Self-Adjusting Grid Networks to Minimize Expected Path Length

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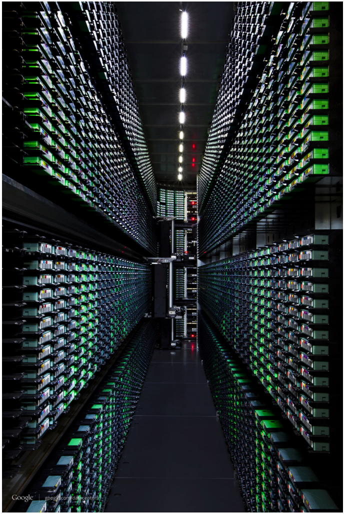
SIROCCO 2013

Motivation - Data Centers

- Energy cost ($50B in US alone 2008, doubles every 5 years!)
- Routing consumes about 20-30%



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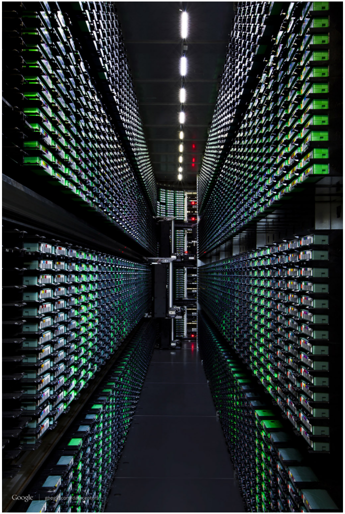

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- Routing consumes about 20-30%
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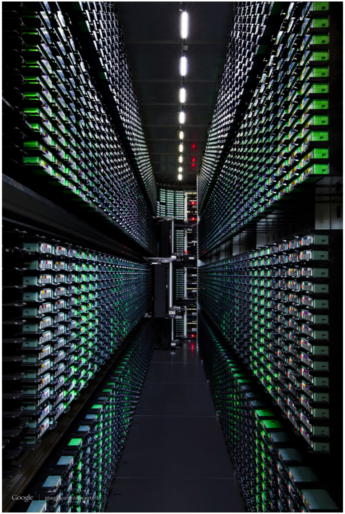
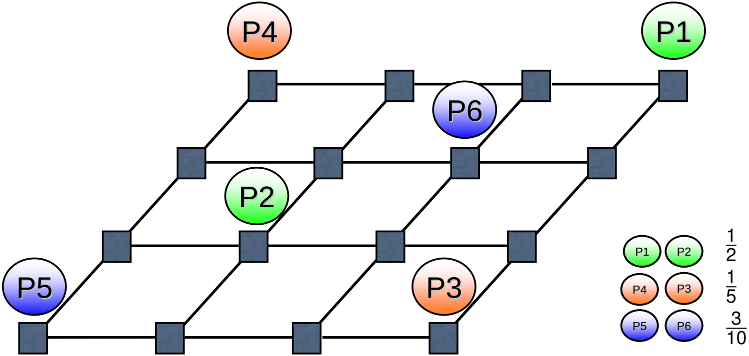
- Fixed infrastructure...
- Move processes (e.g., VM) between machines
- Virtualization and SDN (software defined networks), e.g., OpenFlow enable VM migration



Simple Example



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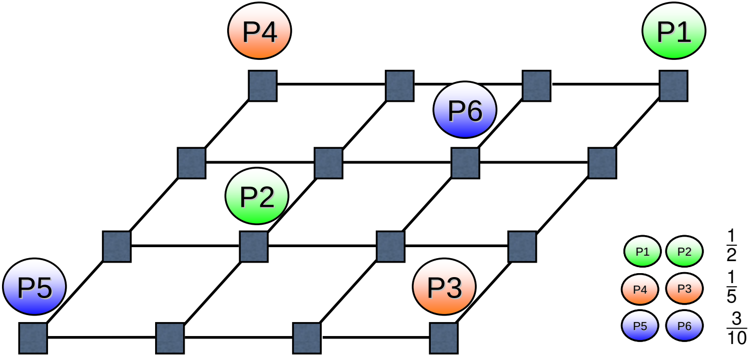

