

A Heuristic Approach towards Drawings of Graphs with High Crossing Resolution

[GD 2018]

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Definition

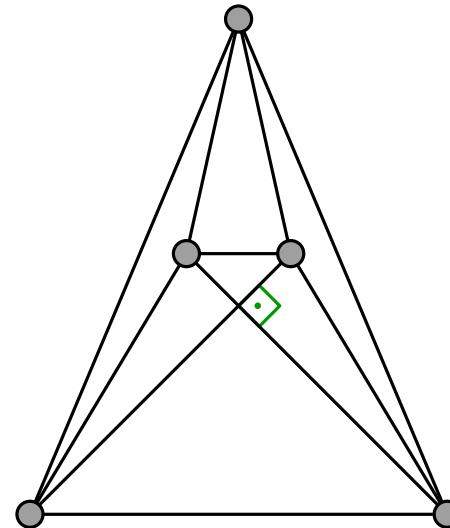
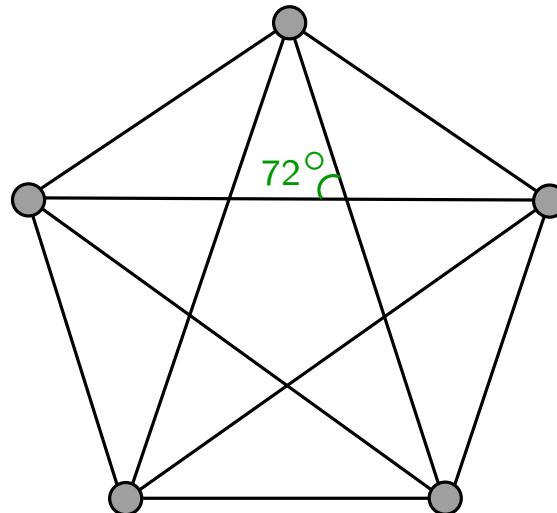
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- **Goal:** A heuristic to maximize the **crossing resolution**



Related Work

- The problem is NP-hard
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- Demel, Dürrschnabel, Mchedlidze, Radermacher, Wulf GD2018
→ next talk

Motivation: GD Live Challenge 2017

Graph	CoffeeVM	TüMidnight
1	90	77
2	88.23	42
3	90	89
4	88.97	89
5	80.4	30
6	90	78
7	56.537	34
8	84.95	61
9	59.885	9
10	20.978	4
11	46.684	6
12	36.47	5
13	25.456	4
14	33.52	5
15	20.512	4

Motivation: GD Live Challenge 2017

Graph	CoffeeVM	TüMidnight	Timed	Our best
1	90	77	89.99	89.99
2	88.23	42	88.21	88.7
3	90	89	87.86	89.95
4	88.97	89	77.13	89.05
5	80.4	30	78.68	86.96
6	90	78	89.96	89.96
7	56.537	34	55.77	63.62
8	84.95	61	81.18	89.28
9	59.885	9	54.63	88.2
10	20.978	4	23.60	23.72
11	46.684	6	57.00	72.00
12	36.47	5	26.24	35.86
13	25.456	4	22.43	33.68
14	33.52	5	29.69	43.08
15	20.512	4	13.51	29.18

Heuristic Approach

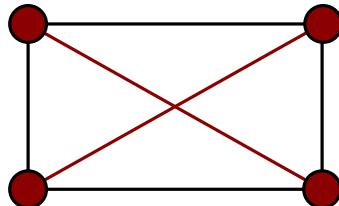
- Method Iterative movement of vertex, $v_i \rightarrow \pi_i$

Heuristic Approach

- **Method** Iterative movement of vertex, $v_i \rightarrow \pi_i$
- **Invariant** Non-decreasing crossing resolution, $c(\Gamma_i) \geq c(\Gamma_{i-1})$

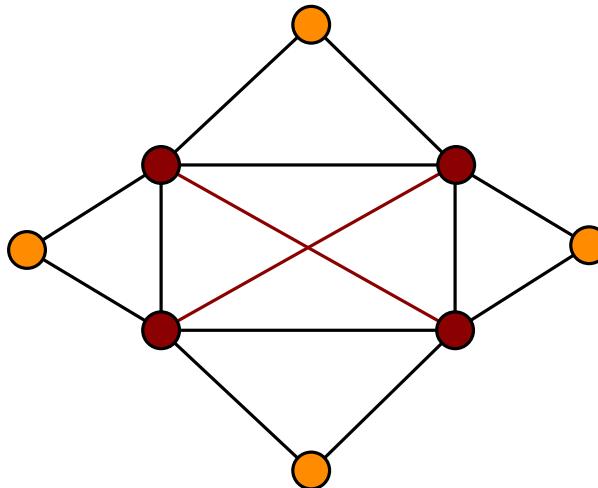
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- **Method** Iterative movement of vertex, $v_i \rightarrow \pi_i$
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- **Vertex Selection**
 - $v_i \in V$ or
 - $v_i \in C$, where C are **critical vertices**
i.e. if $c(e, e') = c(\Gamma_{i-1})$, then endvertices of $e, e' \in C$
 - v_i selected with probability dependent on graph distance from C



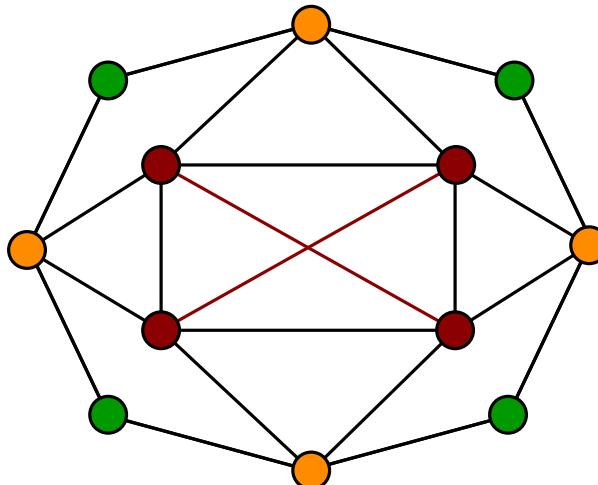
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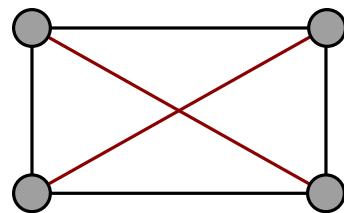


