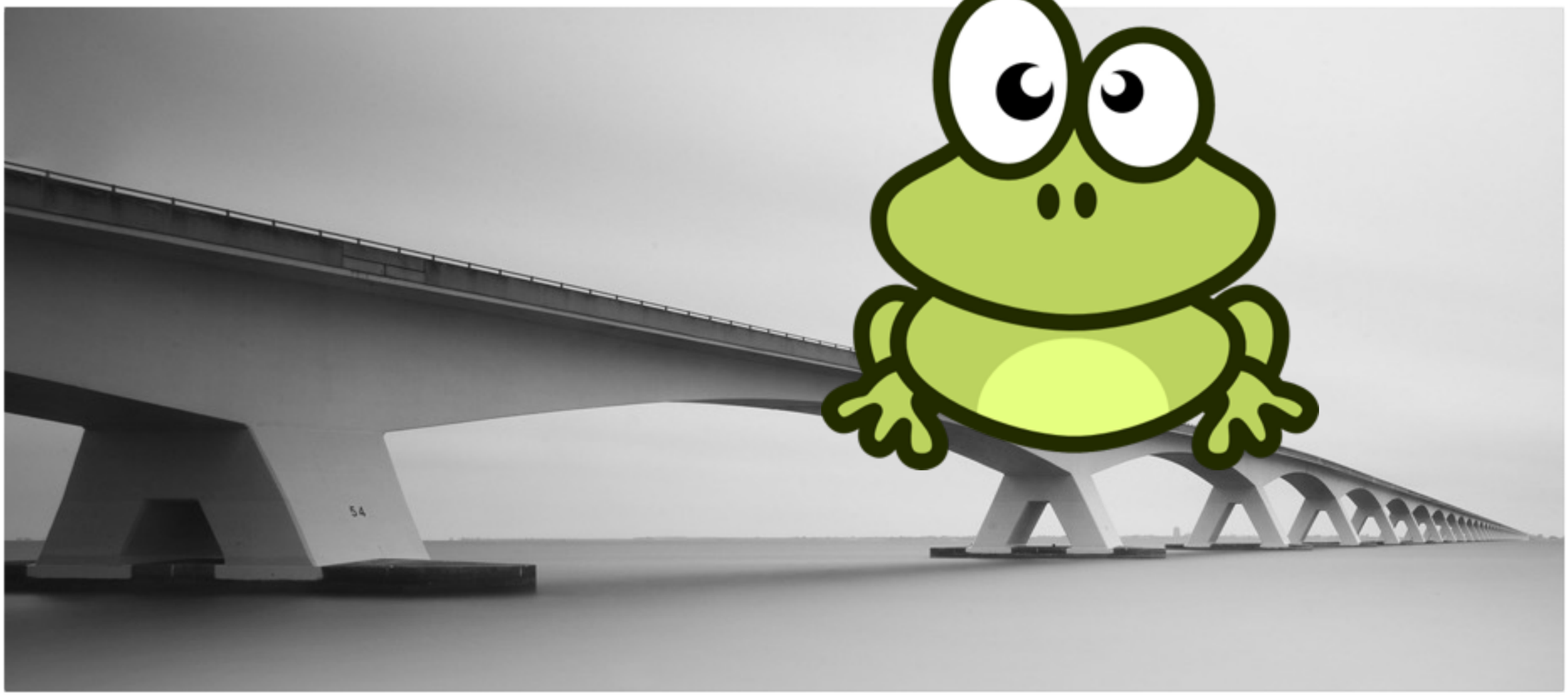


FrogWild!

Fast PageRank Approximations on Graph Engines

Ioannis Mitliagkas
Michael Borokhovich
Alex Dimakis
Constantine Caramanis

VLDB 2015

Contribution

- Fast algorithm for pagerank approximation
  - want quickly identify **top-k pagerank** vertices
- Implemented in GraphLab: 7-10x faster than normal pagerank function
- Provable approximation guarantees for graphs with no bad structure

PageRank [Page et al., 1999]

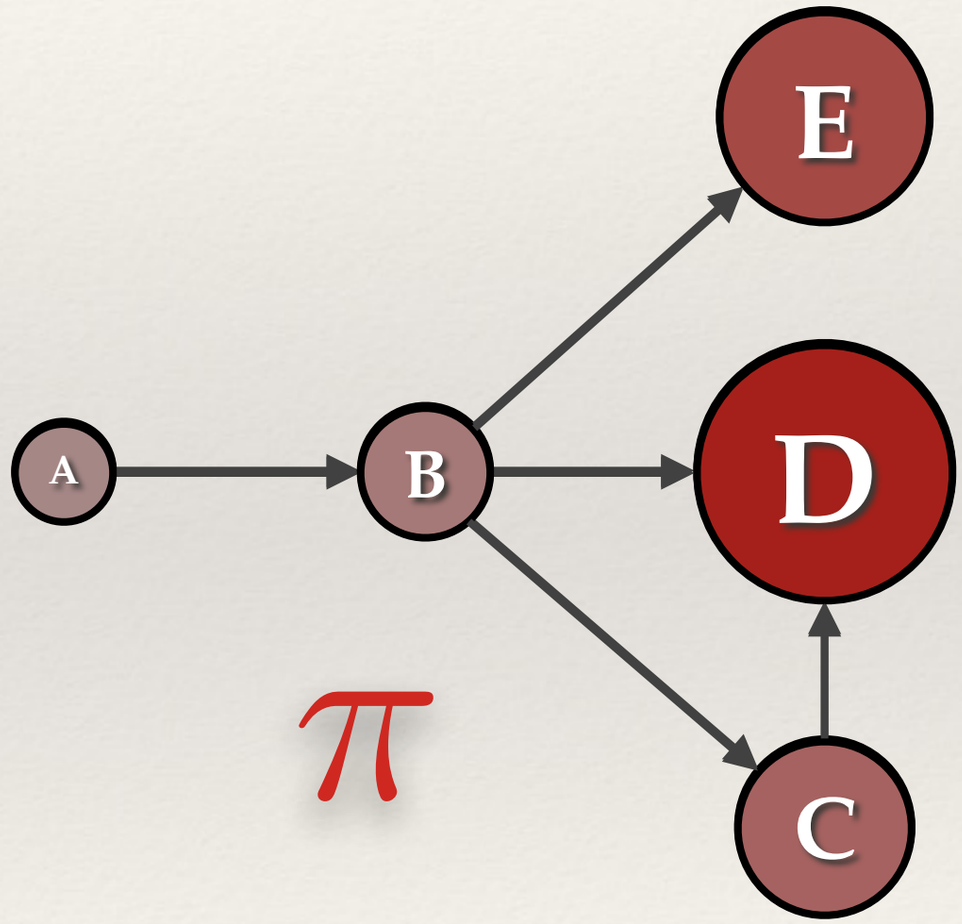
Page Importance

Described by distribution π

Recursive Definition

Important pages are pointed to by

- important pages are pointed to by
  - important pages are pointed to by…



Robust

to manipulation by spammer networks

PageRank Applications

Telco

Find the most influential users in a “calls” graph

Social networks

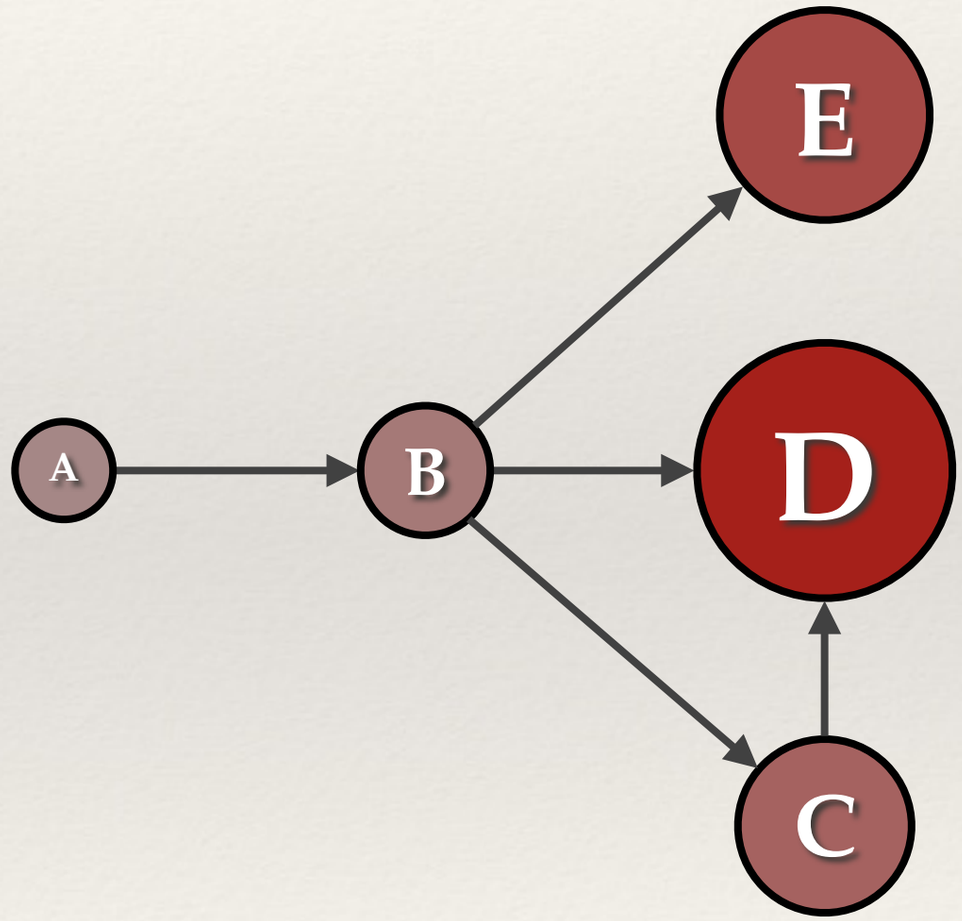
Text analysis

Find keyWords

Recommender systems

booking.com

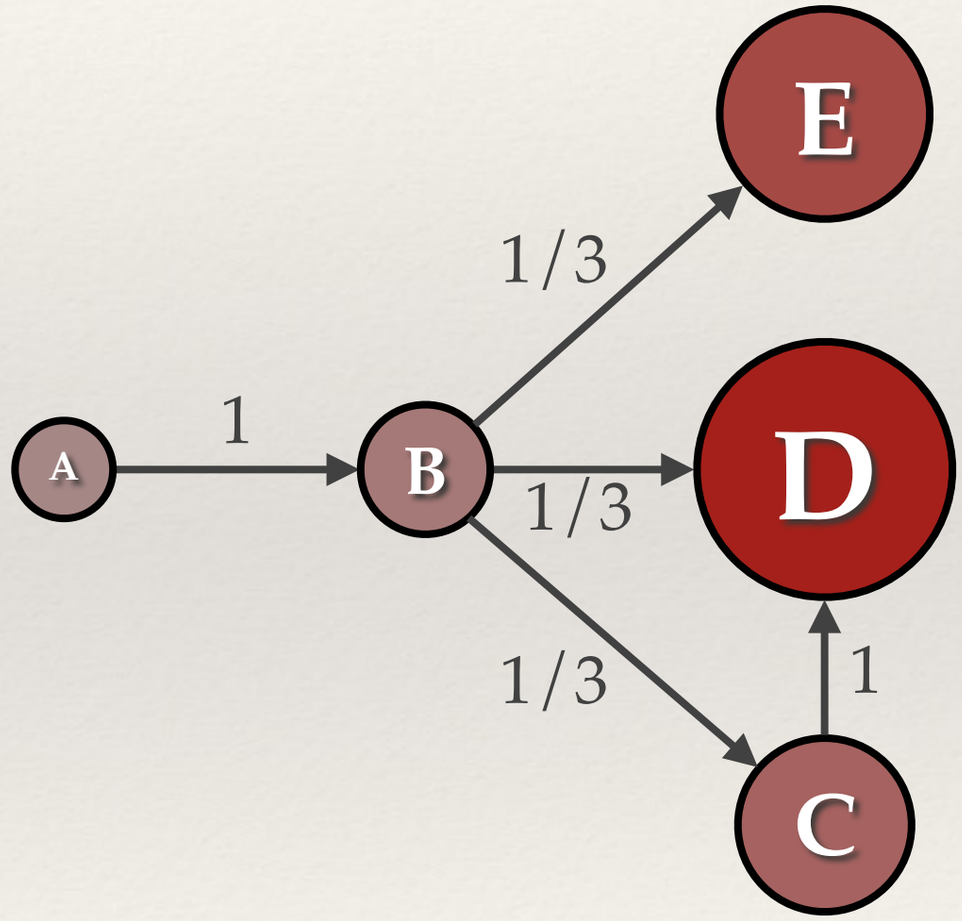
PageRank



PageRank

Normalized Adjacency Matrix

$$P_{ij} = \frac{1}{\mathrm{d}_{\mathrm{out}}(j)}, \quad (j,i) \in G$$



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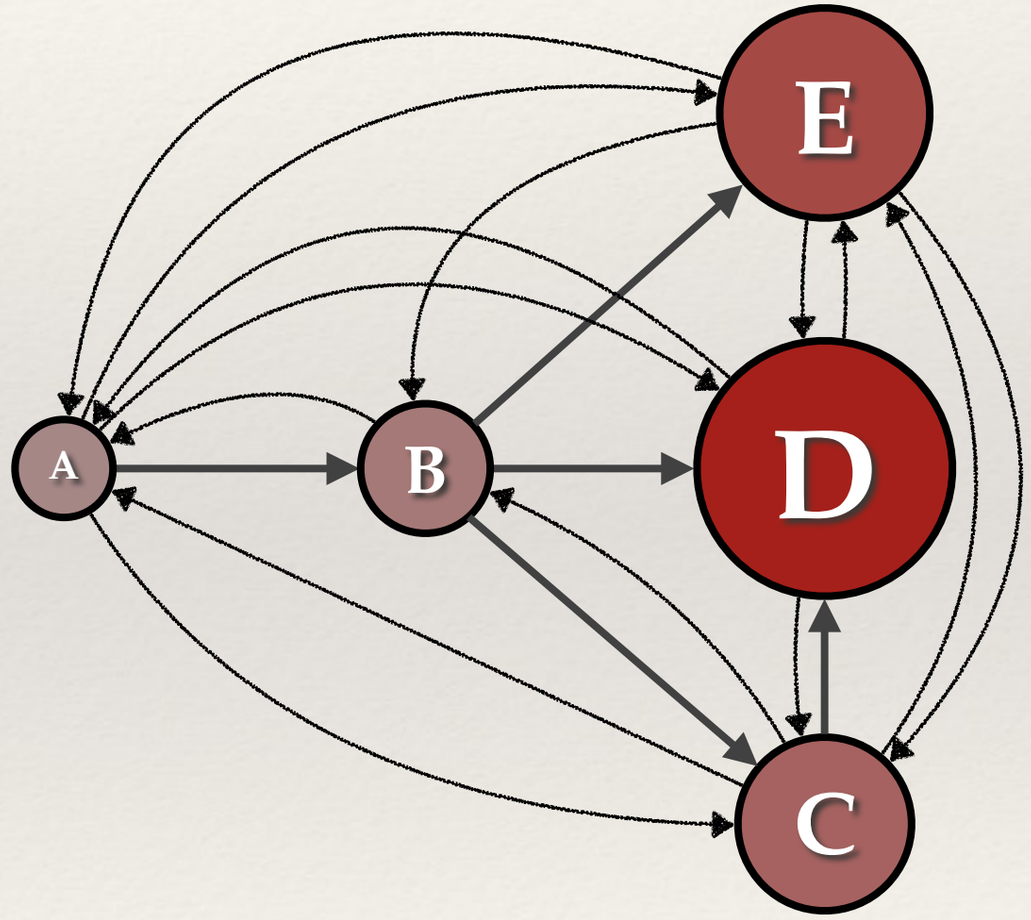
Augmented Matrix

$$Q_{ij} = (1 - p_T)P_{ij} + \frac{p_T}{n}$$



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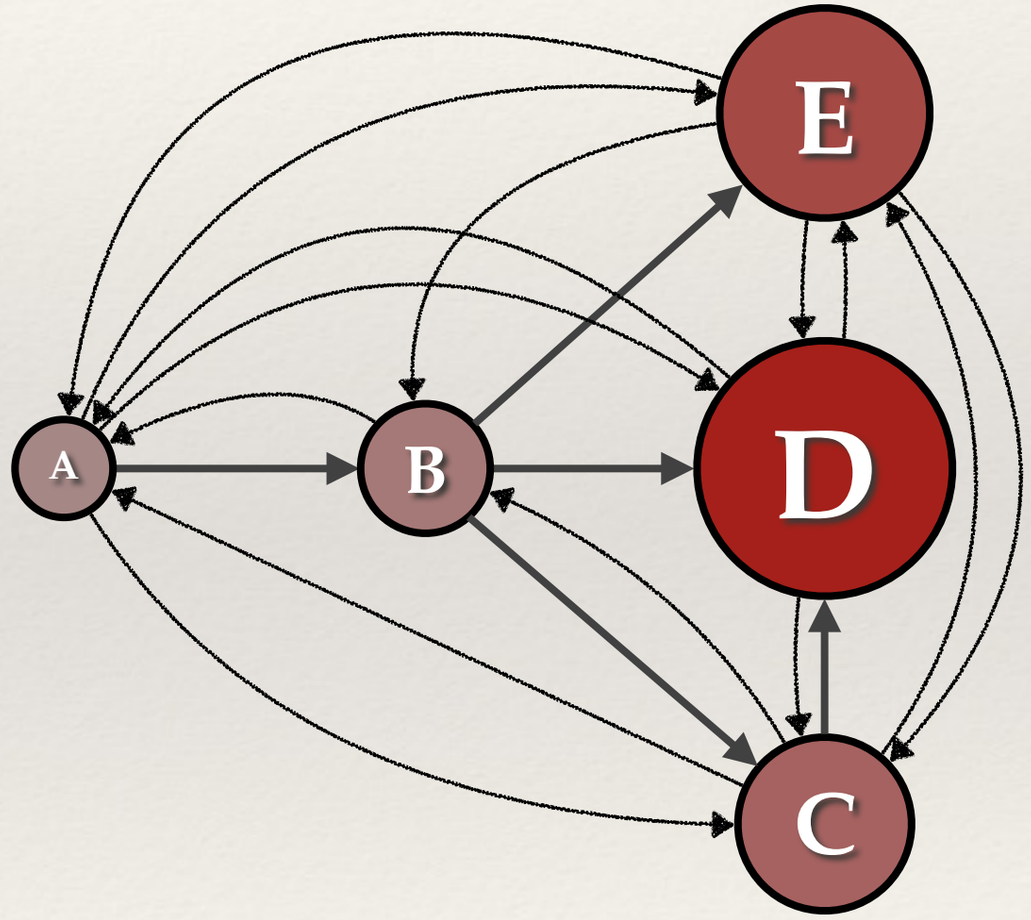
PageRank Vector $\pi = Q\pi$



