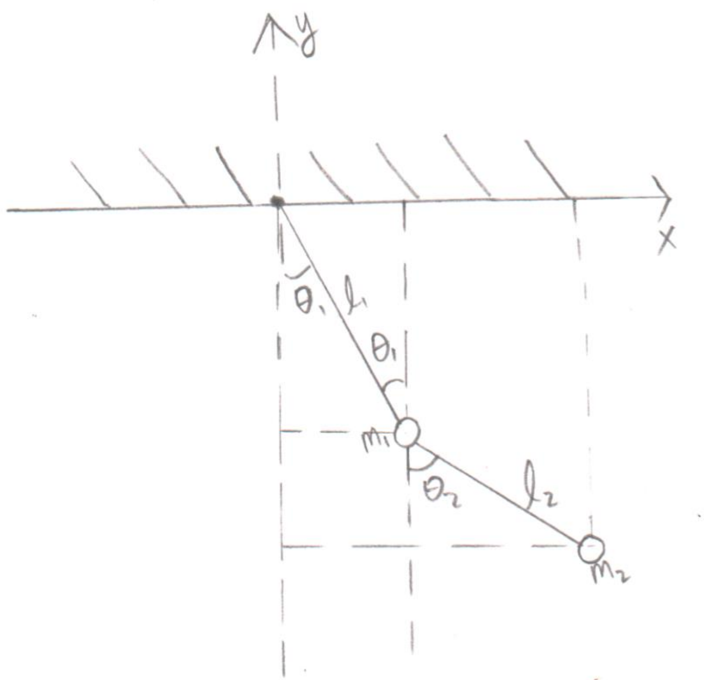
Mathematical model

m_1 and m_2 are located at (x_1, y_1) and (x_2, y_2) respectively.



Rewrite (x, y) in terms of θ and L

$x_1 = L_1 * \sin \theta_1$

$y_1 = - L_1 * \cos \theta_1$

$x_2 = L_1 * \sin \theta_1 + L_2 * \sin \theta_2$

$y_2 = - L_1 * \cos \theta_1 - L_2 * \cos \theta_2$

$(x_1, y_1) = (L_1 * \sin \theta_1, - L_1 * \cos \theta_1)$

$(x_2, y_2) = (L_1 * \sin \theta_1 + L_2 * \sin \theta_2, - L_1 * \cos \theta_1 - L_2 * \cos \theta_2)$

Now, to obtain the velocity, we calculate the derivative of position functions with respect to time

$dx_1/dt = \dot{x}_1 = L_1 * \cos \theta_1 * (d\theta_1/dt)$

$dy_1/dt = \dot{y}_1 = L_1 * \sin \theta_1 * (d\theta_1/dt)$

$dx_2/dt = \dot{x}_2 = L_1 * \cos \theta_1 * (d\theta_1/dt) + L_2 * \cos \theta_2 * (d\theta_2/dt)$

$dx_2/dt = \dot{y}_2 = L_1 * \sin \theta_1 * (d\theta_1/dt) + L_2 * \sin \theta_2 * (d\theta_2/dt)$

To find the lagrangian,

$L = E_k - E_p$

$E_p = m*g*h = g\ (m_1 * y_1 + m_2 * y_2)$

$E_k = (1/2)\ (m * v^2) = (1/2)\ (m_1\ v_1^2 + m_2\ v_2^2)$

Once we solved for L,

Substitute it into the Lagrange’s equation to solve for θ, which is written in the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$