Heuristic Approach

- Vertex Placement
 - $r_0, \dots, r_\rho \leftarrow$ equidistant rays, rotated by an angle in $[0, 2\pi]$
 - $\pi_i \leftarrow$ translate v_i along r_i by $\delta_i \in [\delta_{min}, \delta_{max}]$
 - filter feasible points where $c(\Gamma_i) \geq c(\Gamma_{i-1})$ where $\pi_i \in \Gamma_i$

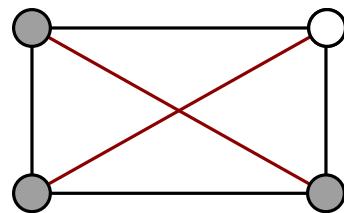
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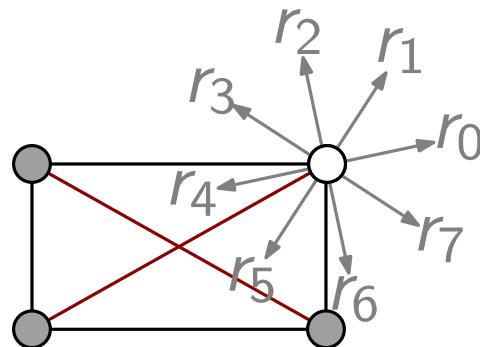
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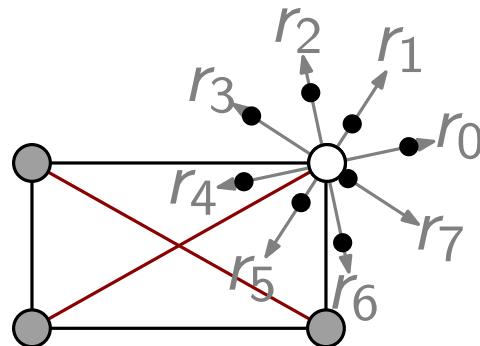
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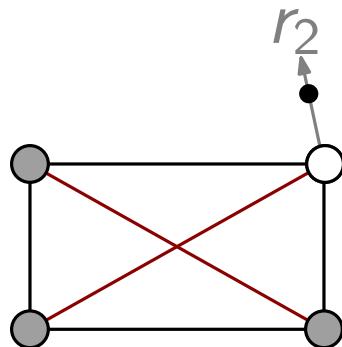
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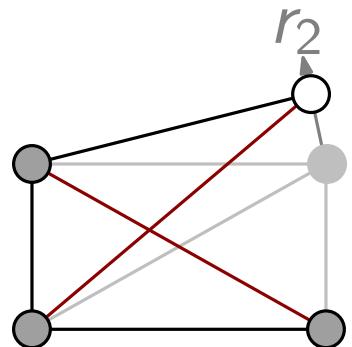
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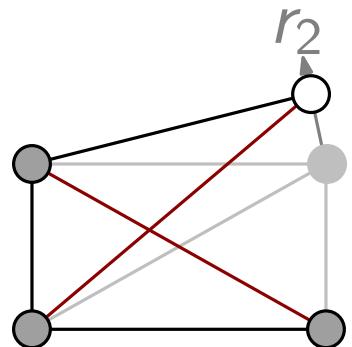
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 - double $\rho, \delta_{min}, \delta_{max}$ for ζ' iterations
 - results in graph drawings with larger area

Complexity Issues

- Major cost is computing crossing points and angles
 - naïve approach: $\mathcal{O}(m^2)$

Complexity Issues

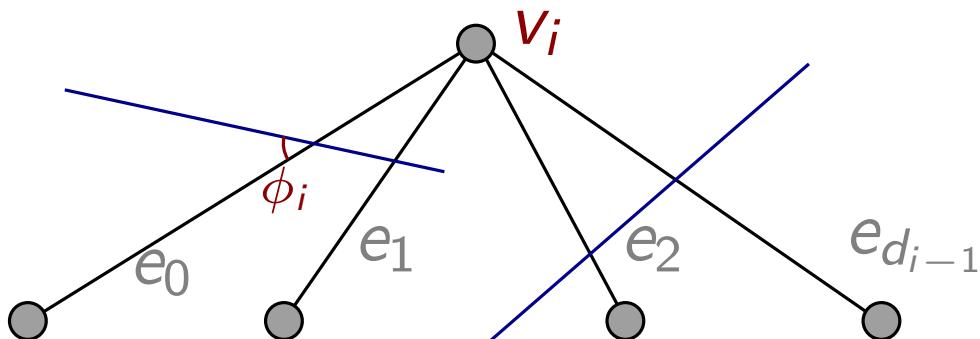
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Complexity Issues

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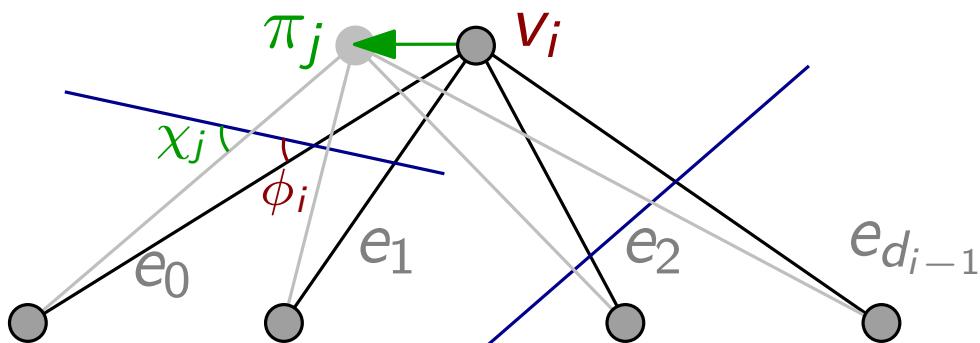
- naïve approach: $\mathcal{O}(m^2)$
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- our approach: $\mathcal{O}(\deg(v_i)m)$