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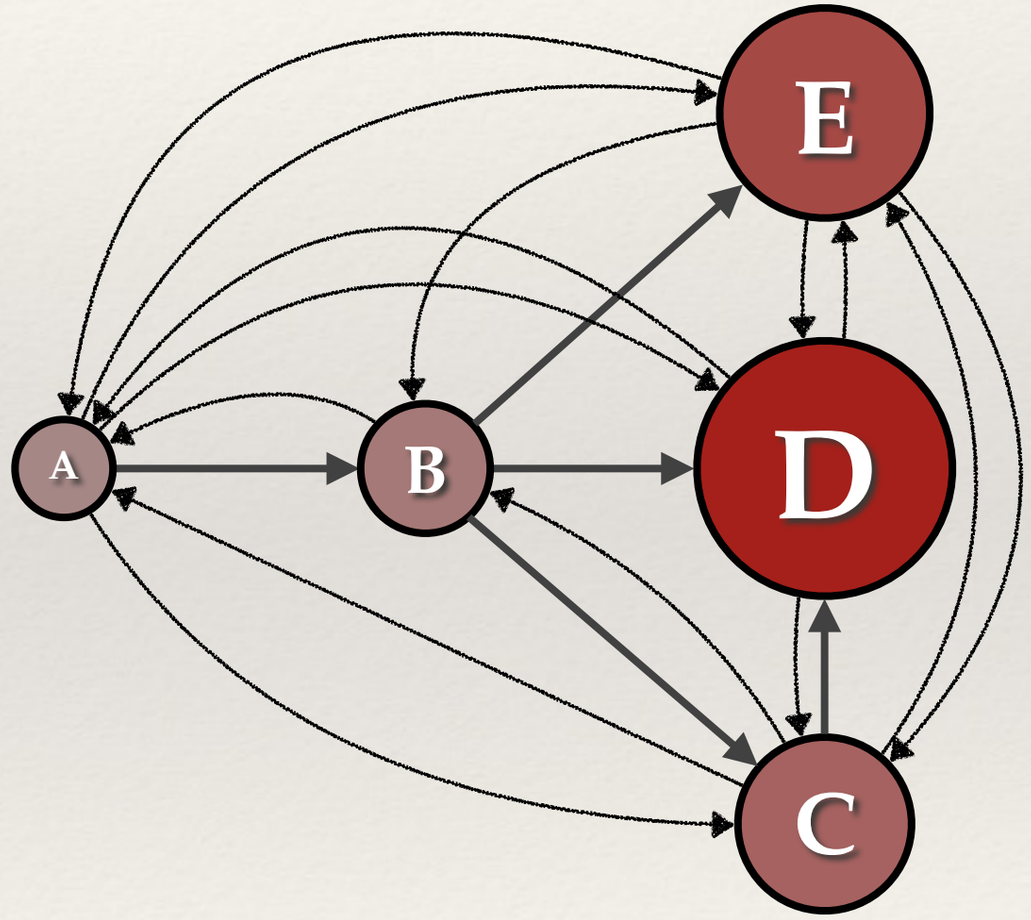
Power Method $Q^t p^0 \to \pi$



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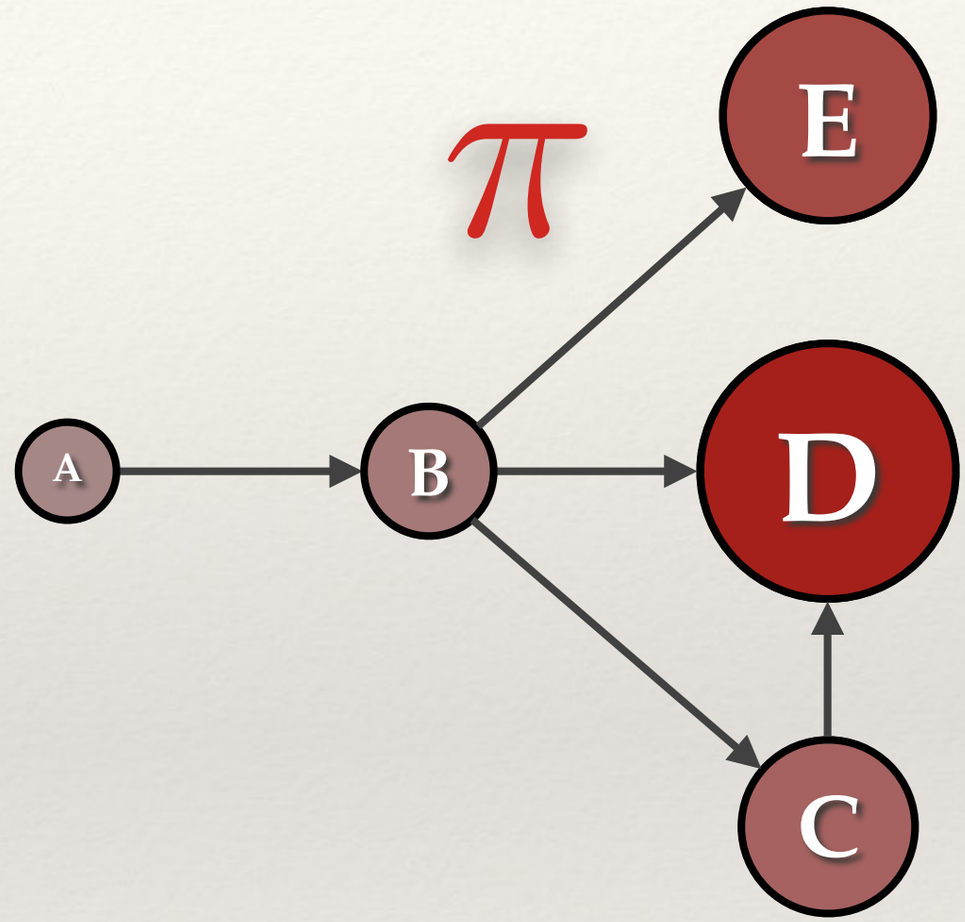
- Show 10-20 hotels with highest pagerank to user

- There are many applications where **graphs are formed at query-time** and high pagerank vertices must be discovered quickly

Top-k PageRank

Looking for k "heavy nodes"

Do not need full PageRank vector



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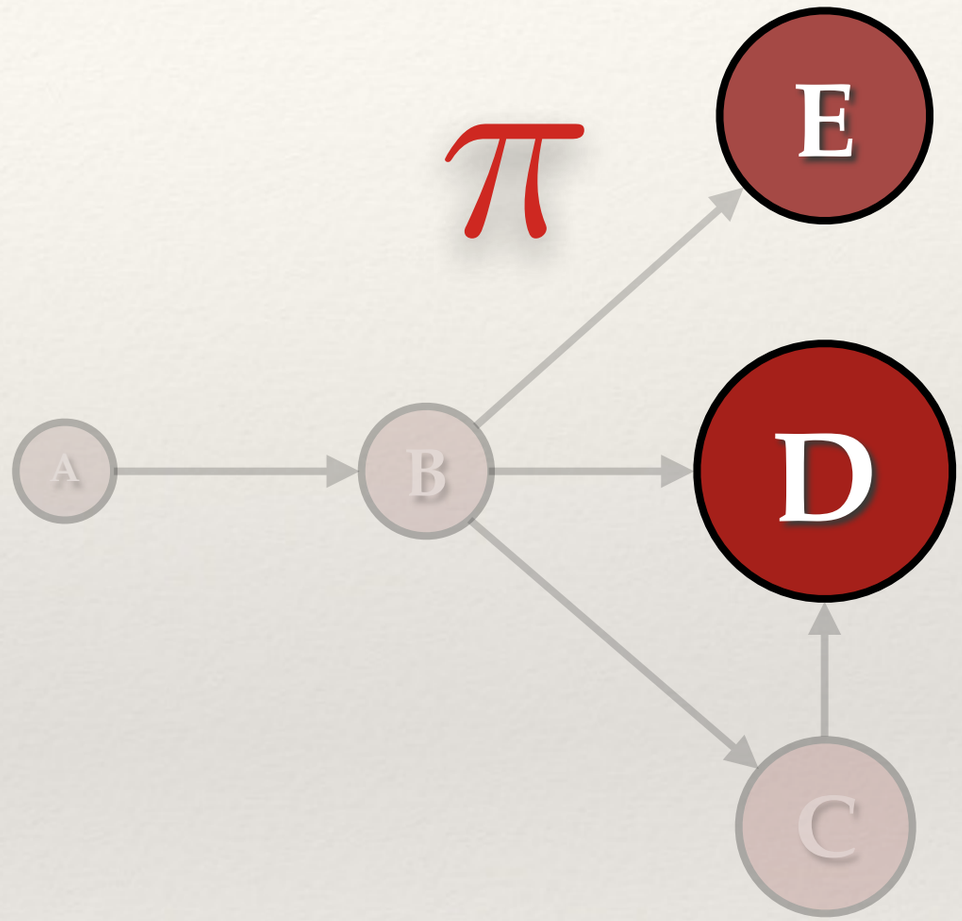

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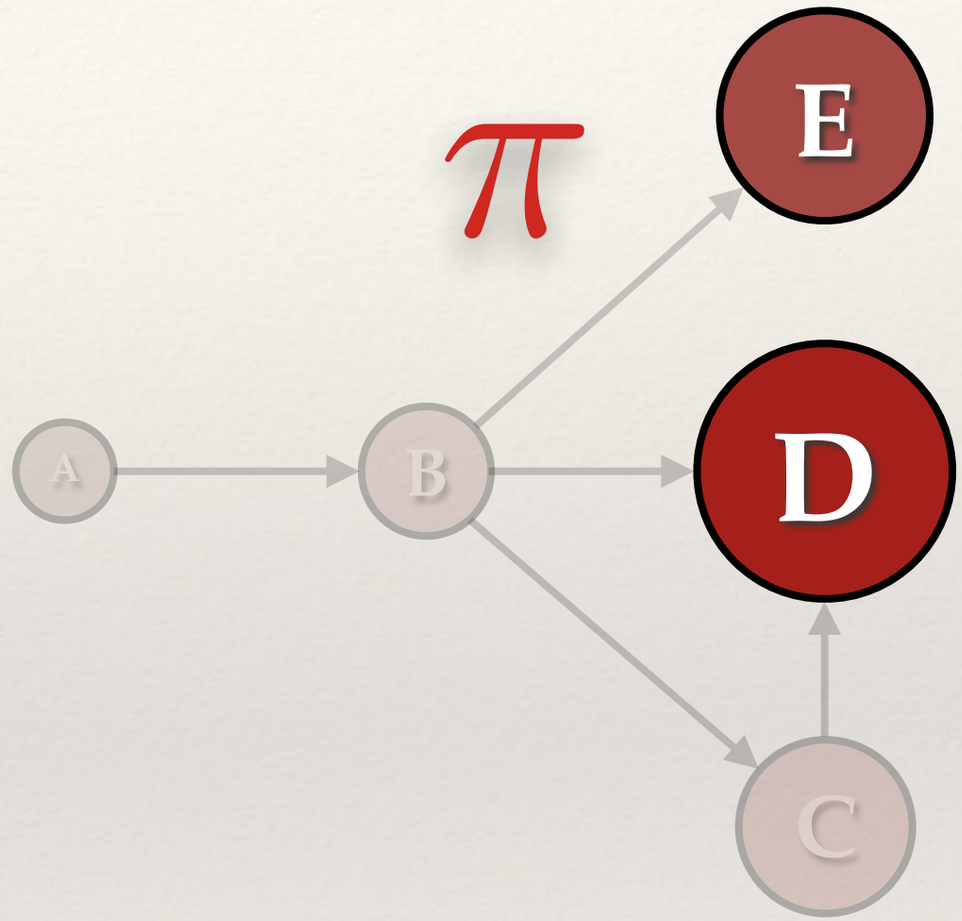
Random Walk Sampling



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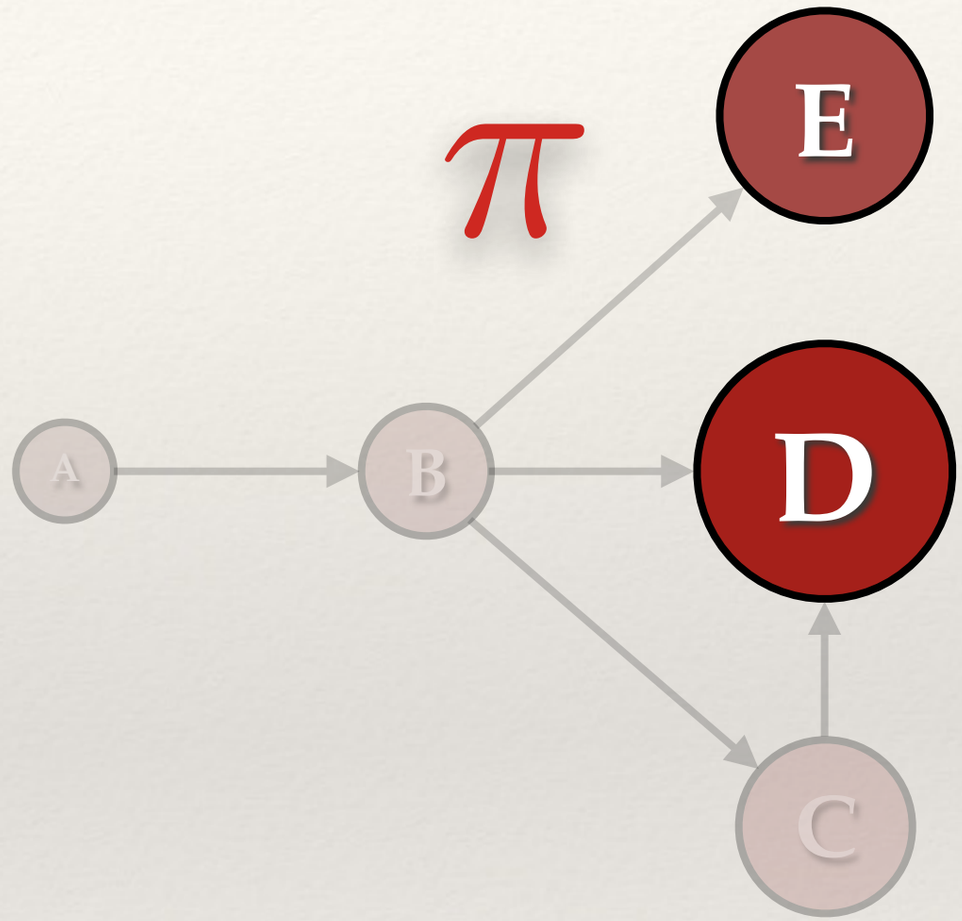


Random Walk Sampling

Captured Mass Metric

Our algorithm ranks k nodes — S_k

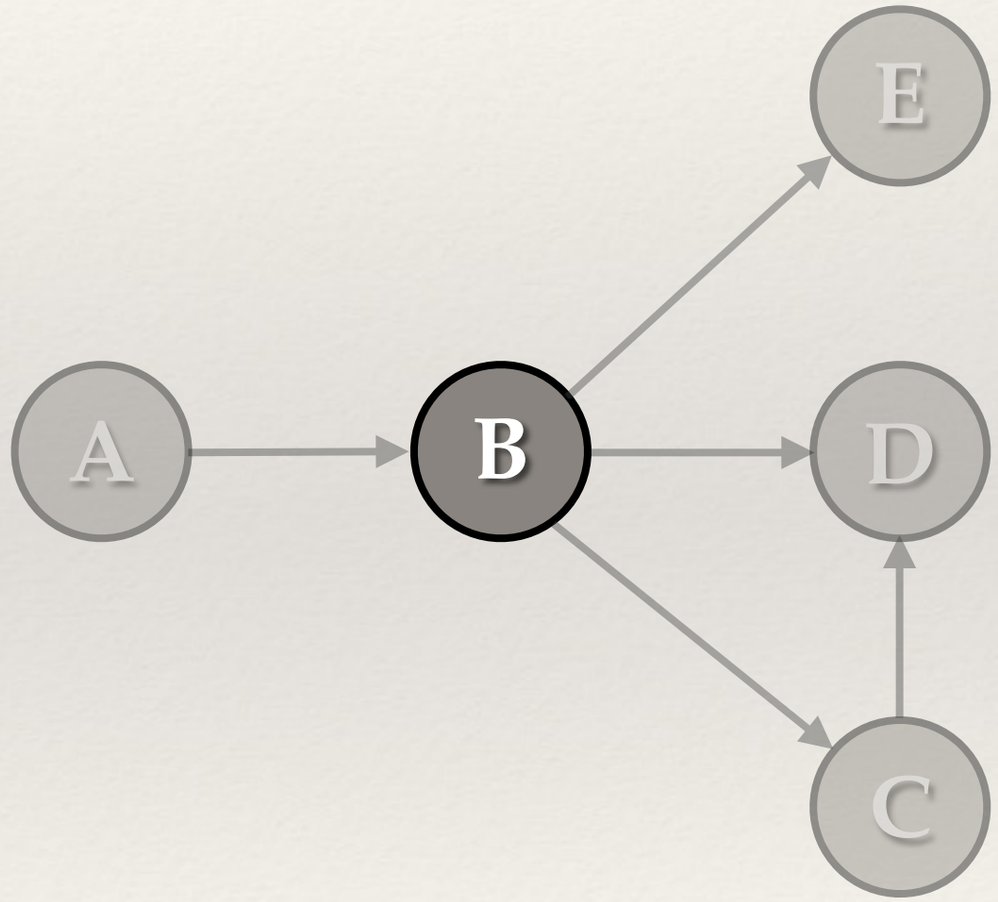
We compare sum of their ranks — $\pi(S_k)$

with the true Top-k PageRank sum

Platform - Graph Engine

- Engine splits graph across cluster
- **Vertex program** describes logic

GAS abstraction



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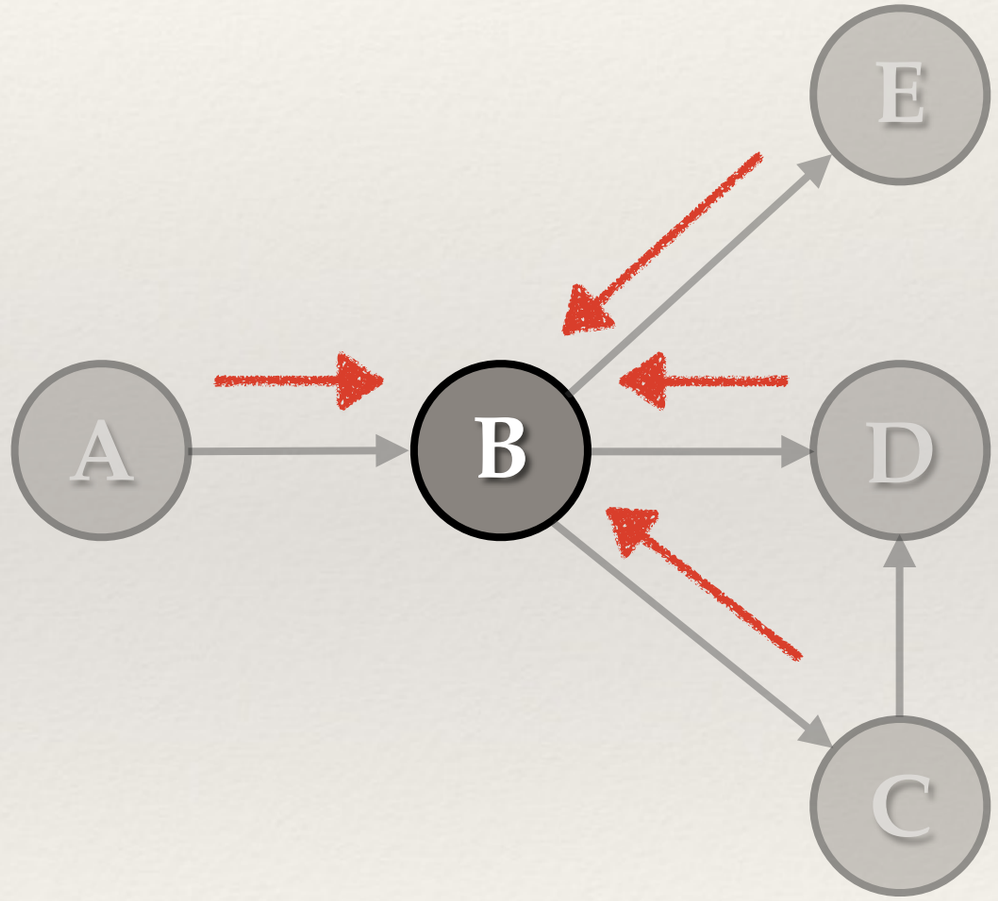
1. Gather



