

# Algorithmen für Kreuzungswinkelmaximierung beim Zeichnen von Graphen



Der Sieger beim Wettbewerb 2018

Amadäus Spallek

Christian Geckeler

Henry Förster

Michael Bekos

Michael Kaufmann

# GRAPH DRAWING CONTEST

## *historic perspective*



**1994** Uwe Wuerker • Stefan Hougardy •  
Petra Mutzel & Thomas Odenthal •  
Georg Sander

**1995** Falk Schreiber & Carsten Friedrich  
• Vladimir Batagelj & Andrej Mrvar •  
Paulis Kikusts & Peteris Rucevskis •  
Georg Sander

**1996** Ulrich Fößmeier & Michael  
Kaufmann • Vladimir Batagelj & Andrej  
Mrvar • Falk Schreiber & Carsten  
Friedrich • Thomas Kamps, Jörg Kleinz &  
Thomas Reichenberger

**1997** B. Bascary, B. Cattan, A.  
Cohen-Solal, M. Philip & H. Szigeti •  
Vladimir Batagelj & Andrej Mrvar •  
Michael Kaufmann & Gunnar Klau •  
Thomas Ziegler & Petra Mutzel • Cristian  
Ghezzi • Falk Schreiber & Carsten  
Friedrich

**1998** Vladimir Batagelj & Andrej Mrvar •  
Petrus Abri Santoso & Andi Surjanto •  
Roland Wiese

**1999** Vladimir Batagelj & Andrej Mrvar •  
Ulrik Brandes • Karlis Freivalds & Paulis  
Kikusts • Rowena Mankelow

**2000** Nikola S. Nikolov • Vladimir  
Batagelj & Andrej Mrvar • Jan Adamec  
• C. Duncan, P. Gajer, M. Goodrich & S.  
Kobourov • Robby Schönfeld & Nikola  
S. Nikolov

**2001** Ulrik Brandes & Marco Gärtler • C.  
Gutwenger, K. Klein, J. Kupke & S.  
Leipert • Roland Wiese • Carsten  
Friedrich • Merijam Percan

**2002** Daniel Gmach • Paul Holleis &  
Thomas Zimmermann • Nikola S. Nikolov &  
Patrick Healey • Christoph Vogt

**2003** Daniel Gmach, Paul Holleis &  
Thomas Zimmermann • Christian Kuklas,  
Dirk Koschützki & Falk Schreiber

**2004** Andrei Grecu & Gunnar Klau •  
Ulrik Brandes, Daniel Fleischer & Thomas  
Puppe

**2005** Markus Chimani, Carsten  
Gutwenger & Karsten Klein • V.  
Batagelj, A. Mrvar, A. Ahmed, X. Fu, S.  
Hong & D. Merrick • Marco Gärtler &  
Markus Krug

**2006** Michael McGuffin • Adel Admed,  
Seok-Hee Hong, Quan Nguyen &  
Donald Taylor • Adel Admed, Xiaoyan  
Fu, Seok-Hee Hong, Quan Nguyen &  
Kai Xu • Fabrizio Frati & Markus Geyer •  
Andreas Gerasch & team

**2007** Wolfgang Brunner & Jens Schmidt  
• Robert Theron, Rodrigo Santamaría,  
Juan Garcia, Diego Gomez & Vadim  
Paz-Madrid • Felix Heinen

**2008** Yifan Hu & Emden Gansner •  
Melanie Badent & Pietro Palladino •  
Melanie Badent, Martin Mader &  
Christian Piech • Hoi-Ming Wong,  
Markus Chimani & Karsten Klein

**2009** Hui Liu • Nicholas Jefferson •  
Melanie Badent & Michael Baur •  
Hoi-Ming Wong & Karsten Klein • Joe  
Fowler

**2010** Quan Nguyen • Sergey Pupyrev •  
M. Baur, M. Siebenhaller, R. Wiese &  
Thomas Wurst • Petra Mutzel &  
Hoi-Ming Wong • Maarten Löffler &  
Martin Nöllenburg

