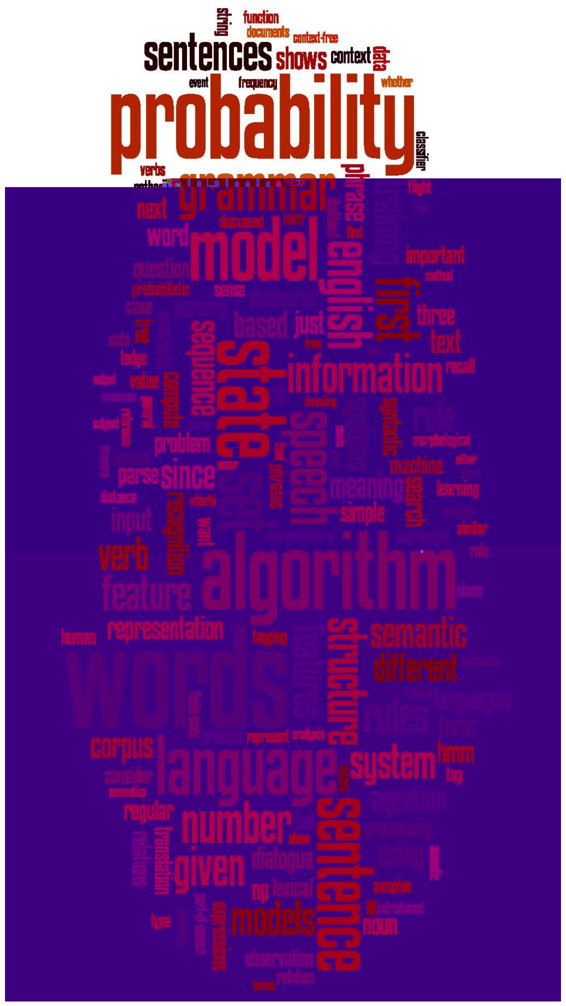

Language Modeling

Probabilistic Language Models

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        - high large
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        - minuets
            - minutes minuets
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Probabilistic Language Modeling

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-

  language model

- **the grammar** **language model** **LM**

How to compute P(W)

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    -
-

Reminder: The Chain Rule

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-
-

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

How to estimate these probabilities

-

$$P(\text{the} \mid \text{its water is so transparent that}) = \frac{Count(\text{its water is so transparent that the})}{Count(\text{its water is so transparent that})}$$

-
-

Markov Assumption

-

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{that})$$

-

$$P(\text{the} \mid \text{its water is so transparent that}) \approx P(\text{the} \mid \text{transparent that})$$

Markov Assumption

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i \mid w_{i-k} \ldots w_{i-1})$$

-

$$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-k} \ldots w_{i-1})$$

Simplest case: Unigram model

$$P(w_1 w_2 \ldots w_n) \approx \prod_i P(w_i)$$

```
fifth, an, of, futures, the, an, incorporated, a,
a, the, inflation, most, dollars, quarter, in, is,
mass

thrift, did, eighty, said, hard, 'm, july, bullish

that, or, limited, the
```