- $\phi_i \leftarrow$ minimum crossing angle in Γ_i
- $e_0, \dots, e_{d_i-1} \leftarrow$ edges incident to v_i
- $\chi_j \leftarrow$ minimum crossing angle for e_0, \dots, e_{d_i-1} when v_i is at π_j
- π_j is feasible $\Leftrightarrow \chi_j \geq \phi_i$



Complexity Issues

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 - i. $\phi_i \leftarrow$ minimum crossing angle in Γ_i
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 - iii. $\chi_j \leftarrow$ minimum crossing angle for e_0, \dots, e_{d_i-1} when v_i is at π_j
 - iv. π_j is feasible $\Leftrightarrow \chi_j \geq \phi_i$

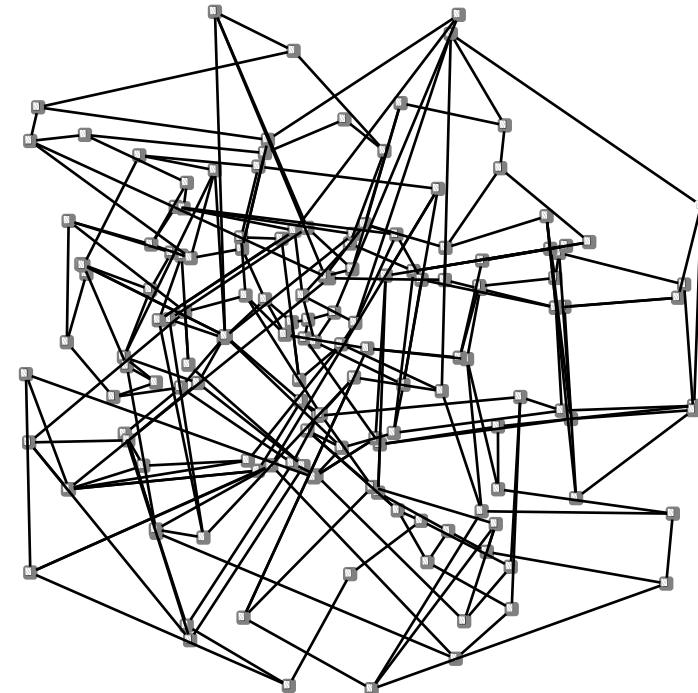
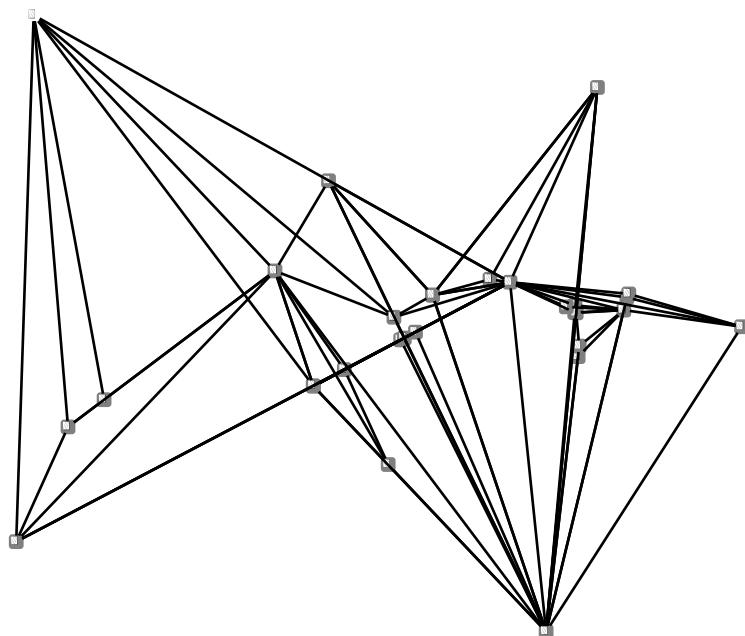


Variants

- what about grid drawings? Round candidate positions.