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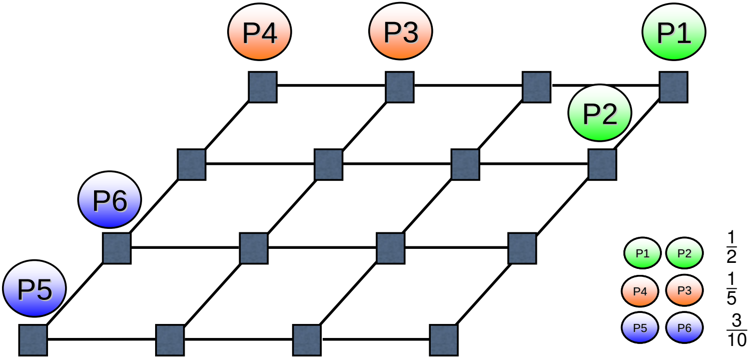

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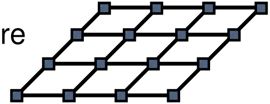
$$\mathbb{E}\left[\text{route length}\right] = \tfrac{1}{2}1 + \tfrac{1}{5}1 + \tfrac{3}{10}1 = 1$$

Model and Problem Definition

- ***Host*** Graph: $H(V, E)$: Physical Infrastructure



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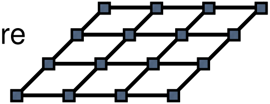

- Routing Requests: $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_m)$ $\quad \sigma_t = (u, v)$

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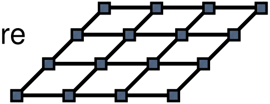

- Routing Requests: $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_m) \qquad \sigma_t = (u, v)$

- We assume the requests are i.i.d. from a given distribution

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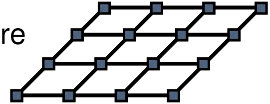
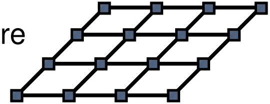


- Requests Distribution

	1	2	3	4	5
1		1/8		1/2	
2			0		
3					0
4		1/9			
5					

- ***Host*** Graph: $H(V, E)$: Physical Infrastructure



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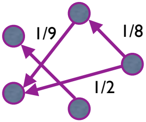
- ***Guest*** Weighted Graph: $G(P, W)$
- $|V| = |P| = n$



$$p(u, v)$$

Expected Path Length

- A placement (arrangement):

$$\varphi : P \to V$$

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- For H, G, φ:

$$\mathrm{EPL}(\varphi) = \sum_{u,v \in P} \Pr(u,v)\mathrm{d}_H(\varphi(u), \varphi(v))$$

Expected Path Length

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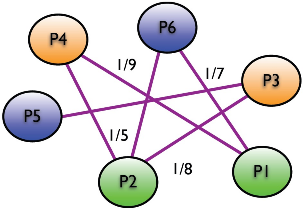
$$\mathrm{EPL}(\varphi) = \sum_{u,v \in P} \Pr(u,v)\, \mathrm{d}_H(\varphi(u), \varphi(v))$$

- **Minimum Expected Path Length Problem**

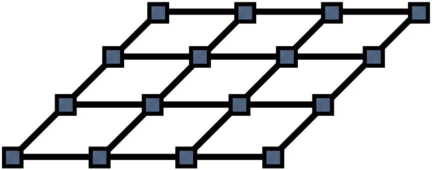
$$\mathrm{MEPL} = \min_{\varphi} \mathrm{EPL}(\varphi)$$

Example



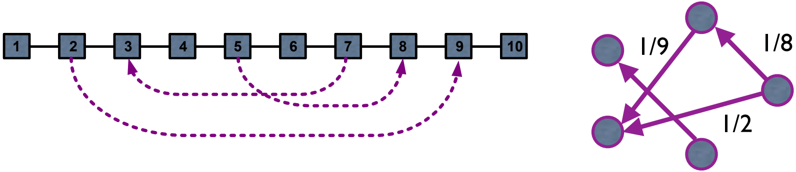
$G(P, W)$

$\varphi : P \to V$ **Find the best way to put processes on the graph to minimize expected path length**