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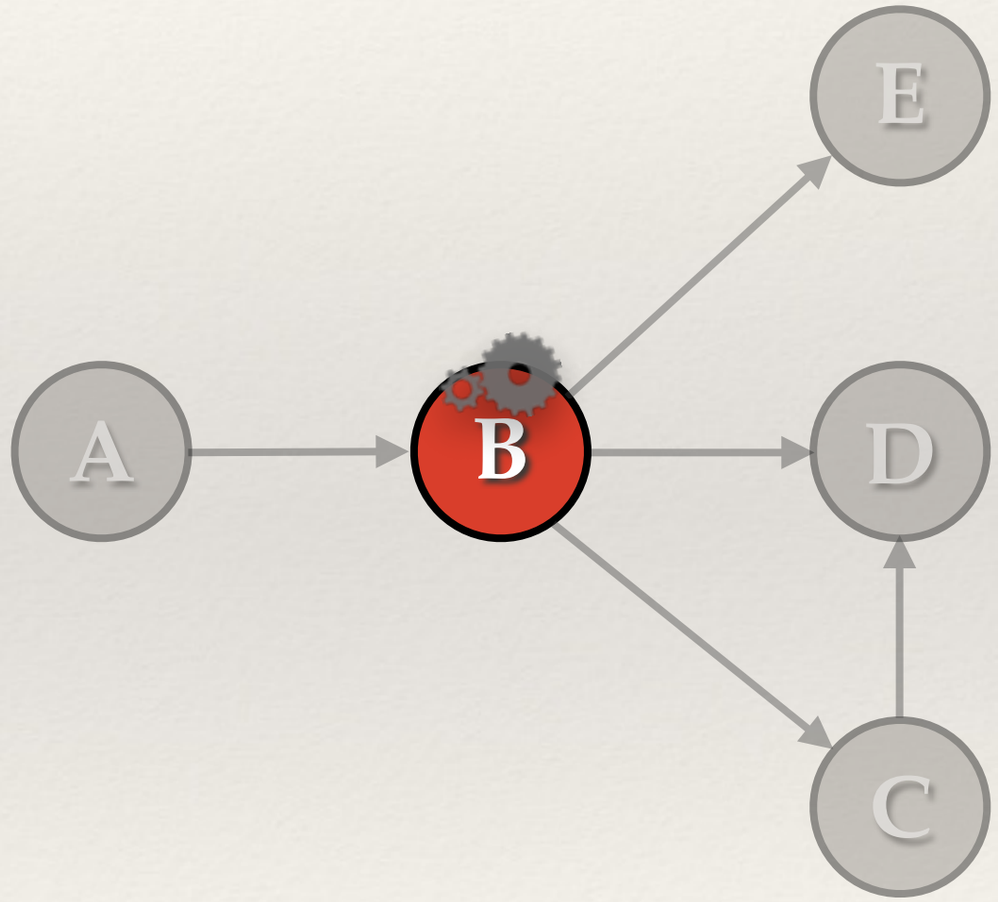
GAS abstraction

1. Gather
2. Apply



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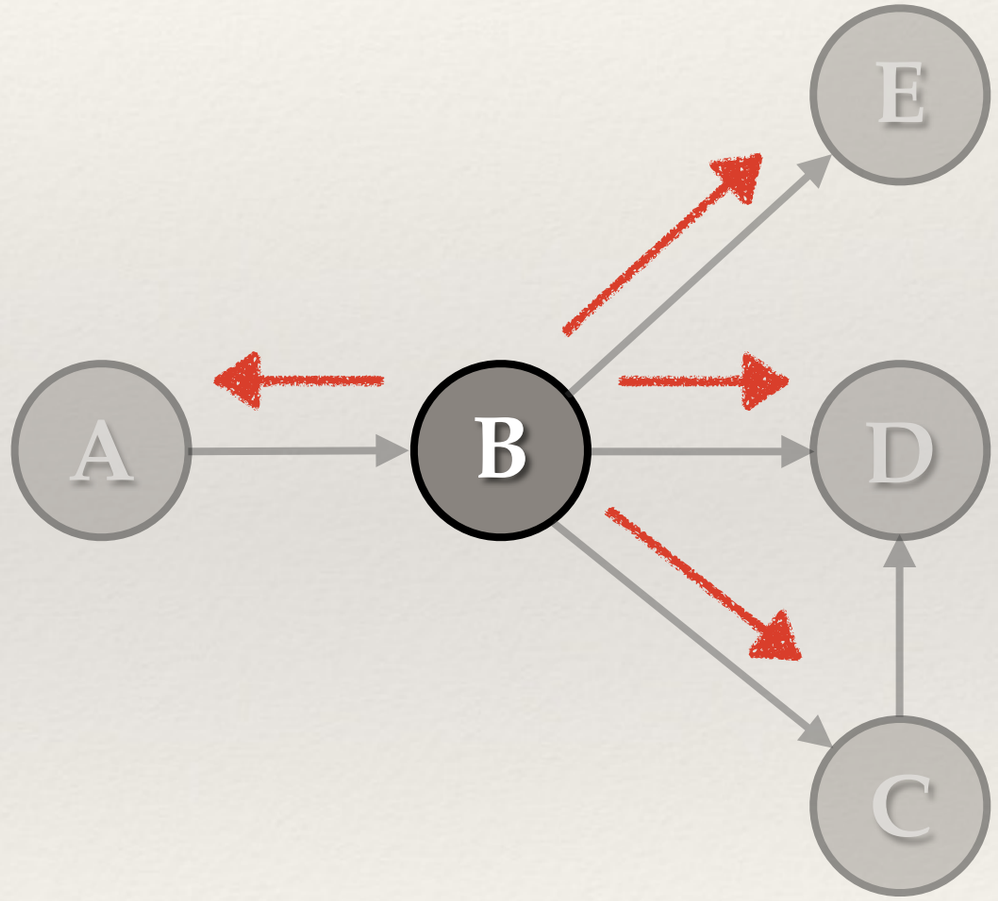
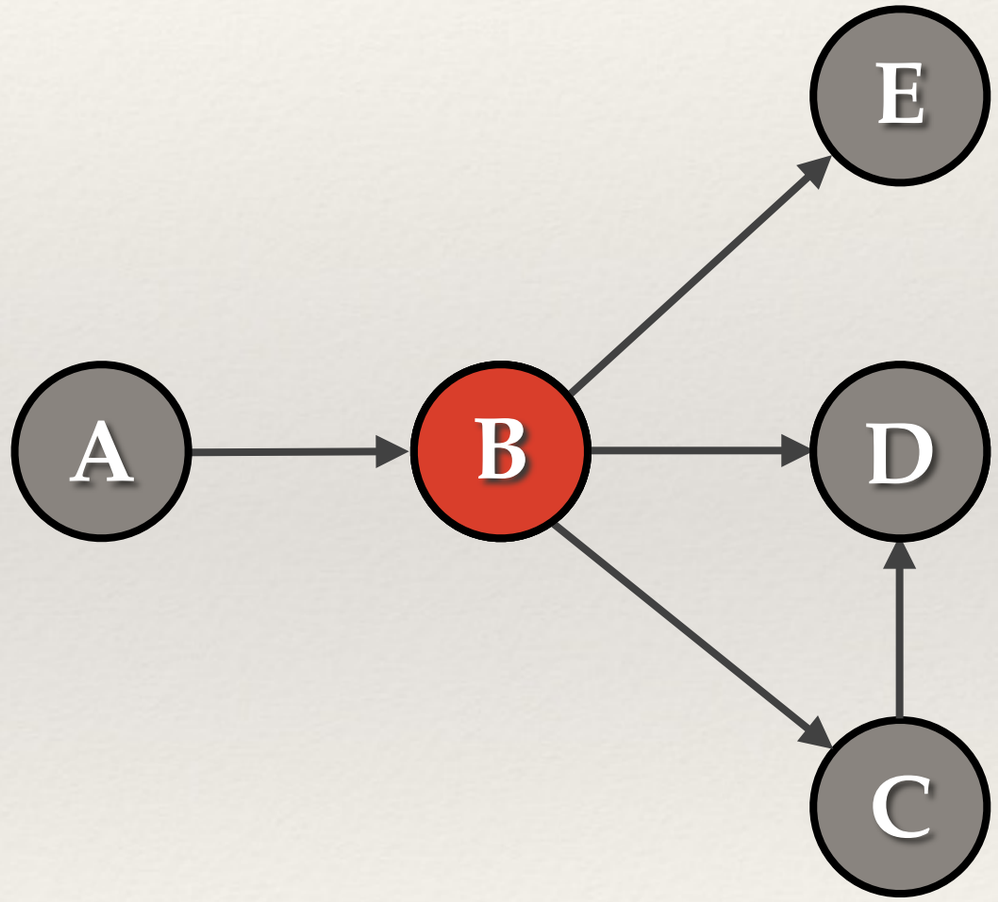


GAS abstraction

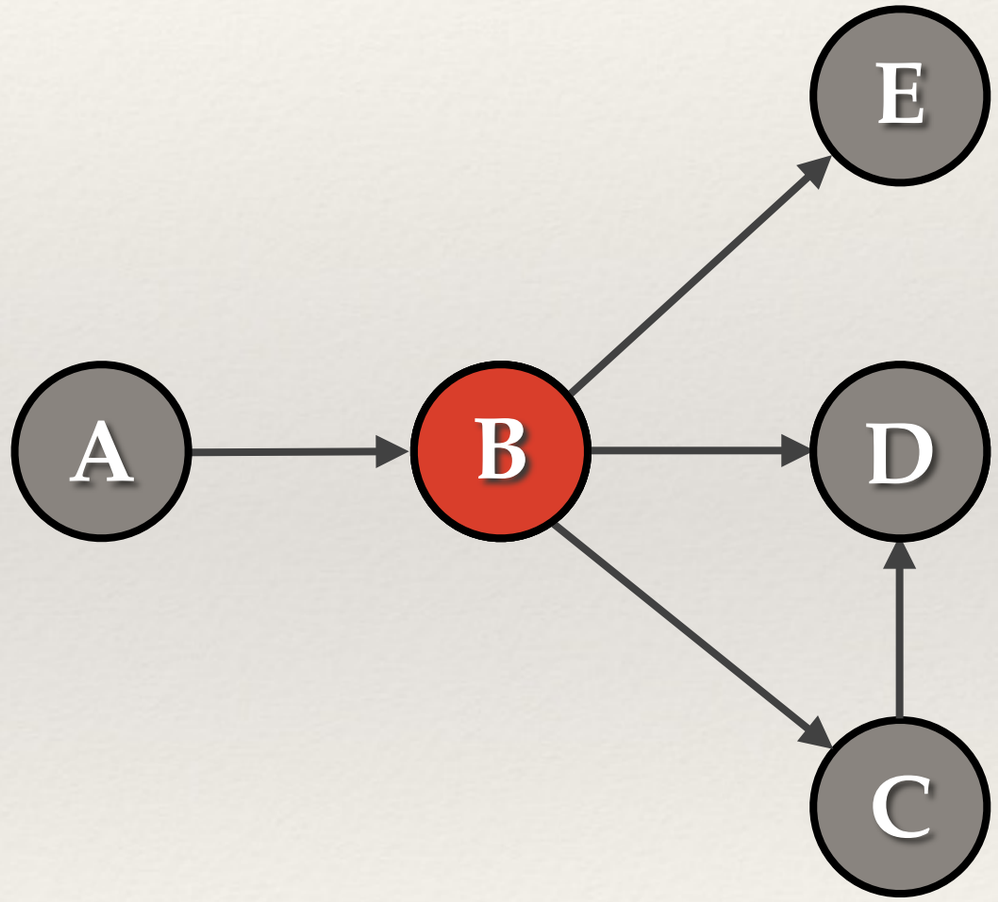
1. Gather
2. Apply
3. Scatter

Vertex Cuts (GraphLab 2.0 - PowerGraph)



Vertex Cuts (GraphLab 2.0 - PowerGraph)

- Assign **edges** to machines



Vertex Cuts (GraphLab 2.0 - PowerGraph)

- Assign **edges** to machines
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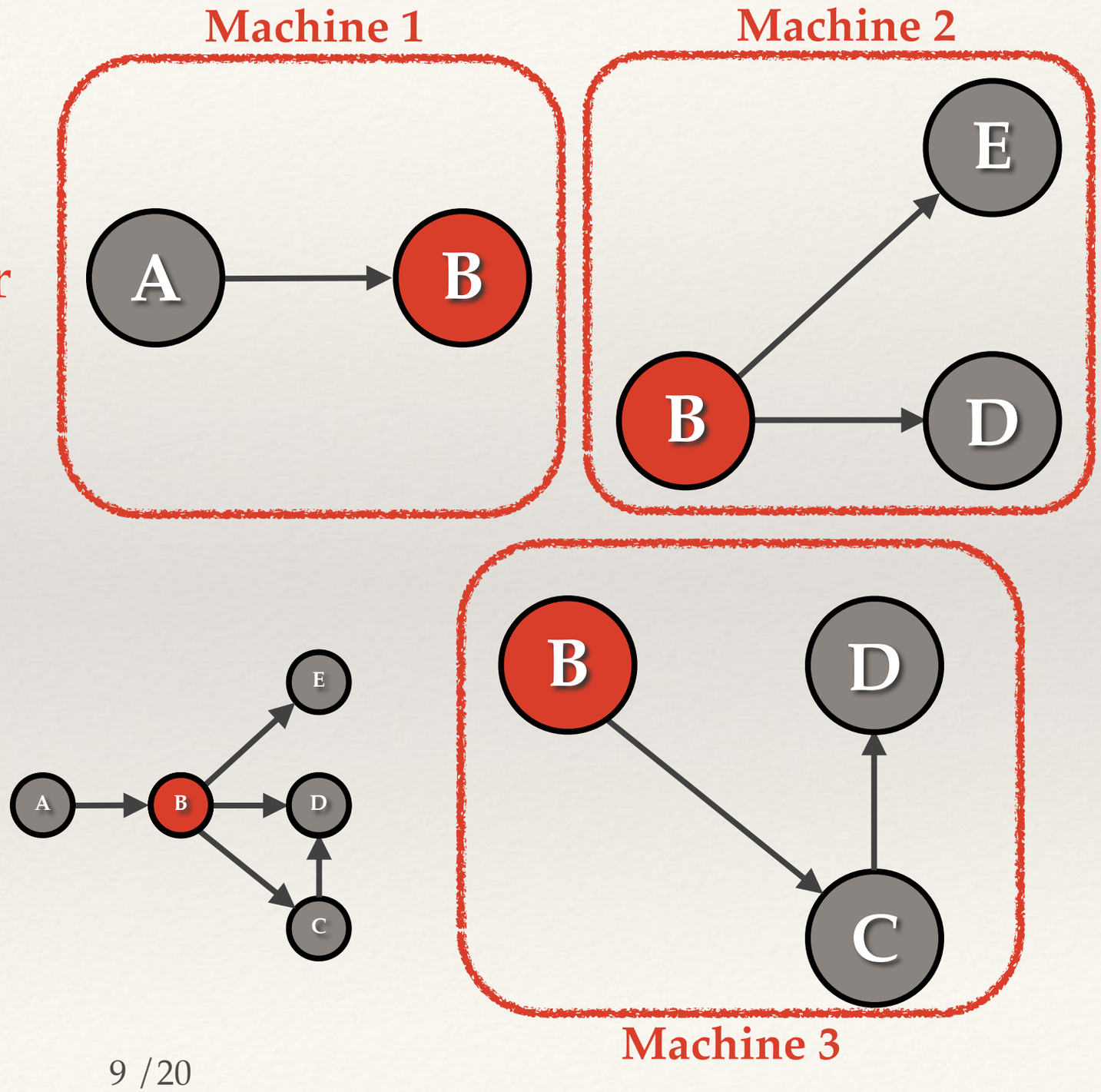

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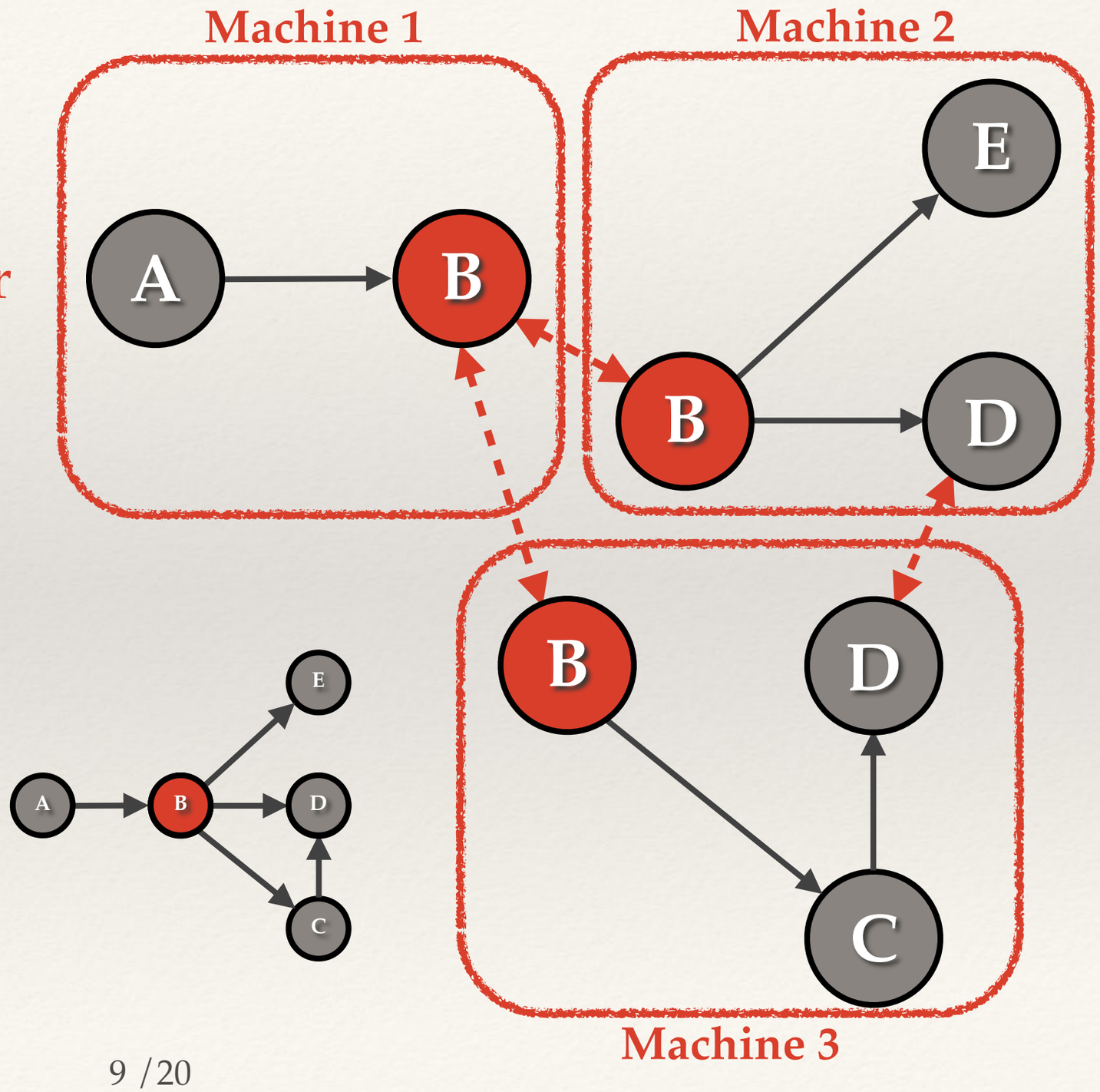