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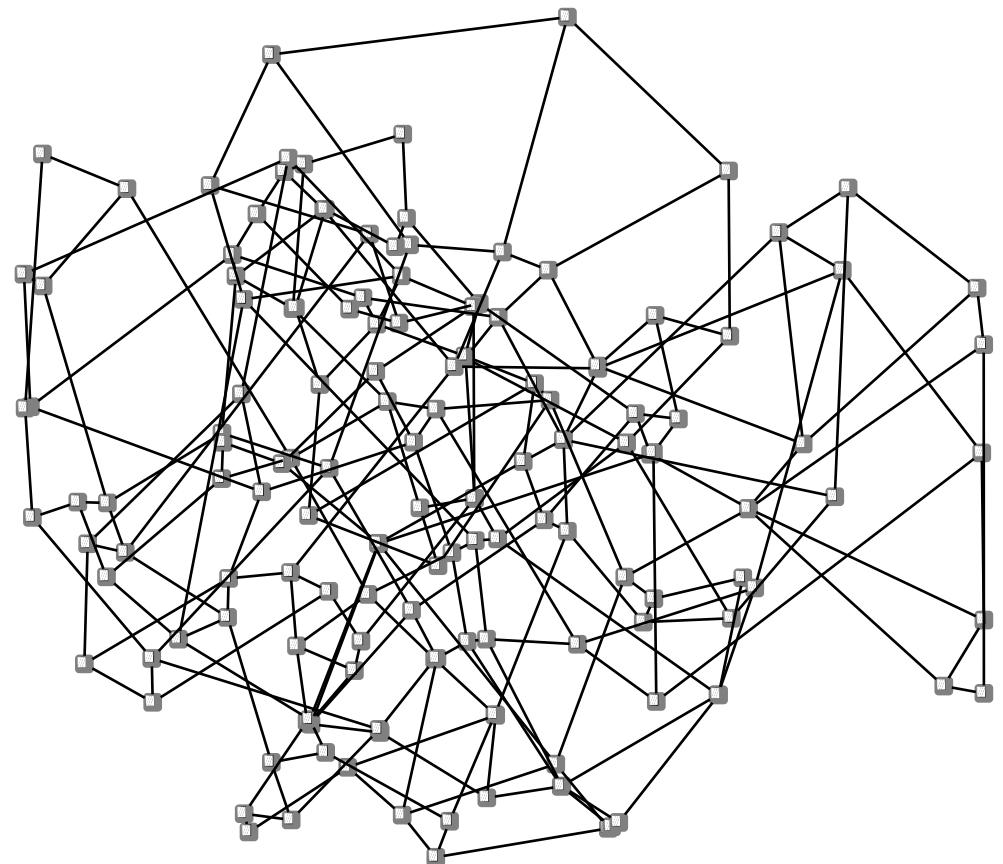
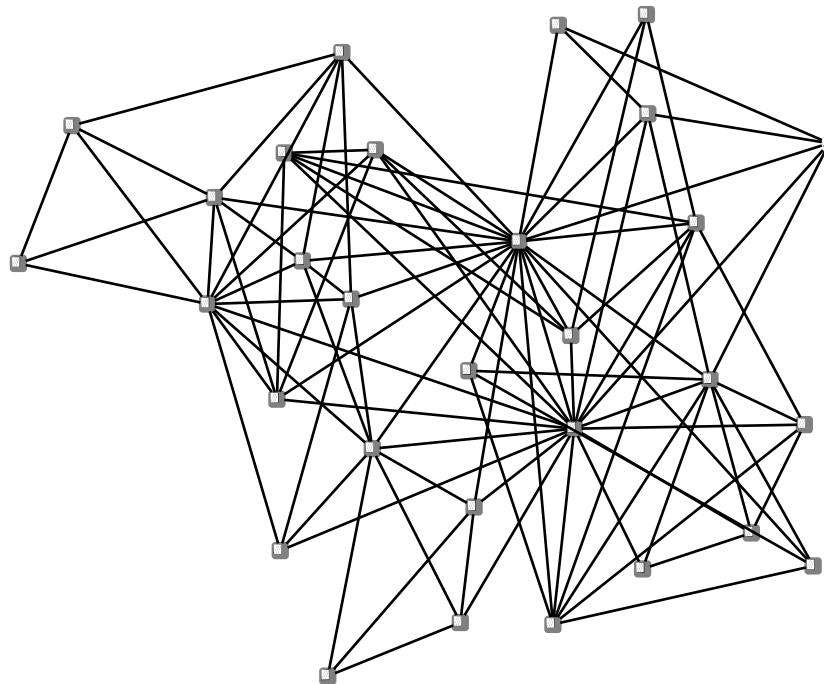
- what about grid drawings? Round candidate positions.
- opt. crossing resolution → poor aspect ratio, angular resolution
- compromise between aesthetic criteria



