

# Homework 3 - Proofs

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1. Use **contra-positive proof** method for each of the following.
  - (a) There are 10 boxes. Prove that if 40 balls are placed in the boxes, then at least one box has four or more balls.
  - (b) Let  $x$  be a real number. Prove that if  $x^2$  is irrational, then  $x$  must be irrational.
2. Use **contrapositive proof** for each of the following, where the domain of  $n$  is positive integers.
  - (a) Prove that if  $n^2$  is not divisible by 3, then  $n$  is not divisible by 3.
  - (b) Prove that if  $n^2$  is divisible by 3, then  $n$  is divisible by 3. (Hint: If  $n$  is not divisible by 3, then  $n = 3k + r$ , where  $k$  is an integer quotient and  $r$  is a non-zero remainder,  $r$

.)  
3. Let  $x$  and  $y$  be two real numbers and let  $A = (x + y)/2$ . We want to formally prove that if  $(x > y)$  then

$$x < A < y$$

You are not allowed to state it as a known fact that the average of two values fall between those two values! Rather, you must provide a formal proof in two ways:

- (a) **Direct Method**; and
- (b) **Contrapositive Method**.

Hints: For direct proof, assume  $x < y$