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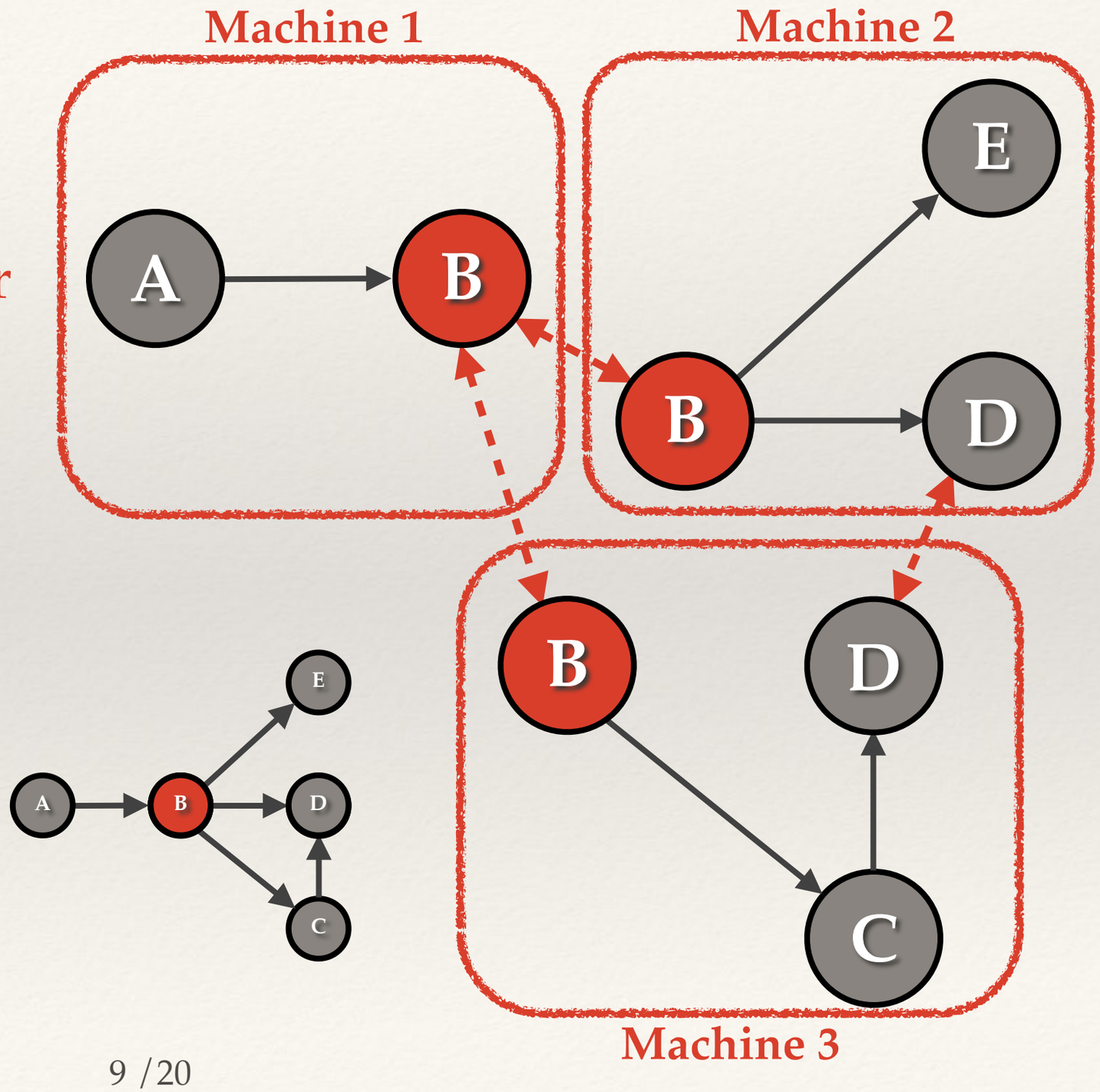
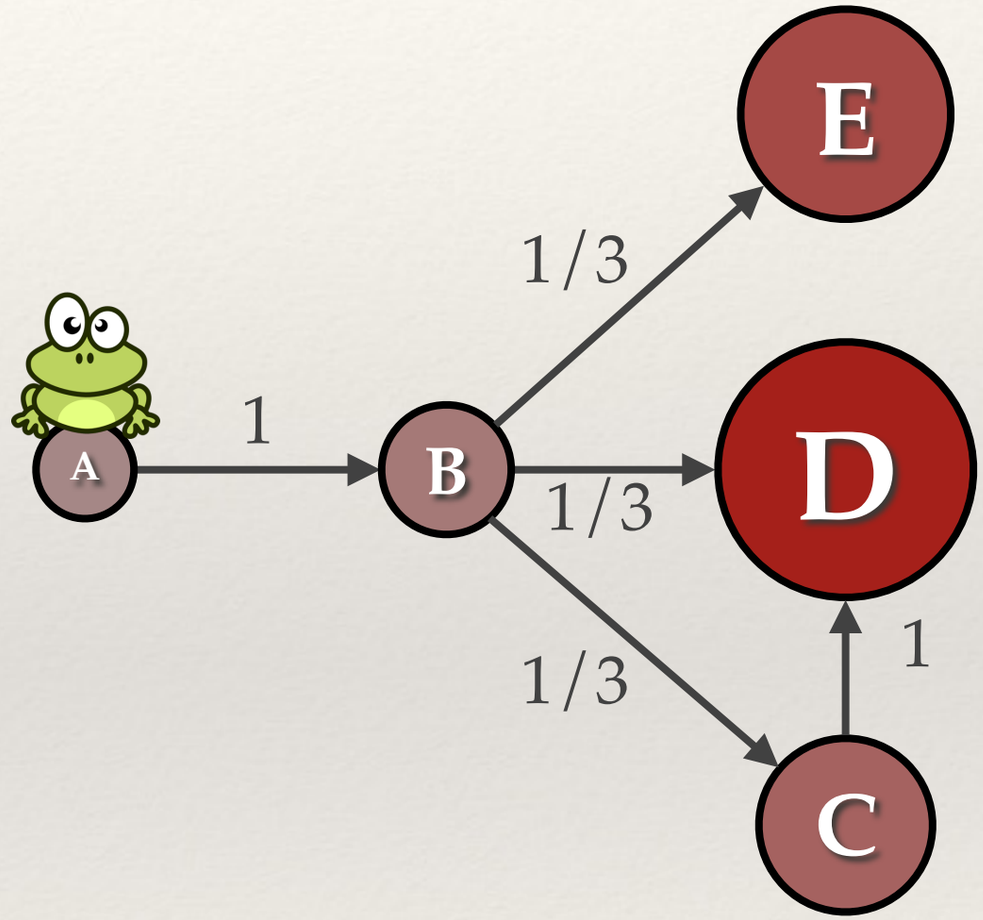
- Network is bottleneck



Random Walks

Frog walks randomly on graph



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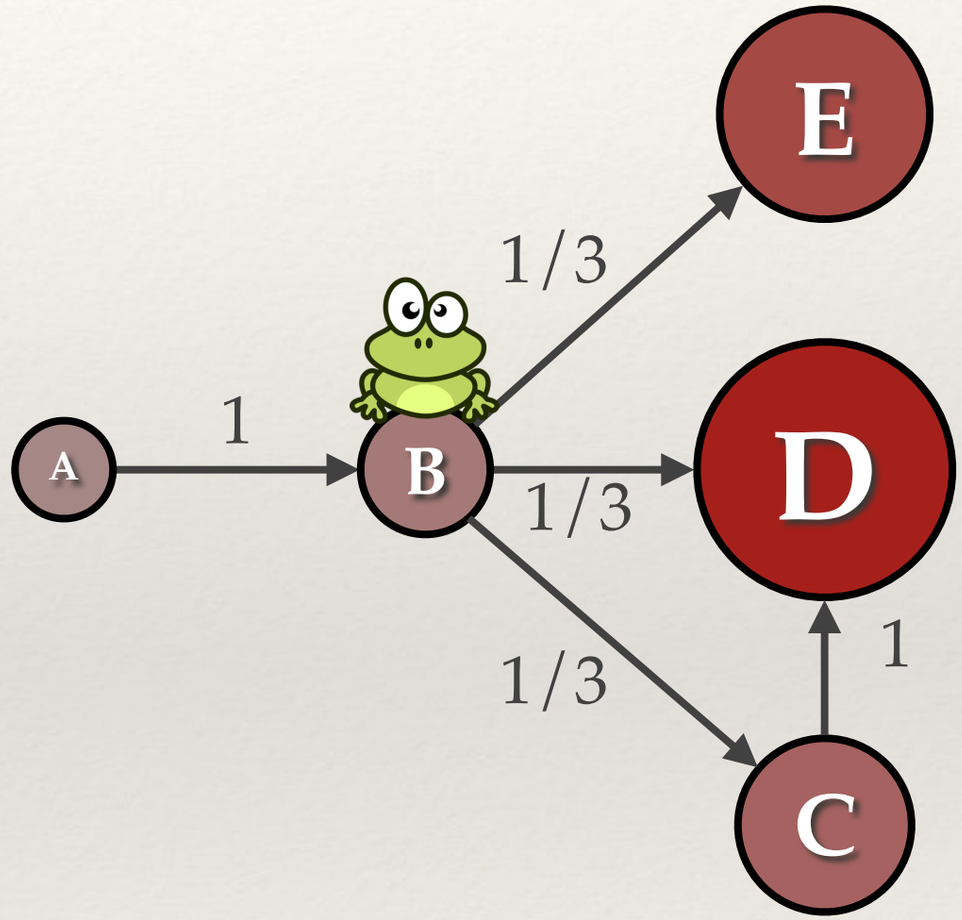

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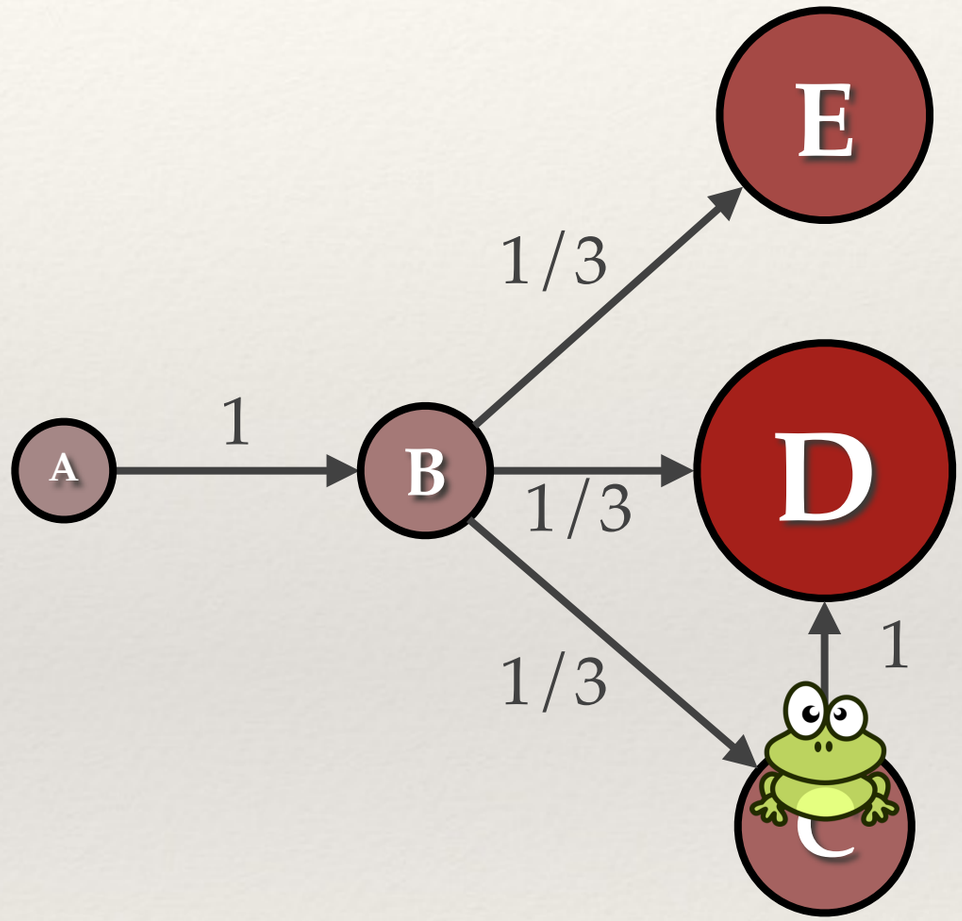

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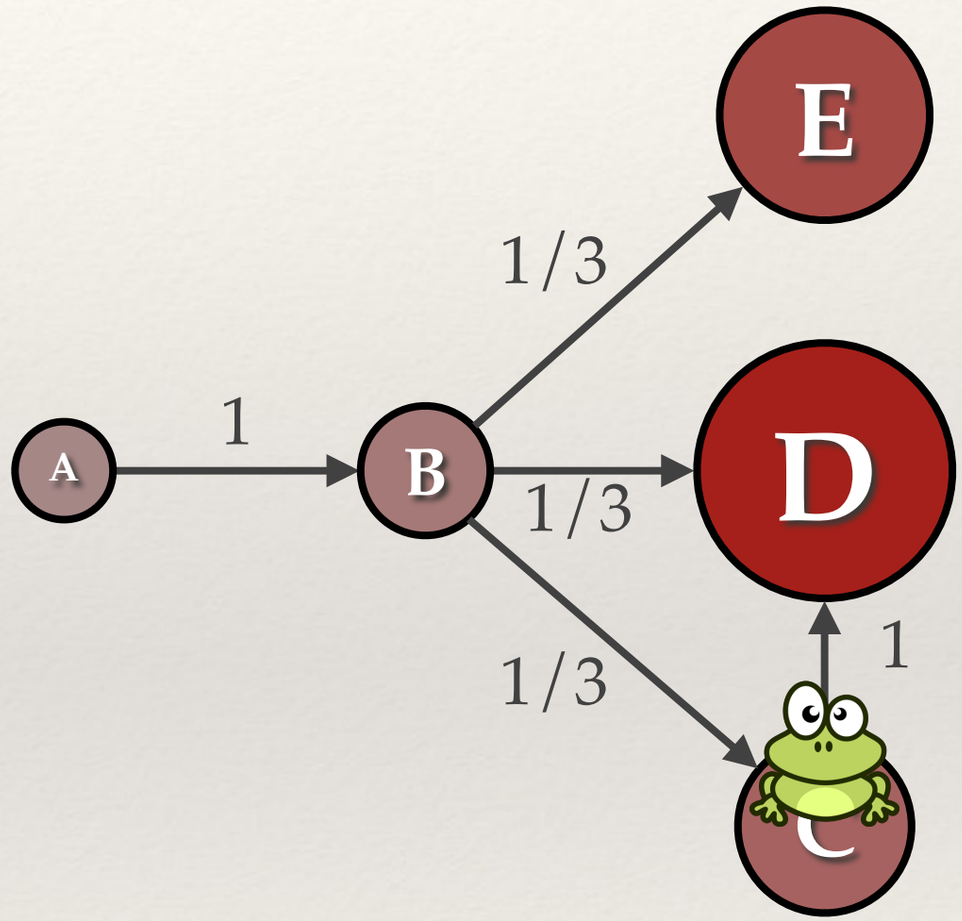

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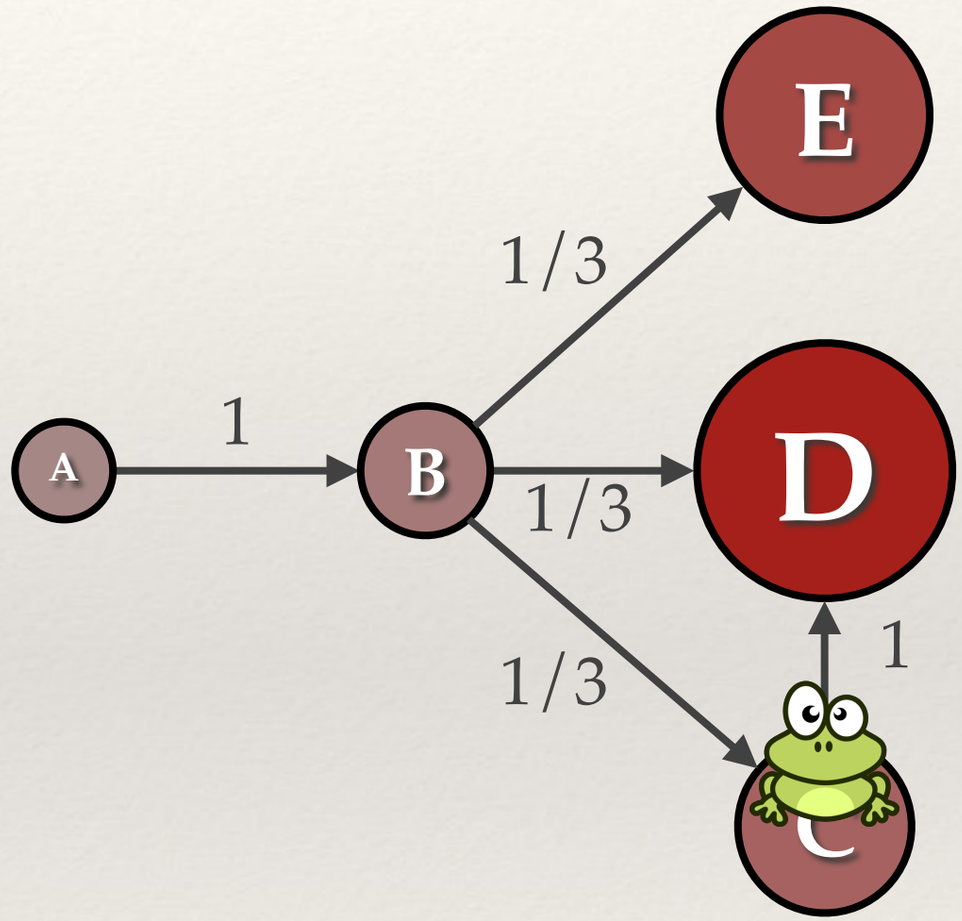
Teleportation w.p. p_T



