For $n=1,2,...$, define $H_1(n)=\sum_{k=1}^{n}\frac{1}{k}$, $H_2(n)=\sum_{k=1}^{n}\frac{1}{kH_1(k)}$, $H_3(n)=\sum_{k=1}^{n}\frac{1}{kH_1(k)H_2(k)}$,

$\cdots$, $H_j(n)=\sum_{k=1}^{n}\frac{1}{kH_1(k)H_2(k)\cdots H_{j-1}(k)}$, $\cdots$. Notice that the ordinary harmonic function H is my function H_1. The idea is that, asymptotically, $H_1(n)$ is like $\ln(n)$, $H_2(n)$ is like $\ln(\ln(n)), \cdots$, and H_r is like $\ln(\ln(\cdots(\ln(n))\cdots)$ where there are r nested parenthesis pairs. Call $H_r(n)$ the nth rth iterated harmonic number. I think that the rth iterated harmonic numbers may be new objects when $r\geq 2$.

The obvious facts are that

(1) For each j, the series $H_j(\infty)$ diverges to positive ∞,

(2) The rate of divergence is slower as j increases,

(3) Each j and each $p>1$ produce a convergent series that is "very close" to the divergent $H_j(\infty)$:

$$\sum_{k=1}^{\infty}\frac{1}{kH_1(k)H_2(k)\cdots H_{j-2}(k)(H_{j-1}(k))^p}.$$

From this we may easily reproduce the usual well known remarks involving examples of very slowly diverging and very slowly converging infinite series built from iterated logarithms without formally introducing logarithms. To see some of these, go to https://condor.depaul.edu/mash/realvita.html and look at the paper