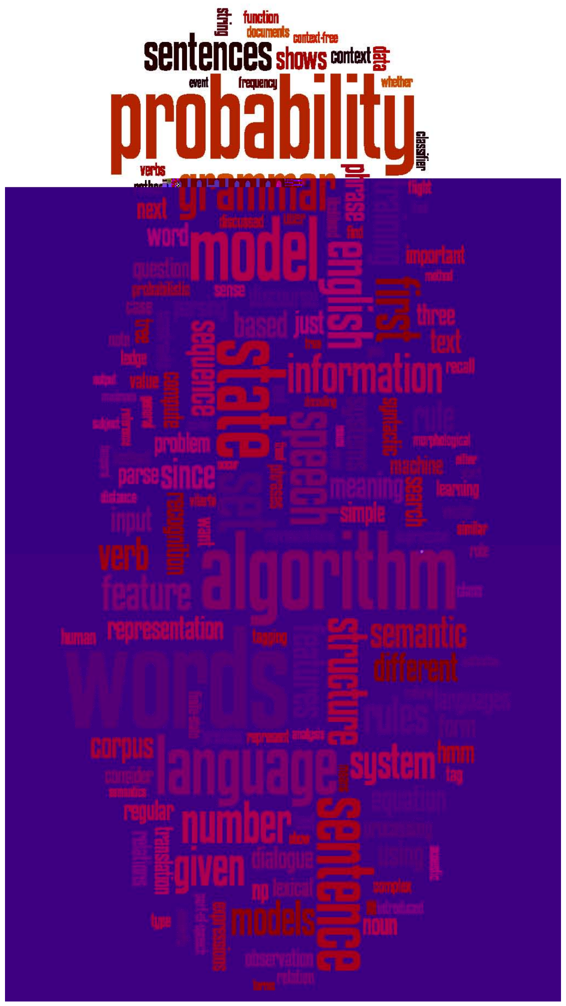
Bigram model

- $$P(w_i \mid w_1 w_2 \ldots w_{i-1}) \approx P(w_i \mid w_{i-1})$$

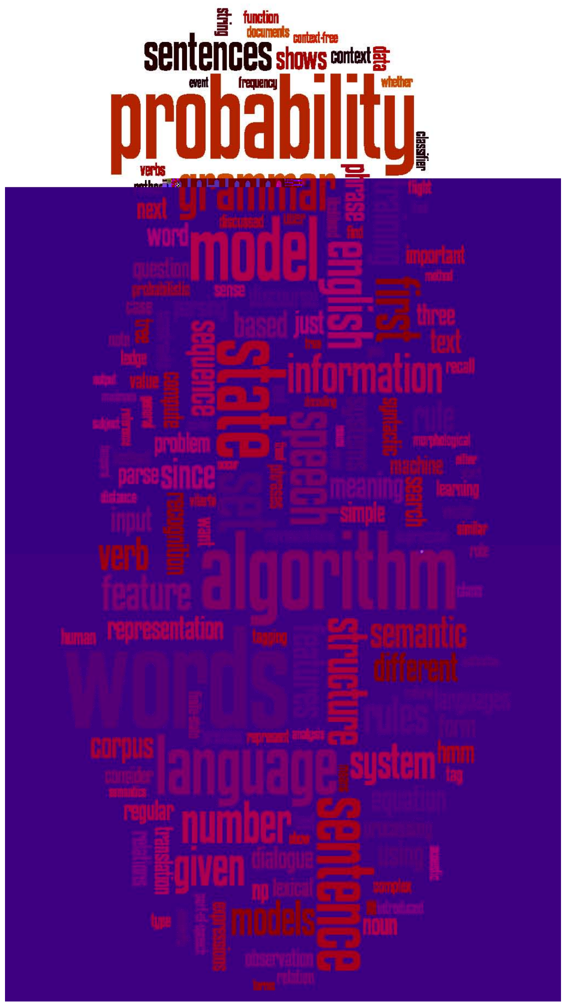
```
texaco, rose, one, in, this, issue, is, pursuing, growth, in,
a, boiler, house, said, mr., gurria, mexico, 's, motion,
control, proposal, without, permission, from, five, hundred,
fifty, five, yen

outside, new, car, parking, lot, of, the, agreement, reached

this, would, be, a, record, november
```

N-gram models

-
-
  - long-distance dependencies
-



Language Modeling



Language Modeling

Estimating bigram probabilities

-

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})}$$

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

$$P(\texttt{I} \mid \texttt{<s>}) = \tfrac{2}{3} = .67 \qquad P(\texttt{Sam} \mid \texttt{<s>}) = \tfrac{1}{3} = .33 \qquad P(\texttt{am} \mid \texttt{I}) = \tfrac{2}{3} = .67$$

$$P(\texttt{</s>} \mid \texttt{Sam}) = \tfrac{1}{2} = 0.5 \qquad P(\texttt{Sam} \mid \texttt{am}) = \tfrac{1}{2} = .5 \qquad P(\texttt{do} \mid \texttt{I}) = \tfrac{1}{3} = .33$$

