An Experiment in Chaos: The Double Pendulum

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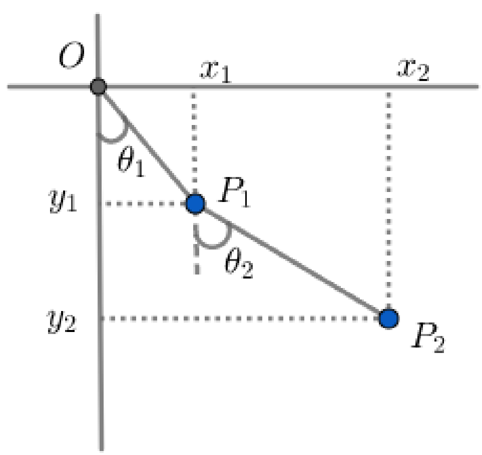
Abstract: We describe an experiment analyzing the motion of a double pendulum. Double pendulums were found to be extremely sensitive to initial conditions. In other words, infinitesimally close initial data would result in arbitrarily large divergences as time evolves. As a result, the motion of such a simple system is exceptionally difficult to predict. By obtaining a dataset using *Logger Pro*- a data analysis software, we can gain some insights into the chaotic behavior in the seemingly excessively deterministic mechanical system.



Introduction

The objective of this experiment is to have a better understanding of the dynamical complexity of a double pendulum system. We want to test our **hypothesis that double pendulums, as opposed to single pendulums, exhibit chaotic behaviors when θ2 starts to deviate from θ1. The motion is chaotic, in the sense that a slight change in initial conditions would result in a dramatic difference in measurements of positions as time progresses.** We intend to test this hypothesis by using a double pendulum consisting of a metal ball with a wire attached to it, recording its trajectory at different angles, tracking its motion using Logger Pro and measuring the various positions of the metal ball in the Cartesian coordinates. The only independent variable in this experiment is θ2 (See Figure 1). The angle is controlled by the experimenter. The variable that is being tested and measured is the vertical position (y component) of the metal ball. As we change the independent variable (θ2), the effect on the vertical position of the metal ball is observed and recorded. We will assume that the length of the wire is constant throughout the experiment.

The double pendulum that will be analyzed consists of two simple pendulums (Figure 1). The upper simple pendulum is suspended at a fixed point *O*. The lower simple pendulum is suspended at the lower end of the upper pendulum, called P_1. The lower end of the lower pendulum is called P_2. The plane in which the pendulums swing will have *O* as the origin, the horizontal line through *O* as the x-axis, and the vertical line through *O* as the y-axis. Let l_1 and l_2 be the lengths of the upper and lower pendulums respectively, m_1 and m_2 be the mass of P_1 and P_2. The angles created by the pendulums and the y-axis are θ1 and θ2. The positions of P_1 and P_2 are at (x_1, y_1) and (x_2, y_2) respectively.

Methods

1. Assemble the double pendulum:

   a. The two masses should ideally be small, spherical masses with a hole through a diameter of the spheres. Tie one mass through the hole of another, leaving approximately 25 cm between the masses.

2. Attach the two-mass system to a stable stand with boom arm, leaving approximately 25 cm between the top mass and stand. The stand should be located on a table or surface with enough space for the pendulum to swing freely.
3. Use the Starrett ASME 1000 aluminum straight edge balanced on its short side as a reference point to maintain the same initial $\theta 1$ for all trials. Place the straight edge approximately 10 cm to the left of the center of the pendulum stand. Adjust the height of the boom arm as needed to allow for the top mass to be comfortably in line with the straight edge.
4. You will be using a video recording device to film approximately 3 seconds of the double pendulum's motion, so create a stable position for the recording device in front of the pendulum setup. The camera should view the motion of the pendulum perpendicular to the plane of motion. Make sure to have the camera in a stable position that allows the entire motion of the pendulum to be observed.
5. Starting with $\theta 2 = 0^\circ$, begin to record the motion of the pendulum for about three seconds after releasing it. Repeat at this initial condition for a total of five trials. Some things to keep in mind when doing trials:

   b. Be mindful of how you release the masses, attempting to be consistent with how you release the pendulum across all trials. Keep the masses swinging in the same plane, parallel to the camera.

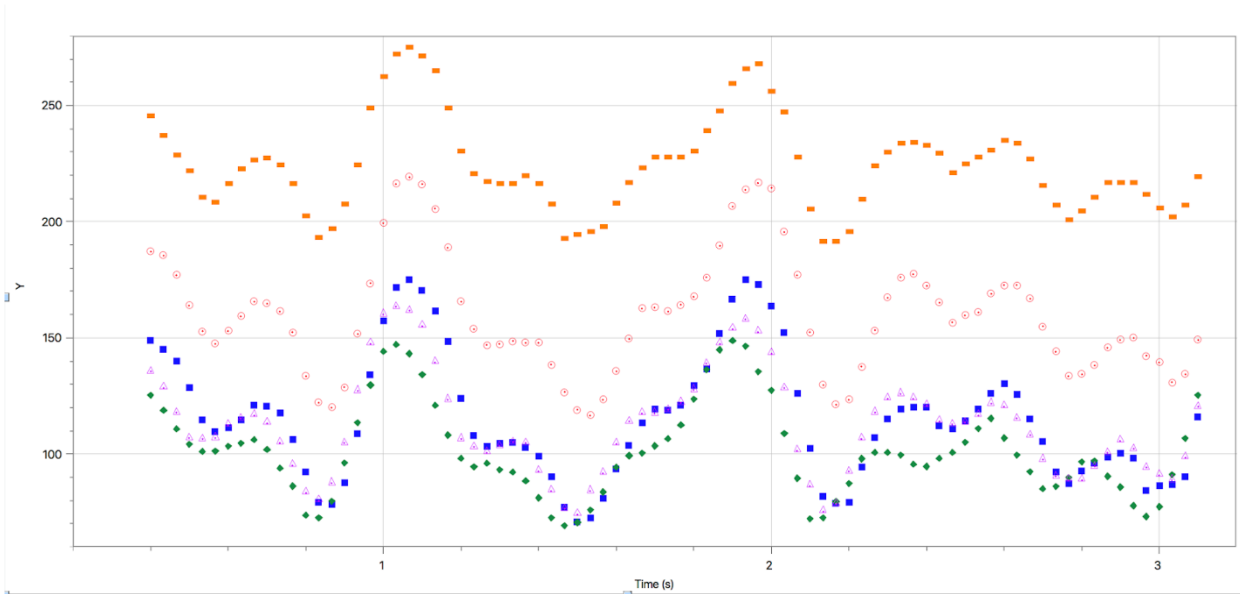
   c. Any trials in which the pendulum hits any surrounding objects, such as the vertical straight edge, table, etc. should be omitted from the data and repeated.
6. Repeat step five at both $\theta 2=\theta 1$ and $\theta 2=90^\circ$.

