Define $o = o(n) = 3n+1$, $e = e(n) = \frac{n}{2}$

~~n~~ pos $n \in \mathbb{N}$ for example $oee = 3(\frac{1}{2}(\frac{n}{2}))+1$

$= \frac{3n}{4}+1$

If a pattern loops, then the pattern must ~~return~~ be able to return to its original #. $f(n) = n$

Ex. $eoe(n) = n$

$\frac{1}{2}(3(\frac{n}{2})+1) = n$

$\frac{3n}{4} + \frac{1}{2} = n$

$3n+2 = 4n$

$2 = n$

2 loops in eoe pattern

In $f(n)=n$, define m & q as m constant multiplying n^1 and q as the ~~...~~ constant multiplying ~~n~~ nothing

Ex. $eo(n) = n$

$\frac{1}{2}(3n+1) = n$

$3n+1 = 2n$

$1 = -1n$

$q=1$ $m=-1$

~~all polynomials functions~~ times this is a constant function so all functions can be generalized to $q = mn$; $\frac{q}{m} = n$

therefore $\frac{q}{m}$ = solution to the loop

~~because to for~~

Define C_E as # of e's and C_O as # of o's

for $O(n)=n$

the left side gets multiplied by 3

$ooo(n)=n$

$3(3(3(n)+1)+1)+1=n$

we only care about the constans multiplying n

$(3\cdot3\cdot3)=27$ on left

$27n+13=n$

we subtract to move it to right

$+13=-26n$

in $e(n)=n$

the left side gets multiplied by $\frac{1}{2}$ or

the right side gets multiplied by 2

$\frac{1}{2}n=n$

$n=2n$

$0=n$

The general form of ~~this~~ w/ (constant on right) would be

$2^{C_E}-3^{C_O}=m$

~~Because~~ Because $\frac{q}{m}$ must be an integer Solution
for the 3n+1 problem, $\bmod_{m} q = 0$ so $q = \bar{q} \cdot m$
$\bar{q}$ = coprime factor between q & m

$$\frac{q}{m} = \frac{\bar{q} \cdot m}{m} = \frac{\bar{q}}{1},$$

this is equilivent to $m = 1$, $\bar{q}$ = answer to loop

~~let~~ $m = 2^{CE} - 3^{Co}$ so $m = 1$ for all unique loop
(a non unique loop would be eoe eoe as it is just the loop of eoe twice)

$m = 2^{CE} - 3^{Co} = 1$ by Catalans conjecture
either $CE > 1$ or $Co > 1$ but not both

$2^{0} - 3^{0} = 0$

case CE & Co both ≤ 1

$2^0 - 3^0 = 0$

$2^1 - 3^0 = 1$

$2^0 - 3^1 = -2$

$2^1 - 3^1 = -1$

case $CE > 1$

$2^2 - 3^1 = 1$ (only one)

$2^2 - 3^0 = 3$ } all > 1

$2^3 - 3^1 = 5$

$2^3 - 3^0 = 7$

case $Co > 1$

$2^1 - 3^2 = -7$

$2^0 - 3^2 = -8$ all < 1

$2^1 - 3^3 = -25$

$2^0 - 3^3 = -26$

$2^0 - 3^0 \neq 0$

$2^0 - 3^1 = -2$

by Catalan's conjecture
either x or $y > 1$ but not both
$2^x - 3^y = 1$

$2^0 - 3^0 = 0$

$2^1 - 3^0 = 1$

$2^1 - 3^1 = -1$ negative, can't work

$2^0 - 3^1 = -2$

$2^2 - 3^1 = 1$

$2^1 - 3^2 = -7$ all < 1

$2^1 - 3^3 = -25$

$-\infty$

$2^3 - 3^1 = 7$ all > 1

$2^4 - 3^1 = 13$

$2^5 - 3^1 = 31$ ∞

therefore $(CE, CO) \in [(1,0), (2,1)]$

permutation $[e]$ permutations $[eeo, eoe, oee]$

$[0,0]$ $[(1,1), (1,2), (1,4)]$

$[0]$ $[1, 2, 4]$

$0 < 1$

the only two cases that work are $(C_E, C_O) \in [(1,0),(2,1)]$

permutations

$[e]$ $\quad$ $[eeo, eoe, oee]$

(m,q) $\quad$ $[(1,0)]$ $\quad$ $[(1,1),(1,2),(1,4)]$

$\left(\frac{q}{m}\right)$ $\quad$ $[0]$ $\quad$ $[1,2,4]$

$0<1$

e is not a solution

Solutions $\in [1,2,4]$

1,2,4 is the only loop

~~If n ∈~~

As 1,2,4 is the only loop

An integer has 3 ~~options~~ for end states

go to infinity, end a pattern, or go in a loop

If an integer, m, is the end of a pattern, then it would not be able to do $O(n)$ or $e(n)$, but that means the integer is neither even or odd so this can not happen.

Either a number goes to infinity or it end in a loop. The only loop is the 1,2,4 loop so a number must go to infinity or end in the 1,2,4 loop