69. Series involving iterated logarithms, College Math. J., 40 (2009), 40-42.

$$H_1(1) := 1; H_1(2) := \sum_{i=1}^{2} \frac{1}{i} = \frac{3}{2} \equiv 3 \times 2$$
$$H_1(3) := \sum_{i=1}^{3} \frac{1}{i} = \frac{11}{2\times 3} \equiv \frac{1}{3}$$
$$H_1(4) := \sum_{i=1}^{4} \frac{1}{i} = \frac{5^2}{2^2 3} \equiv \frac{1}{3}$$
$$H_1(5) := \sum_{i=1}^{5} \frac{1}{i} = \frac{137}{2^2 3\times 5} \equiv \frac{1}{3}$$
$$H_1(6) := \sum_{i=1}^{6} \frac{1}{i} = \frac{7^2}{2^2 5} \equiv 2$$
$$H_1(7) := \sum_{i=1}^{7} \frac{1}{i} = \frac{3\times 11^2}{2^2 5\times 7} \equiv 3 \times 2$$
$$H_1(8) := \sum_{i=1}^{8} \frac{1}{i} = \frac{761}{2^3 5\times 7} \equiv 2$$
$$H_1(9) := \sum_{i=1}^{9} \frac{1}{i} = \frac{7129}{2^3 3^2 5\times 7} \equiv \frac{1}{3^2}$$
$$H_1(10) := \sum_{i=1}^{10} \frac{1}{i} = \frac{11^2\times 61}{2^3 3^2 5\times 7} \equiv \frac{1}{3^2}$$
$$H_1(11) := \sum_{i=1}^{11} \frac{1}{i} = \frac{97\times 863}{2^3 3^2 5\times 7\times 11} \equiv \frac{1}{3^2}$$
$$H_1(12) := \sum_{i=1}^{12} \frac{1}{i} = \frac{13^2\times 509}{2^3 3^2 5\times 7\times 11} \equiv \frac{1}{3^2}$$
$$H_1(13) := \sum_{i=1}^{13} \frac{1}{i} = \frac{29\times 43\times 919}{2^3 3^2 5\times 7\times 11\times 13} \equiv \frac{1}{3^2}$$
$$H_1(14) := \sum_{i=1}^{14} \frac{1}{i} = \frac{1049\times 1117}{2^3 3^2 5\times 7\times 11\times 13} \equiv \frac{1}{3^2}$$
$$H_1(15) := \sum_{i=1}^{15} \frac{1}{i} = \frac{29\times 41\,233}{2^3 3^2 5\times 7\times 11\times 13} \equiv \frac{1}{3^2}$$
$$H_1(16) := \sum_{i=1}^{16} \frac{1}{i} = \frac{17^2\times 8431}{2^4 3^2 5\times 7\times 11\times 13} \equiv \frac{1}{3^2}$$
$$H_1(17) := \sum_{i=1}^{17} \frac{1}{i} = \frac{37\times 1138\,979}{2^4 3^2 5\times 7\times 11\times 13\times 17} \equiv \frac{1}{3^2}$$
$$H_1(18) := \sum_{i=1}^{18} \frac{1}{i} = \frac{19^2\times 39\,541}{2^4 3\times 5\times 7\times 11\times 13\times 17} \equiv \frac{2}{3}$$
$$H_1(19) := \sum_{i=1}^{19} \frac{1}{i} = \frac{37\times 7440\,427}{2^4 3\times 5\times 7\times 11\times 13\times 17\times 19} \equiv \frac{2}{3}$$
$$H_1(20) := \sum_{i=1}^{20} \frac{1}{i} = \frac{5\times 11\,167\,027}{2^4 3\times 7\times 11\times 13\times 17\times 19} \equiv \frac{2}{3}$$
$$H_1(21) := \sum_{i=1}^{21} \frac{1}{i} = \frac{18\,858\,053}{2^4 7\times 11\times 13\times 17\times 19} \equiv 2$$
$$H_1(22) := \sum_{i=1}^{22} \frac{1}{i} = \frac{3\times 23^2 53\times 227}{2^4 7\times 11\times 13\times 17\times 19} \equiv 3$$
$$H_1(23) := \sum_{i=1}^{23} \frac{1}{i} = \frac{761\times 583\,859}{2^4 7\times 11\times 13\times 17\times 19\times 23} \equiv 2$$
$$H_1(24) := \sum_{i=1}^{24} \frac{1}{i} = \frac{5\times 577\times 467\,183}{2^4 3\times 7\times 11\times 13\times 17\times 19\times 23} \equiv \frac{2}{3}$$
$$H_1(25) := \sum_{i=1}^{25} \frac{1}{i} = \frac{109\times 312\,408\,463}{2^4 3\times 5^2 7\times 11\times 13\times 17\times 19\times 23} \equiv \frac{2}{3}$$
$$H_1(26) := \sum_{i=1}^{26} \frac{1}{i} = \frac{34\,395\,742\,267}{2^4 3\times 5^2 7\times 11\times 13\times 17\times 19\times 23} \equiv \frac{2}{3}$$
$$H_1(27) := \sum_{i=1}^{27} \frac{1}{i} = \frac{521\times 2789\times 215\,087}{2^4 3^3 5^2 7\times 11\times 13\times 17\times 19\times 23} \equiv \frac{1}{3^3}$$
$$H_1(28) := \sum_{i=1}^{28} \frac{1}{i} = \frac{29^2\times 375\,035\,183}{2^4 3^3 5^2 7\times 11\times 13\times 17\times 19\times 23} \equiv \frac{1}{3^3}$$
$$H_1(29) := \sum_{i=1}^{29} \frac{1}{i} = \frac{9227\,046\,511\,387}{2329\,089\,562\,800} = \frac{43^2\times 4990\,290\,163}{2^4 3^3 5^2 7\times 11\times 13\times 17\times 19\times 23\times 29} \equiv \frac{1}{3^3}$$
$$H_1(30) := \sum_{i=1}^{30} \frac{1}{i} = \frac{9304\,682\,830\,147}{2329\,089\,562\,800} = \frac{31^2 53\times 10\,273\times 17\,783}{2^4 3^3 5^2 7\times 11\times 13\times 17\times 19\times 23\times 29} \equiv \frac{1}{3^3}$$
$$H_1(31) := \sum_{i=1}^{31} \frac{1}{i} = \frac{290\,774\,257\,297\,357}{72\,201\,776\,446\,800} = \frac{109\times 2667\,653\,736\,673}{2^4 3^3 5^2 7\times 11\times 13\times 17\times 19\times 23\times 29\times 31} \equiv \frac{1}{3^3}$$
$$H_1(33) := \sum_{i=1}^{33} \frac{1}{i} = \frac{53\,676\,090\,078\,349}{13\,127\,595\,717\,600} = \frac{269\times 199\,539\,368\,321}{2^5 3^3 5^2 7\times 13\times 17\times 19\times 23\times 29\times 31} \equiv \frac{1}{3^3}$$
$$H_1(34) = \sum_{i=1}^{34} \frac{1}{i} = \frac{54\,062\,195\,834\,749}{13\,127\,595\,717\,600} \equiv \frac{1}{3^3}$$
$$H_1(35) = \sum_{i=1}^{35} \frac{1}{i} = \frac{54\,437\,269\,998\,109}{13\,127\,595\,717\,600} \equiv \frac{1}{3^3}$$
$$H_1(36) = \sum_{i=1}^{36} \frac{1}{i} = \frac{54\,801\,925\,434\,709}{13\,127\,595\,717\,600} \equiv \frac{1}{3^3}$$
$$H_1(37) = \sum_{i=1}^{37} \frac{1}{i} = \frac{2040\,798\,836\,801\,833}{485\,721\,041\,551\,200} \equiv \frac{1}{3^3}$$
$$H_1(38) = \sum_{i=1}^{38} \frac{1}{i} = \frac{2053\,580\,969\,474\,233}{485\,721\,041\,551\,200} \equiv \frac{1}{3^3}$$
$$H_1(39) = \sum_{i=1}^{39} \frac{1}{i} = \frac{2066\,035\,355\,155\,033}{485\,721\,041\,551\,200} \equiv \frac{1}{3^3}$$

