ECE259
Electricity and Magnetism

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Contents

1 Static Electric Fields

1.1 Fundamental Postulates of Electrostatics in Free Space

Electric field intensity is defined as the force per unit charge that a very small stationary test charge experiences when it is placed in a region where and electric field exists:

$$\vec{E} = \lim_{q \to \infty} \frac{\vec{F}}{q} \quad \left[\frac{V}{m}\right]$$

This is proportional to and in the direction of the force $\vec{F}$. We can express the inverse relationship, which gives the force $\vec{F}$ on a charge q in an electric field $\vec{E}$

The two fundamental postulates of electrostatics in free space specify the **divergence** and **curl** of $\vec{E}$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \vec{\nabla} \times \vec{E} = 0$$

These two expression are point relations, and are referred to as the differential form of the postulates of electrostatics. We can integrate to get the expressions for a total field of an aggregate or a distribution of charges.

$$\oint_S \vec{E} \cdot ds = \frac{Q}{\epsilon_0} \qquad oint_C \vec{E} \cdot dl = 0$$

1.2 Coulomb's Law

We can express the electric field intensity on a point charge as the following:

$$\vec{E} = \vec{a_R} E_R = \vec{a_R} \frac{q}{4\pi\epsilon_0 R^2} \quad \left[\frac{V}{m}\right]$$

Here we assume the charge is located at the origin, but this is often not the case. If we want to account for the changes in $\vec{a_R}$ we can use the vector form of the equation above:

$$\vec{E_p} = \frac{q(\vec{R} - \vec{R}')}{4\pi\epsilon_0 |\vec{R} - \vec{R}'|^3} \quad \left[\frac{V}{m}\right]$$

We are also able to determine the force $\vec{F_{12}}$ experienced by a charge q_2 when placed in the field of another charge q_1. This is in fact the mathematical form of Coulomb's Law:

$$\vec{F_{12}} = q_2 \vec{E_{12}} = \vec{a_R} \frac{q_1 q_2}{4\pi\epsilon_0 R^2} \quad [N]$$

1.2.1 Electric Field Due to a System of Discrete Charges

All of these equations presented so far involve only a single charge, but what if we have a field created from many point charges. Since electric field potential is a liner function, we are able to apply the principle of superposition. The electric field $\vec{E}$ is thus the the vector sum of the fields caused by the individual charges

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^{n} \frac{q_k(\vec{R} - \vec{R'_k})}{|\vec{R} - \vec{R'_k}|^3} \quad \left[\frac{V}{m}\right]$$

1.3 Electric Field Due to a Continuous Distribution of Charge

The electric field from a distribution of charge can be obtained by integrating the contribution of an element charge over the charge distribution. The differential element is the following:

$$d\vec{E} = \vec{a_R}\frac{\rho dv'}{4\pi\epsilon_0 R^2}$$

Integrating this, we get:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{V'} \rho\frac{\vec{R}}{R^3}dv' \quad \left[\frac{V}{m}\right]$$

This is integrating a charge over the volume. If we want to integrate over the surface, then:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{S'} \vec{a_R}\frac{\rho_S}{R^2}ds' \quad \left[\frac{V}{m}\right]$$

Likewise, if we have a line charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{L'} \vec{a_R}\frac{\rho_l}{R^2}dl' \quad \left[\frac{V}{m}\right]$$

1.4 Gauss's Law and Applications

1.5 Electric Potential

1.6 Dielectrics in Static Electric Field

1.7 Electric Flux Density and Dielectric Constant

1.8 Boundary Conditions for Static Electric Fields

1.9 Capacitors and Capacitance

1.10 Electrostatic Energy Forces