easylinear — a LATEX package for Linear Algebra

by Michael Kearney 2023-02-27 v1.1

1 About the easylinear package

The easylinear package was developed by a linear algebra student to make writing linear in LaTeX more easy. It relies heavily on spalign to define a limited set of efficient commands that save keystrokes, improve readability, and match the Octave syntax used in Sage cells more closely. This package is shared as is. The easylinear LaTeX package and associated template are not official course material. Use them at your own risk. The package has been thoroughly tested, but there is no guarantee of functionality. Always check your own work. See easylinear.sty for implementation specifics. Send feedback to michael.andrew.kearney@gmail.com.

2 How to set up easylinear

The easiest way to use easylinear is to make a copy of an Overleaf project that you can edit. Open the project, located at bit.ly/easylinear. Click "Menu" in the upper left corner and "Copy Project". (If you have already opened this template while signed into Overleaf, you can make a new copy directly from your Overleaf Projects page.) Choose a name for your new project (e.g. "Problem Set 1"). Your new project will contain main.tex, README.tex, and easylinear.sty. Write your document in main.tex. Compile and refer to README.tex for command usage. Do not edit easylinear.sty unless you know what you are doing. If you do mess up easylinear.sty, pull a fresh copy of the template. For more help with LATEX see Overleaf's guide to Learn LATEX in 30 minutes.

3 Updates to easylinear

I may update this package with more functionality. Existing commands will remain. Until indicated here, the Overleaf project is the most up-to-date version. To get the latest version of easylinear, simply copy the project. Depending on maintenance capacity, future updates may be distributed over the fledgeling GitHub repository or CTAN.

4 Commands in easylinear

Use easylinear commands inside a math environment (i.e. inline \$\command\$ or display \[\command\]). Pass an argument with curly braces like this: \command{arg}. Pass multiple arguments with multiple curly braces like this: \command{arg1}{arg2}{arg3}

5 Systems of Linear Equations

5.1 Commonly used commands

Input	Math	Input	Math
x^2	x^2	\to	\rightarrow
\in	\in	\implies	\Longrightarrow
\{	{	\mathbb{R}	\mathbb{R}
\}	}	hello	hello

5.2 Linear system: \spalignsys

This command comes directly from the spalign package. It aligns the terms and operators of a system of equations and makes it easy to parse coefficients. \spalignsys{} takes one argument: a system of equations where terms are delimited by spaces or commas and equations are delimited by semicolons. For example:

$$\begin{cases} 2x_1 - 3x_2 + 12x_3 = 3 \\ -4x_1 + 0x_2 + 6x_3 = -10 \end{cases}$$
 \$\spalignsys{2x_1 - 3x_2 + 12x_3 = 3; -4x_1 + 0x_2 + 6x_3 = -10}\$\$

5.3 Vectors and Matrices

Use \vct{...} to make a column vector, \mat{...} to make a matrix, and \amat{...} to make an augmented matrix. Each command takes as an argument the matrix contents delimited like in Octave: delimit elements with spaces and/or commas and delimit rows with semicolons. Spaces collapse into each other and into commas, but commas do not collapse. For example:

```
\int vct \{0 \ 1 \ 2\}
1
2
    1
       2
           3
                                                            $\mat{0 1 2 3; 4 5 6 7}$
   5
       6
   1
       2
           3
                                                            1 \$ \text{mat}\{0,1,2,\dots,2\}
                                                                                   3;4,5,6,7
                                                                                                ;}$
   5
       6
           7
        ^2
           3
                                                            $\amat{0 1 2 3; 4 5 6 7}$
    5
        6
           7
```

5.4 Row operations

There are three row operation symbol that describe row operations above a right arrow: \rowop represents addition/subtraction with optional scaling coefficients, \rowscale represents row scaling in-place, and \rowswap represents swapping two rows. Note that these commands do not perform row operations just like writing "3 × 4" does not actually compute a result. These operation symbols only represent the row operations you have done independently.

\rowop takes five arguments: operated_coefficient, operated_row, operator, operand_coefficient, and operand_row. operated_row is the row to which operand_row is being added or subtracted. The result is stored in operated_row. The arguments are best understood by example:

$$\begin{bmatrix} 0 & 1 & 2 & | & 3 \\ 1 & 2 & 1 & | & -3 \end{bmatrix} \xrightarrow{-3R_1 + 2R_2 \to R_1} \begin{bmatrix} 2 & 1 & -4 & | & -15 \\ 1 & 2 & 1 & | & -3 \end{bmatrix} \xrightarrow{1} \begin{bmatrix} | \langle [\lambda mat\{0\ 1\ 2\ 3;\ 1\ 2\ 1\ -3 \} \rangle \\ | \langle [\lambda mat\{0\ 1\ 2\ 3;\ 1\ 2\ 1\ -3 \} \rangle \\ | \langle [\lambda mat\{0\ 1\ 2\ 3;\ 1\ 2\ 1\ -3 \} \rangle] \end{bmatrix}$$

