Question 1 *(20 points)*

(a) What is the binary representation of forty seven (i.e., 47_{10})? Write it using exactly 8 bits.

(b) Compute the sum of these two binary numbers, in binary, showing the carry bits.

$$\begin{array}{r} 101101 \\ +\ 100111 \\ \hline \end{array}$$

(c) Using two's complement with exactly 8 bits, what is the binary representation of negative twenty three? (i.e. -23_{10}).

(d) Using two's complement with 7 bits, what is the largest positive integer that can be represented? What is the smallest (most negative)? Write your answers in base 10.

(1) (a) 47_{10}

last digit
odd = 1
even = 0

$\frac{0}{2^7}\ \frac{0}{2^6}\ \frac{1}{2^5}\ \frac{0}{2^4}\ \frac{1}{2^3}\ \frac{1}{2^2}\ \frac{1}{2^1}\ \frac{1}{2^0}$

$1\cdot 2^5 + 1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$

$32 + 8 + 4 + 2 + 1 = 47$

$\frac{(47-1)}{2}$

$\frac{(23-1)}{2}$

$\frac{(11-1)}{2}$

$\frac{(5-1)}{2}$

$\frac{2}{2}$

$\frac{1-1}{2}$

(b)

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1\ 1\ \ \ } \\ 101101 \\ +\ 100111 \\ \hline 1010100 \end{array}$$

(c) -23_{10}

① find binary of positive

$\frac{0}{2^7}\ \frac{0}{2^6}\ \frac{0}{2^5}\ \frac{1}{2^4}\ \frac{0}{2^3}\ \frac{1}{2^2}\ \frac{1}{2^1}\ \frac{1}{2^0}$

$1\cdot 2^4 + 1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0$

16	4	2	1
			1
		2	1
	4	2	1
16	4	2	1

23

② flip bits of (1)

11101000

③ add 1 to flip version (step 2)

11101000
1
-23 → 11101001
in 8 bits

---

④ 128 # in 7 bits

64 neg, 64 non-neg (including 0)
(-1, ..., -64), (0, ..., 63)

Smallest: $-2^{7-1} = -2^6$ large: $2^{7-1} - 1$

$2^6 - 1 = 63$

-64

n bits
$L = 2^{n-1} - 1$
$S = -2^{n-1}$

---

②

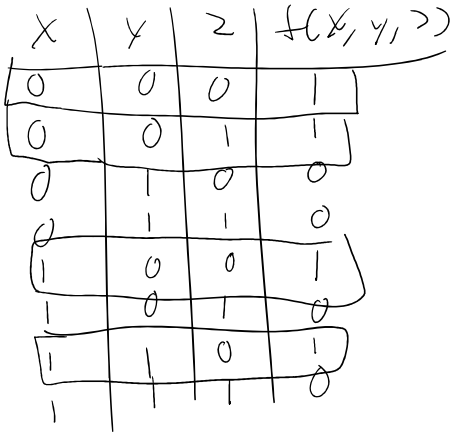
Question 2 *(15 points)*
Here is a table that defines a boolean function f with three inputs.

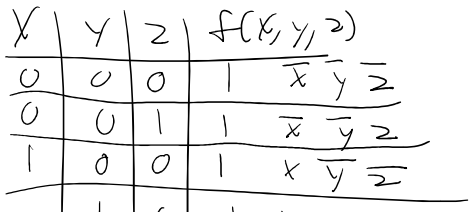
x	y	z	$f(x, y, z)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Complete the following Python implementation of the function, using a single return expression. Your expression should be based on the minterm expansion principle. Use the built-in Python functions "and", "or", "not".

```
def f(x,y,z):

    return ???
```





0	0	1	1	$\bar{x}$	$\bar{y}$	z
1	0	0	1	x	$\bar{y}$	$\bar{z}$
1	1	0	1	x	y	$\bar{z}$

```
def f(x,y,z)
    return (NOT x and NOT y and NOT z) or\
           (NOT x and NOT y and z ) or\
           (x and NOT y and NOT z) or\
           (x and y and NOT z)
```

---

(3)

Question 3 *(15 points)* Using assert statements, implement the following function so that it tests at least four different cases for function `f` from Question 2.

```
def test_f():
```

```
def test_f():
    assert f(0,0,0) == 1
    assert f(0,1,0) == 0
    assert f(1,0,0) == 1
    assert f(1,1,1) == 0
```

---

(4)

Question 4 *(15 points)* What gets printed by this Python code?

```
mem = {('', 'st'): 0, ('', 't'): 0, ('a', ''): 0, ('a', 't'): 0, ('a', 'st'): 0, \
       ('ea', 'est'): 1, ('tea', 'test'): 2}

mem[('a', 't')] = 8

print( ('e', 't') in mem )

print( mem[('ea', 'est')] )

print( mem[('a', 't')] )
```

updates the value

false

1

8

ans: False

1

8

---

(5)