**2011** Hanley Weng • M. Löffler, D.  
Eppstein, M. Goodrich & S. Kobourov •  
R. Zelina, S. Bota, S. Houtman & R. Ban •  
Sergey Pupyrev • Maarten Löffler &  
Martin Nöllenburg

**2012** Roman Prutkin • Sergey Pupyrev •  
Remus Zelina, S. Bota, S. Houtman & R.  
Balaban • Tobias Brinkjost • A. Bauer, I.  
Rutter, J. Vordermeier & M. Kaufmann

**2013** Jos de Jong & Giovanni Pazienza  
• Remus Zelina, Sebastian Bota, Siebren  
Houtman & Radu Balaban • Thomas  
Bläsius & Ignaz Rutter

**2014** Martin Nöllenburg • Remus Zelina,  
Sebastian Bota, Siebren Houtman &  
Radu Balaban • Tamara Mchedlidze,  
Martin Nöllenburg, Alexander  
Khomenko, Igor Karlinsky & Denis  
Knöpfle • Philipp Kindermann, Fabian  
Lipp & Wadim Reimche

**2015** Tamara Mchedlidze • Jennifer  
Hood & Pat Morin • Boris Klemz, Ulf  
Rüegg & Fabian Lipp • Josef Cibulka

**2016** Fabian Klute • Jonathan Klawitter  
& Tamara Mchedlidze • Michael Bekos,  
Thanasis Lianeas & Chrysanthi  
Raftopoulou • Johan de Ruiter

**2017** Steven Shangzhou Wang • Fabian  
Klute & Irene Parada • Theresa Fröschl,  
Jonathan Klawitter & Darren Strash •  
Almut Demel, Dominik Dürrschnabel,  
Tamara Mchedlidze, Marcel  
Radermacher & Lasse Wulf