7. After recording all the data, use logger pro's (or a similar software) video analysis tool to plot points at every frame for three seconds. This points should be plotted on the lower mass of the double pendulum and they denote the position of this mass.
8. Using these data points plot graphs to analyze your results.

Experimental Results

We first took measurements starting at θ2 = 0º , θ2 = θ1 =30º and then θ2 =90º. We repeated all three experiments 5 times and gathered the data and plotted them. We then created the a table with the mean, standard deviation and the standard errors for all the trials and compared them.The results are in the following order - graph for trials followed by the corresponding table.

θ2 = 0º, Trial 1- red, Trail 2- blue, Trial 3- purple, Trial 4- green, Trail 5- yellow

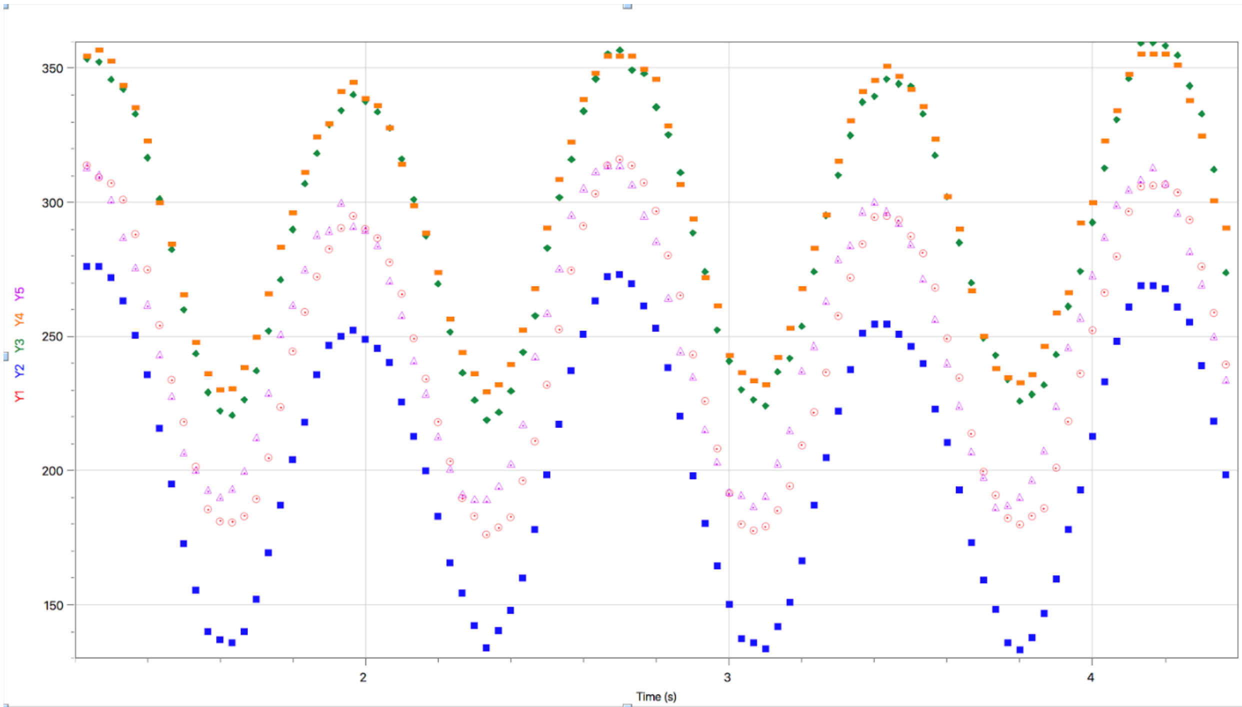


Time(s)	Y1(cm)	Y2(cm)	Y3(cm)	Y4(cm)	Y5(cm)	Mean(cm)	Std. Dev.(cm)	Std. Err.(cm)
0.400	187.282	148.930	125.319	245.757	135.468	168.551	10.921	1.213
0.433	185.368	145.219	118.757	237.389	128.468	163.040	10.830	1.203
0.467	177.067	139.926	110.792	229.147	117.651	154.917	10.865	1.207
0.500	163.926	128.364	104.163	222.311	106.815	145.116	10.964	1.218
0.533	152.735	114.696	101.003	210.971	106.303	137.142	10.217	1.135
0.567	147.450	109.598	101.331	208.846	106.909	134.827	10.047	1.116
0.600	152.934	111.161	103.300	216.698	112.444	139.307	10.532	1.170
0.633	159.317	114.528	104.686	222.905	115.249	143.337	10.943	1.216
0.667	165.739	120.817	106.198	226.975	116.686	147.283	11.120	1.236
0.700	164.669	120.645	101.882	227.620	113.366	145.636	11.474	1.275
0.733	161.423	117.798	93.991	224.745	104.889	140.569	11.906	1.323
0.767	152.399	106.094	86.175	216.706	95.116	131.298	12.029	1.337
0.800	133.469	92.344	73.702	202.725	83.268	117.102	11.780	1.309
0.833	122.192	78.977	72.557	193.702	80.053	109.496	11.339	1.260
0.867	120.001	78.157	79.721	197.507	87.475	112.572	11.201	1.245
0.900	128.747	87.696	96.194	207.764	104.612	125.003	10.830	1.203
0.933	151.794	108.719	113.448	224.760	127.124	145.169	10.564	1.174
0.967	173.434	134.145	129.628	249.436	147.522	166.833	10.939	1.215
1.000	199.286	157.403	144.214	262.839	160.034	184.755	10.722	1.191
1.033	216.223	171.555	147.061	272.487	163.245	194.114	11.280	1.253
1.067	219.301	175.079	143.178	275.503	161.553	194.923	11.797	1.311
1.100	216.165	170.294	134.171	271.432	155.081	189.428	12.191	1.355
1.133	205.532	161.676	120.964	265.292	139.573	178.607	12.849	1.428
1.167	188.840	148.477	108.112	249.053	123.370	163.571	12.603	1.400
1.200	165.563	123.864	98.147	230.550	106.440	144.913	12.106	1.345
1.233	153.911	108.012	94.538	221.128	102.889	136.096	11.741	1.305
1.267	146.864	103.102	96.026	217.772	100.585	132.870	11.488	1.276
1.300	147.352	104.305	93.218	216.678	103.280	132.967	11.384	1.265
1.333	148.512	104.981	92.280	216.678	105.135	133.517	11.364	1.263
1.367	147.876	102.618	88.487	220.159	104.589	132.746	11.931	1.326
1.400	147.876	98.907	81.132	216.718	92.675	127.461	12.450	1.383
1.433	138.333	90.122	72.577	207.718	84.268	118.603	12.392	1.377
1.467	126.520	76.899	69.155	193.003	76.335	108.382	11.678	1.298
1.500	118.923	70.712	70.565	194.725	74.128	105.810	11.944	1.327
1.533	116.563	72.528	75.928	196.151	84.057	109.046	11.493	1.277
1.567	123.505	80.903	83.612	198.296	91.753	115.614	10.941	1.216
1.600	135.719	93.376	94.370	208.436	104.389	127.258	10.781	1.198
1.633	149.696	103.540	99.245	217.151	113.964	136.719	10.921	1.213
1.667	162.657	113.434	100.401	223.288	117.577	143.471	11.203	1.245
1.700	163.094	119.462	103.468	228.249	117.405	146.335	11.324	1.258
1.733	161.364	118.973	106.550	228.249	119.057	146.839	11.117	1.235
1.767	164.118	120.997	112.436	228.249	122.089	149.578	10.745	1.194
1.800	167.649	129.282	123.593	230.495	127.530	155.710	10.098	1.122
1.833	175.962	136.430	136.237	239.589	138.784	165.400	9.947	1.105