Question 5 *(15 points)*

Complete the following function, using recursion on `L`. That means you can only access `L` using the expressions `L[0]`, `L == []`, and `L[1:]`.

```
def dropWhile(f, L):
    '''
    Assume L is a list and f is a function that returns True or False.
    Discard the elements of L that make f true, up to but not including
    the first element that makes f false.'''
```

For example, `dropWhile(odd,[1,3,2,5,3])` is `[2,5,3]` (assuming `odd` does what its name suggests). Also, the following tests should all print True.

```
def testDropWhile():
    '''Prints True for each successful test.'''
    print( dropWhile(lambda x: x>0, [1,7,0,1,7]) == [0,1,7] )
    print( dropWhile(lambda x: x>0, [-2,1,2]) == [-2,1,2] )
    print( dropWhile(lambda x: x>0, []) == [] )
```

```
def dropWhile(f, L):
    if L == []:
        return []
    else:
        if f(L[0]):
            return dropWhile(f, L[1:])
        else:
            return L
```

Question 6 *(20 points)*

This is about memoization of a function called `LAS` which applies to a string `S` and a letter `ltr`. It returns a *longest ascending subsequence of `S` whose elements are all greater than `ltr`*. Reminder: characters are compared according to their numerical encoding, which agrees with alphabetical order in the case of letters. For example, `'a' < 'b'` is true, and `max('a','b')` returns `'b'`. Study these examples, which are all true:

```
LAS("", 'b') == ""
LAS("bcd", 'd') == ""
LAS("bcd", 'a') == "bcd"
LAS("bbccdd", 'a') == "bcd"
```

The last example shows that which shows this is about *subsequences*, not sub-segments. Note that for `LAS("bbccdd", 'a')`, the specification allows both of these: `"bcd"` `"bdf"`

The code below works correctly, if we delete the ??? parts. *YOUR TASK:* replace each ??? with some code, so that it uses a dictionary as a memo table. For keys, the dictionary should use pairs `(S,ltr)`. Hint: the first ??? should initialize the dictionary.

```
def LAS(S, ltr):
    '''Assume S is string of letters and ltr is a letter.
       Return a longest subsequence of S that is increasing and
       whose elements are all greater than ltr.'''

        ???
```

```
    return a longest subsequence of S that is increasing and
    whose elements are all greater than ltr.'''

    ???

    ???


    if (S,ltr) in Tab: return Tab[(S,ltr)]

    if S == "":
        result = ""
    elif S[0] <= ltr:
        result = LAS(S[1:], ltr)
    else:
        use = S[0] + LAS(S[1:], max(ltr, S[0]))
        lose = LAS(S[1:], ltr)
        if len(use) >= len(lose): result = use
        else: result = lose

    ???


    return result
```

① Tab: {}

```
def LAS(S, ltr):
```

②

```
if (S, ltr) in Tab:
    return Tab[(S, ltr)]

if S == "":
    result = ""

else:
    if S[0] + LAS(S[1:], max(ltr, S[0])):
        result = LAS(S[1:], ltr)
    else:
        use = S[0] + LAS(S[1:], ltr), max(ltr, S[0])
```