GD17 Contest: Graph 5, Graph 9 Crossing Resolution

Variant: Angular Resolution

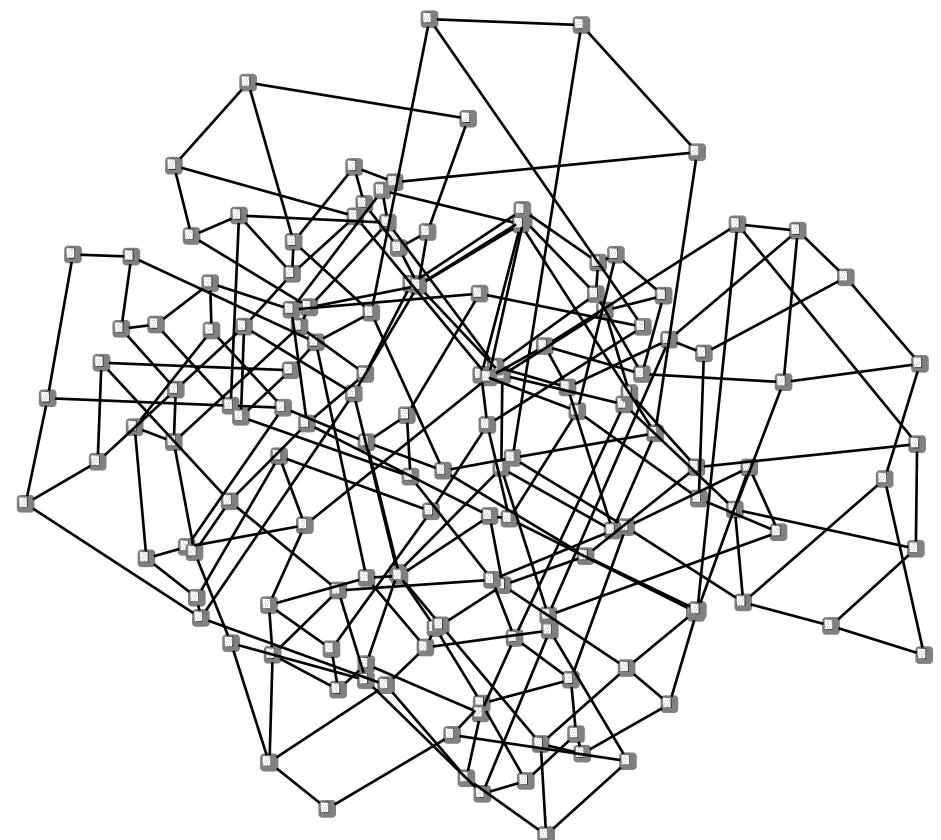
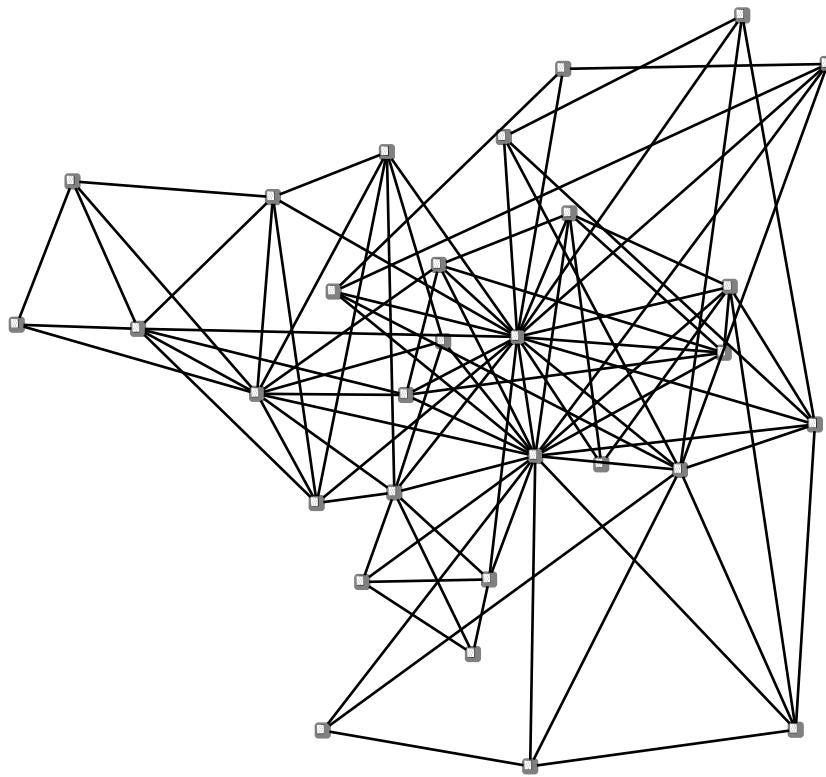
- Idea: Reject candidate positions that decrease the angular resolution
- Critical vertices \leftarrow those defining the angular resolution