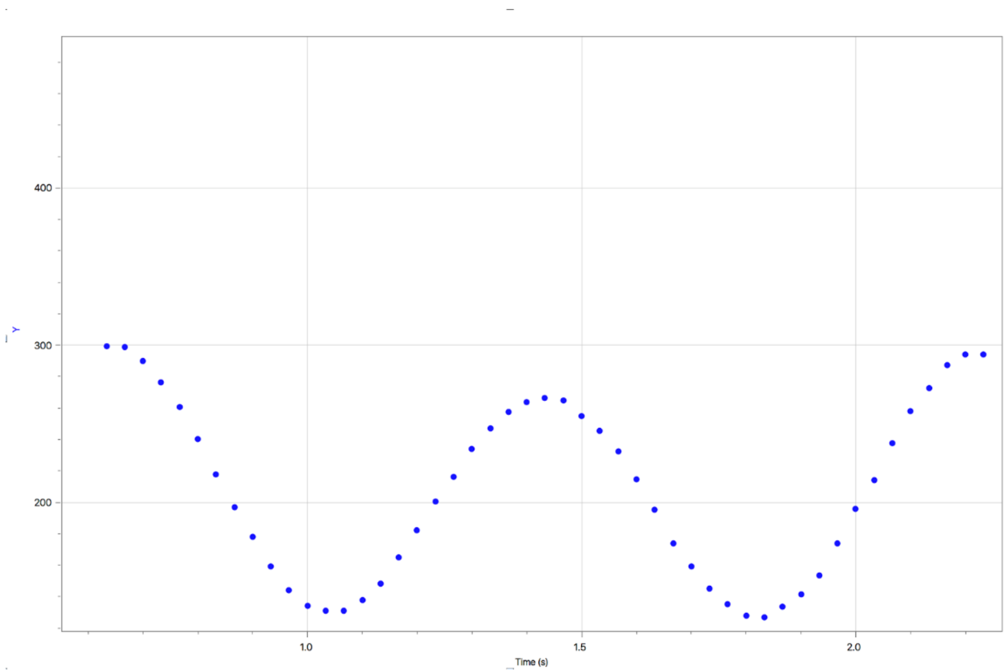
1.867	189.673	151.579	144.854	248.073	147.510	176.338	9.787	1.087
1.900	206.637	166.602	148.803	259.659	153.878	187.116	10.326	1.147
1.933	213.833	174.926	146.346	266.292	157.874	191.854	10.850	1.206
1.967	216.727	173.024	135.526	268.350	152.620	189.250	11.917	1.324
2.000	214.438	163.649	127.417	256.245	143.382	181.026	11.846	1.316
2.033	195.673	152.266	108.901	247.725	128.268	166.567	12.388	1.376
2.067	176.934	125.997	89.534	227.960	101.487	144.382	12.781	1.420
2.100	152.321	102.364	72.053	205.956	86.350	123.809	12.224	1.358
2.133	129.805	81.872	72.608	191.760	75.405	110.290	11.365	1.263
2.167	121.274	78.762	79.553	191.753	78.428	109.954	10.949	1.217
2.200	123.399	79.071	87.253	196.237	92.343	115.660	10.682	1.187
2.233	137.548	94.485	97.999	210.085	106.534	129.330	10.718	1.191
2.267	153.247	107.130	100.733	224.276	117.827	140.642	11.326	1.258
2.300	167.270	115.212	100.733	230.401	123.850	147.493	11.681	1.298
2.333	175.930	119.321	99.542	234.186	125.850	150.966	12.083	1.343
2.367	177.516	119.977	95.659	234.514	123.882	150.310	12.391	1.377
2.400	172.415	119.977	94.589	233.178	120.866	148.205	12.285	1.365
2.433	165.098	112.094	98.093	229.991	114.241	143.903	12.096	1.344
2.467	156.626	110.872	100.604	221.288	112.690	140.416	11.128	1.236
2.500	159.657	114.048	105.022	225.112	113.460	143.460	11.206	1.245
2.533	161.067	119.208	111.018	227.983	116.639	147.183	10.968	1.219
2.567	169.098	125.833	115.288	231.260	121.639	152.624	10.845	1.205
2.600	172.645	130.075	106.846	235.499	120.581	153.129	11.600	1.289
2.633	172.645	125.559	99.608	234.198	115.175	149.437	12.146	1.350
2.667	166.977	114.907	92.374	227.178	108.034	141.894	12.288	1.365
2.700	154.911	105.348	85.112	216.030	97.593	131.799	12.007	1.334
2.733	143.993	92.094	86.046	207.561	90.280	123.995	11.642	1.294
2.767	133.657	87.118	89.811	201.311	88.811	120.142	10.979	1.220
2.800	134.395	92.700	96.546	205.085	88.811	123.507	10.917	1.213
2.833	138.309	96.001	97.061	210.694	94.507	127.314	11.136	1.237
2.867	145.848	98.548	90.397	216.991	100.331	130.423	11.788	1.310
2.900	149.192	100.087	85.741	216.995	105.776	131.558	11.845	1.316
2.933	150.075	98.239	77.729	216.995	102.096	129.027	12.416	1.380
2.967	142.204	84.051	73.077	212.018	94.128	121.096	12.726	1.414
3.000	139.579	86.481	77.288	206.268	90.925	120.108	11.977	1.331
3.033	130.813	86.782	91.050	202.300	88.729	119.935	11.005	1.223
3.067	134.395	90.137	106.811	207.393	98.550	127.457	10.595	1.177
3.100	149.090	115.895	125.385	219.569	120.245	146.037	9.569	1.063

