Session 11: Difference-in-differences

MGT 581 | Introduction to econometrics

Michaël Aklin

PASU Lab | EPFL

Last time...

- Endogeneity
- Instrumental variables

Today:

- Difference-in-differences
- Panel data
- Two-way fixed effects models

Readings:

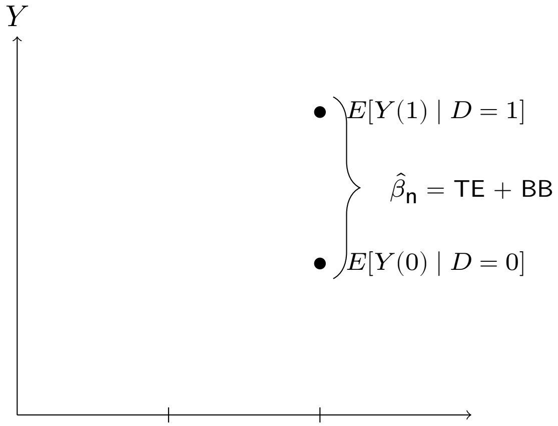
- Stock and Watson (2011), ch 10
- Verbeek (2018), ch 10
- Angrist and Pischke (2008) ch 5
- Morgan and Winship (2014) ch 11

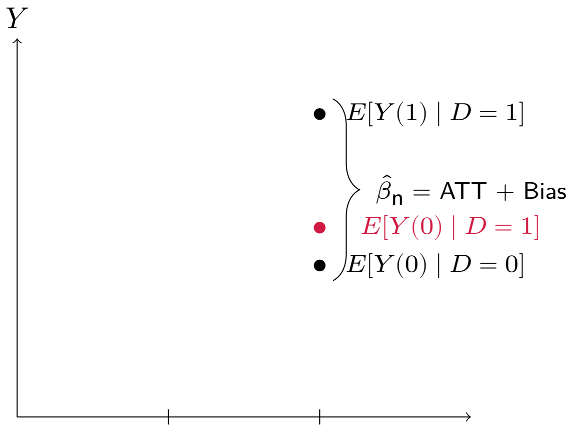
Difference-in-differences

- Idea: if we can have access to data over time, we can leverage it to identify treatment effects of interest
- As before, consider a continuous outcome $Y \in \mathfrak{R}$
- Dichotomous treatment $D \in \{0, 1\}$

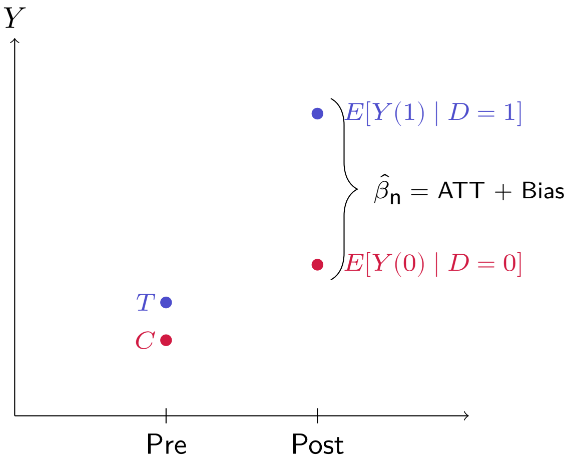
Recall:

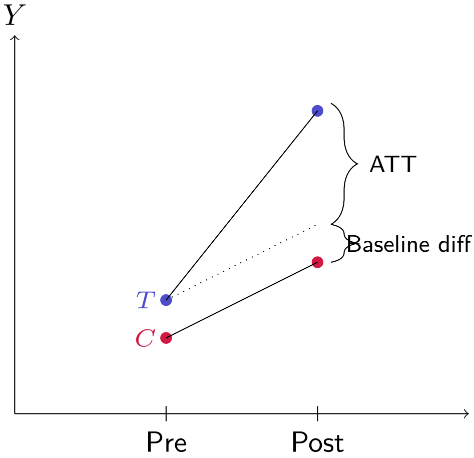
$$
\begin{aligned}
\text{Naïve difference} &= E[Y_i(1)|D_i = 1] - E[Y_i(0)|D_i = 0] \\
&= E[Y_i(1)|D_i = 1] - E[Y_i(0)|D_i = 1] \\
&+ E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 0] \\
&= E[Y_i(1) - Y_i(0)|D_i = 1] \\
&+ E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 0] \\
&\equiv \text{ATT} + \text{Selection (baseline) bias}
\end{aligned}
$$