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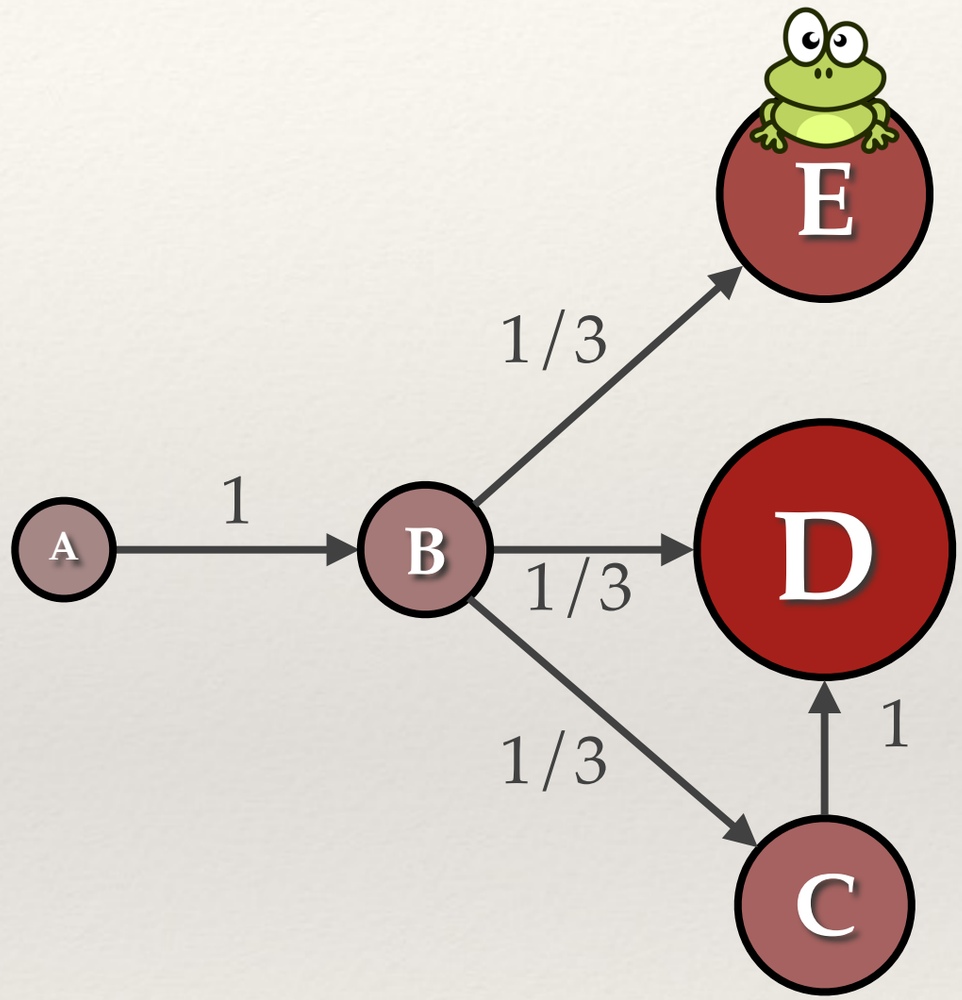

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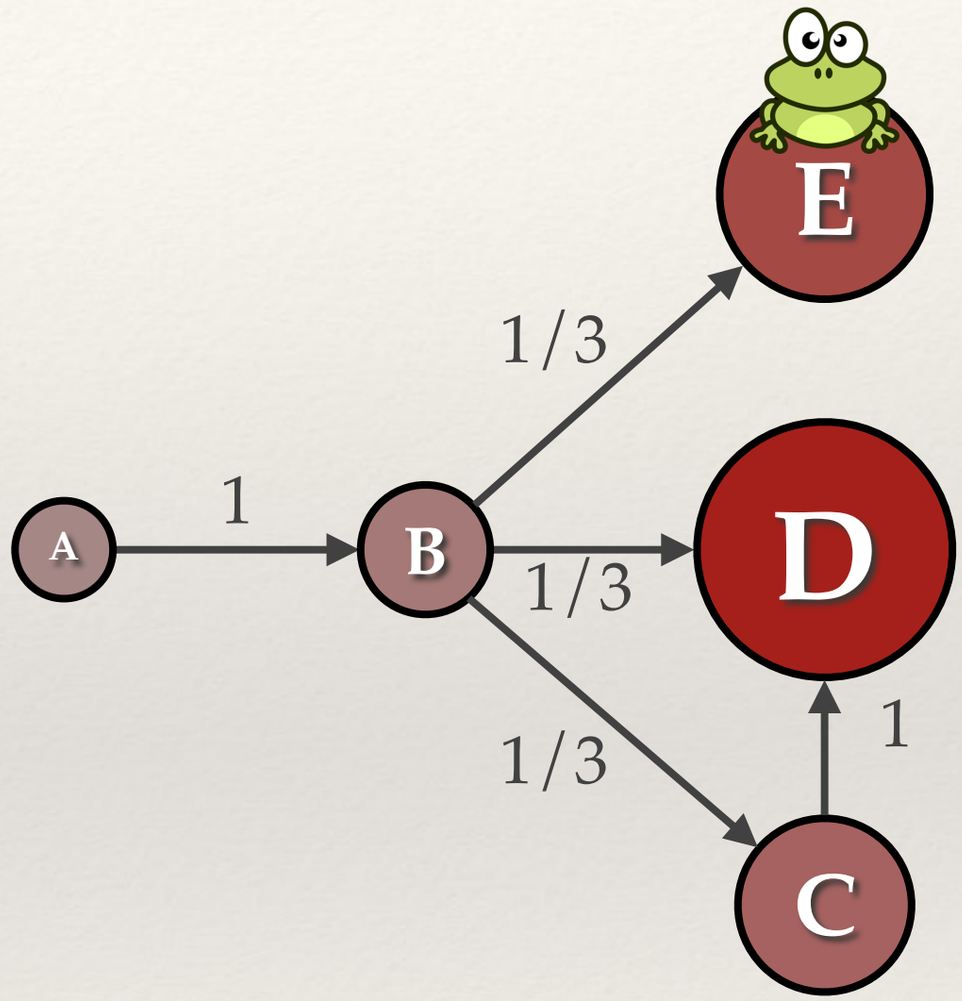
Sampling after t steps

Frog location gives sample from π



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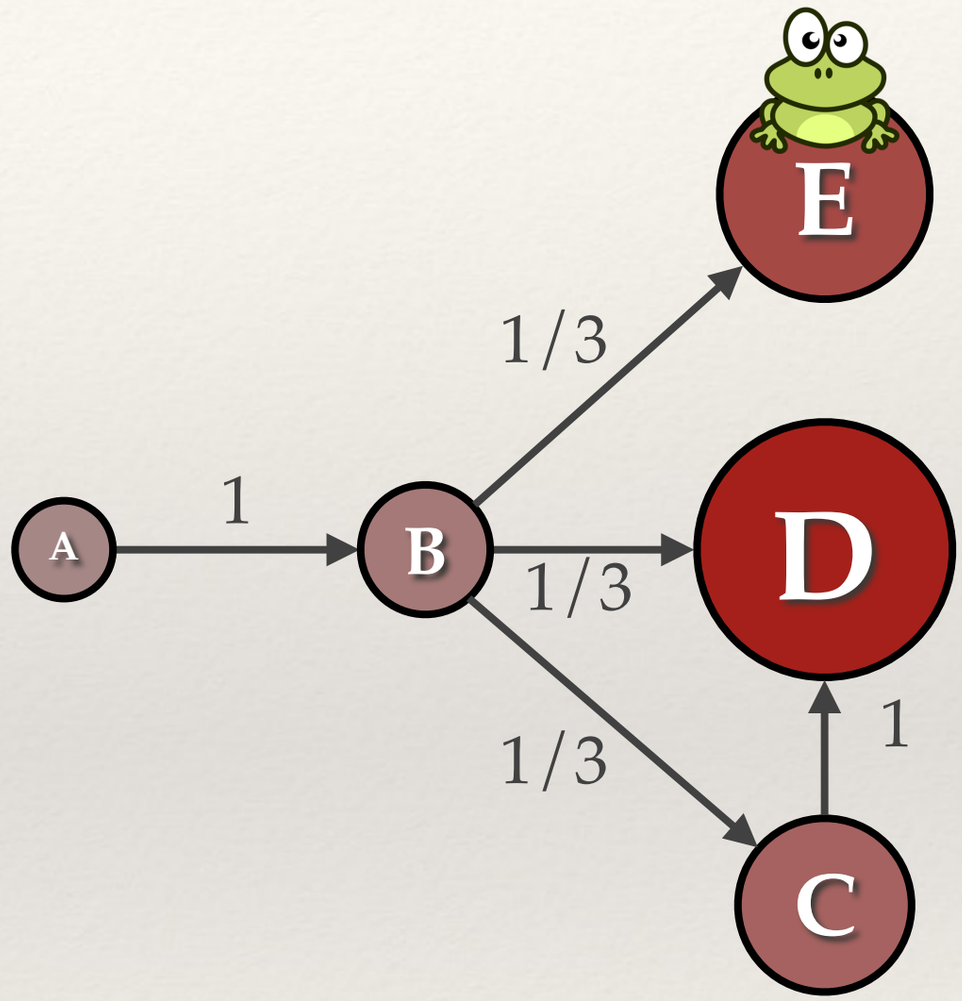
PageRank Vector

Many frogs, estimate vector π



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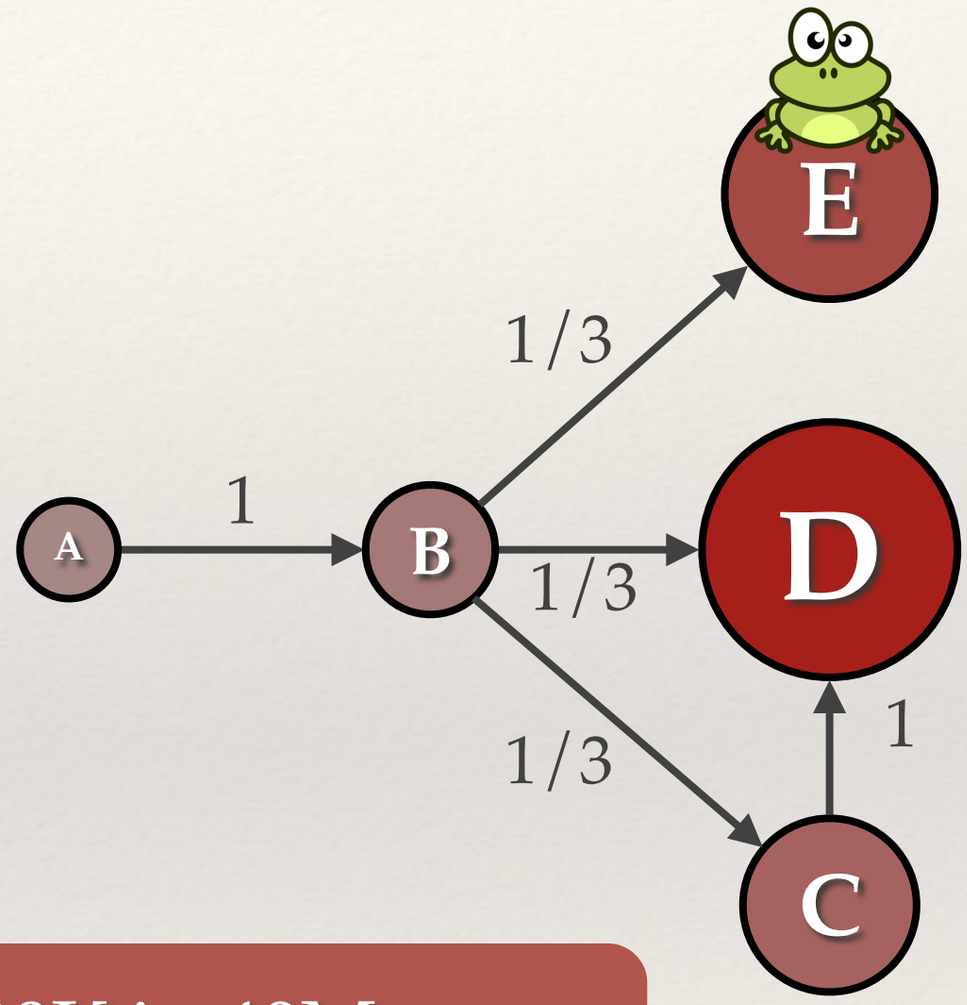
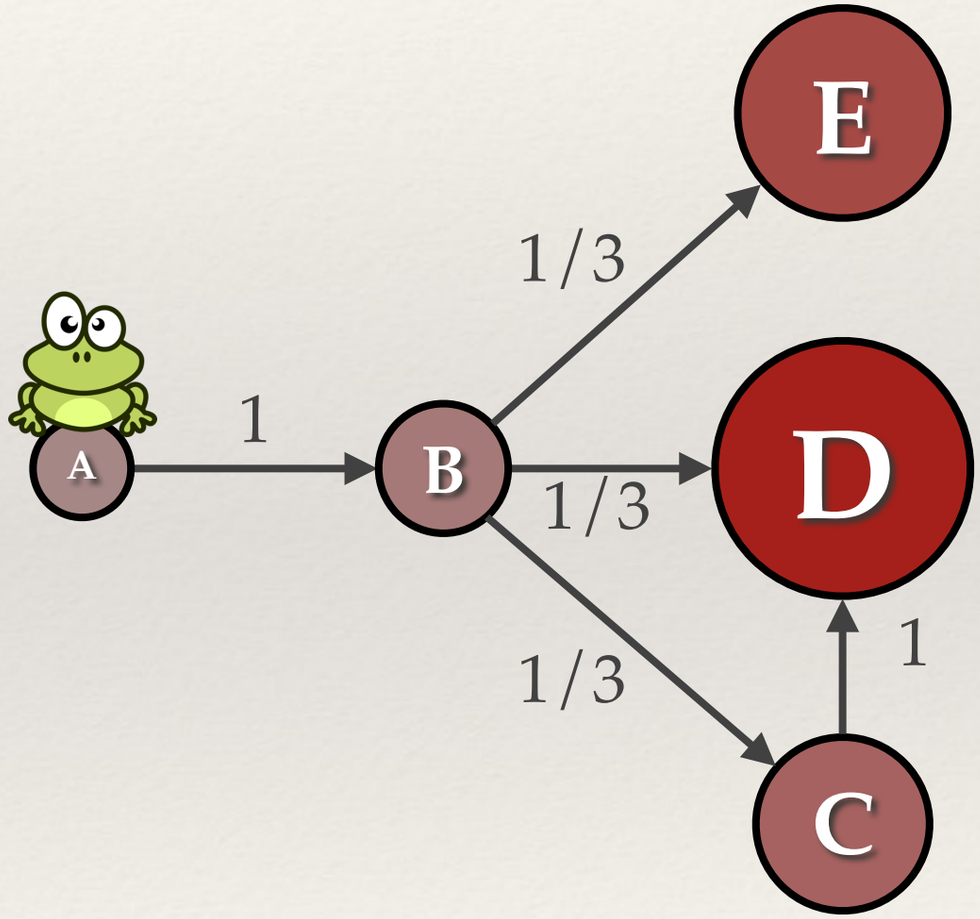


Lemma: A few random-walkers (e.g. 800K in 40M vertex graph) will suffice to estimate large entries of π vector whp. (mixing time and Chernoff bounds)

Problem 1: Teleportation

Requires global communication



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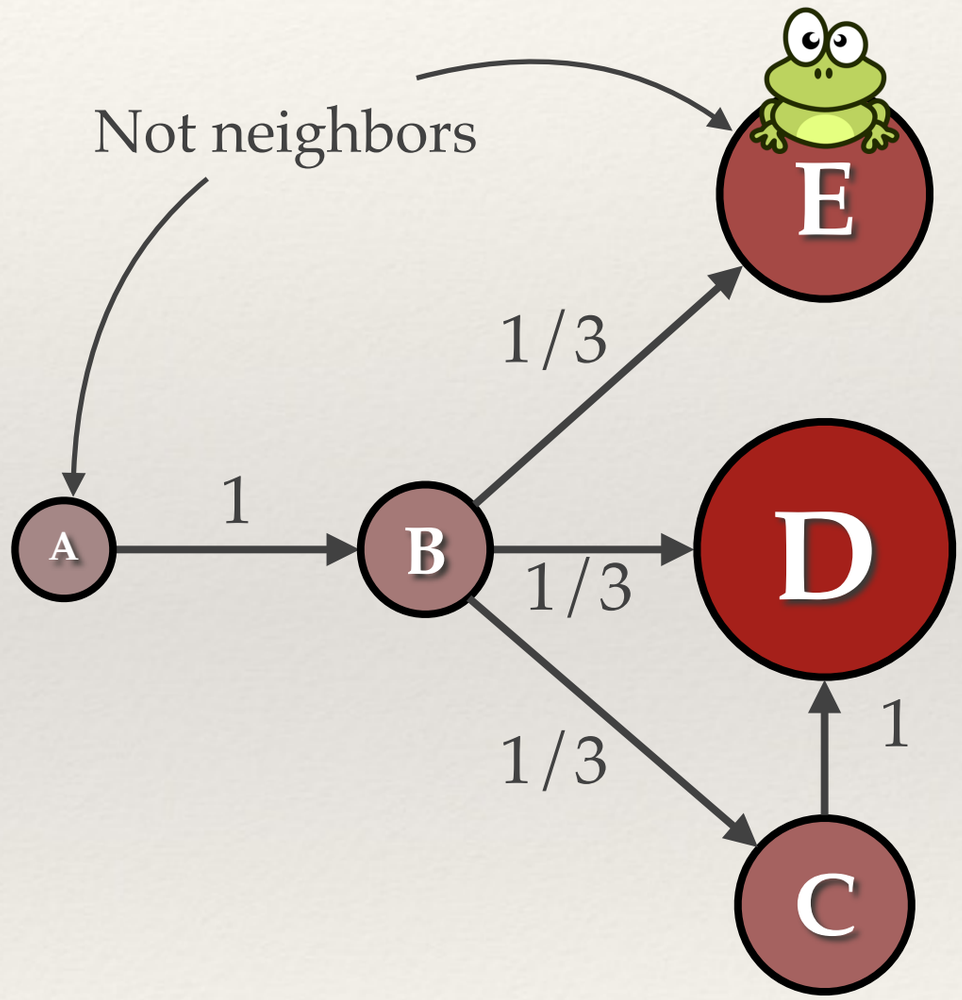

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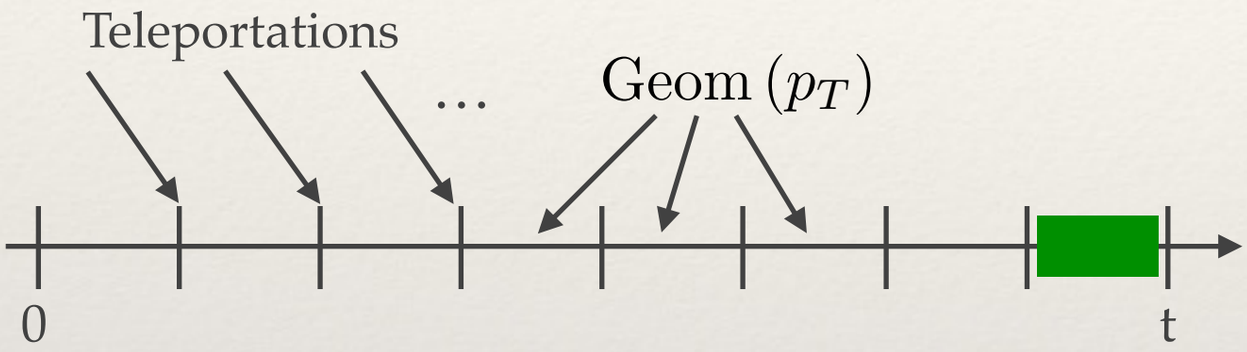