$H_1(40) = \sum_{i=1}^{40} \frac{1}{i} = \frac{2078\,178\,381\,193\,813}{485\,721\,041\,551\,200} \equiv \frac{1}{3^3}$
$H_1(41) = \sum_{i=1}^{41} \frac{1}{i} = \frac{85\,691\,034\,670\,497\,533}{19\,914\,562\,703\,599\,200} \equiv \frac{1}{3^3}$
$H_1(42) = \sum_{i=1}^{42} \frac{1}{i} = \frac{12\,309\,312\,989\,335\,019}{2844\,937\,529\,085\,600} \equiv \frac{1}{3^3}$
$H_1(43) = \sum_{i=1}^{43} \frac{1}{i} = \frac{532\,145\,396\,070\,491\,417}{122\,332\,313\,750\,680\,800} \equiv \frac{1}{3^3}$
$H_1(44) = \sum_{i=1}^{44} \frac{1}{i} = \frac{5884\,182\,435\,213\,075\,787}{1345\,655\,451\,257\,488\,800} \equiv \frac{1}{3^3}$
$H_1(45) = \sum_{i=1}^{45} \frac{1}{i} = \frac{5914\,085\,889\,685\,464\,427}{1345\,655\,451\,257\,488\,800} \equiv \frac{1}{3^3}$
$H_1(46) = \sum_{i=1}^{46} \frac{1}{i} = \frac{5943\,339\,269\,060\,627\,227}{1345\,655\,451\,257\,488\,800} \equiv \frac{1}{3^3}$
$H_1(47) = \sum_{i=1}^{47} \frac{1}{i} = \frac{280\,682\,601\,097\,106\,968\,469}{63\,245\,806\,209\,101\,973\,600} \equiv \frac{1}{3^3}$
$H_1(48) = \sum_{i=1}^{48} \frac{1}{i} = \frac{282\,000\,222\,059\,796\,592\,919}{63\,245\,806\,209\,101\,973\,600} \equiv \frac{1}{3^3}$
$H_1(49) = \sum_{i=1}^{49} \frac{1}{i} = \frac{13\,881\,256\,687\,139\,135\,026\,631}{3099\,044\,504\,245\,996\,706\,400} \equiv \frac{1}{3^3}$
$H_1(50) = \sum_{i=1}^{50} \frac{1}{i} = \frac{13\,943\,237\,577\,224\,054\,960\,759}{3099\,044\,504\,245\,996\,706\,400} \equiv \frac{1}{3^3}$
$H_1(51) = \sum_{i=1}^{51} \frac{1}{i} = \frac{14\,004\,003\,155\,738\,682\,347\,159}{3099\,044\,504\,245\,996\,706\,400} \equiv \frac{1}{3^3}$
$H_1(52) = \sum_{i=1}^{52} \frac{1}{i} = \frac{14\,063\,600\,165\,435\,720\,745\,359}{3099\,044\,504\,245\,996\,706\,400} \equiv \frac{1}{3^3}$
$H_1(53) = \sum_{i=1}^{53} \frac{1}{i} = \frac{748\,469\,853\,272\,339\,196\,210\,427}{164\,249\,358\,725\,037\,825\,439\,200} \equiv \frac{1}{3^3}$
$H_1(54) = \sum_{i=1}^{54} \frac{1}{i} = \frac{250\,503\,836\,021\,181\,200\,128\,409}{54\,749\,786\,241\,679\,275\,146\,400} \equiv \frac{2}{3^2}$
$H_1\,(79)\, \frac{4868\,007\,055\,309\,996\,043\,055\,960\,217\,131\,137}{2^6 3^2 5^2 7^2 13\times17\times19\times23\times29\times31\times37\times41\times43\times47\times53\times59\times61\times67\times71\times73\times79} \equiv \frac{2}{3^2}$
$H_1\,(80)\, \frac{11\times4127\times322\,512\,198\,708\,761\times333\,328\,587\,314\,641}{2^6 3^2 5^2 7^2 13\times17\times19\times23\times29\times31\times37\times41\times43\times47\times53\times59\times61\times67\times71\times73\times79} \equiv \frac{2}{3^2}$
$H_1\,(81)\, \frac{88\,493\times2845\,201\times174\,881\,933\,884\,719\,184\,571\,561}{2^6 3^4 5^2 7^2 13\times17\times19\times23\times29\times31\times37\times41\times43\times47\times53\times59\times61\times67\times71\times73\times79} \equiv \frac{1}{3^4}$