**2018** ?

# LIVE CHALLENGE

gd  
g18

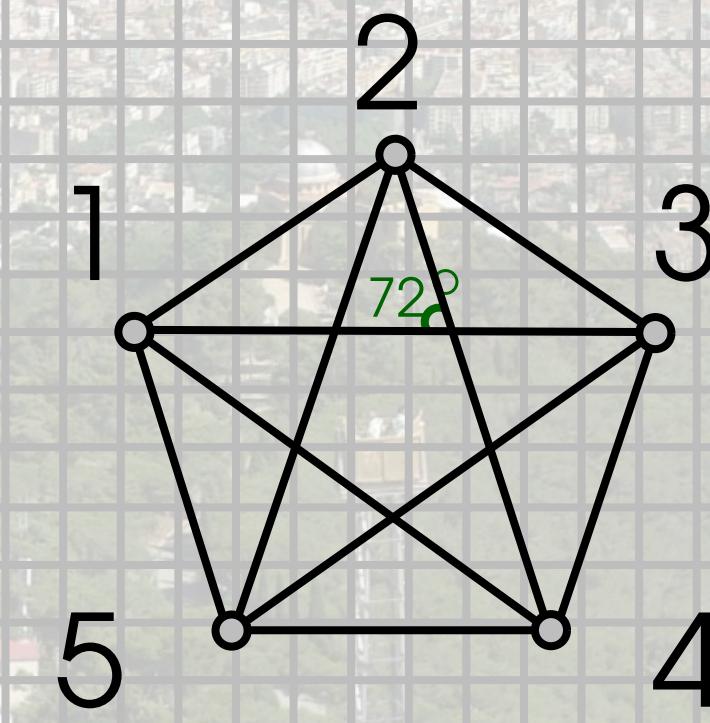
## *Crossing Angle Maximization*

input: arbitrary graph

output: straight-line embedding with  
integer vertex coordinates

must be a proper  
drawing

objective:  
maximize the  
smallest angle  
between any  
crossing edges







# LIVE CHALLENGE 2017

gd  
g18



# LIVE CHALLENGE 2017

gd  
g18

graph

Oh-Hyun Kwon

Arizona Anglers

TuebingenMidnight

CoffeeVM

# LIVE CHALLENGE 2017

gd  
g18

graph 1

Oh-Hyun Kwon	79°
Arizona Anglers	48°
TuebingenMidnight	77°
CoffeeVM	90°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2
Oh-Hyun Kwon	79°	59°
Arizona Anglers	48°	30°
TuebingenMidnight	77°	42°
CoffeeVM	90°	88°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3
Oh-Hyun Kwon	79°	59°	90°
Arizona Anglers	48°	30°	78°
TuebingenMidnight	77°	42°	89°
CoffeeVM	90°	88°	90°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4
Oh-Hyun Kwon	79°	59°	90°	50°
Arizona Anglers	48°	30°	78°	40°
TuebingenMidnight	77°	42°	89°	89°
CoffeeVM	90°	88°	90°	90°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5
Oh-Hyun Kwon	79°	59°	90°	50°	40°
Arizona Anglers	48°	30°	78°	40°	28°
TuebingenMidnight	77°	42°	89°	89°	30°
CoffeeVM	90°	88°	90°	90°	80°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°
Arizona Anglers	48°	30°	78°	40°	28°	36°
TuebingenMidnight	77°	42°	89°	89°	30°	78°
CoffeeVM	90°	88°	90°	90°	80°	90°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6	7
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°	31°
Arizona Anglers	48°	30°	78°	40°	28°	36°	3°
TuebingenMidnight	77°	42°	89°	89°	30°	78°	34°
CoffeeVM	90°	88°	90°	90°	80°	90°	57°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6	7	8
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°	31°	45°
Arizona Anglers	48°	30°	78°	40°	28°	36°	3°	1°
TuebingenMidnight	77°	42°	89°	89°	30°	78°	34°	61°
CoffeeVM	90°	88°	90°	90°	80°	90°	57°	85°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6	7	8	9
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°	31°	45°	28°
Arizona Anglers	48°	30°	78°	40°	28°	36°	3°	1°	7°
TuebingenMidnight	77°	42°	89°	89°	30°	78°	34°	61°	9°
CoffeeVM	90°	88°	90°	90°	80°	90°	57°	85°	60°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6	7	8	9	10
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°	31°	45°	28°	7°
Arizona Anglers	48°	30°	78°	40°	28°	36°	3°	1°	7°	4°
TuebingenMidnight	77°	42°	89°	89°	30°	78°	34°	61°	9°	4°
CoffeeVM	90°	88°	90°	90°	80°	90°	57°	85°	60°	21°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6	7	8	9	10	11
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°	31°	45°	28°	7°	14°
Arizona Anglers	48°	30°	78°	40°	28°	36°	3°	1°	7°	4°	5°
TuebingenMidnight	77°	42°	89°	89°	30°	78°	34°	61°	9°	4°	6°
CoffeeVM	90°	88°	90°	90°	80°	90°	57°	85°	60°	21°	47°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6	7	8	9	10	11	12
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°	31°	45°	28°	7°	14°	11°
Arizona Anglers	48°	30°	78°	40°	28°	36°	3°	1°	7°	4°	5°	0°
TuebingenMidnight	77°	42°	89°	89°	30°	78°	34°	61°	9°	4°	6°	5°
CoffeeVM	90°	88°	90°	90°	80°	90°	57°	85°	60°	21°	47°	36°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6	7	8	9	10	11	12	13
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°	31°	45°	28°	7°	14°	11°	7°
Arizona Anglers	48°	30°	78°	40°	28°	36°	3°	1°	7°	4°	5°	0°	2°
TuebingenMidnight	77°	42°	89°	89°	30°	78°	34°	61°	9°	4°	6°	5°	4°
CoffeeVM	90°	88°	90°	90°	80°	90°	57°	85°	60°	21°	47°	36°	25°