GD17 Contest: Graph 5, Graph 9 Angular Resolution

Variant: Total Resolution

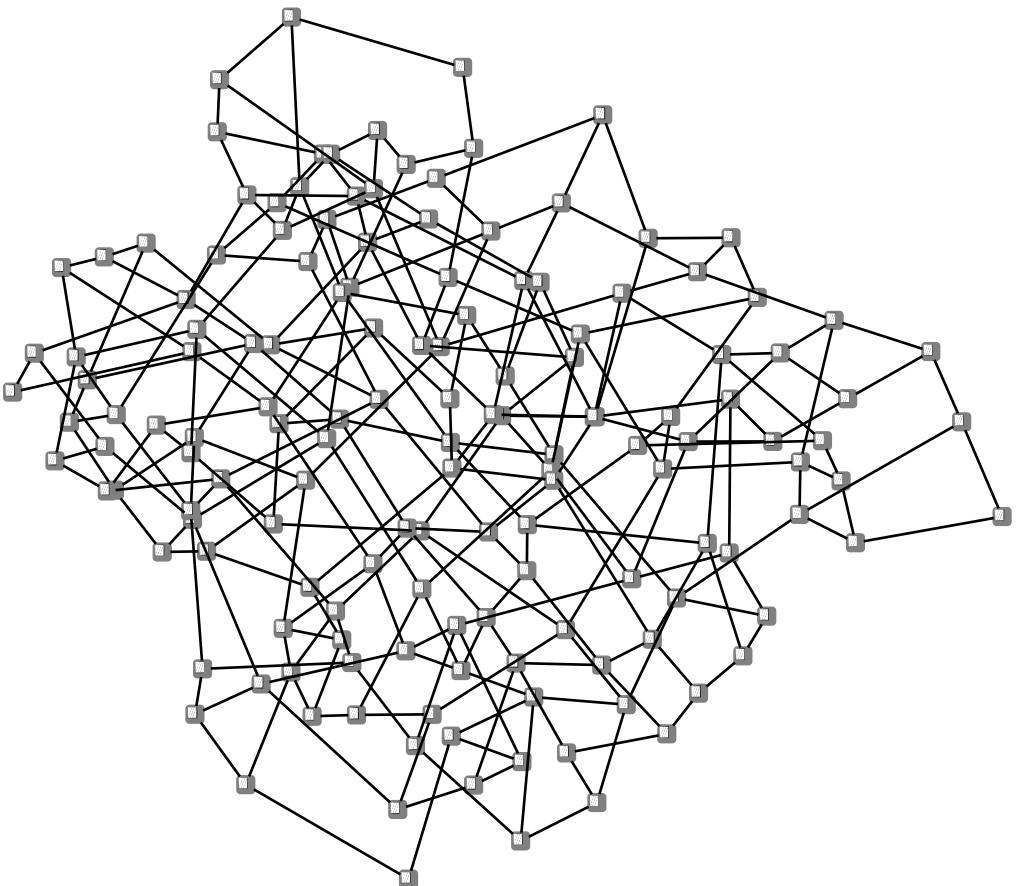
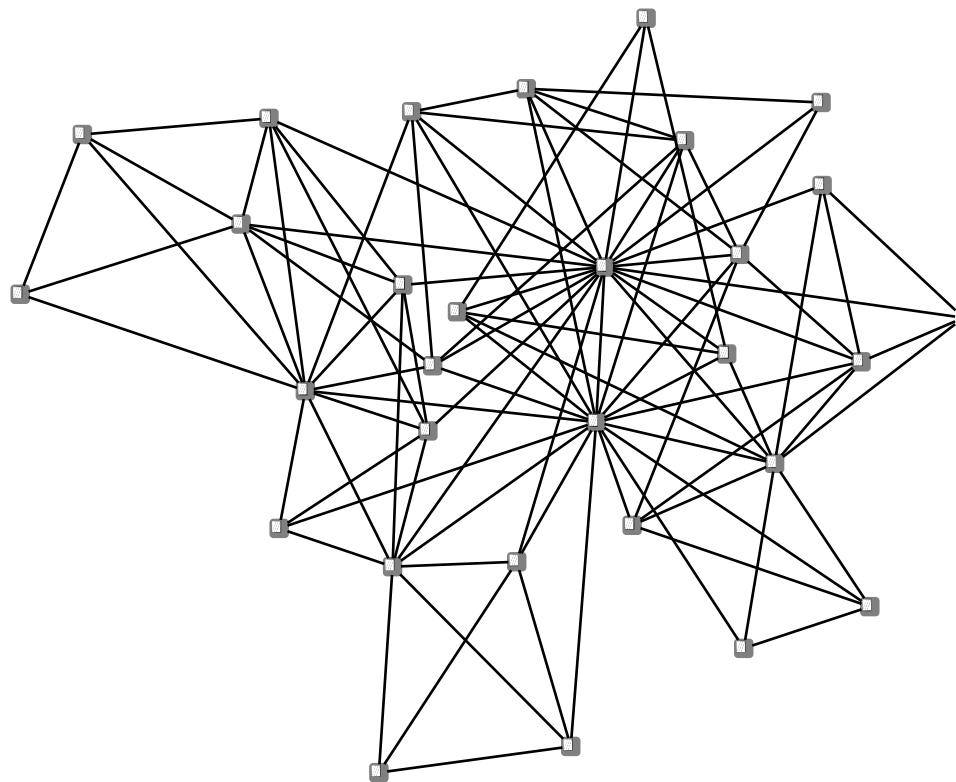
- Idea: Reject potential positions that decrease the minimum of the crossing or the angular resolution
- Critical vertices \leftarrow depend on active criteria