$H_2\left(2\right) := \sum_{n=1}^{2} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{2^2}{3} \equiv \frac{1}{3}$

$H_2\left(3\right) := \sum_{n=1}^{3} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = 2\frac{5^2}{3\times 11} \equiv \frac{1}{3}$

$H_2\left(4\right) := \sum_{n=1}^{4} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = 19\frac{71}{3\times 5^2 11} \equiv \frac{1}{3}$

$H_2\left(5\right) := \sum_{n=1}^{5} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{194\,713}{3\times 5^2 11\times 137} \equiv \frac{1}{3}$

$H_2\left(6\right) := \sum_{n=1}^{6} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{9917\,687}{5538\,225} = 13\frac{762\,899}{3\times 5^2 7^2 11\times 137} \equiv \frac{2}{3}$

$13\frac{762\,899}{5^2 7^2 11\times 137} \bmod 3 = 2$

$H_2\left(7\right) := \sum_{n=1}^{7} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{112\,451\,057}{60\,920\,475} = \frac{112\,451\,057}{3\times 5^2 7^2 11^2 137} \equiv \frac{1}{3}$

$H_2\left(8\right) := \sum_{n=1}^{8} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{87\,707\,471\,002}{46\,360\,481\,475} = 2\times 101\times 18\,427\frac{23\,563}{3\times 5^2 7^2 11^2 137\times 761} \equiv \frac{1}{3}$

$H_2\left(9\right) := \sum_{n=1}^{9} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{638\,247\,495\,586\,258}{330\,503\,872\,435\,275} = 2\times 23\times 131\times 65\,617\frac{1614\,149}{3\times 5^2 7^2 11^2 137\times 761\times 7129} \equiv \frac{1}{3}$

$H_2\left(10\right) := \sum_{n=1}^{10} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{39\,621\,419\,345\,255\,038}{20\,160\,736\,218\,551\,775} = 2\times 223\frac{88\,837\,263\,105\,953}{3\times 5^2 7^2 11^2 61\times 137\times 761\times 7129} \equiv \frac{1}{3}$

$H_2\left(11\right) := \sum_{n=1}^{11} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = 2\times 241\times 2869\,213\frac{2435\,032\,154\,473}{3\times 5^2 7^2 11^2 61\times 97\times 137\times 761\times 863\times 7129} \equiv \frac{1}{3}$

$2\times 241\times 2869\,213 * 2435\,032\,154\,473/3 = \frac{3367\,553\,690\,081\,394\,959\,018}{3} = 1122\,517\,896\,693\,798\,319\,672\frac{2}{3}$

$5^2 7^2 11^2 61\times 97\times 137\times 761\times 863\times 7129/3 = \frac{562\,558\,463\,197\,062\,545\,675}{3} = 187\,519\,487\,732\,354\,181\,891\frac{2}{3}$

$H_2\left(12\right) := \sum_{n=1}^{12} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = 2^3 73\times 157\times 37\,724\,670\,589\frac{84\,876\,376\,979}{3\times 217\,176\,844\,845\,596\,842\,463}$