More examples: Berkeley Restaurant Project sentences

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Raw bigram counts

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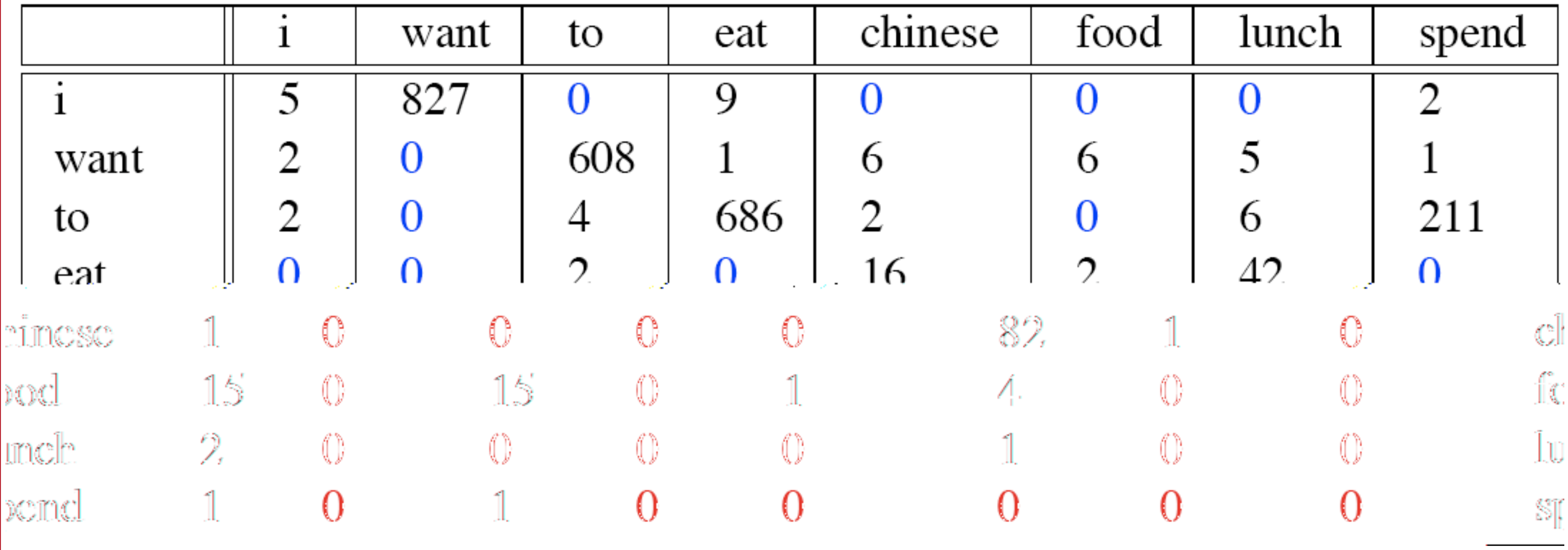

	i	want	to	eat	chinese	food	lunch	spend	
i	5	827	0	9	0	0	0	2	
want	2	0	608	1	6	6	5	1	
to	2	0	4	686	2	0	6	211	
eat	0	0	2	0	16	2	42	0	
rinese	1	0	0	0	0	82	1	0	cl
ood	15	0	15	0	1	4	0	0	fo
unch	2	0	0	0	0	1	0	0	lu
pend	1	0	1	0	0	0	0	0	sp

Raw bigram probabilities

-

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

-

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Bigram estimates of sentence probabilities

What kinds of knowledge?