Y
$E[Y(1) \mid D = 1]$
$\hat{\beta}_{\mathsf{n}} = \mathsf{TE} + \mathsf{BB}$
$E[Y(0) \mid D = 0]$


Y
$E[Y(1) \mid D = 1]$
$\hat{\beta}_{\mathsf{n}} = \text{ATT} + \text{Bias}$
$E[Y(0) \mid D = 1]$
$E[Y(0) \mid D = 0]$

- Thus: the naive difference combines ATT and bias
- Undistinguishable: the treatment effect is not **identified**
- Is there a solution?
- Yes! If we have **pre-treatment** observations, we can remove the baseline difference between treatment and control groups


Y
$E[Y(1) \mid D = 1]$
$\hat{\beta}_{\mathsf{n}} = \text{ATT} + \text{Bias}$
$E[Y(0) \mid D = 0]$
T
C
Pre
Post


Y
ATT
Baseline diff
T
C
Pre
Post

- Simplest: 2 groups ($g \in \{T, C\}$, 2 periods ($t \in \{$pre, post$\}$).
- ATT can be estimated as differences in differences δ:

$$\hat{\delta} = \left(\bar{Y}_T^{\text{post}} - \bar{Y}_T^{\text{pre}}\right) - \left(\bar{Y}_C^{\text{post}} - \bar{Y}_C^{\text{pre}}\right)$$

- In expectation:

$$\begin{aligned}
\hat{\delta} &= [E[Y_T(1)|\text{post(T)}] - E[Y_T(0)|\text{post(T)}]] \\
&- [E[Y_C(0)|\text{post(C)}] - E[Y_C(0)|\text{post(C)}]] \\
&= [E[Y_T(1)|\text{post(T)}] - E[Y_T(0)|\text{post(T)}]] \\
&- [E[Y_C(0)|\text{post(C)}] - E[Y_C(0)|\text{post(C)}]] \\
&+ E[Y_T(0)|\text{post(T)}] - E[Y_T(0)|\text{post(T)}] \\
&= \underbrace{E[Y_T(1)|\text{post(T)}] - E[Y_T(0)|\text{post(T)}]}_{ATT} \\
&+ [E[Y_T(0)|\text{post(T)}] - E[Y_T(0)|\text{pre(T)}]] \\
&- [E[Y_C(0)|\text{post(C)}] - E[Y_C(0)|\text{pre(C)}]]
\end{aligned}$$

- ATT + **parallel trends**

- Can be computed by hand or with OLS:

$$Y_{i,t} = \alpha + \beta D_g + \tau T_t + \delta D_g T_t + \varepsilon_{i,t}$$

- ... with δ being the estimated ATT: additional effect of treatment in the post period
- $\alpha = E[Y|D = 0, pre]$: $\bar{Y}$ for untreated, pre-treat
- $\alpha + \tau = E[Y|D = 0, post]$: $\bar{Y}$ for untreated, post
- $\alpha + \beta = E[Y|D = 1, pre]$: $\bar{Y}$ for treated, pre
- $\alpha + \beta + \tau + \delta = E[Y|D = 1, post$: $\bar{Y}$ for treated, post

Assumption and inference

- *Usual assumptions*: random sampling, SUTVA
- *Specific to DiD*: **parallel trends**. Treatment can be non-random, but it should not correlate with the trend.
  - Sometimes: can sometimes be more likely to hold if confounders $\mathbf{X}$ are adjusted for
- *Specific to DiD*: no **anticipation effect**
- *Specific to panel data*: risk of **serial correlation**:
  - Correlation of error term across time
  - Solution: cluster standard errors by units (eg by individual) (Bertrand, Duflo, and Mullainathan 2004)

Example

- Effect of minimum wages on unemployment (Card and Krueger 1993)
- Concern: increase minimum wage $\rightarrow$ more unemployment
- Naive regression: unlikely to be informative, since treated jurisdictions are probably very different pre-treatment
- Card and Krueger (1993)'s idea: compare 2 US states pre/post
  - New Jersey: increase in minimum wage from \$4.25 to \$5.05/hour in 1992
  - Pennsylvania: no change at \$4.25
- Sample fastfood restaurants and compare total employment pre/post in T/C

ABSTRACT

On April 1, 1992 New Jersey's minimum wage increased from \$4.25 to \$5.05 per hour. To evaluate the impact of the law we surveyed 410 fast food restaurants in New Jersey and Pennsylvania before and after the rise in the minimum. Comparisons of the changes in wages, employment, and prices at stores in New Jersey relative to stores in Pennsylvania (where the minimum wage remained fixed at \$4.25 per hour) yield simple estimates of the effect of the higher minimum wage.