$= 2^3 73\times 157\times 37\,724\,670\,589\frac{84\,876\,376\,979}{217\,176\,844\,845\,596\,842\,463} \bmod 3 = 1/3$

$2^3 73\times 157\times 37\,724\,670\,589 * 84\,876\,376\,979/3 = \frac{293\,578\,866\,124\,447\,319\,211\,215\,128}{3} = 97\,859\,622\,041\,482\,439\,737\,071\,709\frac{1}{3}$

$217\,176\,844\,845\,596\,842\,463/3 = 72\,392\,281\,615\,198\,947\,487\frac{2}{3}$

$H_2\left(13\right) := \sum_{n=1}^{13} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = 2^6 179\times 601\times 227\,693\frac{217\,176\,844\,845\,596\,842\,463}{3\times 5^2 7^2 11^2 13^2 29\times 43\times 61\times 97\times 137\times 509\times 761\times 863\times 919\times 7129}$

$H_2\left(14\right) := \sum_{n=1}^{14} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = 2^2 2357\,639\times 63\,016\,452\,625\,793\frac{678\,492\,741\,526\,289\,329}{3\times 5^2 7^2 11^2 13^2 29\times 43\times 61\times 97\times 137\times 509\times 761\times 863\times 919\times 104}$

$H_2\left(15\right) := \sum_{n=1}^{15} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = 2^2 33\,305\,468\,561\frac{126\,009\,694\,348\,928\,734\,063\,694\,078\,017\,349}{3\times 5^2 7^2 11^2 13^2 29\times 43\times 61\times 97\times 137\times 509\times 761\times 863\times 919\times 1049\times 1117\times 7129\times 4}$

$2^2 33\,305\,468\,561 * 126\,009\,694\,348\,928\,734\,063\,694\,078\,017\,349/3 = \frac{16\,787\,247\,654\,077\,861\,265\,551\,571\,547\,714\,793\,328\,259\,1}{3}$

$= 5595\,749\,218\,025\,953\,755\,183\,857\,182\,571\,597\,776\,086\,385\frac{1}{3}$

$H_2\left(16\right) := \sum_{n=1}^{16} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{41\,265\,191\,869\,346\,609\,110\,070\,477\,712\,048\,816\,327\,162\,225\,083\,329}{3\times 6528\,368\,218\,193\,266\,623\,358\,749\,186\,625\,804\,353\,558\,475\,999\,5253}$

$H_2\left(17\right) := \sum_{n=1}^{17} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{1753\,122\,294\,522\,440\,418\,772\,782\,970\,599\,045\,853\,686\,401\,441\,104\,977\,274\,367}{3\times 275\,119\,949\,277\,213\,299\,140\,041\,417\,223\,853\,264\,622\,032\,139\,112\,130\,444\,075}$

$H_2\left(18\right) := \sum_{n=1}^{18} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{25\,211\,863\,968\,897\,979\,340\,587\,398\,121\,880\,863\,268\,484\,578\,613\,617\,063\,163\,147\,113\,467}{3\times 3927\,144\,967\,087\,675\,073\,127\,992\,341\,919\,865\,879\,047\,537\,985\,360\,422\,709\,990\,216\,575}$

$H_2\left(19\right) := \sum_{n=1}^{19} \frac{1}{n\sum_{i=1}^{n} \frac{1}{i}} = \frac{188\,887\,474\,547\,260\,288\,326\,769\,800\,221\,233\,943\,076\,354\,459\,335\,669\,569\,434\,494\,661\,826\,224\,409}{3\times 29\,219\,635\,446\,033\,248\,981\,328\,488\,676\,613\,801\,922\,844\,035\,909\,801\,293\,862\,824\,377\,140\,477\,525}$

$$H_2\left(20\right) := \sum_{n=1}^{20} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i}} = \frac{2122\,915\,755\,706\,566\,059\,523\,784\,884\,015\,045\,293\,812\,016\,880\,172\,173\,289\,532\,654\,971\,139\,191\,768\,696}{3\times 326\,296\,457\,956\,010\,334\,272\,217\,728\,920\,940\,594\,645\,051\,265\,793\,720\,613\,201\,094\,115\,785\,895\,314\,56}$$