$H(V, E)$

- VLSI layout
- Minimum Linear Arrangement (MLA)
- Known to be hard (NP-Complete)



$$MLA = \min_{\varphi} \sum_{u,v,\in P} w(u,v)|\varphi(u) - \varphi(v)|$$

Hardness of MEPL

- **Host Graph – Grid**
- **Guest Graph – Symmetric Product Distribution**
  - Activity level: $\mathrm{p}(u)$
  - Probability of request: $\mathrm{p}(u, v) = \mathrm{p}(u) \cdot \mathrm{p}(v)$

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Lemma

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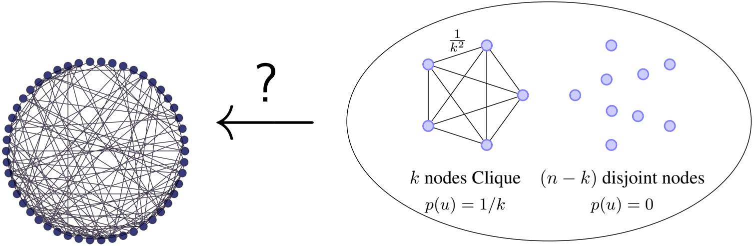

Host H
arbitrary graph

Guest G
symmetric product distribution

$$\mathrm{p}(u, v) = \mathrm{p}(u) \cdot \mathrm{p}(v)$$

Is there a CLIQUE of size k in H ?

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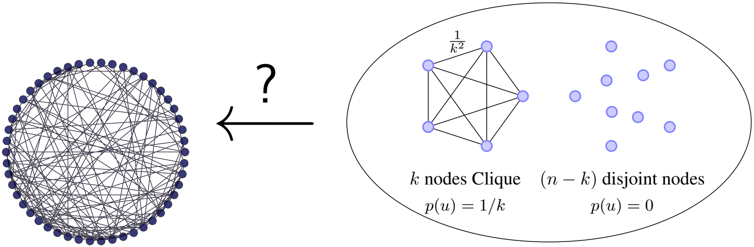


Host H
arbitrary graph

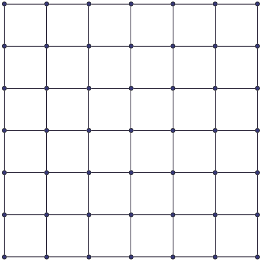
Guest G
symmetric product distribution
$\mathrm{p}(u, v) = \mathrm{p}(u) \cdot \mathrm{p}(v)$

H has a clique of size k if and only if $MEPL = \frac{k(k-1)}{k^2} = 1 - \frac{1}{k}$

Can we embed Tree into Grid?

Lemma

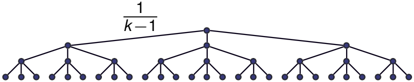
*If H is a **2-dimensional grid**, MEPL is still hard.*



Host H

grid (k^2 nodes)

$\xleftarrow{?}$



Guest G

tree (k nodes)

- Can we embed G to H?
- This is hard [Bhatt et al., 1987]

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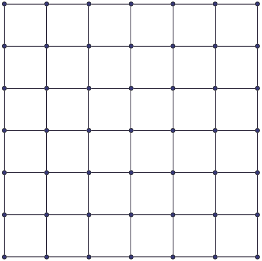

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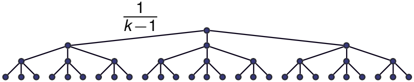

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tree (k nodes)

- Can we embed G to H?
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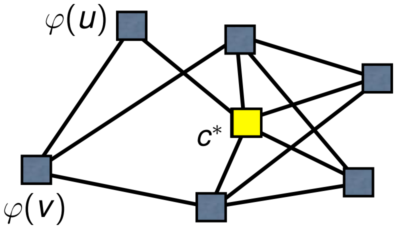
G **can be embedded into** ***H*** **if and only if** $MEPL = \frac{k-1}{k-1} = 1$

Main Result

Theorem

*For **d-dimensional grid** (H) and a **symmetric product distribution** (G) there is a **simple distributed algorithm** with a local switching policy between processes and their neighbors that achieves a **constant** approximation to* MEPL