Standard Error mean: 1.265 mm

$\theta 2 = 30^{o}$, Trial 1- red, Trail 2- blue, Trial 3- purple, Trial 4- green, Trail 5- yellow



The graph of a simple pendulum for comparison

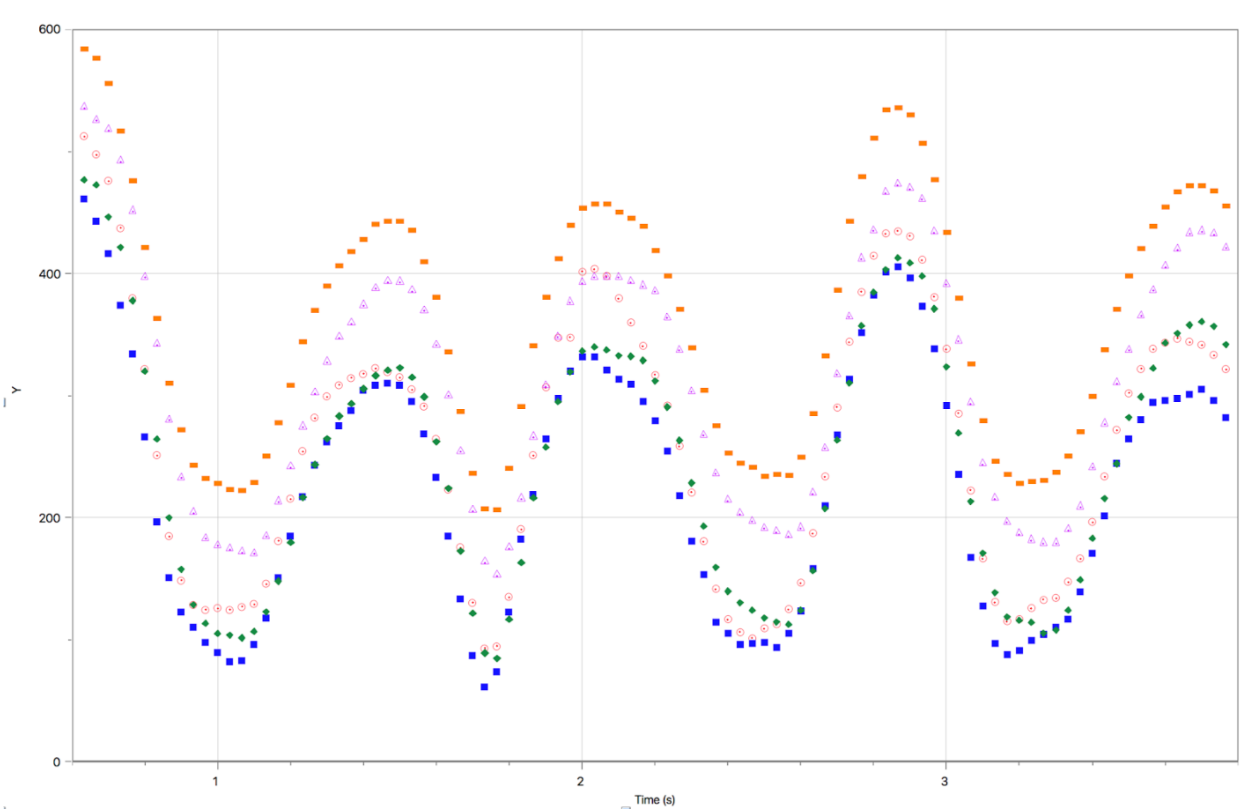


time(s)	Y1(mm)	Y2(mm)	Y3(mm)	Y4(mm)	Y5(mm)	Mean(mm)	Std.dev	Std.err
1.2333	313.7272	276.0475	353.5358	354.7545	312.4733	322.1077	6.8298	0.7044
1.2667	309.0905	276.0045	352.2975	356.8444	309.4538	320.7381	7.0052	0.7225
1.3000	307.0475	271.8522	345.7350	352.8795	300.1647	315.5358	6.9739	0.7193
1.3333	300.8170	263.1022	342.2506	343.6842	286.3444	307.2397	7.3176	0.7548
1.3667	288.0475	250.3014	332.9772	335.5866	274.9967	296.3819	7.7087	0.7951
1.4000	274.8561	235.6139	316.5983	323.1022	261.1803	282.2702	7.7058	0.7948
1.4333	254.0358	215.6139	301.1686	300.0553	242.4147	262.6577	7.7452	0.7989
1.4667	233.8639	194.9069	282.4928	284.7038	226.9694	244.5873	7.9917	0.8243
1.5000	218.1569	172.9108	259.9694	265.8092	206.1178	224.5928	8.0349	0.8287
1.5333	201.4538	155.4147	243.5827	248.0514	199.5631	209.6131	7.8538	0.8101
1.5667	185.4342	140.0280	229.1334	236.4459	192.1647	196.6413	8.0261	0.8278
1.6000	181.1842	136.8913	222.3248	230.4225	189.5241	192.0694	7.7293	0.7972
1.6333	180.8327	135.6452	220.5358	230.9108	192.5436	192.0936	7.7800	0.8024
1.6667	183.1217	139.9694	226.3795	238.4967	199.0514	197.4038	8.0547	0.8308
1.7000	189.4381	152.0983	237.2663	249.9577	211.5788	208.0678	8.0984	0.8353
1.7333	204.6413	169.3366	252.0944	266.3131	228.1881	224.1147	8.0054	0.8257
1.7667	223.6373	186.9264	271.1373	283.7077	250.0319	243.0881	8.0438	0.8297
1.8000	244.4967	203.8873	289.8639	296.2467	261.1061	259.1202	7.7543	0.7998
1.8333	258.9264	217.9342	306.8678	311.3991	274.2116	273.8678	7.9308	0.8180
1.8667	272.1647	235.8913	318.2623	324.5670	287.2155	287.6202	7.4912	0.7727
1.9000	282.5397	246.7038	328.8913	329.5631	288.7311	295.2858	7.2363	0.7464
1.9333	290.2116	249.8600	334.1569	341.6373	299.0944	302.9920	7.6666	0.7908
1.9667	294.7506	252.4264	340.1647	344.8014	290.2741	304.4834	7.9685	0.8219
2.0000	290.1608	248.9616	337.6608	338.7741	289.0241	300.9163	7.8561	0.8103
2.0333	286.6764	245.4850	333.6998	336.3561	283.3444	297.1123	7.9261	0.8175
2.0667	277.6530	240.2389	327.6178	327.8795	269.9733	288.6725	7.9453	0.8195
2.1000	265.8913	225.5241	316.0827	314.3288	257.1608	275.7975	8.0859	0.8340
2.1333	249.1139	212.8092	301.0202	299.0006	240.1842	260.4256	7.9926	0.8244
2.1667	234.1295	199.9186	287.6139	288.7780	228.0553	247.6991	8.1212	0.8376
2.2000	218.1842	183.0553	269.6608	273.9889	212.1061	231.3991	8.1398	0.8396
2.2333	203.4616	165.7155	251.6100	256.8405	200.0866	215.5428	7.9474	0.8197
2.2667	189.7077	154.4850	236.4264	244.4381	190.6569	203.1428	7.7022	0.7944
2.3000	183.0123	142.3795	226.3209	236.4694	188.5514	195.3467	7.7974	0.8042
2.3333	176.0553	134.0944	218.8678	229.5358	188.6373	189.4381	7.8402	0.8087
2.3667	178.7194	140.3952	221.6647	232.0788	193.4381	193.2592	7.5633	0.7801
2.4000	182.6413	147.8952	229.6217	239.6959	201.9772	200.3663	7.6711	0.7912
2.4333	196.3248	159.9498	244.1491	252.6295	216.5631	213.9233	7.7886	0.8033
2.4667	210.9186	178.1569	257.7311	268.0397	241.6491	231.2991	7.6166	0.7856
2.5000	231.8092	198.5006	282.9420	290.7780	258.1022	252.4264	7.8749	0.8122
2.5333	252.7194	217.2858	301.7623	308.8600	274.6530	271.0561	7.7684	0.8013
2.5667	274.6491	237.0748	316.0123	322.6920	294.6998	289.0256	7.1838	0.7410
2.6000	291.2780	250.8639	333.9538	338.6881	304.5319	303.8631	7.3951	0.7627
2.6333	303.2272	263.3561	345.9967	348.3405	310.7897	314.3420	7.2539	0.7482