$$H_2\left(21\right) := \sum_{n=1}^{21} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i}} = \frac{40\,275\,198\,749\,193\,957\,779\,775\,881\,538\,252\,646\,150\,124\,550\,729\,833\,431\,601\,641\,478\,054\,280\,904\,250\,19}{3\times 6153\,315\,897\,846\,714\,552\,253\,198\,359\,530\,730\,543\,667\,892\,958\,055\,070\,390\,938\,732\,493\,478\,550\,494\,5}$$

$$H_2\left(22\right) := \sum_{n=1}^{22} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i}} : \frac{257\,774\,349\,957\,654\,543\,546\,322\,866\,761\,227\,097\,008\,166\,783\,562\,130\,063\,724\,062\,318\,600\,633\,853\,006\,53}{3\times 39\,162\,157\,546\,939\,732\,249\,645\,703\,386\,199\,021\,941\,389\,394\,274\,352\,731\,941\,020\,078\,142\,762\,393\,289\,1}$$

$$H_2\left(27\right) := \sum_{n=1}^{27} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i}} = \frac{15\,207\,734\,170\,546\,299\,313\,830\,398\,249\,673\,228\,203\,580\,981\,888\,727\,724\,355\,725\,663\,680\,240\,113\,602\,80}{3\times 2256\,273\,837\,172\,179\,862\,542\,501\,097\,381\,198\,384\,166\,119\,778\,189\,608\,196\,461\,823\,963\,135\,102\,938\,5}$$

:

$$H_2\left(54\right) = \sum_{n=1}^{54} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i}} \bmod 3$$

9+2+5+6+8+7+1+6+6+0+2+2+6+2+1+6+7+6+6+2+5+7+8+6+ 6+7+1+3+7+7+7+2+8+1+3+9+4+5+0+1+7+3+2+0+9+7+1+9+5+5+ 0+6+0+1+4+1+0+8+9+8+9+8+2+7+9+2+8+8+3+4+9+3+7+9+0+1+ 4+2+6+9+6+7+5+5+2+1+2+3+3+4+7+9+9+4+3+1+3+4+8+4+7+2+ 3+0+4+1+8+5+8+6+9+0+2+5+8+4+1+3+9+2+4+3+6+0+3+9+7+9+ 1+5+5+7+1+9+8+7+5+2+5+4+5+3+0+7+3+7+7+0+7+1+2+4+3+0+ 3+7+2+4+4+1+4+8+4+0+6+7+6+0+0+2+0+1+6+5+5+2+3+1+9+7+ 1+8+5+0+7+2+5+0+0+4+3+7+3+7+6+7+6+8+9+7+6+2+6+7+6+7+ 3+5+2+6+8+6+6+3+0+7+0+4+8+8+7+4+4+3+8+5+4+1+5+8+2+3+ 5+9+2+5+2+7+8+1+6+5+3+2+7+0+5+3+2+1+3+1+8+2+3+6+3+8+ 1+1+8+1+4+6+4+4+6+0+7+8+1+6+3+5+9+0+7+7+0+0+0+3+9+3+ 4+6+8+8+2+9+0+0+8+7+2+2+5+8+9+9+3+4+3+7+1+5+9+2+9+ 1+4+3+3+3+1+0+8+7+2+0+7+2+1+7+7+9+2+1+5+3+0+2+0+4+ 9+2+7+9+1+7+2+9+1+1+2+8+5+2+1+1+2+3+4+8+8+6+0+9+9+ 1+6+8+7+8+9+3+8+1+4+9+2+7+2+4+6+6+4+7+9+5+4+1+6+0+ 3+9+7+9+3+9+5+3+2+5+8+4+7+3+1+5+7+9+2+0+3+2+0+2+0+ 5+3+8+5+8+6+3+1+6+0+8+8+8+7+8+3+8+4+6+8+6+4+3+2+8+ 1+5+5+1+3+5+5+5+2+4+4+4+2+5+3+9+6+9+1+1+3+2+9+5+9+ 3+3+2+7+8+7+9+4+8+1+3+0+6+0+5+9+0+0+4+4+8+2+4+7+5 = 2180

$H_3(2) := \sum_{n=1}^{2} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{13}{10} = \frac{13}{2\times 5}$

$H_3(3) := \sum_{n=1}^{3} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{125}{86} = \frac{5^3}{2\times 43}$

$H_3(4) := \sum_{n=1}^{4} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{3406\,225}{2197\,042} = 5^2 19 \times 71 \frac{101}{2\times 43\times 59\times 433}$

