	0.0189	0.0177	0.0059	0.0028	0.0070	1.4269	1935
$\text{Ret}^{5}_{t\to t+1}$	0.0123	0.0192	0.0170	0.0055	0.0016	0.0063	1.5005	1935
$\text{Ret}^{6}_{t\to t+1}$	0.0048	0.0181	0.0164	0.0053	0.0026	0.0072	1.4861	1935
$\text{Ret}^{7}_{t\to t+1}$	0.0003	0.0165	0.0130	0.0052	0.0018	0.0048	1.1791	1935
$\text{Ret}^{8}_{t\to t+1}$	0.0057	0.0156	0.0128	0.0048	0.0054	0.0087	1.2847	1935
$\text{Ret}^{9}_{t\to t+1}$	0.0087	0.0141	0.0133	0.0047	0.0026	0.0068	1.3342	1935
$\text{Ret}^{10}_{t\to t+1}$	0.0017	0.0131	0.0101	0.0046	0.0041	0.0061	1.0333	1935
$\text{Ret}^{11}_{t\to t+1}$	0.0079	0.0135	0.0122	0.0046	0.0014	0.0052	1.2894	1935
$\text{Ret}^{12}_{t\to t+1}$	0.0107	0.0129	0.0127	0.0044	0.0030	0.0075	1.3806	1935
Panel B: Change of PCs								
$\Delta\text{PC}^{1}_{t\to t+1}$	0.0056	-0.0014	0.0013	0.0004	0.1119	0.1260	2.2738	1935
$\Delta\text{PC}^{2}_{t\to t+1}$	−-0.0109	0.0025	−-0.0015	0.0006	0.1605	0.1718	2.2360	1935
$\Delta\text{PC}^{3}_{t\to t+1}$	0.0000	0.0004	0.0001	0.0001	0.0529	0.0522	0.4937	1935

Table A.7
Model Fit of Alternative Models. We report the fit of the unspanned model, the Lucia and Schwartz model, and the Villaplana model. For each model, we report both the root mean squared error (RMSE) and relative root mean squared error (RRMSE), defined as follows:

$$\mathrm{RMSE} = \sqrt{\tfrac{1}{n}\textstyle\sum_{i=1}^{n}(\hat{\mathrm{F}}^i - \mathrm{F}^i)^2} \qquad \mathrm{RRMSE} = \sqrt{\tfrac{1}{n}\textstyle\sum_{i=1}^{n}\tfrac{(\hat{\mathrm{F}}^i - \mathrm{F}^i)^2}{(\mathrm{F}^i)^2}}$$

where $\hat{\mathrm{F}}^i$ is the fitted futures price and F^i is the realized futures price. Overall RMSE and RRMSE are calculated as simple averages of the RMSE and RRMSE over all futures contracts. The sample period is from May 2003 to May 2014.

	The Unspanned Model		The Lucia and Schwartz Model		The Villaplana Model	
	RMSE	RRMSE	RMSE	RRMSE	RMSE	RRMSE
Spot	6.6906	0.0595	8.5740	0.0798	8.8579	0.0632
F^{DA}	11.6164	0.0893	13.5178	0.1163	17.1724	0.1581
F^1	6.0108	0.0877	6.2719	0.0959	6.1703	0.0945
F^2	3.7946	0.0579	5.0239	0.0729	4.9588	0.0741
F^3	4.2285	0.0592	4.4362	0.0606	4.5075	0.0646
F^4	4.3773	0.0582	3.9428	0.0509	4.0862	0.0564
F^5	3.9984	0.0558	3.4399	0.0469	3.8090	0.0559
F^6	3.9192	0.0578	3.0811	0.0437	3.8390	0.0572
F^7	4.1716	0.0627	3.3695	0.0493	3.8758	0.0593
F^8	4.6189	0.0669	4.1008	0.0593	4.3935	0.0656
F^9	4.5299	0.0651	4.5610	0.0651	4.7674	0.0701
F^10	3.9168	0.0559	4.5116	0.0632	4.7354	0.0681
F^11	4.0623	0.0600	4.5251	0.0649	4.7148	0.0693
F^12	4.6704	0.0722	4.6058	0.0699	4.9181	0.0753
Overall	5.0433	0.0649	5.2830	0.0671	5.7719	0.0737

where $K_0^Q(i)$ is the ith element of the column vector K_0^Q, $K_1^Q(i, \mathrm{PC1})$, $K_1^Q(i, \mathrm{PC2})$, and $K_1^Q(i, \mathrm{PC3})$ are the first, second, and third elements of the i row of the matrix K_1^Q respectively.