GD17 Contest: Graph 5, Graph 9 Total Resolution

Variant: Aspect Ratio

- Idea: Reject potential positions that decrease the aspect ratio



GD17 Contest: Graph 5, Graph 9 Aspect Ratio with Total Resolution

Show Case

Experiments

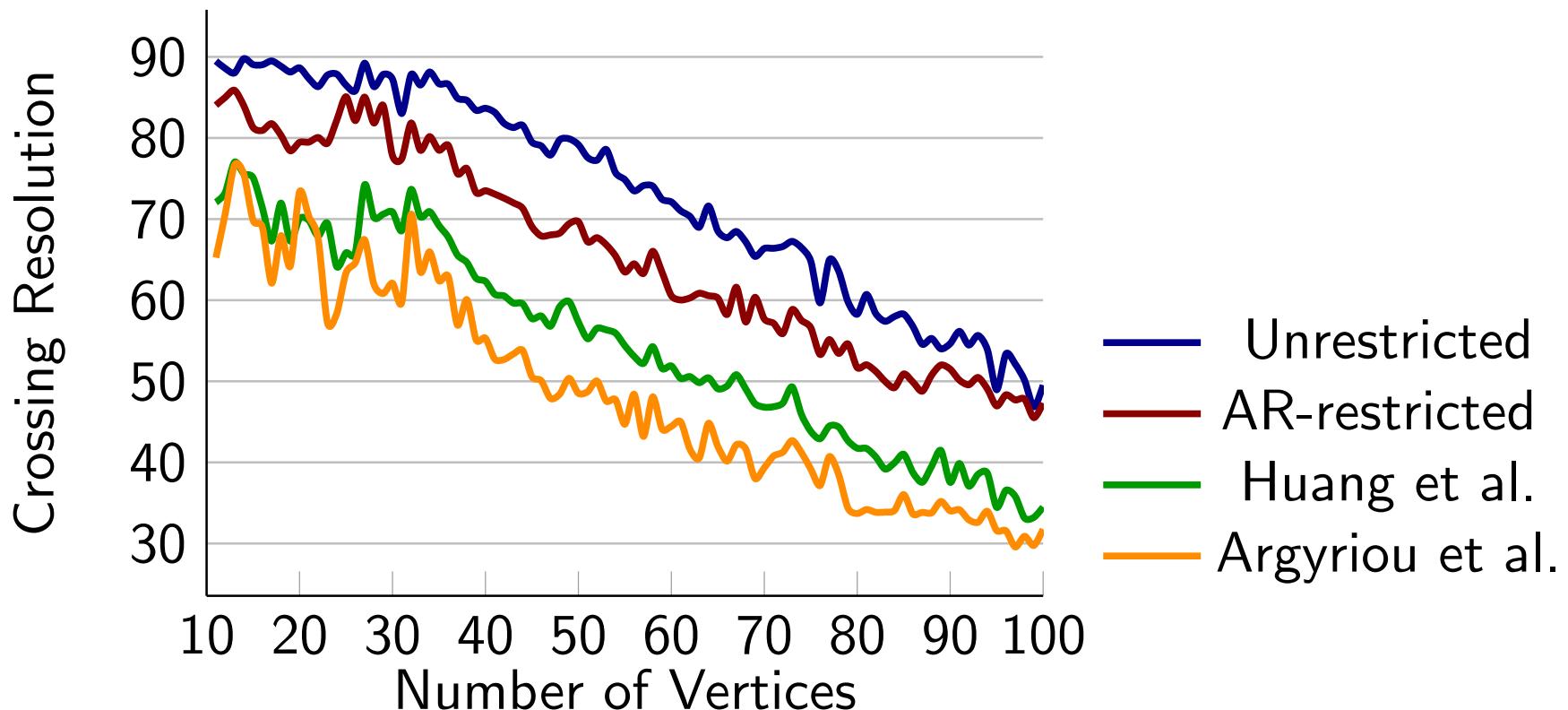
- Testsets
 - Non-planar Rome graphs: 8100 graphs
 - Non-planar AT&T graphs: 424 graphs
- Initial drawings \leftarrow SmartOrganic from yFiles
- Evaluated algorithms
 - Our algorithm (with and w.o. aspect ratio restrictions)
 - Force-directed algorithm by Argyriou et al.
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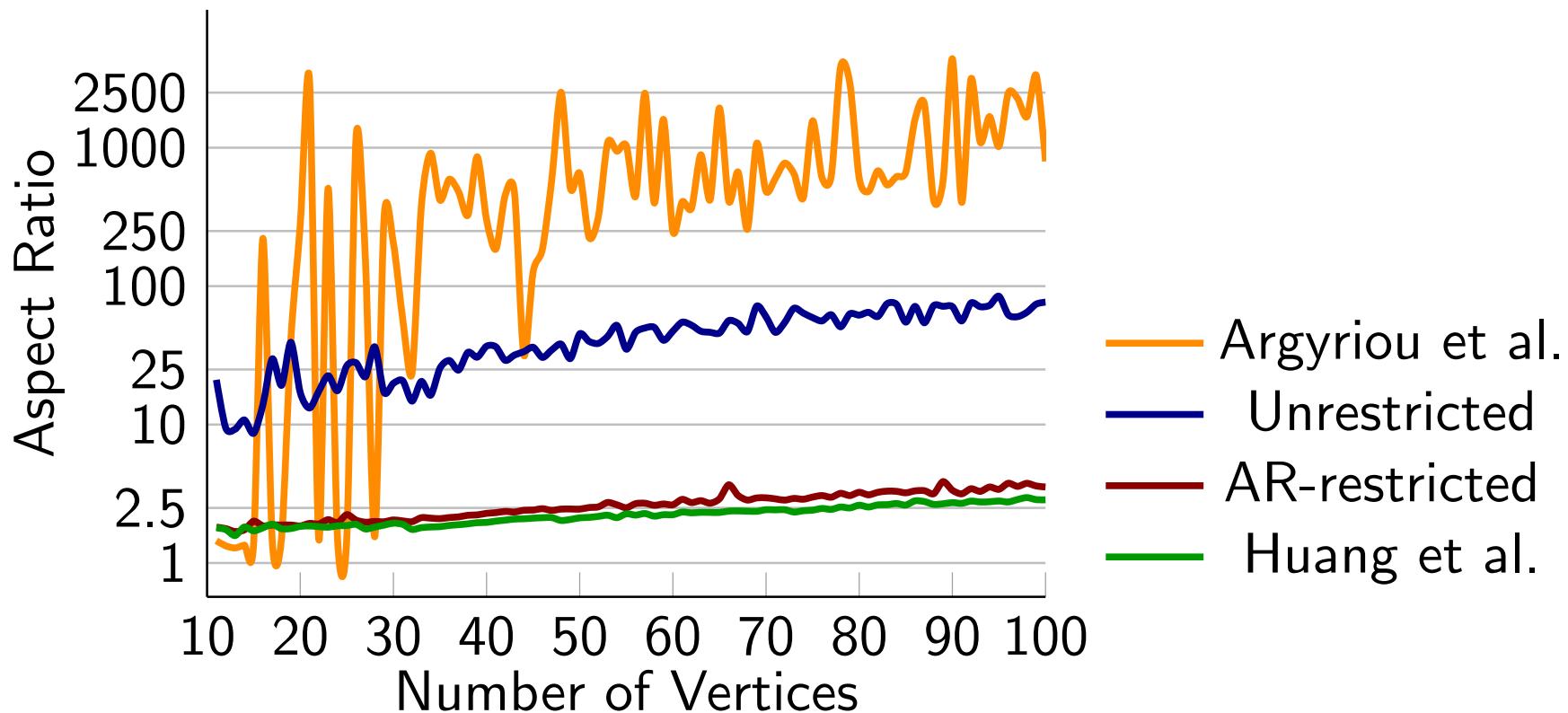
Crossing Resolution Results