-
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Practical Issues

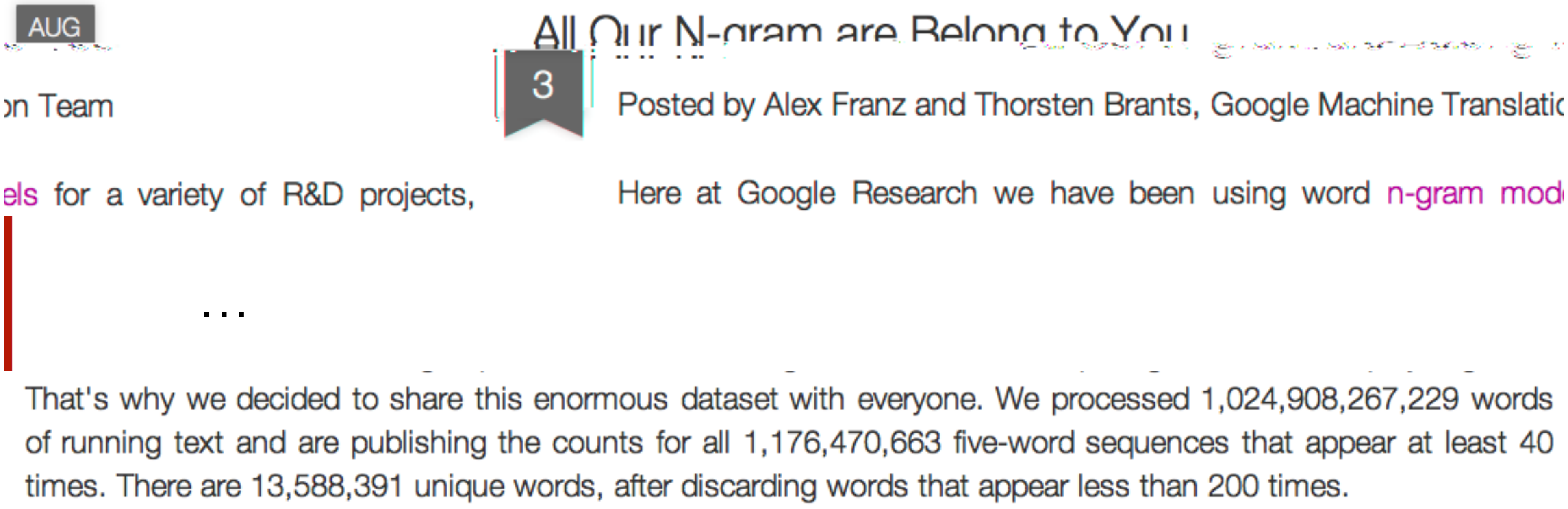
-
    -
    -

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Language Modeling Toolkits

-
    -

Google N-Gram Release, August 2006



AUG

3

All Our N-gram are Belong to You

Posted by Alex Franz and Thorsten Brants, Google Machine Translatic

on Team

Here at Google Research we have been using word n-gram mod

els for a variety of R&D projects,

…

That's why we decided to share this enormous dataset with everyone. We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.