Only the row and operator arguments are mandatory. Coefficient arguments can be independently omitted (implying a coefficient of 1), but their curly brackets must remain. Any operator can be used. For example:

$$\begin{bmatrix} 0 & 1 & 2 & | & 3 \\ 1 & 2 & 1 & | & -3 \end{bmatrix} \xrightarrow{R_1 - R_2 \to R_1} \begin{bmatrix} -1 & -1 & 1 & | & 6 \\ 1 & 2 & 1 & | & -3 \end{bmatrix} \xrightarrow{1} \begin{bmatrix} | \langle x | | \rangle \\ | \langle x |$$

5.4.2 \rowscale{}{}

\rowscale takes two arguments: operand and operated_row. operated_row is scaled by operand and stored back in operated_row.

```
\begin{bmatrix}
0 & 1 & 2 & | & 3 \\
1 & 2 & 1 & | & -3
\end{bmatrix}
\xrightarrow{3R_1 \to R_1}
\begin{bmatrix}
0 & 3 & 6 & | & 9 \\
1 & 2 & 1 & | & -3
\end{bmatrix}

\begin{bmatrix}
1 & \text{Amat}\{0 \ 1 \ 2 \ 3; \ 1 \ 2 \ 1 \ -3\} \\
\text{Amat}\{0 \ 3 \ 6 \ 9; \ 1 \ 2 \ 1 \ -3\} \\
\text{Amat}\{0 \ 3 \ 6 \ 9; \ 1 \ 2 \ 1 \ -3\} \\
\end{bmatrix}
```

5.4.3 \rowswap{}{}

\rowswap takes two arguments: first_row and second_row. first_row and second_row are swapped.

```
\begin{bmatrix} 0 & 1 & 2 & | & 3 \\ 1 & 2 & 1 & | & -3 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 2 & 1 & | & -3 \\ 0 & 1 & 2 & | & 3 \end{bmatrix} \xrightarrow{1} \begin{bmatrix} | \langle \text{namt}\{0 \ 1 \ 2 \ 3; \ 1 \ 2 \ 1 \ -3 \} \\ | \langle \text{rowswap}\{1\}\{2\} \\ | \langle \text{amat}\{1 \ 2 \ 1 \ -3; \ 0 \ 1 \ 2 \ 3 \} \rangle \end{bmatrix}
```

5.5 Sets and solution sets

5.5.1 \bracket{} and \set{}

\bracket{} and \set{} surround an arbitrary argument with appropriately sized brackets. \bracket{} displays the contents as given, while \set{} will clean and comma-delimit the contents as much as possible.

```
 \left\{ \text{Big brackets! } \begin{bmatrix} 2 & 4 \\ 3 & 2 \\ 7 & 6 \end{bmatrix} \right\} \quad \left\{ 0, 1, \begin{bmatrix} 9 \\ 8 \end{bmatrix}, 3, \begin{bmatrix} 0 & 1 \\ 9 & 6 \end{bmatrix} \right\} 
 \left\{ 0, 1, \begin{bmatrix} 9 \\ 8 \end{bmatrix}, 3, \begin{bmatrix} 0 & 1 \\ 9 & 6 \end{bmatrix} \right\} 
 \left\{ 0, 1, \begin{bmatrix} 9 \\ 8 \end{bmatrix}, 3, \begin{bmatrix} 0 & 1 \\ 9 & 6 \end{bmatrix} \right\} 
 \left\{ 0, 1, \begin{bmatrix} 9 \\ 8 \end{bmatrix}, 3, \begin{bmatrix} 0 & 1 \\ 9 & 6 \end{bmatrix} \right\} 
 \left\{ 0, 1, \begin{bmatrix} 9 \\ 8 \end{bmatrix}, 3, \begin{bmatrix} 0 & 1 \\ 9 & 6 \end{bmatrix} \right\} 
 \left\{ 0, 1, \begin{bmatrix} 9 \\ 8 \end{bmatrix}, 3, \begin{bmatrix} 0 & 1 \\ 9 & 6 \end{bmatrix} \right\}
```

$5.5.2 \setminus setwhere{}{}$

\setwhere adds a qualifying 'such that' statement to \set as a second argument. This 'such that' statement is not affected by the comma delimitation of \set.

```
\left\{ \begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} c \\ d \end{bmatrix} | a, b, c, d \in \mathbf{R} \right\} $\setwhere{\vct{a b} \vct{c d}}{\text{ab} \vct{c d}}$ \mathbf{R}}$$
```

5.5.3 \emptyset

For inconsistent systems with no solution, the solution set is the empty set.