Our empirical findings challenge the prediction that a rise in the minimum reduces employment. Relative to stores in Pennsylvania, fast food restaurants in New Jersey increased employment by 13 percent. We also compare employment growth at stores in New Jersey that

Figure 1: Source: Card and Krueger (1993)

Generalizing: fixed effects models

- We often generalize the DiD model to multiple periods, units, treatment time
- This is the **(twoway) fixed effects** model (TWFE) (with time-varying treatment)

$$Y_{i,t} = \alpha_i + \tau_t + \beta D_{i,t} + \mathbf{X}'\gamma + \varepsilon_{i,t}$$

- Note: α_i stands for a dummy for each i units (no constant). These are **unit fixed effects**.
- τ_t stands for a dummy for each t time period. These are **time fixed effects**.
- Interpretation: we are estimating the effect of D removing effects that are unit- and time-specific. AKA **within-estimator**.

Advantages:

- Units FE remove all effects that are unit-invariant
  - Time-invariant differences across units
  - Eg: Sales $_{f,t}$ = a + b*Ads $_{f,t}$ + Firm $_{f} + \varepsilon_{f,t}$
- Time FE remove all effects that affect the units at given time period
  - Time effects that affect all units
  - Eg: a Covid dummy
- From design viewpoint: makese sense to identify treatment effect based on changes within units, within time
  - We are our best counterfactuals

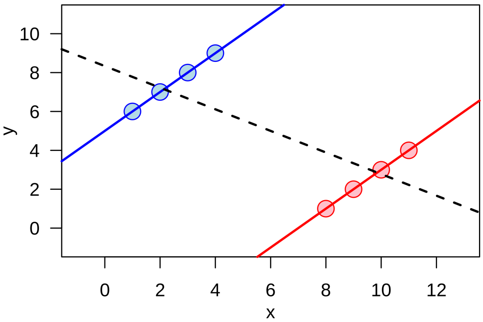


Figure 2: Simpson's Paradox. Source: wikipedia.

Fixed effects help address **Simpson's paradox**

- All great, then? “Yes” according to Angrist and Pischke (2008)...
- “No” according to recent work (Callaway and Sant’Anna 2021; De Chaisemartin and d’Haultfoeuille 2020; Roth et al. 2023)
- Two challenges..

1. DiD mixes several comparisons

- Consider three groups:
    - Never treated (a)
    - Treated at time t1 (b)
    - Treated at time t2 (c)
- DiD will combine four comparisons!
    - (a) with (b) at t1
    - (a) with (c) at t2
    - (b) with (c) at t1.
    - (b) with (c) at t2. But (b) is not a control!

2. Complication from heterogeneous treatment effects

Conclusion

- Diff-in-diff has become a common workhorse quasi-experimental approach
- Initially thought to 'easily' generalize to two-way fixed effects model
- Ongoing econometric research shows problems when doing the latter. Very much work in progress.

Questions?

References

Angrist, Joshua, and Steffan J. Pischke. 2008. *Mostly Harmless Econometics*. Princeton: Princeton University Press.

Bertrand, Marianne, Esther Duflo, and Sendhil Mullainathan. 2004. "How Much Should We Trust Differences-in-Differences Estimates?" *Quarterly Journal of Economics* 119 (1): 249–75.

Callaway, Brantly, and Pedro HC Sant'Anna. 2021. "Difference-in-Differences with Multiple Time Periods." *Journal of Econometrics* 225 (2): 200–230.

Card, David, and Alan B. Krueger. 1993. "Minimum Wages and Employment."

De Chaisemartin, Clément, and Xavier d'Haultfoeuille. 2020. "Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects." *American Economic Review* 110 (9): 2964–96.

Morgan, Steven L., and Christopher Winship. 2014. *Counterfactuals and Causal Inference: Methods and Principles for Social Research*. 2nd Edition. Cambridge: Cambridge University Press.

Roth, Jonathan, Pedro HC Sant'Anna, Alyssa Bilinski, and John Poe. 2023. "What's Trending in Difference-in-Differences? A Synthesis of the Recent Econometrics Literature." *Journal of Econometrics* 235 (2): 2218–44.

Stock, James H., and Mark W. Watson. 2011. *Introduction to Econometrics, 3rd Edition*. Pearson.

Verbeek, Marno. 2018. *A Guide to Modern Econometrics 5th Edition*. Wiley.