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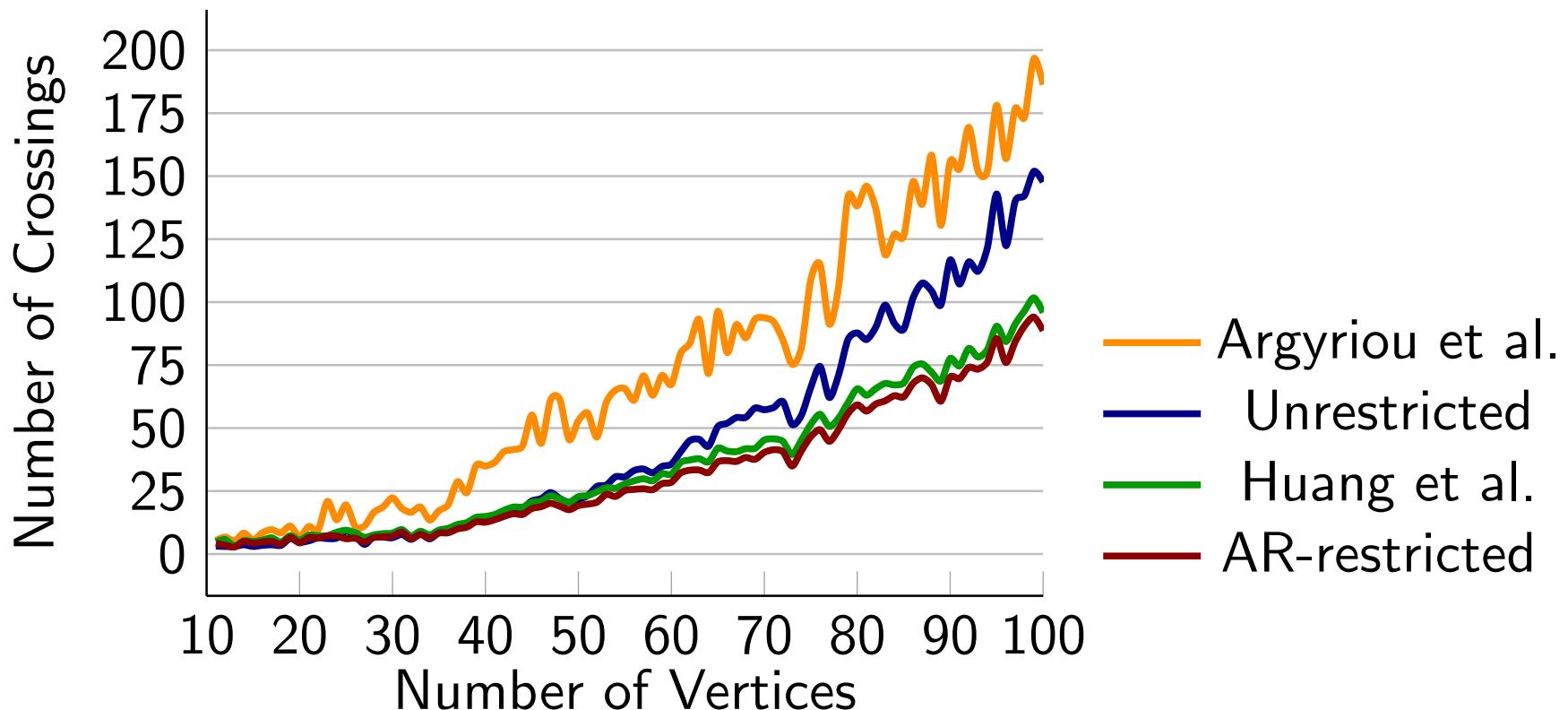
Aspect Ratio Results

- Observations:
 - Our algorithm w.o. aspect ratio restrictions, Argyriou et al.
→ bad aspect ratio
 - Our algorithm with aspect ratio restrictions, Huang et al.
→ similar aspect ratio



Number of Crossings Results

- Hierarchy (from best to worst):
 - Our algorithm with aspect ratio restrictions
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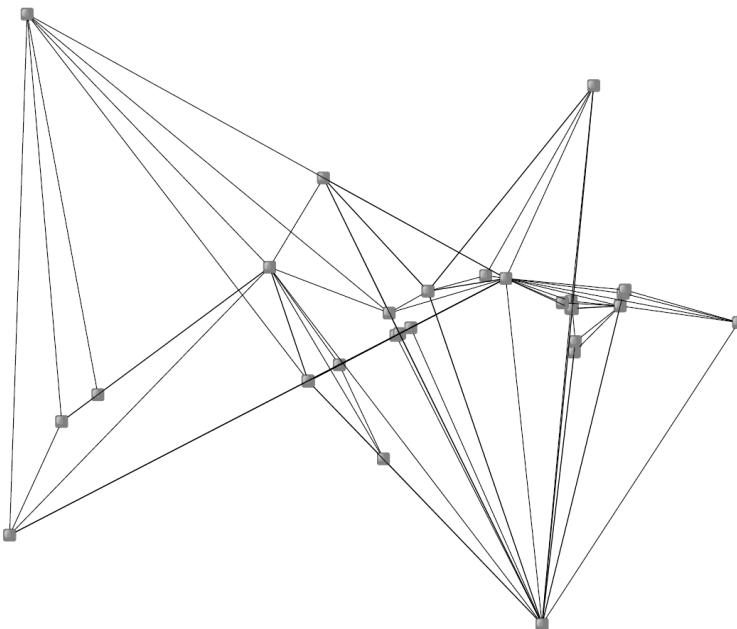


Concluding Remarks

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 - Drawings that are optimized only in terms of the crossing angles might be bad and may have several undesired properties
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 - Evaluate the technique with other objectives

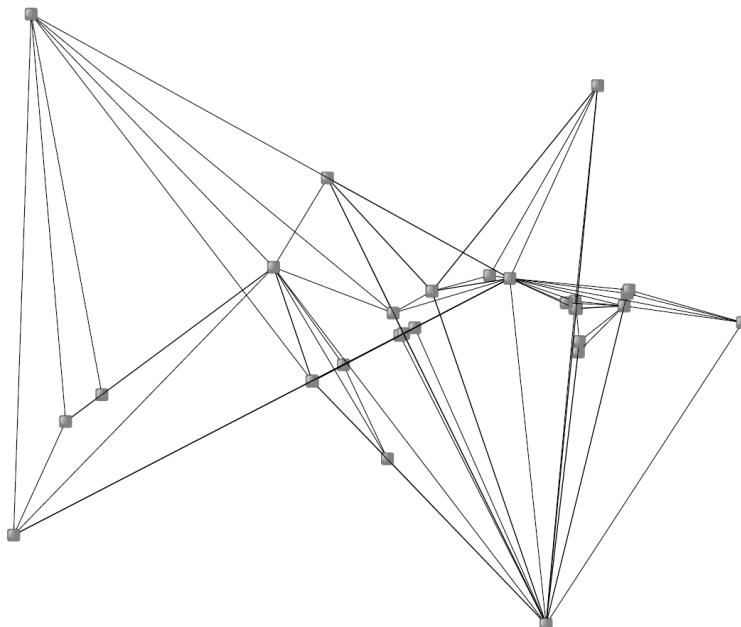
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