$5.5.4 \setminus unisolset\{\}$

Use \unisolset{} for consistent systems with a unique solution. It takes one argument: the terms of the solution vector delimited with spaces. For example:

$$\begin{bmatrix}
1 & 0 & 0 & | & 1 \\
0 & 1 & 0 & | & 2 \\
0 & 0 & 1 & | & 3
\end{bmatrix}$$
 has the solution set
$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}.$$

 $\hat{1001}; 0102; 0013$ has the solution set $\hat{123}$.

$5.5.5 \setminus infsolset{\}{}}$

Use \infsolset{}{} for consistent systems with infinitely many solutions. The command takes two arguments. Delimit the terms of the solution vector with spaces in the first argument. Delimit the unbound variables with commas in the second argument. For example:

$$\begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ sol'n set } \left\{ \begin{bmatrix} 2a - 3b \\ a \\ b \end{bmatrix} \middle| a, b \in \mathbb{R} \right\}.$$

5.6 Putting it all together

This simple example shows how to combine easylinear commands. It may or may not be an example of good writing by this course's standards.

Problem 1. Find the solution set of the linear system $\begin{cases} 4x_1 + 2x_2 = 6 \\ 2x_1 + 0x_2 = 4 \end{cases}$

From the linear system we derive the augmented matrix below. Using row operations, we transform it into RREF:

$$\begin{bmatrix} 4 & 2 & | & 6 \\ 2 & 0 & | & 4 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 4 & 2 & | & 6 \\ 1 & 0 & | & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 4 & 2 & 6 \end{bmatrix} \xrightarrow{R_2 - 4R_1 \to R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & -2 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2 \to R_2}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

Each variable is bound and there are no contradictions. There is one unique solution. This system of equations has the solution set $\left\{\begin{bmatrix}2\\-1\end{bmatrix}\right\}$.

\begin{problem}Find the solution set of the linear\\ system \$\spalignsys{4x_1 + 2x_2 = 6; 2x_1 + 0x_2 = 4}\$. \end{problem}

From the linear system we derive the augmented matrix below. Using row operations, we transform it into RREF:

\begin{align*}
&\amat{4 2 6; 2 0 4}
\rowscale{\frac{1}{2}}{2}
\amat{4 2 6; 1 0 2}
\rowswap{1}{2}\^{\circ}\

2 \amat{1 0 2; 0 2 -2} 3 \rowscale{\frac{1}{2}}{2}\\^\\\
4 \&\amat{1 0 2; 0 1 -1}

 $\ensuremath{\backslash \mathbf{end}} \{ \mathrm{align*} \}$

Each variable is bound and there are no contradictions. There is one unique solution. This system of equations has the solution set $\$ unisolset $\{2-1\}$ \$.

6 Vector Spaces

6.1 Commonly used commands

Input	Math	Input	Math
\oplus	\oplus	\bigoplus	\oplus
\odot	\odot	\bigodot	\odot

6.2 Spans and Linear Combinations

Use \vlist to display a comma-separated list of vectors.

$\textbf{6.2.2} \quad \setminus \texttt{vset}\{\}$

Use \vset to display a comma-separated set of vectors.

$$\left\{ \begin{bmatrix} 0 \\ 5 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 4 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 9 \\ 9 \\ 0 \\ 4 \end{bmatrix} \right\}$$
 \$\square\$ \$\vset{0 5 6 3; 5 4 2 7; 9 9 0 4}\$\$

$6.2.3 \quad \text{vspan} \}$

Use \vspan to display the span of a linear combination of a set of vectors.

$$\operatorname{span}\left\{\begin{bmatrix} 0\\ 5\\ 6\\ 3 \end{bmatrix}, \begin{bmatrix} 5\\ 4\\ 2\\ 7 \end{bmatrix}, \begin{bmatrix} 9\\ 9\\ 0\\ 4 \end{bmatrix}\right\}$$

$$\operatorname{\$} \operatorname{vspan}\{0\ 5\ 6\ 3;\ 5\ 4\ 2\ 7;\ 9\ 9\ 0\ 4\}\$$$

6.2.4 \rspace{}

Use \rspace\ to display a Euclidean vector space of a certain dimension.

7 Changelog

- 1.0 Published 2/14/23. First release. Added \mat, \amat, \rowop, \rowscale, \rowswap, \unisolset, and \infsolset commands.
- 1.1 Published 2/27/23. Added changelog. Added \vct, \set, \setwhere, \rspan, \lincom, \vecset, and \vspan commands. Added \bracket and \commadelim helper commands. Updated \mat and \amat to reference spalign commands directly. Updated \infsolset to use helper functions. Updated {} as default spalign delimiters. Updated README.

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