<u>Reframing the Graph Isomorphism Problem using Symmetric Groups</u>

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This paper is not meant to be a rigorous proof, and as such, it is assumed that the reader is familiar with modern algebra , linear algebra, boolean algebra and basic computer science theory. With that out of the way, lets get started.

Graph Isomorphism(GI) is an open problem in computer science. It is currently unknown whether or not this problem is np-complete. In this paper I am going to demonstrate a method of mapping a certain class of isomorphism problems(in which GI is included) into a homogeneous equation.

The specific kinds of isomorphisms we will investigate are those in which the transformation matrices that satisfy them are members of the symmetric group(SYM(I,N)) of the identity matrix for some n. To be clear, the transformations these matrices perform is that of a permutation function, which effectively permutes either the rows(TxM) or the columns(MxT) of a matrix M.

SYM(I,N) of the N by N identity matrix has several properties, but we are only interested in a few:

1- For all Y in SYM(I,N) , Y x Y(inverse) = I
2- Y(inverse) = Y(transpose)
3- For any matrix M, and any Y in SYM(I,N) , the entries of Y that contain a value of 1 at row i and column j permutes row j in M to row I in M as a result of the expression Y x M.
4- For any matrix M, and any Y in SYM(I,N) , the entries of Y that contain a value of 1 at row i and column j permutes column i in M to column j in M as a result of the expression
M x Y.

GI is typically posed as an algebra problem by mapping the two graphs we want an isomorphism for into adjacency matrices M1 and M2. Then the following expression is set up:

T x M1 x T(transpose) = M2* , where T is a transformation matrix. What's interesting is that the set of all possible solutions of T is equal the set of all matrices in SYM(I,N) , because any valid solution to GI consists of a permutation of M1 such that M1 = M2.

Consider the properties of SYM(I,N) mentioned earlier. In the expression above, we are first permuting the rows of M1 using T, and then permuting the columns of M1 using T(inverse), which is equivalent to M2. It stands to reason then, that the matrix given by permuting the rows of M1 with T is equivalent to the matrix given by permuting the columns of M2 using

T(effectively permuting the columns of M2 according to the inverse of the permutation function represented by T). The above statement is also true for the columns of M1 and the rows of M2. Therefore, we find the following expressions equivalent:

(T x M1 x T(transpose) = M2) =

(M1 x T(transpose) = T(transpose)x M2) =

(T x M1 = M2 x T) =

(T(transpose) x M1 x T= M2) =

(M1 x T = T x M2) =

(T(transpose) x M1 = M2 x T(transpose))

*- Routinely, the equation for graph isomorphism is written as TxM1xT = M2, but this is incorrect. That expression permutes the rows according to some permutation T , and permutes the columns according to T's inverse. The transformation of M1 is intended to be transpose symmetric, as represented above.