$H_3(5) := \sum_{n=1}^{5} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{70\,546\,582\,127\,075}{43\,581\,076\,569\,542} = 5^2 \frac{2821\,863\,285\,083}{2\times 41\times 43\times 59\times 433\times 483\,811}$

$H_3(6) := \sum_{n=1}^{6} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{77\,979\,813\,085\,378\,664\,380\,175}{46\,682\,008\,275\,896\,562\,178\,598} = 5^2 53 \times 83 \frac{709\,068\,543\,626\,993\,993}{2\times 41\times 43\times 59\times 433\times 483\,811\times 1071\,153\,169}$

$H_3(7) := \sum_{n=1}^{7} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{7128\,449\,092\,095\,277\,541\,932\,683\,361\,233\,775}{4165\,242\,215\,111\,272\,787\,202\,405\,031\,818\,934} = 5^2 29 \times 131 \times 313 \times 379 \times 599 \frac{1056\,271\,058\,212\,849\,490\,813}{2\times 17\times 41\times 43\times 59\times 433\times 483\,811\times 1071\,153\,169\times 5248\,579\,849}$

$H_3(8) := \sum_{n=1}^{8} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{526\,989\,794\,634\,578\,777\,891\,199\,786\,263\,994\,125\,437\,656\,937\,075}{302\,003\,051\,625\,131\,760\,659\,744\,290\,875\,698\,500\,845\,598\,190\,212} = 5^2 \frac{21\,079\,591\,785\,383\,151\,115\,647\,991\,450\,559\,765\,017\,506\,277\,483}{2^2 17\times 41\times 43\times 59\times 347\times 433\times 483\,811\times 5248\,579\,849\times 1071\,153\,169\times 104\,474\,812\,297}$

$H_3(9) := \sum_{n=1}^{9} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{439\,126\,511\,592\,756\,077\,178\,537\,561\,374\,764\,543\,989\,045\,671\,632\,801\,387\,065\,664\,888\,97}{247\,647\,062\,568\,971\,810\,742\,417\,642\,895\,581\,969\,566\,352\,064\,336\,068\,871\,343\,683\,395\,31}$

$21\,739\times 104\,915\,827 \frac{6521\,074\,782\,141\,741\,330\,781\,570\,338\,034\,070\,419\,235\,655\,155\,363}{2^2 17\times 41\times 43\times 59\times 229\times 347\times 433\times 483\,811\times 1658\,009\times 1071\,153\,169\times 2159\,730\,113\times 5248\,579\,849\times 104\,474\,812\,297}$

$H_3(10) := \sum_{n=1}^{10} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{1375\,069\,473\,096\,738\,882\,297\,180\,024\,549\,608\,867\,753\,502\,890\,257\,340\,096\,207\,344\,473}{765\,041\,073\,424\,132\,856\,964\,262\,504\,806\,019\,218\,679\,380\,800\,421\,541\,642\,044\,781\,292}$

$5^2 9824\,327\times 8109\,151\times 42\,779\,778\,269 \frac{16\,138\,676\,116\,592\,075\,359\,289\,378\,400\,724\,971\,355\,701\,419\,551\,7}{2^2 17\times 41\times 43\times 59\times 229\times 347\times 433\times 483\,811\times 1658\,009\times 5248\,579\,849\times 351\,290\,039\times 1071\,153\,169}$

$H_3(11) := \sum_{n=1}^{11} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{70\,227\,448\,648\,824\,293\,811\,250\,501\,439\,520\,538\,095\,822\,508\,097\,249\,456\,777\,688\,999\,13}{38\,619\,564\,266\,611\,655\,169\,802\,437\,232\,167\,511\,191\,398\,093\,850\,417\,562\,734\,402\,008\,29}$

$$H_3\left(12\right) := \sum_{n=1}^{12} \frac{1}{n \sum_{i=1}^{n} \frac{1}{i} \sum_{k=1}^{i} \frac{1}{k \sum_{j=1}^{k} \frac{1}{j}}} = \frac{320\,098\,070\,046\,556\,973\,550\,952\,646\,357\,634\,878\,558\,967\,398\,408\,375\,454\,814\,956\,728}{174\,248\,175\,152\,236\,682\,912\,350\,020\,582\,544\,826\,860\,073\,606\,221\,365\,421\,580\,662\,091}$$