2.6667	313.6569	272.3561	355.2936	354.7506	312.9381	321.7991	7.1820	0.7408

2.7000	315.9147	273.1178	356.7545	354.7506	312.9381	322.6952	7.1742	0.7400
2.7333	313.7780	269.7897	349.3600	354.7506	305.9069	318.7170	7.1990	0.7425
2.7667	307.2819	261.4577	348.0631	349.8327	294.3600	312.1991	7.7734	0.8018
2.8000	296.8092	253.1686	335.4694	345.9577	284.7975	303.2405	7.8646	0.8112
2.8333	280.0397	238.2155	325.2233	328.7975	263.6998	287.1952	8.1512	0.8407
2.8667	265.1998	220.3639	311.0866	307.0709	243.8873	269.5217	8.1846	0.8442
2.9000	243.0748	198.2077	288.6842	293.9225	234.2663	251.6311	8.2893	0.8550
2.9333	225.8873	180.4772	274.0944	272.2233	214.5709	233.4506	8.2794	0.8540
2.9667	208.1256	164.4772	252.4967	261.5592	202.5788	217.8475	8.2223	0.8481
3.0000	191.6139	149.9733	240.8209	243.3366	191.1842	203.3858	8.1239	0.8379
3.0333	180.0788	137.4850	230.1413	236.9303	190.2428	194.9756	8.3899	0.8654
3.0667	177.5709	135.6452	226.4264	233.7897	186.0202	191.8905	8.2640	0.8524
3.1000	179.0827	133.5084	224.1842	232.1178	189.7389	191.7264	8.1900	0.8447
3.1333	185.0983	141.7858	236.8092	242.3639	202.0592	201.6233	8.5205	0.8788
3.1667	194.1256	150.8170	241.9186	253.3014	214.3561	210.9038	8.4645	0.8731
3.2000	209.4108	166.2311	253.8248	267.9538	236.5475	226.7936	8.3537	0.8616
3.2333	221.6100	187.2702	274.0827	283.0592	245.7545	242.3553	8.1252	0.8380
3.2667	236.6139	204.8834	295.1061	295.6100	262.5006	258.9428	8.0895	0.8344
3.3000	257.6686	222.0553	310.1647	315.4889	278.1100	276.6975	8.0152	0.8267
3.3333	271.9420	237.6959	324.9030	330.5045	283.2233	289.6538	8.0094	0.8261
3.3667	284.4538	251.2897	337.3678	341.4108	295.9108	302.0866	7.8429	0.8089
3.4000	294.4498	254.6139	339.5123	345.6491	299.3717	306.7194	7.6975	0.7939
3.4333	294.9772	254.6139	345.9381	351.1256	295.9069	308.5123	8.3368	0.8599
3.4667	293.3678	250.8131	344.1647	347.1217	291.5163	305.3967	8.3991	0.8663
3.5000	287.3639	246.2584	343.2428	342.3288	283.6569	300.5702	8.6602	0.8932
3.5333	281.1256	239.9538	332.9538	335.7663	270.7350	292.1069	8.5960	0.8866
3.5667	268.1647	223.0045	317.4264	324.0397	255.6530	277.6577	8.8562	0.9134
3.6000	249.2897	210.5944	302.0163	302.3014	239.3131	260.7030	8.3832	0.8647
3.6333	234.4069	192.6452	284.9264	290.3248	223.4967	245.1600	8.6534	0.8925
3.6667	213.7194	173.2623	269.9967	267.3756	206.3405	226.1389	8.6547	0.8927
3.7000	199.5788	159.0709	249.5045	250.4850	196.9655	211.1209	8.0760	0.8330
3.7333	190.6881	148.4264	242.9616	238.1803	185.5319	201.1577	8.1995	0.8457
3.7667	182.3405	135.6959	233.9264	234.7819	186.4928	194.6475	8.5803	0.8850
3.8000	179.9498	133.3014	225.8952	233.0905	189.2663	192.3006	8.3183	0.8580
3.8333	183.0358	137.8131	228.3952	236.1452	195.8405	196.2459	8.1746	0.8431
3.8667	185.8131	146.7506	231.9303	246.5319	206.8522	203.5756	8.1680	0.8425
3.9000	200.9108	159.4069	243.1920	258.9655	223.1959	217.1342	8.0731	0.8327
3.9333	218.2858	177.8991	261.2116	266.6334	245.3327	233.8725	7.5690	0.7807
3.9667	236.2545	192.8170	274.3327	292.6061	256.2702	250.4561	7.9680	0.8218
4.0000	252.1491	212.5670	292.4733	300.1217	272.1295	265.8881	7.2953	0.7525
4.0333	266.3678	233.0475	312.7545	322.9498	286.3014	284.2842	7.5176	0.7754
4.0667	279.8014	248.2780	330.8522	334.5280	298.4186	298.3756	7.4894	0.7725
4.1000	296.4655	261.1295	346.1139	347.9498	304.1022	311.1522	7.5804	0.7819
4.1333	305.9655	268.7428	359.3170	355.4928	307.6452	319.4327	7.8842	0.8132
4.1667	306.1881	268.7428	359.5866	355.4928	312.1881	320.4397	7.8322	0.8078

