| CS 506, Online<br>Foundations of CS | **Homework 2**<br>**Logic** | Dr. David Nassimi |
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# Study Module 2: Logic

1. (a) Use a truth-table to show that the following propositions are logically equivalent.
   i. $X \rightarrow Y$
   ii. $\neg Y \rightarrow \neg X$ (This implication form is the *contrapositive* of the first form.)
   iii. $\neg(X \wedge \neg Y)$
   iv. $\neg X \vee Y$

   (b) Use a truth-table to show that the following propositions are logically equivalent.
   i. $X \equiv Y$ (equivalence)
   ii. $X \leftrightarrow Y$ (if and only if)
   iii. $(X \rightarrow Y) \wedge (Y \rightarrow X)$
   iv. $(X \rightarrow Y) \wedge (\neg X \rightarrow \neg Y)$
   v. $(X \wedge Y) \vee (\neg X \wedge \neg Y)$

2. Prove that for every integer $n$, $n^2$ is odd **if and only if** $n$ is odd.
   Break the proof in two parts:

   (a) Prove that if $n$ is odd, then $n^2$ is odd.

   (b) Prove that if $n$ is not odd, then $n^2$ is not odd.

3. Let $A$ and $B$ be two sets.

   (a) Use algebraic method to prove the following set equality.

   $$(A \cup B) - B = A - B.$$

   (b) Prove the following set equality is valid **if and only if** sets $A$ and $B$ are disjoint.

   $$(A \cup B) - B = A.$$

4. Let $S$ be the following implication. This problem is to find the simplest equivalent form for $\neg S$.

   $$S: \quad (P \rightarrow Q)$$

   (a) First, use a truth table to show that the following proposition is not equivalent to $\neg S$.

   $$P \rightarrow \neg Q$$

   (In fact, $\neg S$ cannot be expressed in the form of another implication!)

   (b) Find an equivalent expression for $\neg S$. This can be done by first expressing the implication $S$ in terms of $\{\wedge, \vee, \neg\}$ operations, and then finding the negation.

5. (a) Determine the truth value of each of the following propositions, where the domain of discourse is intergers. Justify your answers.
   i. $\forall x \exists y, \ [x + y = 0]$
   ii. $\exists x \forall y, \ [x + y = 0]$

(b) Express the negation of each of the above propositions in symbolic form. What is the truth value of each resulting proposition and why.

6. (a) Determine the truth value of each of the following propositions, where the domain of discourse is real numbers. Justify your answers.

   i. $\forall x \forall y,\ [(x < y) \rightarrow (x^2 < y^2)]$
   ii. $\forall x \exists y,\ [(x < y) \rightarrow (x^2 < y^2)]$
   iii. $\exists x \forall y,\ [(x < y) \rightarrow (x^2 < y^2)]$
   iv. $\exists x \exists y,\ [(x < y) \rightarrow (x^2 < y^2)]$

   (b) Express the negation of each of the above propositions in symbolic form.
   Hint: To find the negation of an implication (if-then statement), first express the implication in one of its equivalent forms using only AND, OR, and NOT.

$$\neg(A \rightarrow B) = \neg(\neg A \vee B) = A \wedge \neg B.$$

7. Let $A(S, C)$ be the propositional function (predicate) " student $S$ who takes course $C$ receives an A grade." Let the domain of discourse be the set of NJIT students and courses, respectively.

   (a) Express each of the following propositions in symbolic form.

   i. There are NJIT students with all $A$'s.
   ii. There are NJIT courses that give all $A$'s.
   iii. Every NJIT student gets some $A$'s.
   iv. Every NJIT course gives some $A$'s.
   v. There are NJIT students with no $A$'s.

   (b) Express the negation of each of the above propositions, both in words and in symbolic form.