# LIVE CHALLENGE 2017

gd  
g18

graph 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Oh-Hyun Kwon 79° 59° 90° 50° 40° 84° 31° 45° 28° 7° 14° 11° 7° 5°

Arizona Anglers 48° 30° 78° 40° 28° 36° 3° 1° 7° 4° 5° 0° 2° 2°

TuebingenMidnight 77° 42° 89° 89° 30° 78° 34° 61° 9° 4° 6° 5° 4° 5°

CoffeeVM 90° 88° 90° 90° 80° 90° 57° 85° 60° 21° 47° 36° 25° 34°

# LIVE CHALLENGE 2017

gd  
g18

graph 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Oh-Hyun Kwon 79° 59° 90° 50° 40° 84° 31° 45° 28° 7° 14° 11° 7° 5° 6°

Arizona Anglers 48° 30° 78° 40° 28° 36° 3° 1° 7° 4° 5° 0° 2° 2° 0°

TuebingenMidnight 77° 42° 89° 89° 30° 78° 34° 61° 9° 4° 6° 5° 4° 5° 4°

CoffeeVM 90° 88° 90° 90° 80° 90° 57° 85° 60° 21° 47° 36° 25° 34° 21°

# LIVE CHALLENGE 2017

gd  
g18

graph	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Oh-Hyun Kwon	79°	59°	90°	50°	40°	84°	31°	45°	28°	7°	14°	11°	7°	5°	6°	2°
Arizona Anglers	48°	30°	78°	40°	28°	36°	3°	1°	7°	4°	5°	0°	2°	2°	0°	0°
TuebingenMidnight	77°	42°	89°	89°	30°	78°	34°	61°	9°	4°	6°	5°	4°	5°	4°	3°
CoffeeVM	90°	88°	90°	90°	80°	90°	57°	85°	60°	21°	47°	36°	25°	34°	21°	20°

# LIVE CHALLENGE 2018

gd  
g18



# LIVE CHALLENGE 2018

gd  
g18

graph

Arizona Anglers  
TübingenColdShower  
CoffeeVM+  
TKMKN

# LIVE CHALLENGE 2018

gd  
g18

graph

1

Arizona Anglers	49°
TübingenColdShower	87°
CoffeeVM+	58°
TKMKN	

# LIVE CHALLENGE 2018

gd  
g18

graph

1 2

Arizona Anglers	49° 86°
TübingenColdShower	87° 90°
CoffeeVM+	58° 90°
TKMKN	15°

# LIVE CHALLENGE 2018

gd  
g18

graph 1 2 3

Arizona Anglers	49°	86°	87°
TübingenColdShower	87°	90°	88°
CoffeeVM+	58°	90°	85°
TKMKN	15°	87°	

# LIVE CHALLENGE 2018

gd  
g18

graph	1	2	3	4
Arizona Anglers	49°	86°	87°	37°
TübingenColdShower	87°	90°	88°	88°
CoffeeVM+	58°	90°	85°	60°
TKMKN	15°	87°		

# LIVE CHALLENGE 2018

gd  
g18

graph

1 2 3 4 5

Arizona Anglers	49°	86°	87°	37°	30°
TübingenColdShower	87°	90°	88°	88°	71°
CoffeeVM+	58°	90°	85°	60°	48°
TKMKN	15°	87°			