Standard Error mean: 0.7607 mm

θ2 = 90º, Trial 1- red, Trail 2- blue, Trial 3- purple, Trial 4- green, Trail 5- yellow



time(s)	Y1	Y2	Y3	Y4	Y5	mean	std.dev	std.err
0.6333	512.2389	461.2780	476.7428	584.3444	535.5983	514.0405	10.1524	1.0471
0.6667	497.4811	442.3209	472.3913	576.5280	525.1217	502.7686	10.6482	1.0983
0.7000	475.7545	416.3873	446.2702	555.9264	517.2389	482.3155	11.5130	1.1875
0.7333	437.0748	374.1842	421.3366	516.8248	491.4030	448.1647	11.7773	1.2147
0.7667	379.5045	334.0084	377.6373	476.3913	450.6217	403.6327	12.0960	1.2476
0.8000	321.9420	265.7350	319.7702	421.4967	396.6686	345.1225	13.1052	1.3517
0.8333	251.0475	196.5866	264.1686	364.0045	341.5436	283.4702	14.2296	1.4677
0.8667	184.7702	151.0514	200.1256	311.0553	279.7428	225.3491	13.9536	1.4392
0.9000	148.4069	122.6178	157.7506	272.3952	232.4889	186.7319	13.0554	1.3466
0.9333	128.1920	109.8170	128.5553	243.9264	203.7116	162.8405	12.0183	1.2396
0.9667	124.6413	97.6569	113.5084	232.5397	182.4811	150.1655	11.6315	1.1997
1.0000	125.5709	89.0397	105.0748	228.1803	176.8248	144.9381	11.8389	1.2211
1.0333	124.6842	81.8795	103.7545	223.8834	174.0358	141.6475	11.8719	1.2245
1.0667	126.8873	82.9889	101.5553	222.6413	171.2936	141.0733	11.6874	1.2055
1.1000	129.2233	95.9850	106.8014	229.5905	170.0553	146.3311	11.3047	1.1660
1.1333	145.4616	117.7233	122.7584	250.9303	184.3092	164.2366	11.4324	1.1792
1.1667	180.8756	150.5944	148.0709	278.3952	212.6452	194.1163	11.1856	1.1537
1.2000	215.2741	185.0123	179.5944	309.1959	240.9694	226.0092	10.9221	1.1265
1.2333	254.0905	217.2311	216.5163	344.7116	274.0163	261.3131	10.9342	1.1278
1.2667	281.7311	242.6881	243.3600	370.0670	301.5827	287.8858	10.8779	1.1220
1.3000	299.2623	261.5319	264.6803	390.6491	327.1647	308.6577	11.0270	1.1373
1.3333	308.6647	275.3561	283.1803	406.9967	347.2389	324.2873	11.2143	1.1567
1.3667	314.5123	287.9264	293.5123	418.7311	358.7077	334.6780	11.3257	1.1682
1.4000	317.6920	304.0084	305.5553	428.6920	373.5436	345.8983	11.2627	1.1617
1.4333	322.3913	307.9928	316.2506	441.0475	387.5514	355.0467	11.9309	1.2306
1.4667	318.9498	309.8444	320.8092	443.2155	392.8327	357.1303	12.1364	1.2518
1.5000	314.9459	308.2233	322.6920	443.1022	392.5631	356.3053	12.2697	1.2655
1.5333	304.6920	294.8600	314.9381	436.0436	385.3952	347.1858	12.6743	1.3073
1.5667	291.2663	268.7936	298.8834	409.9616	369.0358	327.5881	12.3146	1.2702
1.6000	264.1491	233.1373	262.0944	381.0514	340.8092	296.2483	12.8503	1.3254
1.6333	223.0241	184.9420	223.9655	336.0123	299.5983	253.5084	12.8827	1.3287
1.6667	175.3561	133.1959	172.5280	287.4069	253.7350	204.4444	13.2200	1.3635
1.7000	130.1920	86.6100	121.7663	237.2975	205.4420	156.2616	12.9956	1.3404
1.7333	92.9264	61.6530	89.1334	207.8991	163.3170	122.9858	12.5502	1.2945
1.7667	94.7272	73.3873	84.7545	207.1295	152.4147	122.4827	11.6698	1.2036
1.8000	135.0202	122.8131	116.7155	241.3444	174.6725	158.1131	10.7237	1.1061
1.8333	190.3483	182.1295	163.0514	291.5748	215.1569	208.4522	10.3883	1.0715
1.8667	250.8717	218.8053	216.2780	341.7272	265.7819	258.6928	10.5729	1.0905
1.9000	306.5045	264.3678	257.7311	380.9498	307.3678	303.3842	10.1857	1.0506
1.9333	347.5905	297.7702	295.0514	413.0280	347.6608	340.2202	9.9746	1.0288
1.9667	347.5905	320.3834	319.3209	439.8405	376.0123	360.6295	10.3732	1.0699
2.0000	401.5358	331.2038	336.3561	454.4733	392.1178	383.1373	10.5716	1.0904
2.0333	403.5123	331.2038	339.8248	457.3366	396.3756	385.6506	10.6949	1.1031
2.0667	397.7389	320.7389	337.3170	457.6491	396.6061	382.0100	11.3276	1.1684