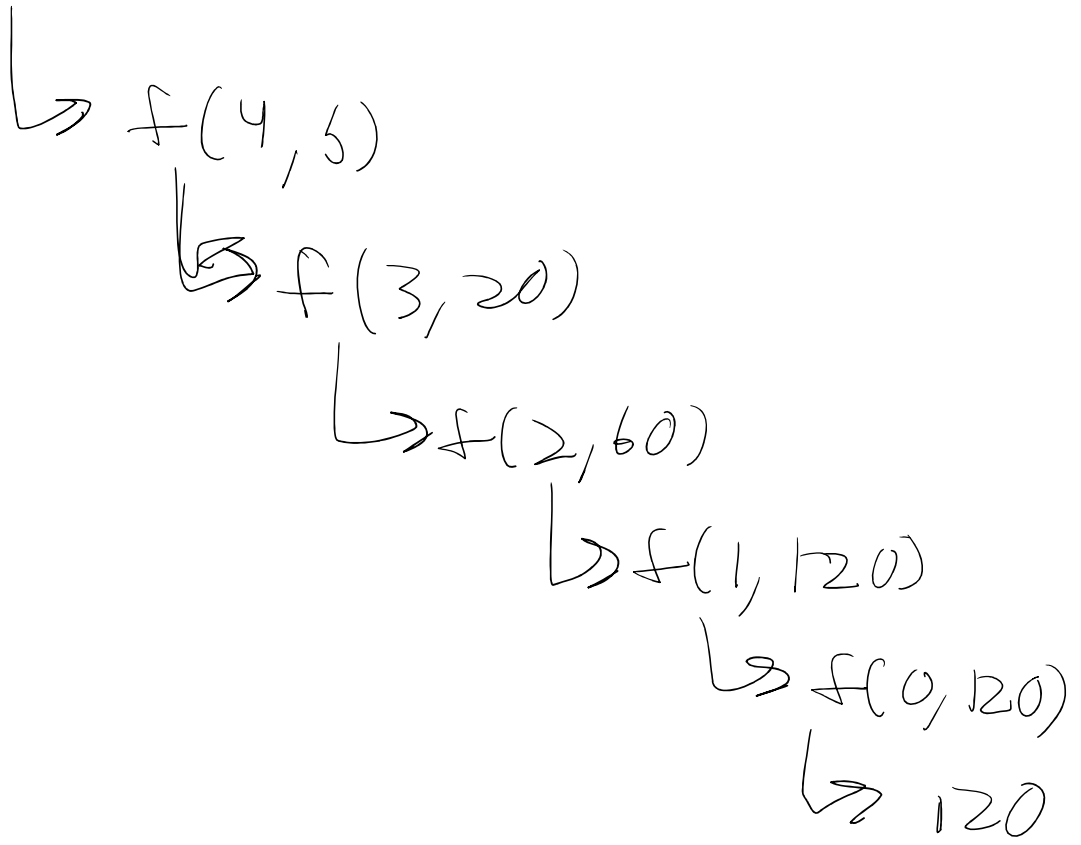
```
lose = LAS(S[:1], ltr)
if len(use) >= len(lose):
        result = use
else:
③       result = lose
Tab L(s, ltr) = result
return result
```

---

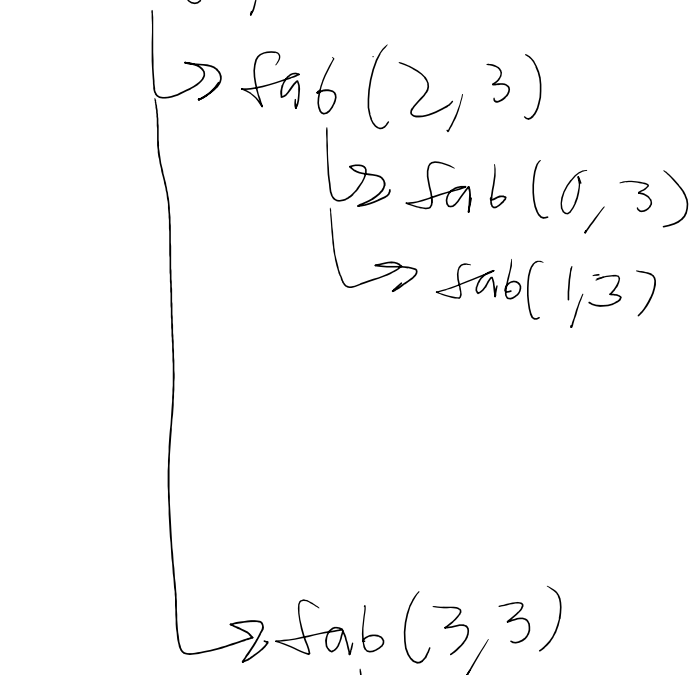
tail recursion $n! = n(n-1) \cdot n(n-2) \cdots n(n-n)$

```
def factorial(n, a=1):
    if n == 0:
        return a
    else:
        return factorial(n-1, a*n)
```

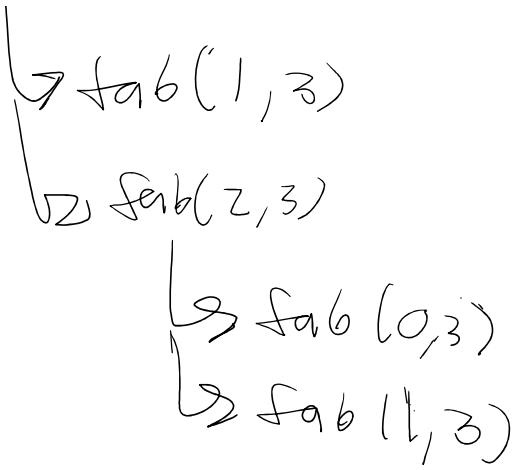
f(5,1)



Trace



See practice test Python file


fab(1,3)
fab(2,3)
fab(0,3)
fab(1,3)