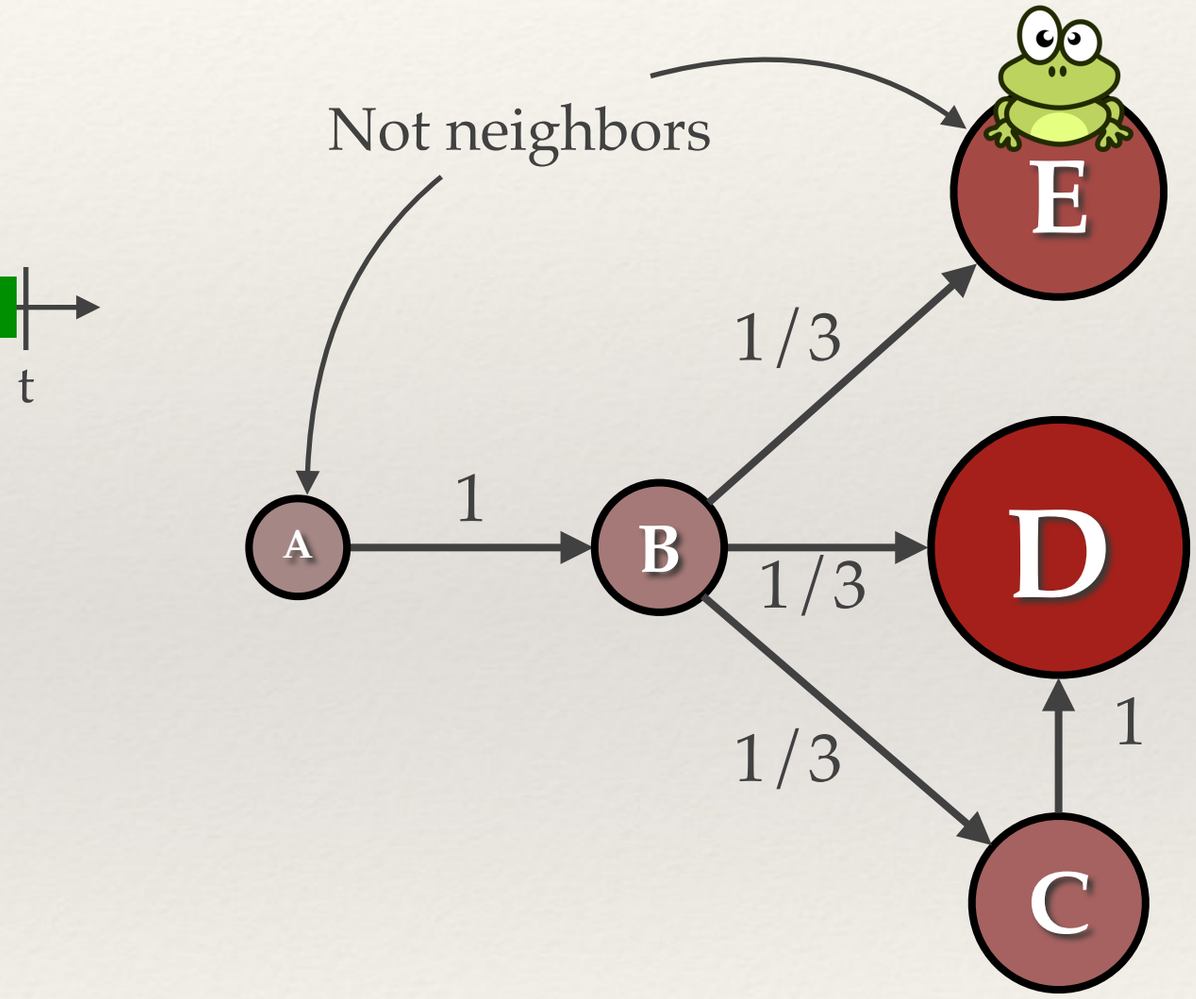

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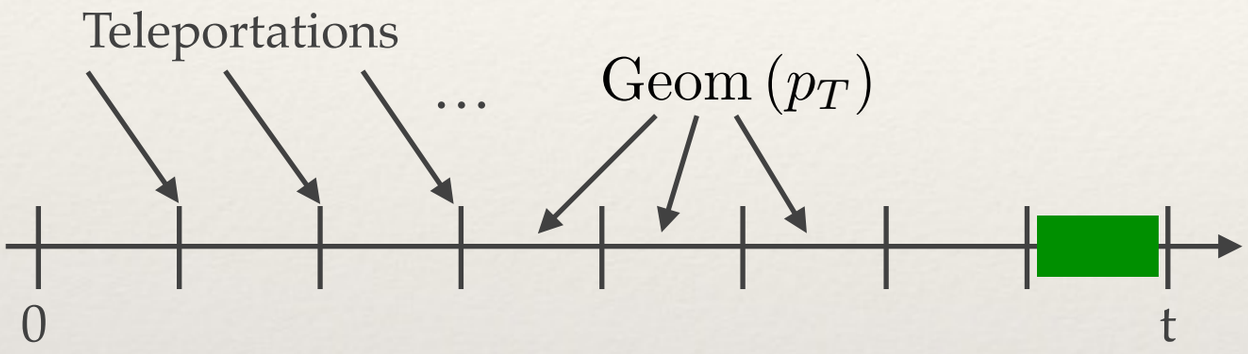

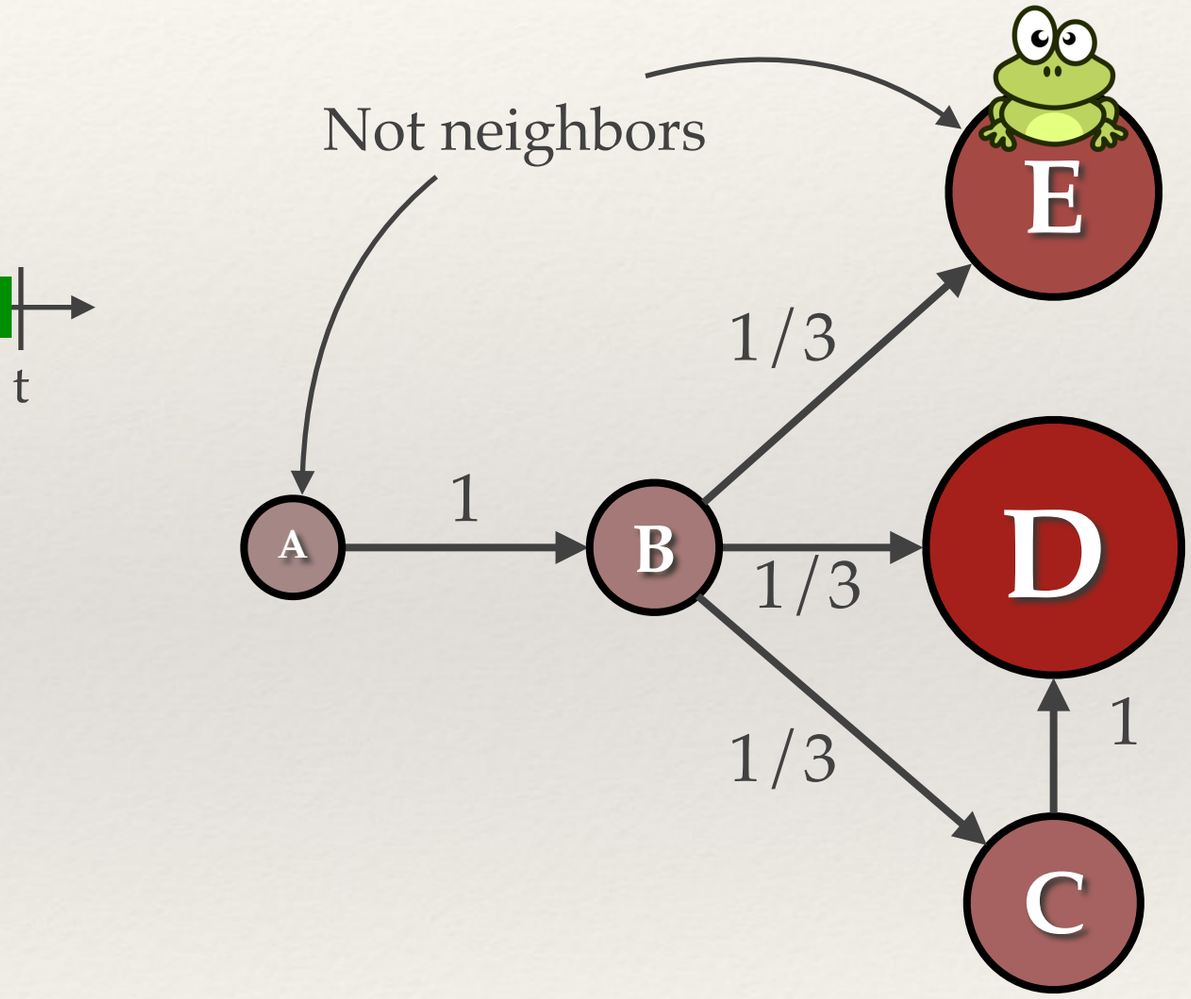

Solution

1. Give them random life
2. Do no teleportation

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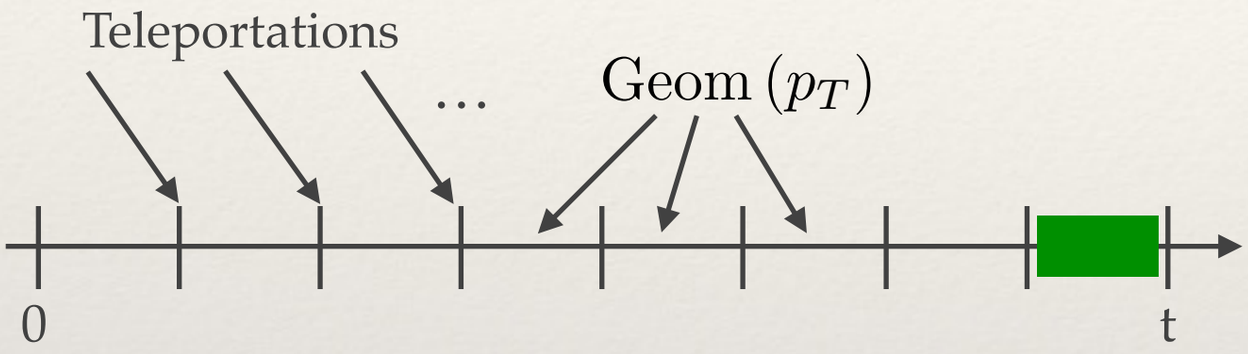

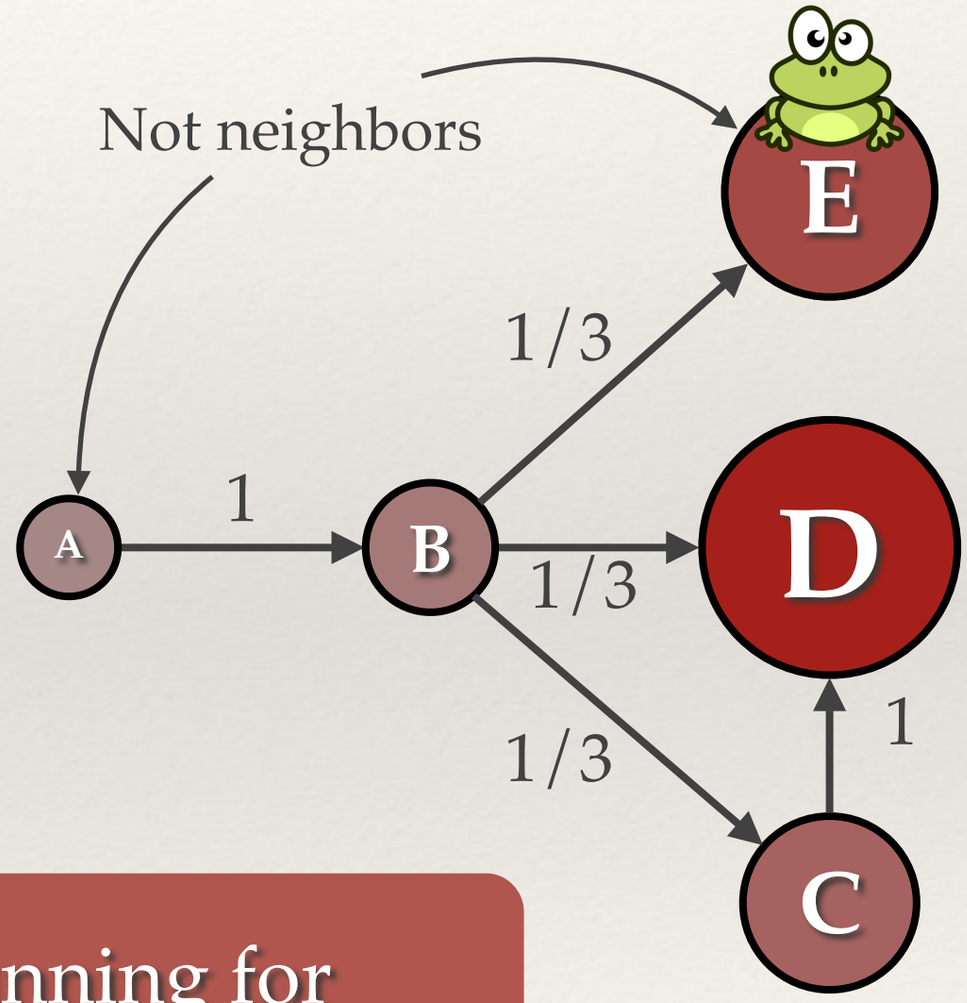

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Lemma: This process is the same as running for mixing time + teleportations

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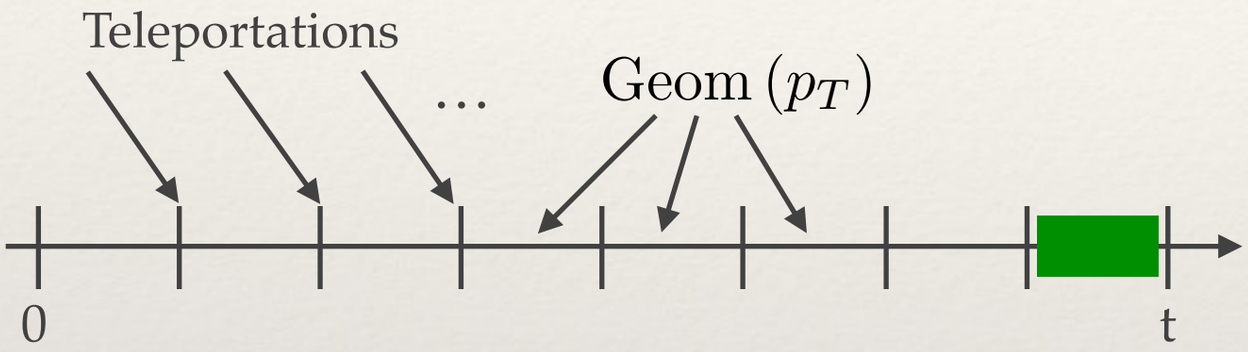

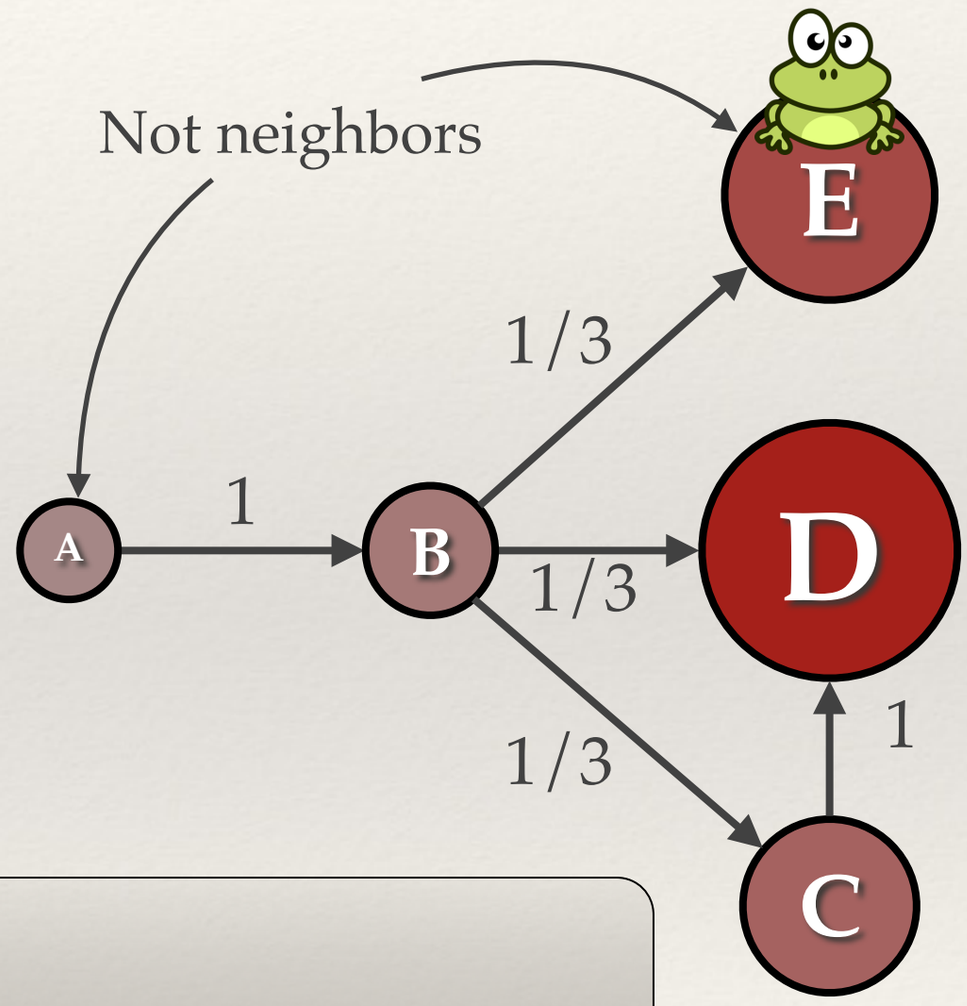

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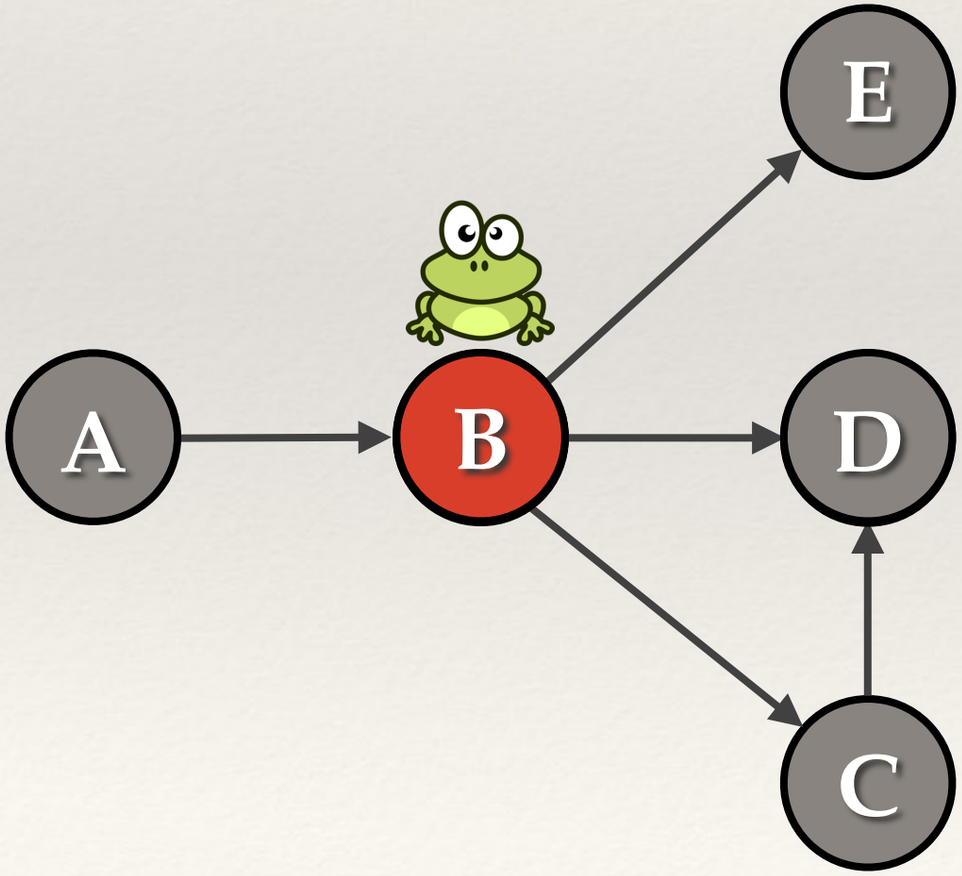
Used already before:
"Monte Carlo Methods in PageRank Computation: When One Iteration is Sufficient"
K. Avrachenkov, N. Litvak, D. Nemirovsky, N. Osipova (2007)

Problem 2: Synchronization traffic

A lot of communication
even for 1 random walk!