- Expected center: $c^*(\varphi) = \arg\min_x \sum_u \mathrm{p}(u)\mathrm{d}(\varphi(u), \varphi(x))$



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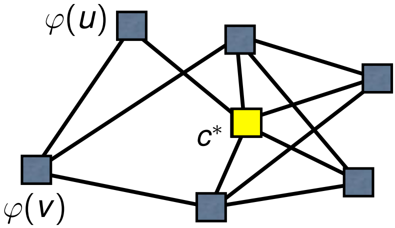

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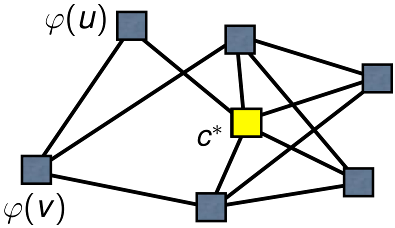
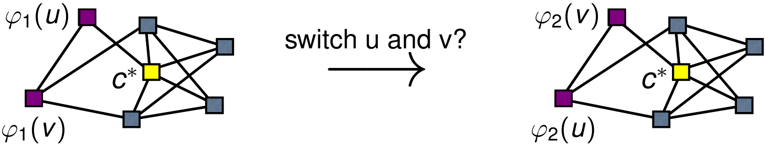
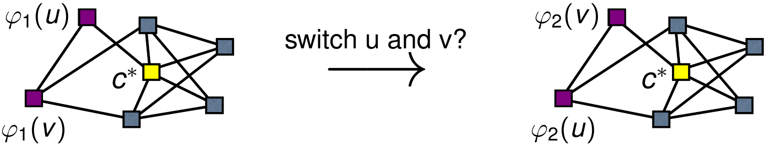
- Minimum expected distance: $C_{\min} = \min_\varphi C(\varphi)$



Switching Rule – Optimize Expected Distance to the Center



Switch only if: $C(\varphi_2) \leq C(\varphi_1)$



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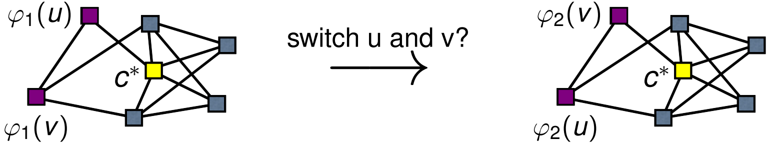

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  - centralized directory

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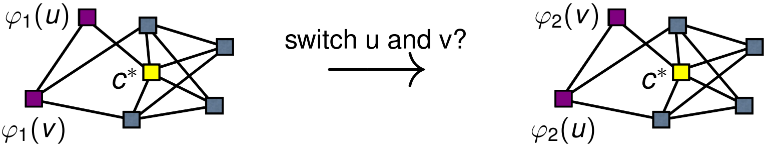

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- Every node knows activity level $\mathrm{p}(u)$ of all nodes
  - observing requests over time

- Greedy approach

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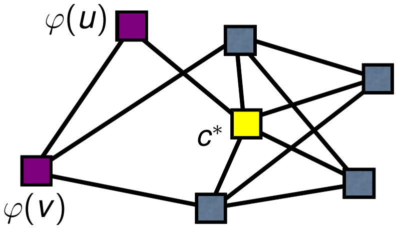
- How far local minimum $C(\widehat{\varphi})$ from global minimum C_{min}?
- What can we say about $\mathrm{EPL}(\widehat{\varphi})$?
- We show:

$$\frac{C(\widehat{\varphi})}{C_{\text{min}}} = O(1) \quad \text{and} \quad \frac{\mathrm{EPL}(\widehat{\varphi})}{\mathrm{MEPL}} = O(1)$$

MEPL and Minimum Expected Distance to Center

Lemma

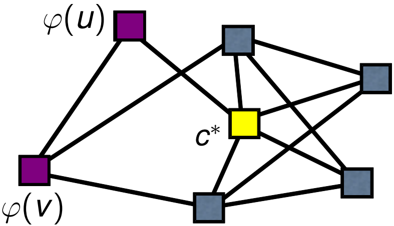
$$\forall \varphi : \quad C(\varphi) \leq \mathrm{EPL}(\varphi) \leq 2C(\varphi)$$



