MAT336
Elements of Analysis

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January 14, 2021

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1 Logic and Proof

1.1 Logical Connectives

Mathematics consists of declarative sentences. A **statement** is a sentence that can be classified as either true or false; we don't need to know if it is true or false, just that it's one or the other.

ex. "*Every continuous function is differentiable*" is a FALSE statement
ex. "*Two plus two equals four*" is a TRUE statement

The words *not*, *and*, *or*, *if* $\cdots$ *then*, and *if and only if* are called **sentential** connectives. The meanings of these are all summarized below:

- **not**: Represents the logical opposite or **negation** of a statement p. This is denoted by $\sim p$
- **and**: Represents the **conjunction** of two statements p and q. This is used in the same way as in English and is denoted by $p \wedge q$
- **or**: Represents the **disjunction** of two statements p and q. This is used only in the inclusive context where there is the possibility of having both statements and is denoted by $p \vee q$
- **if $\cdots$ then**: Represents an **implication** or **conditional** statement. If we say "if p, then q", the if-statement p in the implication is called the **antecedent** and the then-statement q is called the **consequent**. This is denoted by $p \Rightarrow q$. In words, this can be represented in many forms
  - if p, then q
  - p implies q
  - p only if q
  - q if p
  - q provided that p
  - q whenever p
  - p is a sufficient condition for q
  - p is a necessary condition for q
- **if and only if**: Represents the conjunction of two conditional statements, $p \Rightarrow q$ and $q \Rightarrow p$. This is considered a **biconditional** and is denoted by $p \Leftrightarrow q$ which is equivalent to $(p \Rightarrow q) \wedge (q \Rightarrow p)$. If a compound statement is true in all cases, this is called a **tautology**, which shows that the two parts of the biconditional are logically equivalent. That is, the two component statements have the same truth tables.

  ex. $\sim (p \wedge q) \Leftrightarrow [(\sim p) \vee (\sim q)]$
  ex. $\sim (p \vee q) \Leftrightarrow [(\sim p) \wedge (\sim q)]$
  ex. $\sim (p \Rightarrow q) \Leftrightarrow [p \wedge (\sim q)]$

1.2 Quantifiers

Certain sentences need to be considered within a particular context in order to become a statement. When a sentence involves a variable such as x, it is customary to use functional notation when referring to it.

ex. $p(x)$: $x^2 - 5x + 6$

2 The Real Numbers

3 Sequences

4 Limits and Continuity