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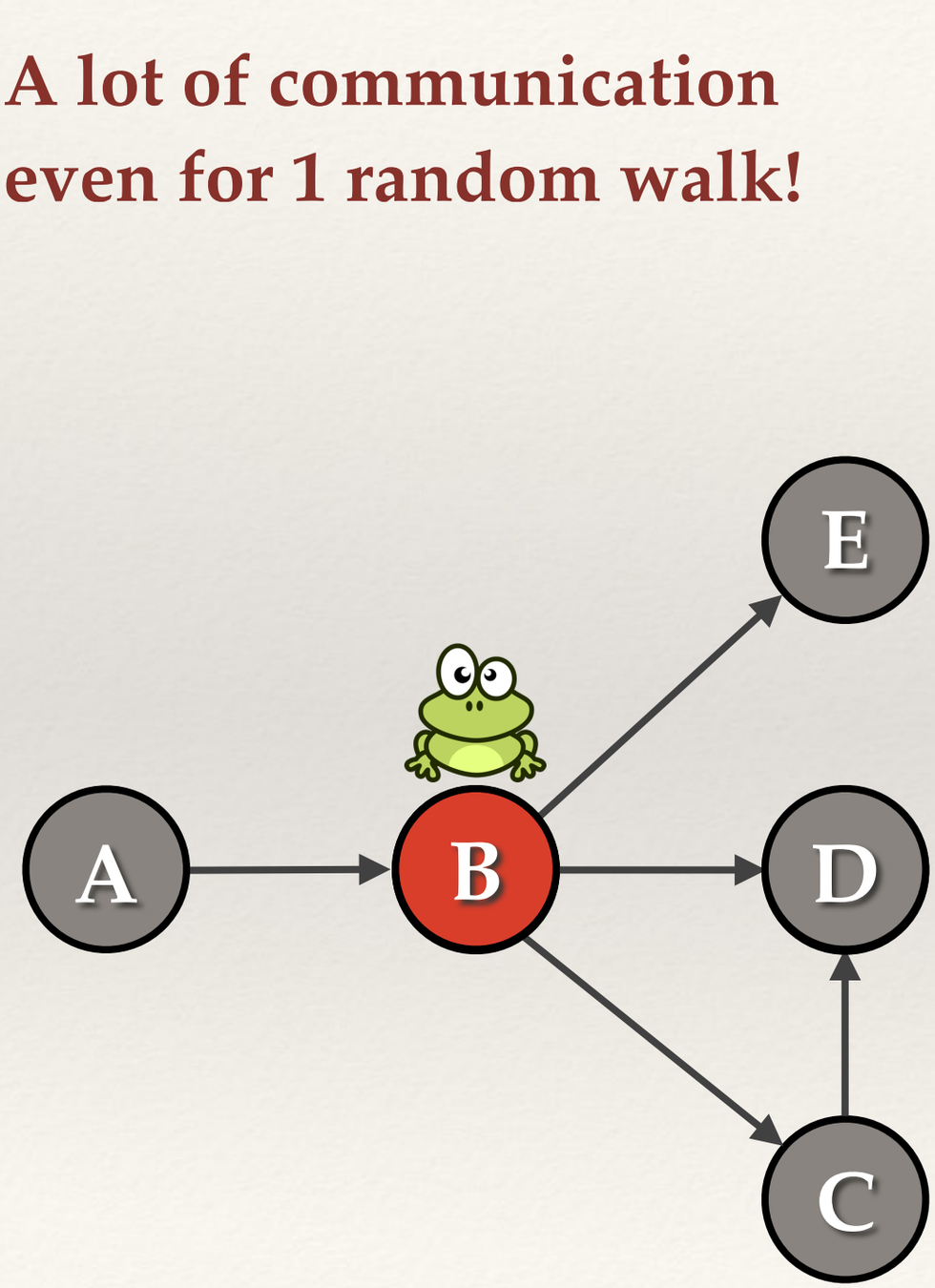
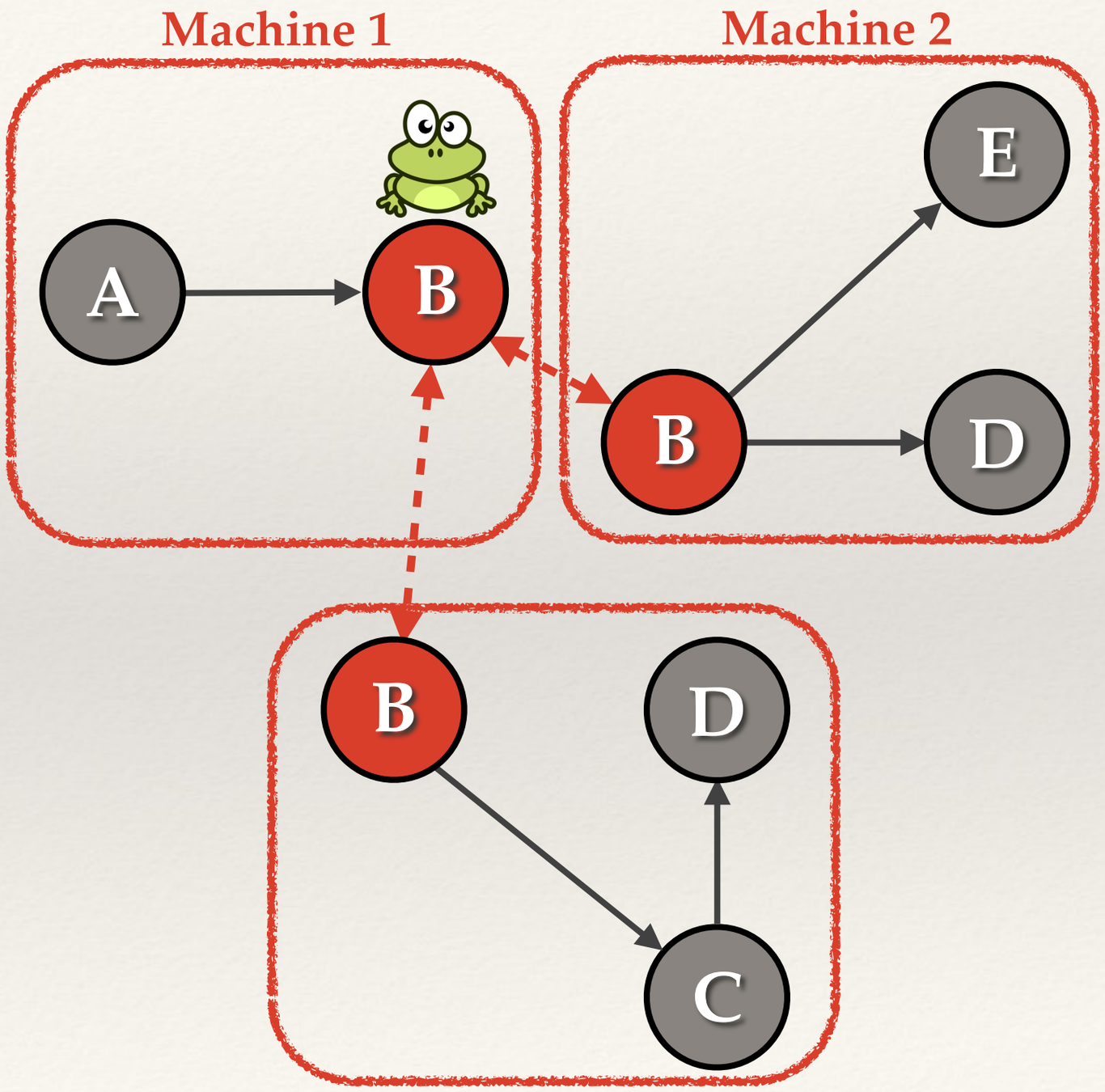
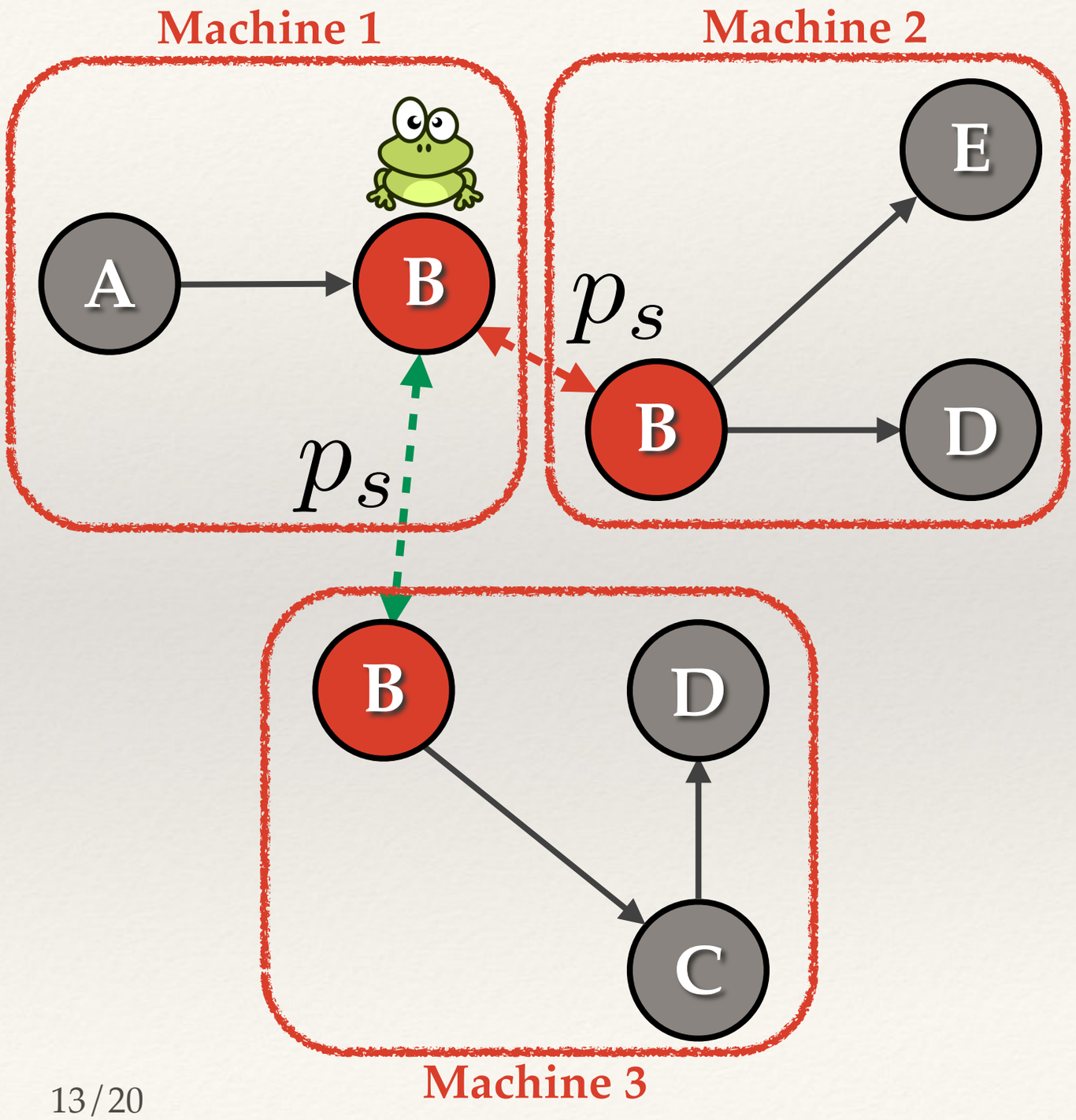




Solution: Randomized Synchronization

Modify GraphLab to expose one parameter p_s to programmer

Flip a coin and synchronize each mirror vertex only w.p. p_s



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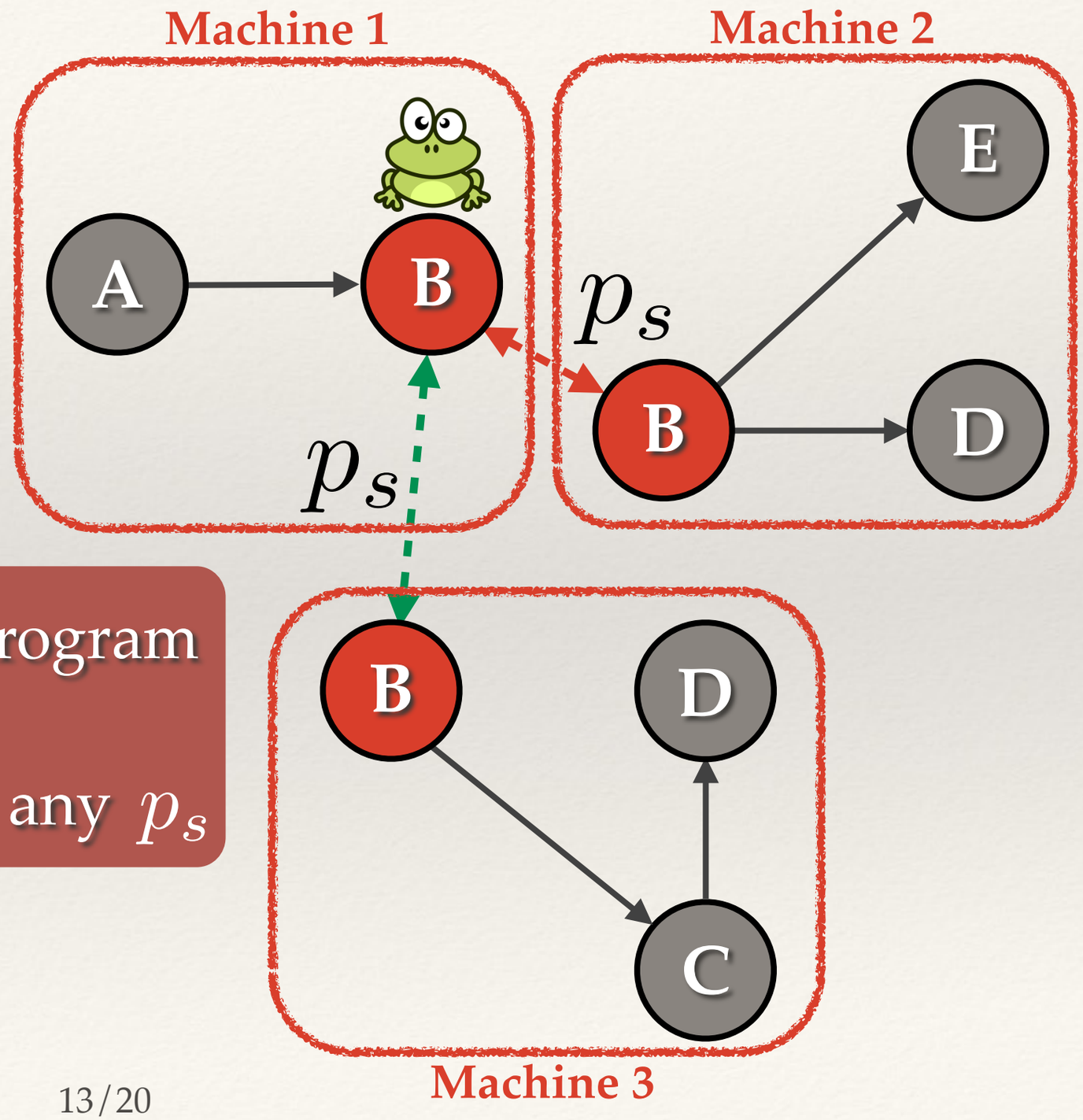

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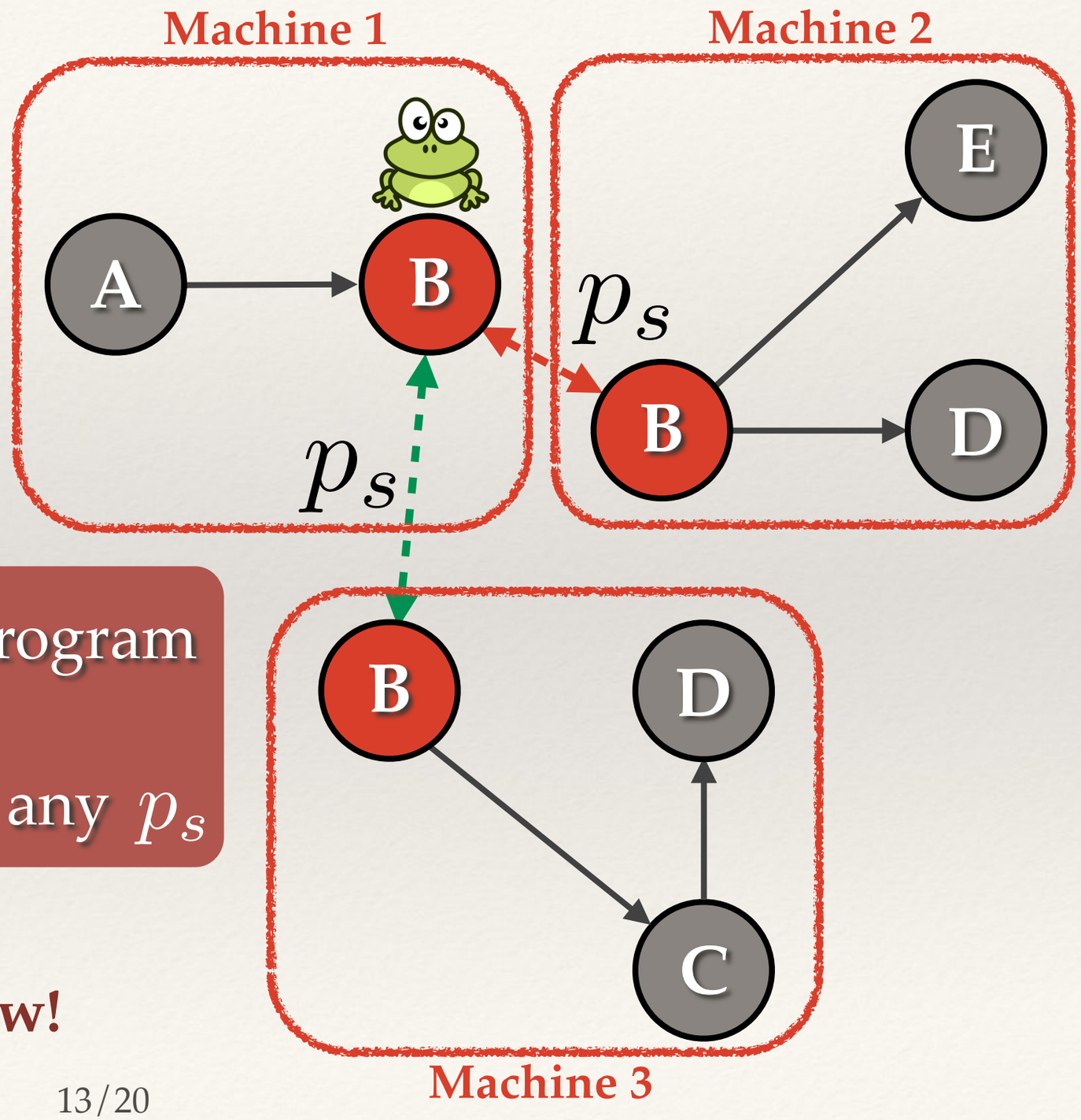