# LIVE CHALLENGE 2018

gd  
g18

graph 1 2 3 4 5 6

Arizona Anglers 

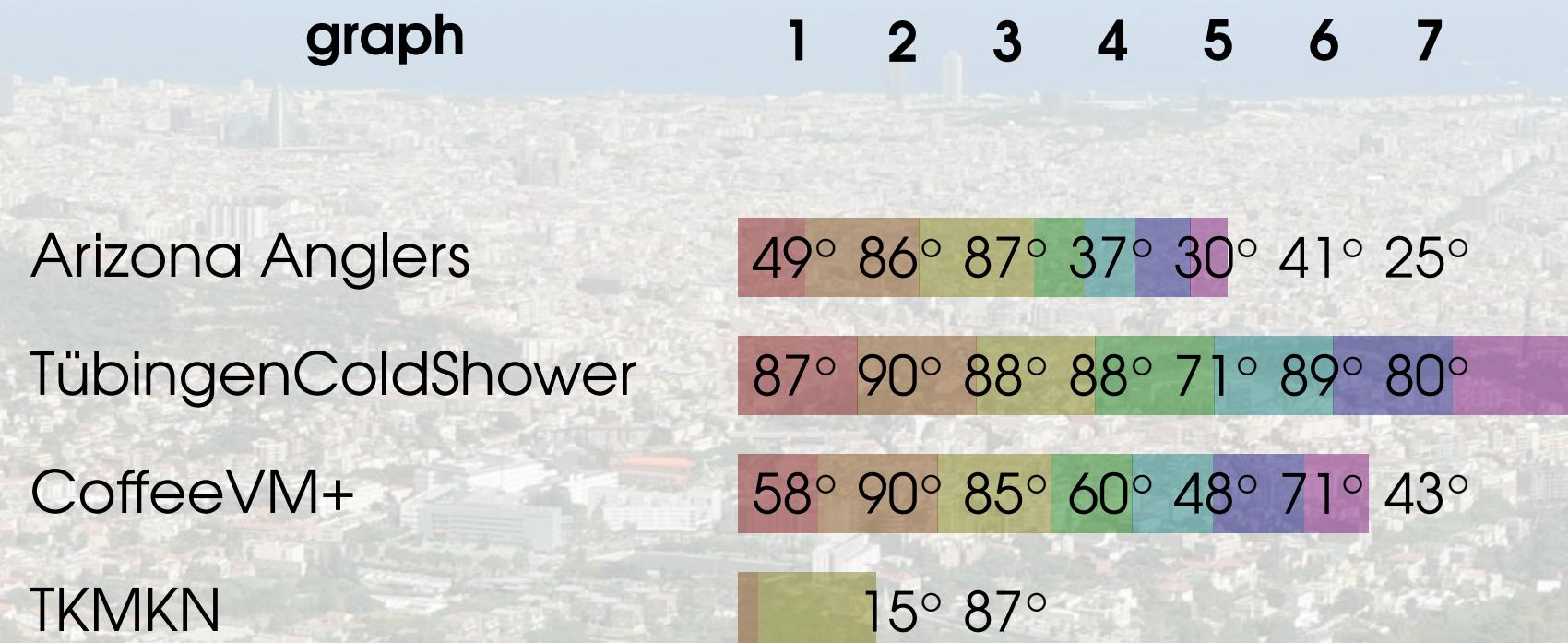
TübingenColdShower 

CoffeeVM+ 

TKMKN 

# LIVE CHALLENGE 2018

gd  
g18



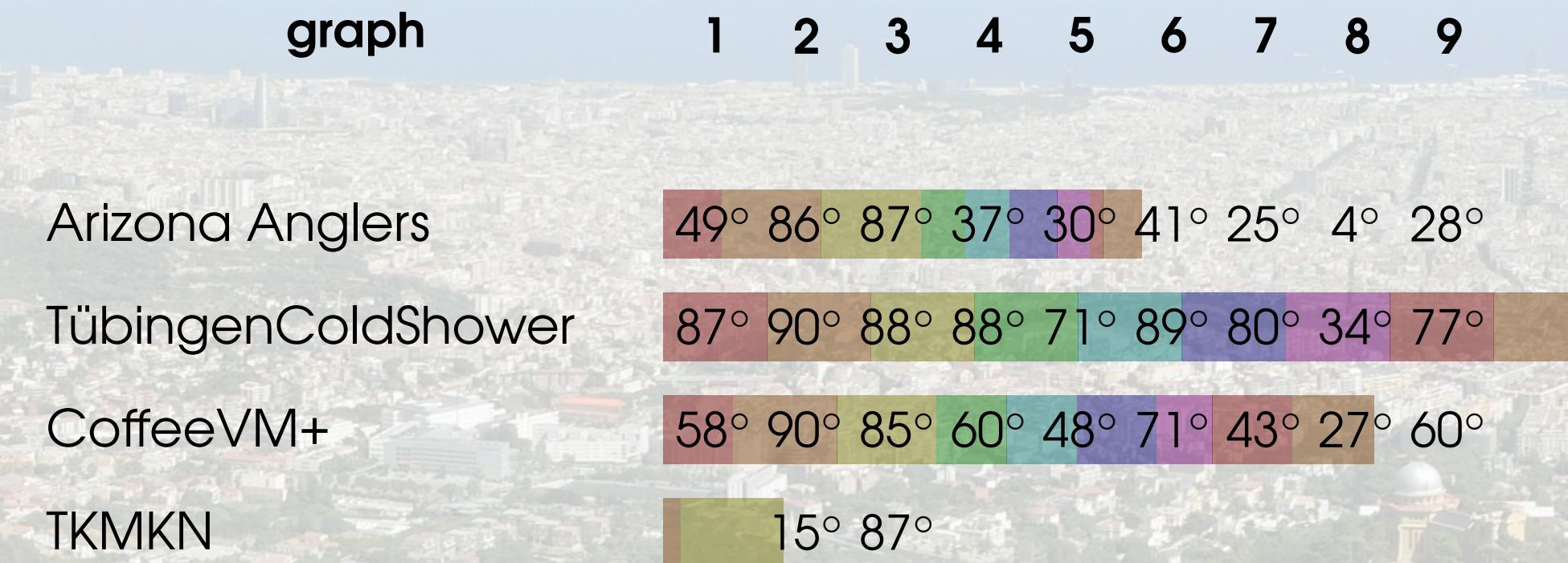
# LIVE CHALLENGE 2018

gd  
g18

graph	1	2	3	4	5	6	7	8
Arizona Anglers	49°	86°	87°	37°	30°	41°	25°	4°
TübingenColdShower	87°	90°	88°	88°	71°	89°	80°	34°
CoffeeVM+	58°	90°	85°	60°	48°	71°	43°	27°
TKMKN	15°	87°						