2.1000	379.6959	313.3561	332.7936	450.7858	396.6061	374.6475	11.2704	1.1625
2.1333	359.8444	309.2623	332.0788	445.7936	392.8952	367.9748	11.1097	1.1459
2.1667	340.7389	295.4225	328.7233	439.5241	389.3209	358.7459	11.6837	1.2051
2.2000	316.9928	279.1217	311.8873	419.5748	384.7936	342.4741	11.9739	1.2350
2.2333	291.7858	254.2116	290.5475	398.6920	363.4655	319.7405	12.3013	1.2688
2.2667	258.3131	217.7506	263.2741	371.5436	336.8873	289.5538	13.0281	1.3437
2.3000	220.6881	180.5202	228.3913	339.7272	302.9381	254.4530	13.4835	1.3907
2.3333	180.2584	153.4147	193.0163	305.2428	267.2194	219.8303	13.2133	1.3628
2.3667	141.6881	114.3639	159.3092	275.6998	235.5866	185.3295	14.0316	1.4472
2.4000	116.6764	104.8756	139.7350	253.1373	213.8561	165.6561	13.4170	1.3839
2.4333	106.3522	96.2116	130.4108	245.0827	202.9694	156.2053	13.4548	1.3878
2.4667	101.1803	96.8991	124.1491	242.1217	196.7897	152.2280	13.3288	1.3748
2.5000	109.2116	97.7233	117.7623	234.7936	190.9928	150.0967	12.4028	1.2793
2.5333	112.5514	93.6608	114.5827	236.0944	188.1530	149.0084	12.5697	1.2965
2.5667	124.8522	105.5045	112.4616	234.9303	184.5631	152.4623	11.5465	1.1909
2.6000	146.3483	123.7741	124.1725	249.8405	191.0709	167.0413	11.1544	1.1505
2.6333	186.9811	158.2741	156.5241	285.5983	219.5827	201.3920	11.1208	1.1470
2.6667	233.5045	209.3444	207.5631	333.3639	256.4030	248.0358	10.7249	1.1062
2.7000	290.3444	268.0045	263.3952	387.3522	316.9967	305.2186	10.4932	1.0823
2.7333	344.0436	313.2350	310.2311	443.7506	364.1920	355.0905	11.2784	1.1633
2.7667	384.9733	351.0788	357.1452	480.1608	411.6334	396.9983	10.8616	1.1203
2.8000	414.4498	382.3014	384.4342	511.6842	434.7428	425.5225	10.9681	1.1313
2.8333	432.5983	401.5084	403.1334	534.9967	465.7038	447.5881	11.4969	1.1858
2.8667	434.6920	405.6256	412.8014	535.8756	472.6100	452.3209	11.0940	1.1443
2.9000	430.4186	396.5280	408.4733	530.3561	469.2272	447.0006	11.2398	1.1593
2.9333	411.1608	373.0163	397.8209	507.3913	460.3209	429.9420	11.1424	1.1493
2.9667	380.7389	337.9694	371.0358	477.6452	433.6491	400.2077	11.4622	1.1822
3.0000	337.9459	292.0983	323.4772	434.3600	390.7467	355.7256	11.7383	1.2107
3.0333	285.4069	235.7155	269.2506	380.3288	344.4420	303.0288	12.1252	1.2506
3.0667	222.2155	167.7077	213.0944	326.5827	293.3288	244.5858	13.3206	1.3739
3.1000	166.5866	127.5553	171.0866	280.3522	243.7311	197.8623	12.9294	1.3336
3.1333	130.9381	97.0631	138.6061	247.2077	215.4069	165.8444	13.0209	1.3430
3.1667	114.8913	87.9850	118.7584	236.3600	195.9850	150.7959	12.9615	1.3369
3.2000	116.4694	91.0280	115.7623	228.5358	186.5084	147.6608	11.9351	1.2310
3.2333	125.7975	99.5709	114.3600	230.2311	181.1295	150.2178	11.2665	1.1621
3.2667	132.8600	104.1413	105.5280	230.8991	178.6842	150.4225	11.2360	1.1589
3.3000	134.1256	109.9108	107.8834	237.4889	178.7077	153.6233	11.3809	1.1738
3.3333	147.4498	116.7741	124.2389	251.3834	189.8366	165.9366	11.5329	1.1895
3.3667	166.4967	139.2897	148.9694	271.2545	208.5983	186.9217	11.2203	1.1573
3.4000	196.2194	170.5514	182.9342	299.7350	240.5397	217.9959	10.9466	1.1291
3.4333	233.6100	201.6217	215.6334	337.9655	276.4889	253.0639	11.4454	1.1805
3.4667	271.5553	244.6530	244.2155	371.0592	310.0748	288.3116	11.0969	1.1446
3.5000	302.1334	264.0397	282.1686	398.8170	336.8717	316.8061	11.0365	1.1383
3.5333	321.9069	279.8561	298.9850	420.7038	364.8834	337.2670	11.6959	1.2063
3.5667	338.0631	294.5006	322.3913	439.6256	385.6530	356.0467	11.8712	1.2244

Standard Error mean:1.222 mm

The results shown above give a basic idea of what we recorded. We performed 5 trials of each initial angle and then recorded the positions of each trial. Then for an initial angle, we calculated the mean, standard deviation and standard error using the 5 trials. Each graph shown contains 5 different measurements take from the trail. We decided to overlap the 5 trail to show how visually, the graphs a the initial angles somewhat appear the same. For $\theta1 = \theta2 = 30^o$,we also attached a graph of a simple pendulum for comparison. This is because, according to our hypothesis, the behavior for $\theta1 = \theta2 = 30^o$ should be simple harmonic.

