Homework 2 - Logic

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CS 506 - Online Foundations of CS

January 30, 2018

1.

(a) Use a truth-table to show that the follwing propositions are logically equivalent.

i. $X \rightarrow Y$

X	Y	X → Y
T	T	T
T	F	F
F	T	T
F	F	T

ii. $\neg Y \rightarrow \neg X$

X	Y	¬ Y → ¬ X
T	T	T
T	F	F
F	T	T
F	F	T

iii. $\neg(X \wedge \neg Y)$

X	Y	¬(X ∧¬ Y)
T	T	T
T	F	F
F	T	T
F	F	T

iv. $\neg X \vee Y$

X	Y	¬ X ∨ Y
T	T	T
T	F	F
F	T	T
F	F	T

(b) Use truth-table to show that the following propositions are logically equivalent.
i. $X \equiv Y$

X	Y	X ≡ Y
T	T	T
T	F	F
F	T	F
F	F	T

ii. $X \leftrightarrow Y$

X	Y	X ↔ Y
T	T	T
T	F	F
F	T	F
F	F	T

iii. $(X \rightarrow Y) \wedge (Y \rightarrow X)$

X	Y	$(X \rightarrow Y) \wedge (Y \rightarrow X)$
T	T	T
T	F	F
F	T	F
F	F	T

iv.$(X \rightarrow Y) \wedge (\neg X \rightarrow \neg Y)$

X	Y	$(X \rightarrow Y) \wedge (\neg X \rightarrow \neg Y)$
T	T	T
T	F	F
F	T	F
F	F	T

v. $(X \wedge Y) \vee (\neg X \wedge \neg Y)$

X	Y	$(X \wedge Y) \vee (\neg X \wedge \neg Y)$
T	T	T
T	F	F
F	T	F
F	F	T

2. Prove that for every interger n, n^2 is odd **if and only if** n is odd. Break the proof in two parts:

(a) Prove that if n is odd, then n^2 is odd.
Suppose that n is odd. Then $n = 2k + 1$ for some interger k

$$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$$

Since k is an integer, k is odd.

(b) Prove that if n is not odd, then n^2 is not odd.
Suppose that n is not odd. Then $n = 2k$ for some integer k

$$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$$

Since k is an integer, k is not odd.

3. Let A and B be two sets.
(a) Use algebraic method to prove the following set equality.

$$(A \cup B) - B = A - B$$

Set Difference Rule

$$(A \cup B) \cap \overline{B} = A \cap \overline{B}$$

Distributive Law

$$(A \cap \overline{B}) \cup (B \cap \overline{B} = A \cap \overline{B}$$

Complement Law

$$(A \cap \overline{B}) \cup (\emptyset) = A \cap \overline{B}$$

Bound Law

$$A \cap \overline{B} = A \cap \overline{B}$$

(b) Prove the following set equality is valid **if and only if** sets A and B are disjoint.

$$(A \cup B) - B = A$$

i.In the case of sets A and B being disjointed, the union of said sets would include no initally shared elements, and thus, subtractions of the B set thereafter would result again in and exclusive A set.

ii.In the case of sets A and B are not disjointed, there would exist an X $\in (A \cap B)$, and the removal of B from the union of $A \cup B$ would remove element x. Therefore the resulting set would not include all elements of the inital A set.

4. Let S be the following implication. This problem is to find the simplest equivalent form for $\neg S$.

$$S : (P \rightarrow Q)$$

(a) First, use a truth table to show that the following proposition is not equivalent to $\neg S$.

$$P \rightarrow \neg Q$$

P	Q	$P \rightarrow Q$	$P \wedge \neg Q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

(In fact, $\neg S$ cannot be expressed in the form of another implication!)

(b) Find an equivalent expression for $\neg S$. This can be done by first expressing the implication S in terms of $\{\wedge, \vee, \neg\}$ operations, and then finding the negation.

P	Q	$P \to Q$	$\neg(P \wedge \neg Q)$	$\neg P \vee Q$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

5. (a) Determine the truth value of each of the following propositions, where the domain of discourse is intergers. Justify your answers.

i.$\forall x \forall y, [x + y = 0]$
This statement is false, because given $x = 0$ every $y > 0$ will not fulfill the conditions of [x + y = 0]
Counterexample: $x = 1$ and $y = 2$, $[1 + 2 \neq 0]$

ii. $\exists x \forall y, [x + y = 0]$
This is false because if there exists one x, only one y will satisfy the equation, specifically $x = -y$

(b) Express the negation of each of the above propositions in symbolic form. That is the truth value of each resulting proposition and why.

i.$\exists x \exists y, [x + y = 0]$
This statement is true, because there exists one x that is sated by one y
Example: $x = -3$ and $y = 3$, $[(-3) + (3) = 0]$

ii. $\forall x \exists y, [x + y = 0]$
This statement is true, because for all x there exists only one cooresponding y to make the statement true $y = -x$

6. (a) Determine the truth value of each of the following propositions, where the domain of discourse is real numbers. Justify your answers.

i. $\forall x \forall y, [(x < y) \to (x^2 < y^2)]$
This statement is false given $x < 0$ and y is $x < y < 0$
Example: $x = -3$, $y = -2$ $[(-3 < -2) \to (9 < 4)]$

ii. $\forall x \exists y, [(x < y) \to (x^2 < y^2)]$
This statement is true, because for every given x, there can always be a y given that the $y = \mid x \mid +1$
Example: $x = 2$, $y = \mid 2 \mid +1 = 3$ $[(2 < 3) \to (4 < 9)]$

iii. $\exists x \forall y, [(x < y) \rightarrow (x^2 < y^2)]$
This statement is true because given $x > 0$ all y will fulfill $x^2 < y^2$.
Example: $x = 3$, $y = 4$, $[(3 < 4) \rightarrow (9 < 16)]$

iv. $\exists x \exists y, [(x < y) \rightarrow (x^2 < y^2)]$
This statement is true because given that $x = 2$ and $y = 4$ we meet the criterion.
Example: $x = 2$, $y = 4$, $[(4 < 16) \rightarrow (4 < 16)]$

7. Let $A(S, C)$ be the propositional function (predicate) "student S who takes course C recieves and A grade." Let the domain of discourse be the set of NJIT students and courses, respectively.
(a) Express each of the following propositions in symbolic form.

i. There are NJIT students with all A's.
$\exists S \forall C, A(S, C)$

ii. There are NJIT courses that give all A's.
$\forall S \exists C, A(S, C)$

iii. Every NJIT student gets some A's.
$\forall S \forall C, \neg A(S, C)$

iv. Every NJIT course gives some A's.
$\forall S \forall C, \neg A(S, C)$

v. There are NJIT students with no A's.
$\exists S, \forall C, \neg A(S, C)$

(b) Express the negation of each of the above propositions, both in words and in symbolic form
i.In some courses, every student doesn't get an A $\forall S, \exists C, \neg A(S, C)$

ii. There are NJIT courses that give all A's.
Some students in all their courses get no A's $\exists S, \forall C, \neg A(S, C)$

iii. Every NJIT student gets some A's.
In some courses, some students get no A's $\exists S, \exists C, A(S, C)$

iv. Every NJIT course gives some A's.
A few students get A's in a couple of courses $\exists S, \exists C, A(S, C)$

v. There are NJIT students with no A's.
All students get A's in all courses! $\forall S, \exists C, A(S, C)$