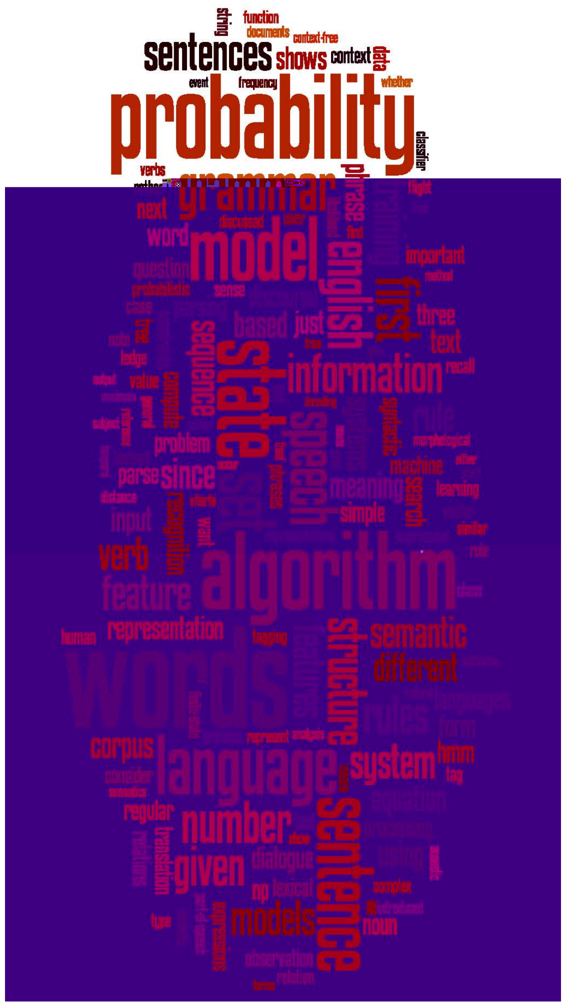
Google N-Gram Release

- serve as the incoming 92
- serve as the incubator 99
- serve as the independent 794
- serve as the index 223
- serve as the indication 72
- serve as the indicator 120
- serve as the indicators 45
- serve as the indispensable 111
- serve as the indispensible 40
- serve as the individual 234

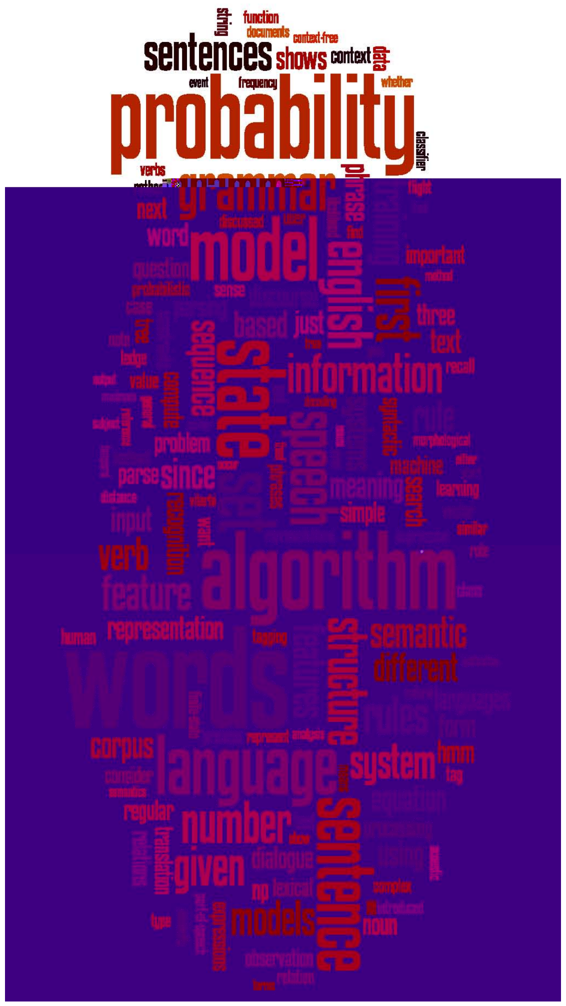
http://googleresearch.blogspot.com/2006/08/all-our-n-gram-are-belong-to-you.html

Google Book N-grams

-



Language Modeling



Language Modeling

Evaluation: How good is our model?

- training set
-
    - test set
    - evaluation metric

Extrinsic evaluation of N-gram models

Difficulty of extrinsic (in-vivo) evaluation of N-gram models

-
  -
-
  - intrinsic perplexity
  -
    - just
    - generally only useful in pilot experiments
  -

Intuition of Perplexity

m shrooms 0.1

pepperoni 0.1

ancho ies 0.01

….

fried rice 0.0001

….

