

## Study Module 1: Sets

- Let  $A = \{1, 3, 5\}$  and  $B = \{2, 3, 4\}$ . Determine each of the following sets.
  - $A \cup B$
  - $A \cap B$
  - $A - B$
  - $B - A$
  - $(A \cup B) - (A \cap B)$
  - $(A - B) \cup (B - A)$
- Prove the following set-equality, first by Venn Diagram, and then by algebraic method.

$$(A \cup B) \cap \overline{(A \cap B)} = (A \cap \overline{B}) \cup (\overline{A} \cap B)$$

The algebraic proof uses set-algebraic manipulations to prove the equality, by using valid algebraic rules already proven. The basic rules listed in the table below are assumed to have been proven, and thus are valid rules to use.

Basic Rules (Equalities) in Set Algebra	
Associative Laws:	$(X \cup Y) \cup Z = X \cup (Y \cup Z)$ $(X \cap Y) \cap Z = X \cap (Y \cap Z)$
Commutative Laws:	$X \cup Y = Y \cup X$ $X \cap Y = Y \cap X$
Distributive Laws:	$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
De Morgan's Laws:	$\overline{X \cap Y} = \overline{X} \cup \overline{Y}$ $\overline{X \cup Y} = \overline{X} \cap \overline{Y}$
Complement Laws:	$X \cup \overline{X} = U$ $X \cap \overline{X} = \phi$ $\overline{\overline{X}} = X$
Repetition:	$X \cup X = X$ $X \cap X = X$
0/1 Laws:	$\overline{\phi} = U$ $\overline{U} = \phi$
Identity:	$X \cup \phi = X$ $X \cap U = X$
Bound Laws:	$X \cup U = U$ $X \cap \phi = \phi$

3. Prove each of the following set equalities by algebraic method. In the proofs, you may use the basic rules listed in the table above.

- (a) Combination Rule:

$$(A \cap B) \cup (A \cap \overline{B}) = A$$

- (b) Absorption Rule:

$$A \cup (A \cap B) = A$$

Hint: If you start by distributive law, you will not find it useful. Instead, first replace the leftmost  $A$  by  $A \cap U$  and then apply distributive law. Alternatively, you may find the above "combination rule" useful.

- (c)

$$A \cup (\overline{A} \cap B) = A \cup B$$

$$A \cap (\overline{A} \cup B) = A \cap B$$

- (d)

$$(A \cap \overline{B}) \cup (A \cap B) \cup (\overline{A} \cap B) = A \cup B$$

4. (a) Use Venn Diagrams to prove each of the following.

i.

$$A - B = A \cap \overline{B}$$

ii.

$$(A - B) - C = A \cap \overline{B} \cap \overline{C}$$

- (b) Use algebraic method to prove each of the following set equalities.

**Hint:** The rules stated in the above table, such as Associate law, DeMorgan law, and Distributive law, apply only to intersection and union operations. They do NOT apply to other operations such as set difference. To deal with set difference, it is helpful to first replace set-difference with the corresponding rule that uses set complement.

i.

$$A - (B - C) = (A \cap \overline{B}) \cup (A \cap C)$$

ii.

$$\overline{(A - B)} = \overline{A} \cup B$$

iii.

$$A \cap \overline{(A \cap B)} = A - B$$

5. Prove each of the following set equalities both by Venn Diagram and by algebraic method.

- (a)

$$A - (B \cap C) = (A - B) \cup (A - C)$$

- (b)

$$A - (B \cup C) = (A - B) \cap (A - C)$$

- (c)

$$A \cap (B - C) = (A \cap B) - C = (A \cap B) - (A \cap C)$$

Hint: To prove the last form, use the equality  $A \cap \overline{C} = A \cap (\overline{A} \cup \overline{C})$ .

- (d)

$$A \cup (B - C) = (A \cup B) \cap (A \cup \overline{C}) = (A \cup B) - (\overline{A} \cap C)$$