Lemma

$$\forall \varphi : \quad C(\varphi) \leq \mathrm{EPL}(\varphi) \leq 2C(\varphi)$$

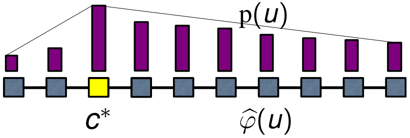
$$\overbrace{\mathrm{d}(\varphi(u), \varphi(v)) \leq \mathrm{d}(\varphi(u), c^*) + \mathrm{d}(c^*, \varphi(v))}$$



Expected Rank

- Rank of a node $\mathrm{r}(u)$ is the position of the node in the ordered list of nodes' activity levels.
- Node with the highest activity level has rank 0.
- $\mathbb{E}[R] = \sum_{u} \mathrm{p}(u)\mathrm{r}(u)$

For any local optimum $\widehat{\varphi}$: $C(\widehat{\varphi}) \leq \mathbb{E}[R]$



$$\mathrm{d}(\widehat{\varphi}(u), c^*) \leq \mathrm{r}(u)$$

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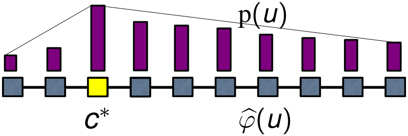
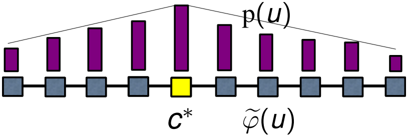


$$\mathrm{d}(\widehat{\varphi}(u), c^*) \leq \mathrm{r}(u)$$

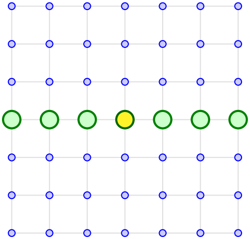
For the global optimum $\widetilde{\varphi}$: $C_{\min} \geq \frac{1}{2}\mathbb{E}[R]$



$$\mathrm{d}(\widetilde{\varphi}(u), c^*) \geq \mathrm{r}(u)/2$$

2-Dimensional Grid

$$C(\widehat{\varphi}) \approx \sqrt{n}$$



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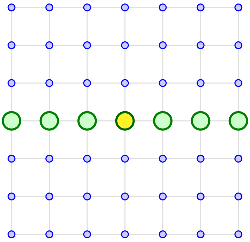
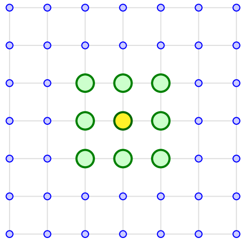


$C_{\min} \approx \sqrt[4]{n}$



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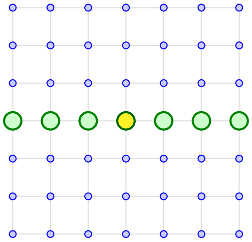

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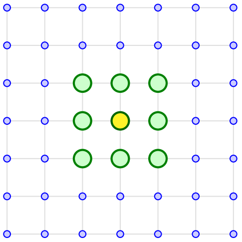

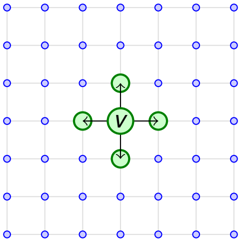

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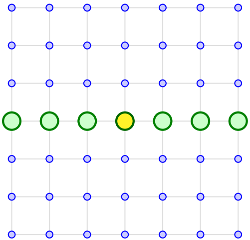

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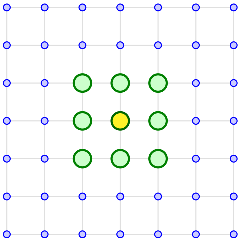
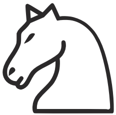
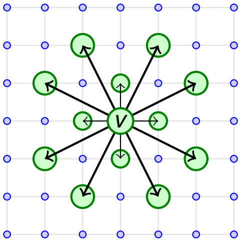
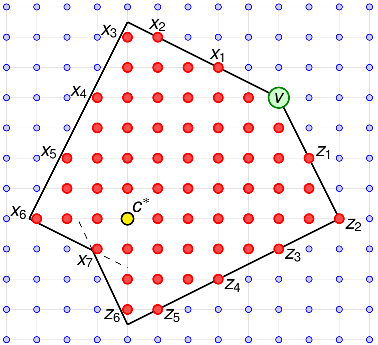