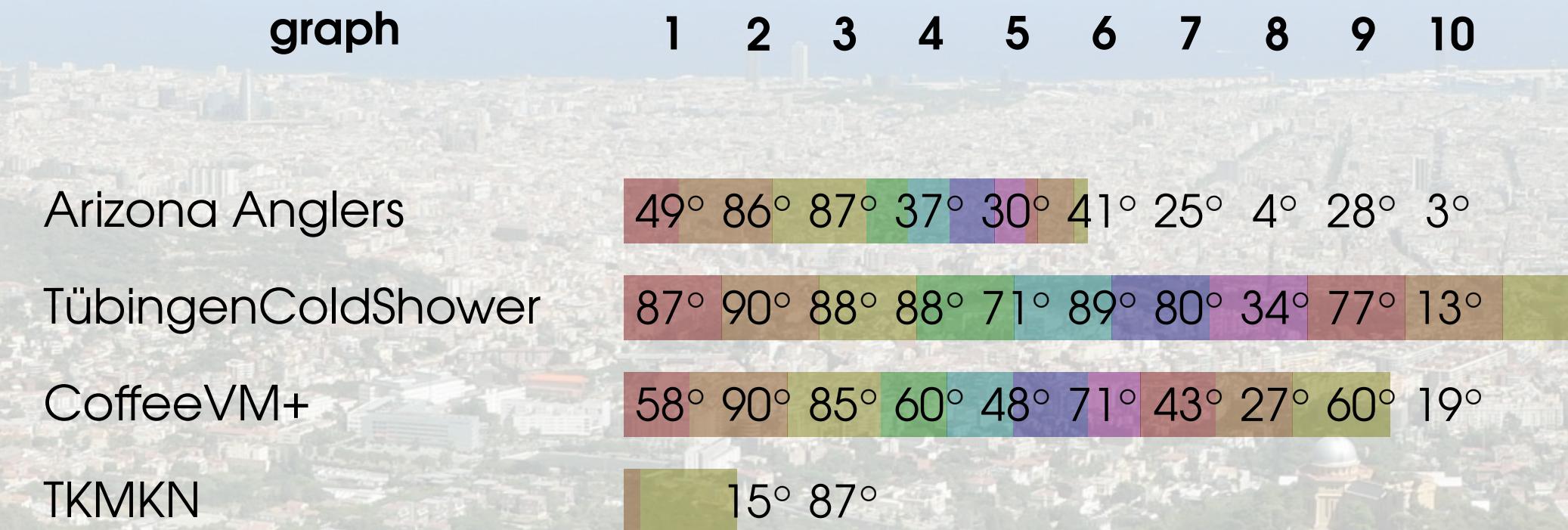
# LIVE CHALLENGE 2018

gd  
g18



# LIVE CHALLENGE

## 2018



# LIVE CHALLENGE 2018

gd  
g18

graph 1 2 3 4 5 6 7 8 9 10 11

Arizona Anglers 49° 86° 87° 37° 30° 41° 25° 4° 28° 3° 1°

TübingenColdShower 87° 90° 88° 88° 71° 89° 80° 34° 77° 13° 11°

CoffeeVM+ 58° 90° 85° 60° 48° 71° 43° 27° 60° 19° 22°

TKMKN 15° 87°

# LIVE CHALLENGE

## 2018



**graph** 1 2 3 4 5 6 7 8 9 10 11 12

Entity	Temperature Range (°)	Count
Arizona Anglers	1° - 15°	1
Arizona Anglers	16° - 25°	1
Arizona Anglers	26° - 35°	1
Arizona Anglers	36° - 45°	1
Arizona Anglers	46° - 55°	1
Arizona Anglers	56° - 65°	1
TübingenColdShower	1° - 15°	1
TübingenColdShower	16° - 25°	1
TübingenColdShower	26° - 35°	1
TübingenColdShower	36° - 45°	1
TübingenColdShower	46° - 55°	1
TübingenColdShower	56° - 65°	1
CoffeeVM+	1° - 15°	1
CoffeeVM+	16° - 25°	1
CoffeeVM+	26° - 35°	1
CoffeeVM+	36° - 45°	1
CoffeeVM+	46° - 55°	1
CoffeeVM+	56° - 65°	1
TKMKN	1° - 15°	1
TKMKN	16° - 25°	1
TKMKN	26° - 35°	1
TKMKN	36° - 45°	1
TKMKN	46° - 55°	1
TKMKN	56° - 65°	1

# LIVE CHALLENGE

## 2018



# graph

1 2 3 4 5 6 7 8 9 10 11 12 13

# Arizona Anglers

49° 86° 87° 37° 30° 41° 25° 4° 28° 3° 1°

TübingenColdShower

87° 90° 88° 88° 71° 89° 80° 34° 77° 13° 11° 74° 2°

# CoffeeVM+

58° 90° 85° 60° 48° 71° 43° 27° 60° 19° 22° 9° 2°

TKMKN

15° 87° 72°

# LIVE CHALLENGE 2018



graph 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Arizona Anglers	49°	86°	87°	37°	30°	41°	25°	4°	28°	3°	1°			
TübingenColdShower	87°	90°	88°	88°	71°	89°	80°	34°	77°	13°	11°	74°	2°	3°
CoffeeVM+	58°	90°	85°	60°	48°	71°	43°	27°	60°	19°	22°	9°	2°	11°
TKMKN		15°	87°									72°		

# 2019 live challenge

## *Unser neues Praktikum*



*Upward drawings on a fixed grid*

input: arbitrary directed graph and  
numbers  $w$  and  $h$

- output: straight-line upward embedding  
with integer vertex  
coordinates

must be a proper  
drawing

objective: minimize the  
number of crossings