The tables below show the maximum and minimum Y position of the lower mass within the time interval of 2.6-3.1 seconds. In looking at our data tables, we realized that, due to differing placement of the camera, the Y component of position between trials is shifted up or down. Because of this, we decided to take the difference of the maximum and minimum Y position within the given time interval, 2.6-3.1 seconds. This negates the error in the shifted translation of the position vs. time graphs seen above.

θ2=0

	Y, trial 1 (mm)	Y, trial 2 (mm)	Y, trial 3 (mm)	Y, trial 4 (mm)	Y, trial 5 (mm)
Max. Y (mm)	172.645	130.075	125.385	235.499	120.581
Min. Y (mm)	130.813	84.051	73.077	201.311	88.789

ΔY (mm)	41.832	46.024	52.308	34.188	31.789

Mean **Δ**Y: 41.228±3.774 mm

Standard Deviation: 8.438 mm

θ2=θ1=30º

	Y, trial 1 (mm)	Y, trial 2 (mm)	Y, trial 3 (mm)	Y, trial 4 (mm)	Y, trial 5 (mm)
Max. Y (mm)	315.915	273.118	356.755	354.751	312.938
Min. Y (mm)	177.571	133.508	224.184	232.118	186.020
ΔY (mm)	138.344	139.571	132.571	122.633	126.981

Mean **Δ**Y: 132.015±3.256 mm

Standard Deviation: 7.281 mm

θ2=90º

	Y, trial 1 (cm)	Y, trial 2 (cm)	Y, trial 3 (cm)	Y, trial 4 (cm)	Y, trial 5 (cm)
Max. Y (mm)	434.690	405.626	412.801	535.876	469.227

Min. Y (mm)	146.348	123.774	124.173	249.841	191.071
ΔY (mm)	288.342	281.852	288.628	286.035	278.156

Mean **Δ**Y:284.6026 ± 2.01696mm

Standard Deviation: 4.5101 mm

Uncertainty

There are multiple sources of uncertainty within this experiment. One form of this is in the plane of motion of the pendulum. In order to get precise measurements from the video footage, the pendulum should ideally be swinging in a plane perpendicular to the camera, where there is no motion towards or away from the camera. This is something that we recognized in designing the experiment, but due to limitations in our equipment and in human error we cannot be certain that the pendulum swung in this plane. One way we attempted to control this source of uncertainty is by using a stationary straight edge to maintain an approximately uniform starting position each trial. Another way we controlled this uncertainty is by analyzing the Y component of position, not the X component. The plane of motion of the pendulum is fixed vertically, due to the force of gravity being in a constant downward direction, so the only uncertainty related to the plane of motion is in the XZ plane. Analyzing the Y component of the position mitigates uncertainty of this kind.

Human error in the way the experiment is performed is also another form of uncertainty. The way the weights are held, exactly how the weight is released, and the exact initial conditions will vary slightly from trial to trial due to human error. However, in this experiment, this kind of error may be useful in some sense. If we expect a system to be chaotic, meaning a slight change in an initial condition will result in a large change in the behavior of the system, then even the small human error that occurs between trials should yield noticeably different results. This difference should be noticeable in the position of the mass, and as a result, across multiple trials of "identical" initial conditions, a chaotic system should have more variation than a non-chaotic system. From this understanding of human error causing uncertainty, we can further apply our hypothesis to say that, for repeated trials of "identical" initial conditions, we would expect a more chaotic system to have a greater standard deviation than a non-chaotic system.

One form of random error would be air flow and resistance. Air resistance will dampen the motion of the pendulum, and air flow near the experiment may be varying between trials. Someone walking nearby or a door shutting could cause air flow that may affect the consistency of the results. However, the effects of air should be very slight in this experiment, as the masses were spherical and relatively small, so small changes in air currents would not greatly affect the results.

Another form of uncertainty is due to the quality of video of the data. The motion of the mass at high speeds appears blurred in the video, and as a result the position of the mass in the computer software used to plot the position is not exactly clear. To attempt to maintain consistency, we aimed the cursor in the approximate center of the blurry mass to negate the variation that might occur in this process. However, because much of the time the velocity of the mass is horizontal in the X direction, the blur will have mostly X-biased components. Since we looked at the Y component of position, much of the uncertainty related to the motion blur is avoided. Still, there is some amount of uncertainty related to this, we estimate to be less than one centimeter in the Y direction.

There is also a source of systematic error due to the filming angle which causes a shift in the Y direction. This is noticeable prominently in the first graph under the results. Since the experiment was recorded by hand, the angle at which the camera faced the instrument were not consistent. This kind of error could possibly reduced by using a tripod with our camera or a better filming device.

Conclusions

We have described an experiment in which the results indicate the dynamical complexity of such simple mechanical system. By analyzing the data, we affirmed many aspects of our hypothesis. By repeating the same experiment five times at $\theta 2 = 0°, 30°, 90°$, we found that the pattern appears to be sinusoidal like that of a simple pendulum when $\theta 1 \simeq \theta 2 \simeq 30°$. However, the motion of the system shows no particular patterns when $\theta 2$ deviates from $\theta 1$ by a huge amount. In other words, the double pendulum is extremely sensitive to small perturbations when the difference between $\theta 1$ and $\theta 2$ is increasing. In addition, by taking the difference of the maximum and minimum y position within the given time interval 2.6 – 3.1 seconds, we observed a relatively high standard deviation among the data. A high standard deviation indicates that the data points are spread out over a wider range of values even though the initial conditions are remarkably similar in all five trials. Therefore, the double pendulum is observed to be a system of deterministic chaos due to the fact that any remotely small change in the initial states of the system would result in a completely different result as time progresses.