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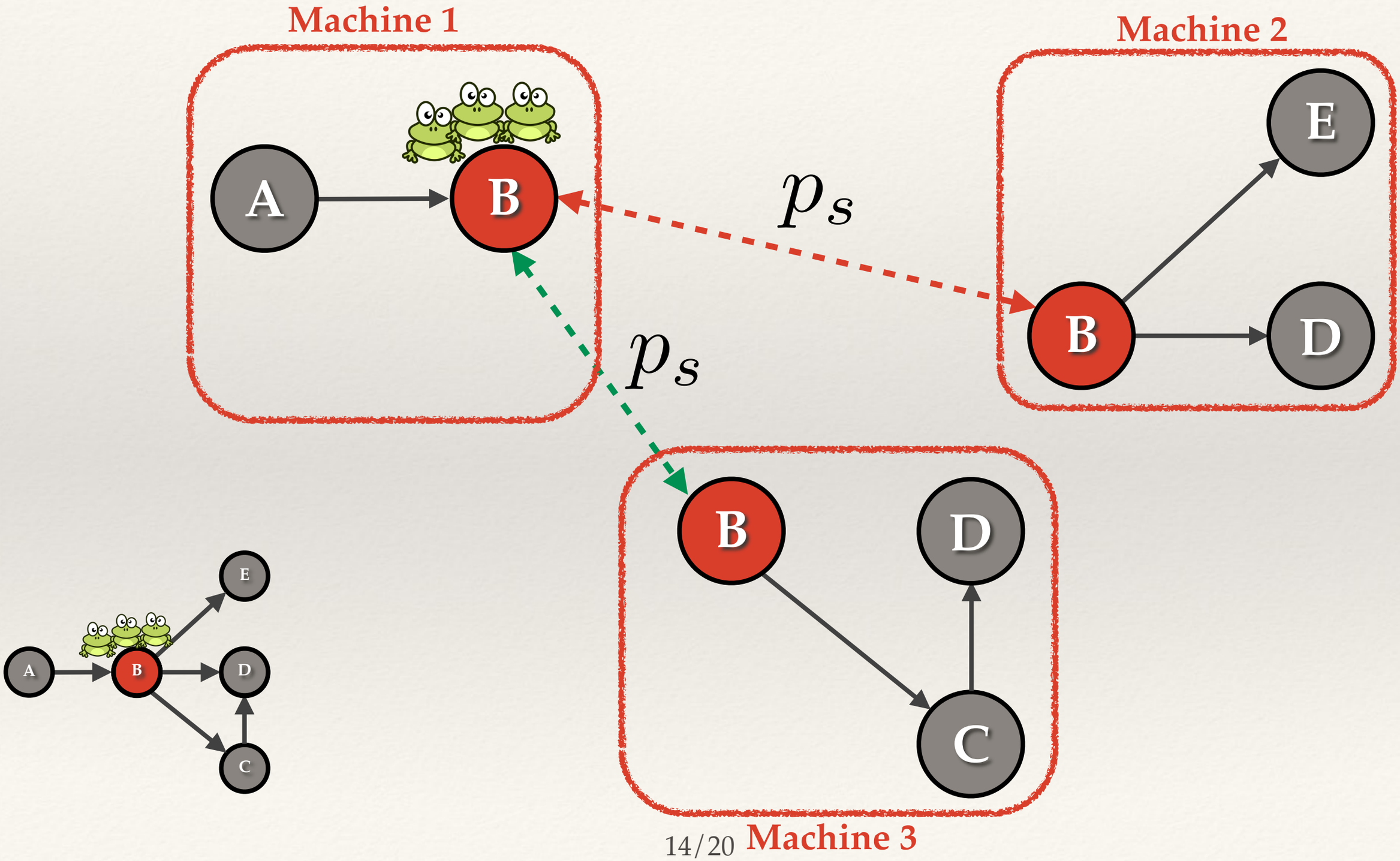
But walks have dependencies now!

Problem 3: Why dependencies?



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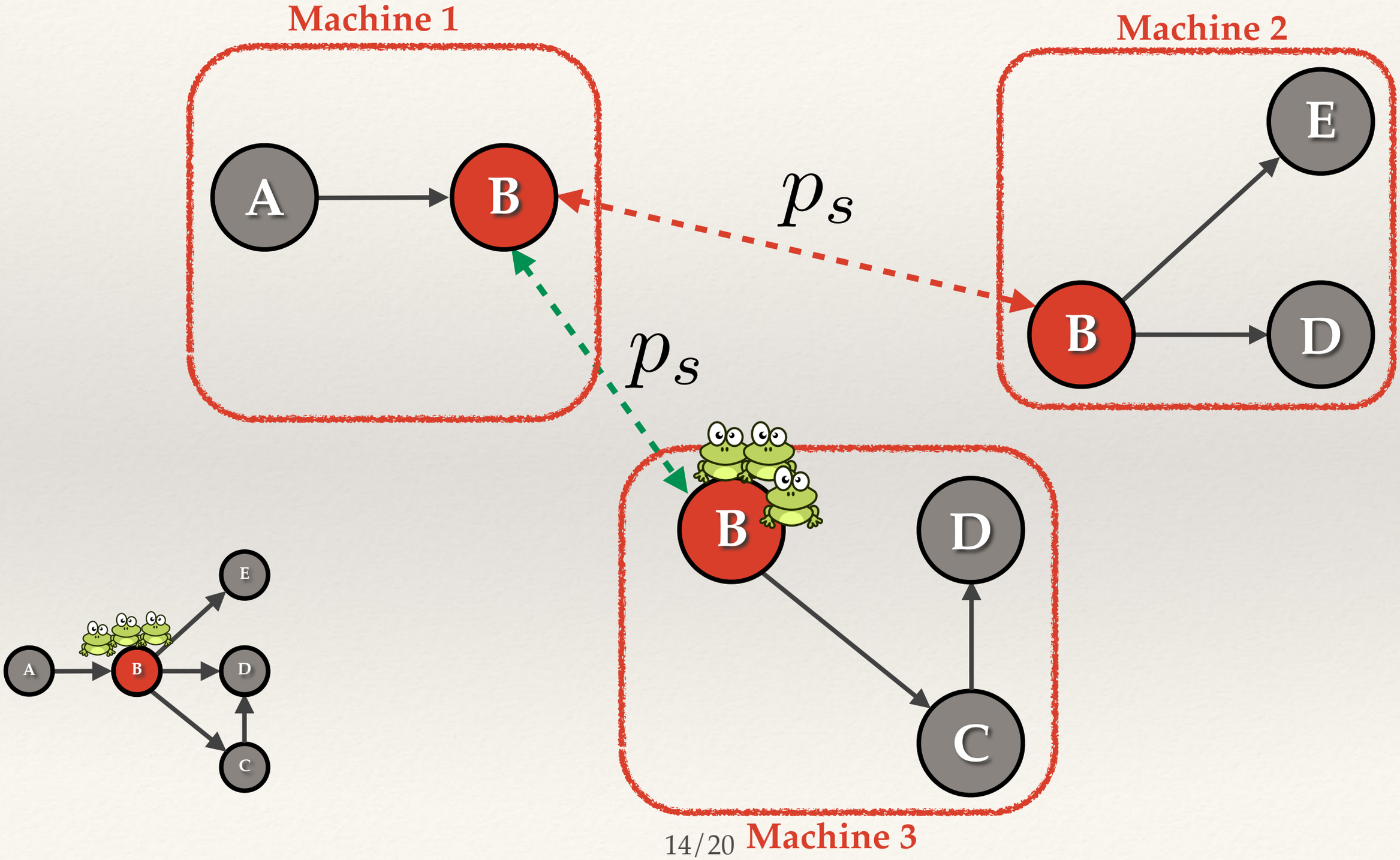

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$$\epsilon < \sqrt{\frac{(1-p_T)^{t+1}}{p_T}} + \sqrt{\frac{k}{\delta}\left[\frac{1}{N} + (1-p_s^2)p_\cap(t)\right]}$$

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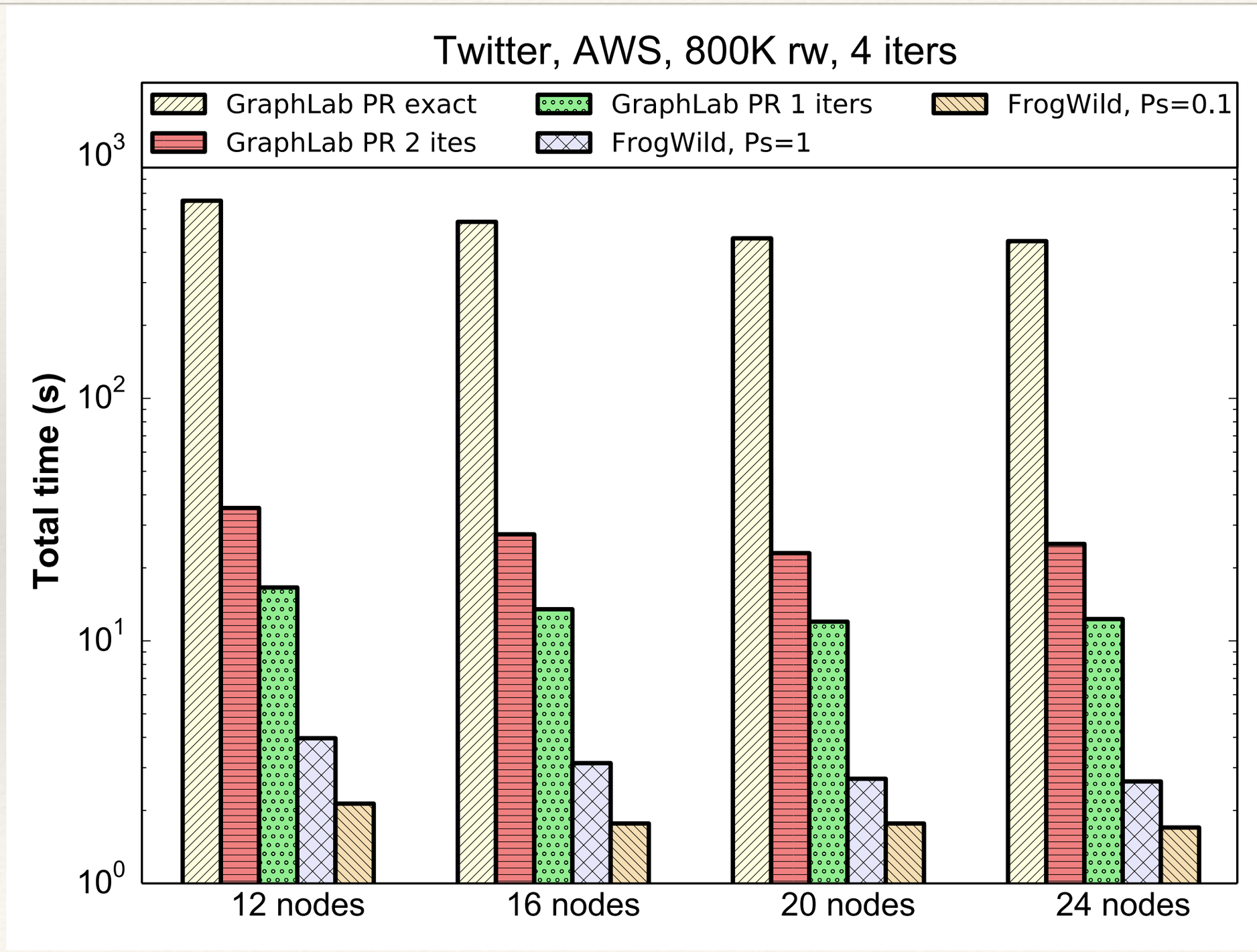
Bad graph: Star

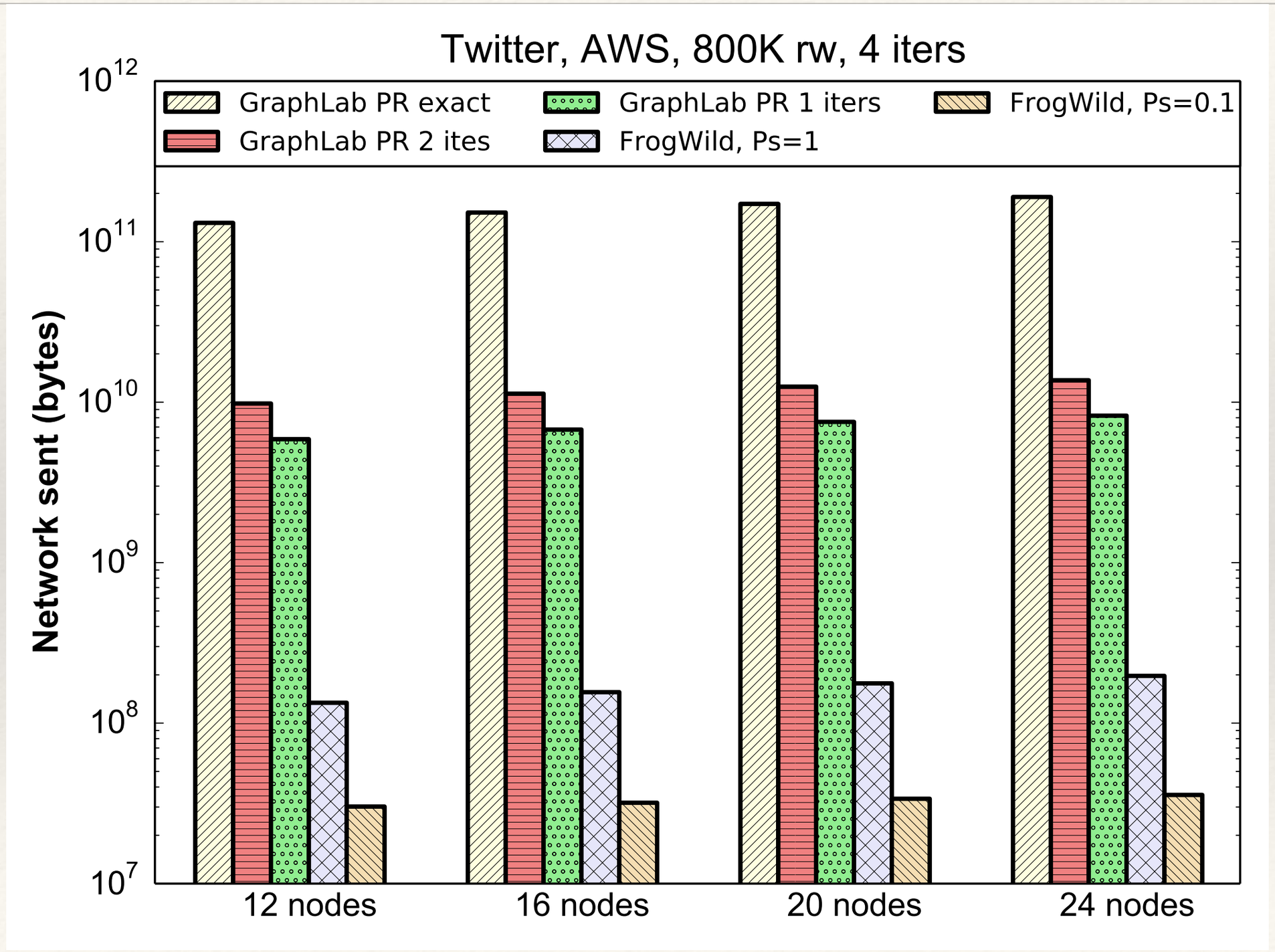
Good graph: Clique, Expander, Power-law

Experiments

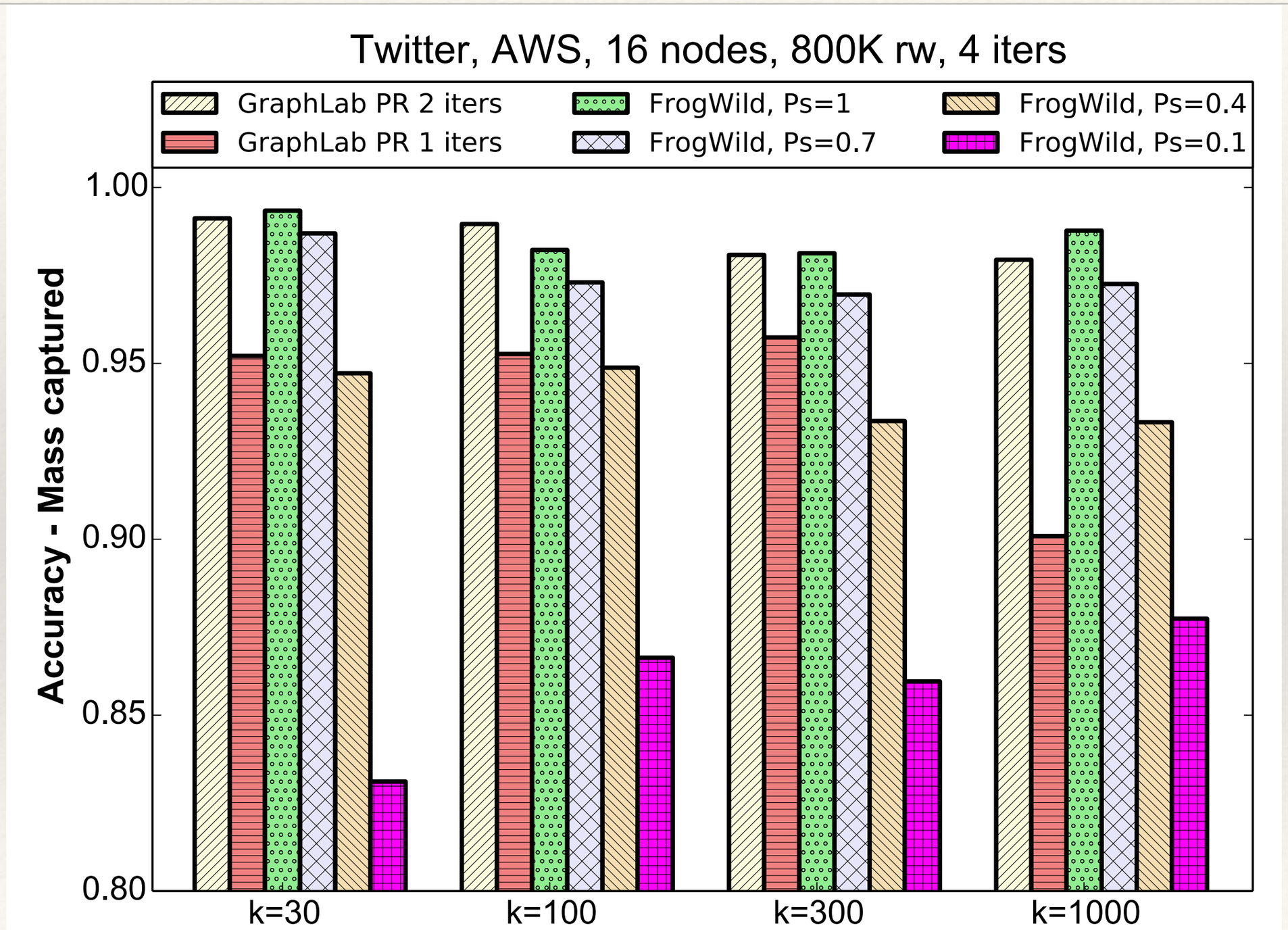
Total Runtime
Twitter Graph - 40M nodes, 1.4B edges



Network Usage
Twitter Graph - 40M nodes, 1.4B edges



Accuracy
Twitter Graph - 40M nodes, 1.4B edges



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Thank you!