Substituting Eqs. (A.1.3) and (A.1.4) into Eq. (A.1.2) and, we get

$$\begin{aligned}
&\text{Spot Premium}_t \\
&= E_t^P[s_{t+1}] - E_t^Q[s_{t+1}] - \tfrac{1}{2}\sigma_s^2 \\
&= \mathrm{Seas}_{s,t+1} + \rho_0 \\
&\quad + \textstyle\sum_{i=1}^{5} \rho_1(i) \times \Big(K_0^P(i) + K_1^P(i,\mathrm{PC1}) \times \mathrm{PC1}_t + K_1^P(i,\mathrm{PC2}) \times \mathrm{PC2}_t \\
&\quad + K_1^P(i,\mathrm{PC3}) \times \mathrm{PC3}_t + K_1^P(i,\mathrm{PX}) \times \mathrm{PX}_t + K_1^P(i,\mathrm{T}) \times \mathrm{T}_t\Big) \\
&\quad - (\mathrm{Seas}_{s,t+1} + \rho_0 \\
&\quad + \textstyle\sum_{i=1}^{3} \rho_1(i) \times \Big(K_0^Q(i) + K_1^Q(i,\mathrm{PC1}) \times \mathrm{PC1}_t + K_1^Q(i,\mathrm{PC2}) \times \mathrm{PC2}_t \\
&\quad + K_1^Q(i,\mathrm{PC3}) \times \mathrm{PC3}_t\Big)) - \tfrac{1}{2}\sigma_s^2 \\
&= \underbrace{\sum_{i=1}^{5} \rho_1(i) \times K_0^P(i) - \sum_{i=1}^{3} \rho_1(i) \times K_0^Q(i) - \frac{1}{2}\sigma_s^2}_{\text{Constant}} \\
&\quad + \underbrace{\left(\sum_{i=1}^{5} \rho_1(i) \times K_1^P(i,\mathrm{PC1}) - \sum_{i=1}^{3} \rho_1(i) \times K_1^Q(i,\mathrm{PC1})\right) \times \mathrm{PC1}_t}_{\text{Spot premium associated with the PC1}} \\
&\quad + \underbrace{\left(\sum_{i=1}^{5} \rho_1(i) \times K_1^P(i,\mathrm{PC2}) - \sum_{i=1}^{3} \rho_1(i) \times K_1^Q(i,\mathrm{PC2})\right) \times \mathrm{PC2}_t}_{\text{Spot premium associated with the PC2}} \\
&\quad + \underbrace{\left(\sum_{i=1}^{5} \rho_1(i) \times K_1^P(i,\mathrm{PC3}) - \sum_{i=1}^{3} \rho_1(i) \times K_1^Q(i,\mathrm{PC3})\right) \times \mathrm{PC3}_t}_{\text{Spot premium associated with the PC3}} \\
&\quad + \underbrace{\sum_{i=1}^{5} \rho_1(i) \times K_1^P(i,\mathrm{PX}) \times \mathrm{PX}_t}_{\text{Spot premium associated with the PX}} + \underbrace{\sum_{i=1}^{5} \rho_1(i) \times K_1^P(i,\mathrm{T}) \times \mathrm{T}_t}_{\text{Spot premium associated with the T}}
\end{aligned} \tag{A.1.5}$$

B2. Alternative De-Seasonalization methods

This section provides details about the three alternative de-seasonalization methods in Eq. (30).

B2.1. Non-Parametric regression with a gaussian kernel

We model the temperature as a function of the day of the year. For a given day t, denote the day-number for that day as d(t). The temperature on day t is given by:

$$\mathrm{T}_t = f(d(t)) + \epsilon_t \tag{A.2.1}$$

where

$$f(d(t)) = \frac{\sum_{i=1}^{n} K\left(\frac{d(t_i)-d(t)}{h}\right) \times T_{t_i}}{\sum_{i=1}^{n} K\left(\frac{d(t_i)-d(t)}{h}\right)} \tag{A.2.2}$$

and i = 1, ..., n denotes the n observations that used in the estimation. We use data for the past 10 years in estimation. Using the past 5 years yields similar results. The Gaussian kernel function is defined as follows:

$$K(x) = \frac{1}{\sqrt{2\pi}} exp(-x^2/2) \tag{A.2.3}$$

We set h = 5 in the estimation. Other values yield similar results.

B2.2. Non-parametric regression with an epanechnikov kernel

The non-parametric regression with Epanechnikov kernel also uses Eqs. (A.2.1) and (A.2.2) but with the following kernel:

$$K(x) = \begin{cases} \frac{3}{4}(1-x^2) & \text{if } |x| \le 1 \\ 0 & \text{else} \end{cases} \tag{A.2.4}$$

B2.3. Parametric regression with trigonometric functions

The model is given by Eq. (A.2.1) with the following f:

$$f(d(t)) = a \times \cos((d(t) + b) \times (2\pi/365)) + c \tag{A.2.5}$$

where a, b, c are parameters. This model is the one in Lucia and Schwartz (2002).