Allow *chess knight* moves





For any local optimum $\widehat{\varphi}$

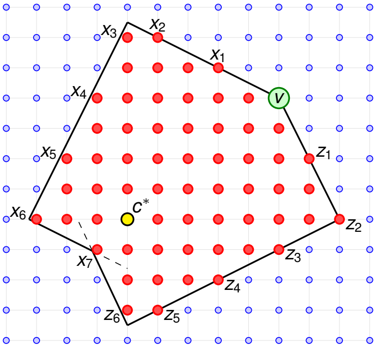


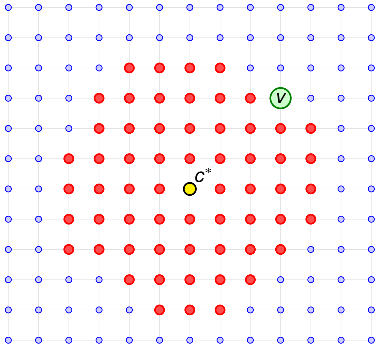
$$C(\widehat{\varphi}) \leq \frac{4}{\sqrt{6}}\mathbb{E}[\sqrt{R}]$$

$$\mathrm{d}(\widehat{\varphi}(v), c^*) \leq \frac{4}{\sqrt{6}}\sqrt{\mathrm{r}(v)}$$

For any local optimum $\widehat{\varphi}$

For the global optimum $\widetilde{\varphi}$





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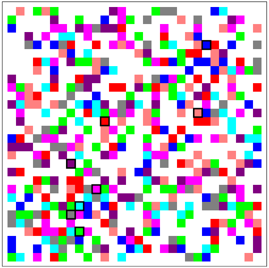
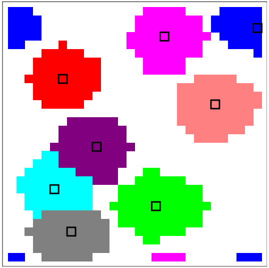
$$C_{\min} \geq \frac{1}{\sqrt{2}}\mathbb{E}[\sqrt{R}]$$

$$\mathrm{d}(\widehat{\varphi}(v), c^*) \leq \frac{4}{\sqrt{6}}\sqrt{\mathrm{r}(v)}$$

$$\mathrm{d}(\widetilde{\varphi}(v), c^*) \geq \sqrt{\frac{\mathrm{r}(v)}{2}}$$

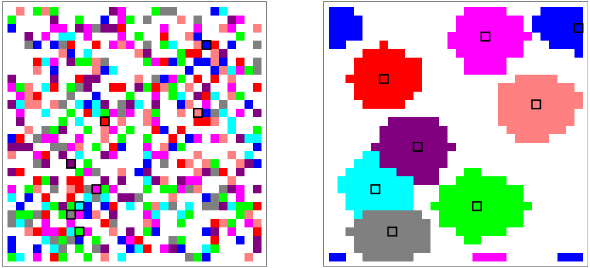
Simulations – Clustered Requests





900 nodes
50% inactive
8 clusters

Simulations – Clustered Requests



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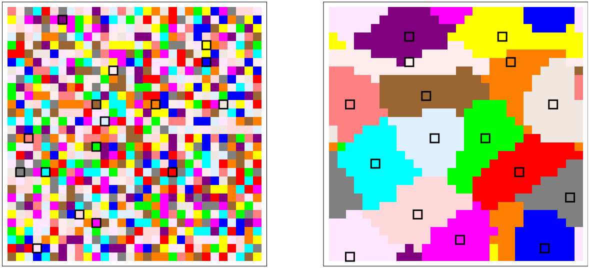
8 clusters



900 nodes
16 clusters

Animation

- MEPL is hard for general graphs and requests patterns
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Summary

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THANK YOU!