Table A.8

Comparing Model Risk Premiums. We report the daily average of the spot premium and the log forward bias for the latent model, the unspanned model, the Lucia and Schwartz model, and the Villaplana model. The spot premium is defined as follows:

Spot $\text{Premium}_t = E_t^P[\text{Log}(S_{t+1})] - E_t^Q[\text{Log}(S_{t+1})]$

And the log forward bias of ith-month futures is defined as follows:

Log Forward $\text{Bias}_t^i = E_t^P[\text{Log}(S_i)] - E_t^Q[\text{Log}(S_i)]$For each model, we report the average risk premium as well as the contributions (where applicable) from a constant, the PCs of electricity prices, economic variables, short-term and long-term components X and ϵ, and jumps. The economic variables used to calculate the risk premium of the unspanned model are the natural gas prices and temperature. The reported numbers are raw daily log returns. The sample period is from May 2003 to May 2014.

	SP	LogFB1	LogFB2	LogFB3	LogFB4	LogFB5	LogFB6	LogFB7	LogFB8	LogFB9	LogFB10	LogFB11	LogFB12
Nobs	2767	2767	2767	2767	2767	2767	2767	2767	2767	2767	2767	2767	2767
The Spanned Model with Latent Factors													
Constant	−0.0058	−0.0007	7.56E−05	0.0002	0.0001	0.0001	8.14×10^{-5}	5.28×10^{-5}	2.91×10^{-5}	9.58×10^{-6}	-6.41×10^{-6}	-1.96×10^{-5}	-3.05×10^{-5}
PCs	−0.0108	−0.0028	−0.0019	−0.0014	−0.0011	−0.0009	−0.0008	−0.0007	−0.0006	−0.0005	−0.0005	−0.0004	−0.0004
Total	−0.0166	−0.0035	−0.0018	−0.0012	−0.0010	−0.0008	−0.0007	−0.0006	−0.0006	−0.0005	−0.0005	−0.0004	−0.0004
The Unspanned Model													
Constant	−0.0195	−0.0012	5.37E-05	0.0002	0.0001	0.0001	7.57×10^{-5}	4.78×10^{-5}	2.46×10^{-5}	5.55×10^{-6}	-1.01×10^{-5}	-2.30×10^{-5}	-3.37×10^{-5}
PCs	−0.0108	−0.0028	−0.0019	−0.0014	−0.0011	−0.0009	−0.0008	−0.0007	−0.0006	−0.0005	−0.0005	−0.0004	−0.0004
Ecs	0.0137	0.0005	1.10×10^{-5}	3.96×10^{-6}	2.69×10^{-6}	2.02×10^{-6}	1.59×10^{-6}	1.30×10^{-6}	1.09×10^{-6}	9.25×10^{-7}	7.97×10^{-7}	6.95×10^{-7}	6.12×10^{-7}
Total	−0.0167	−0.0035	−0.0018	−0.0013	−0.0010	−0.0008	−0.0007	−0.0006	−0.0006	−0.0005	−0.0005	−0.0004	−0.0004
The Lucia and Schwartz Model													
X	−0.0121	−0.0036	−0.0009	−0.0005	−0.0004	−0.0003	−0.0002	−0.0002	−0.0002	−0.0002	−0.0001	−0.0001	−0.0001
e	0.0000	-5.31×10^{-6}	-5.32×10^{-6}	-5.36×10^{-6}	-5.47×10^{-6}	-5.42×10^{-6}	-5.48×10^{-6}	-5.32×10^{-6}	-5.29×10^{-6}	-5.25×10^{-6}	-5.16×10^{-6}	-5.22×10^{-6}	-5.16×10^{-6}
Total	−0.0121	−0.0036	−0.0009	−0.0005	−0.0004	−0.0003	−0.0002	−0.0002	−0.0002	−0.0002	−0.0001	−0.0001	−0.0001
The Villaplana Model													
X	−0.0123	−0.0014	−0.0007	−0.0004	−0.0003	−0.0003	−0.0002	−0.0002	−0.0002	−0.0001	−0.0001	−0.0001	−0.0001
ϵ	0.0000	-5.24×10^{-6}	-5.25×10^{-6}	-5.29×10^{-6}	-5.40×10^{-6}	-5.35×10^{-6}	-5.41×10^{-6}	-5.25×10^{-6}	-5.22×10^{-6}	-5.18×10^{-6}	-5.08×10^{-6}	-5.14×10^{-6}	-5.09×10^{-6}
Jump	−0.0059	−0.0005	−0.0002	−0.0001	−0.0001	-8.68×10^{-5}	-7.23×10^{-5}	-6.24×10^{-5}	-5.49×10^{-5}	-4.90×10^{-5}	-4.32×10^{-5}	-3.89×10^{-5}	-3.54×10^{-5}
Total	−0.0181	−0.0019	−0.0009	−0.0006	−0.0004	−0.0003	−0.0003	−0.0003	−0.0002	−0.0002	−0.0002	−0.0002	−0.0001

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