and 1e-100

Perplexity

•

$$PP(W) = P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}}$$

$$PP(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}$$

$$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability

The Shannon Game intuition for perplexity

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    -
-

Perplexity as branching factor

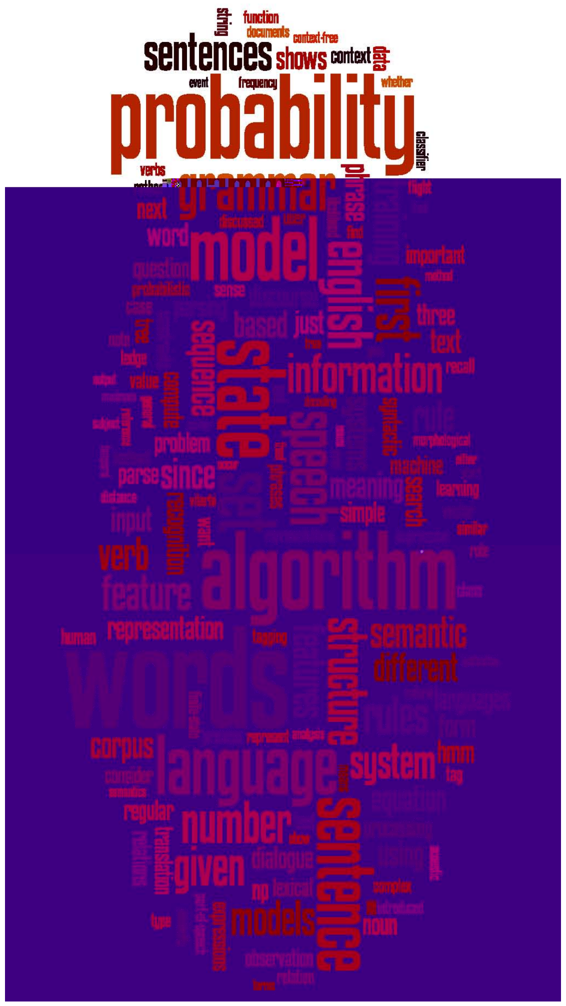
-
-

$$\begin{aligned} \mathrm{PP}(W) &= P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}} \\ &= (\frac{1}{10}^{N})^{-\frac{1}{N}} \\ &= \frac{1}{10}^{-1} \\ &= 10 \end{aligned}$$

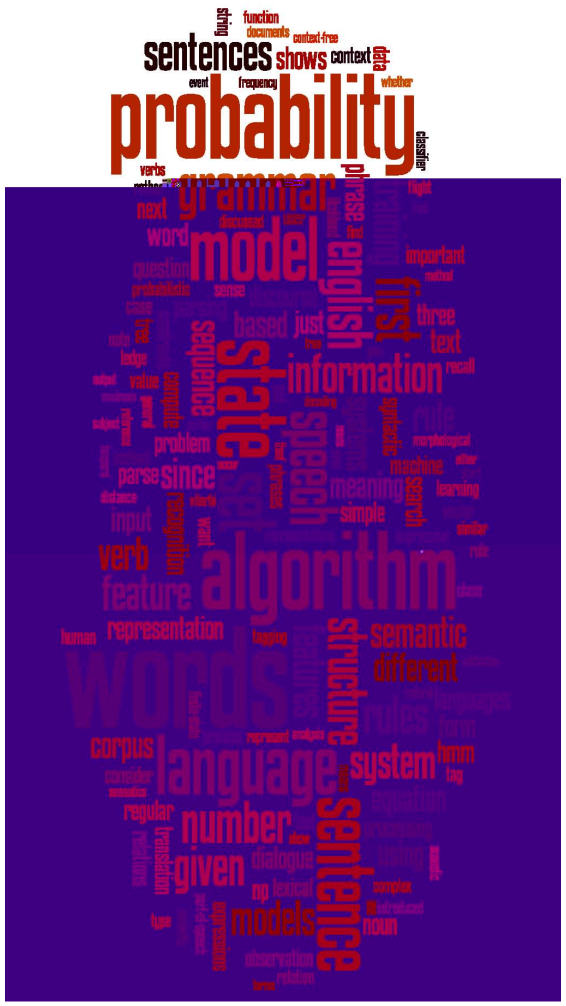
Lower perplexity = better model

-

N-gram Order	Unigram	Bigram	Trigram



Language Modeling

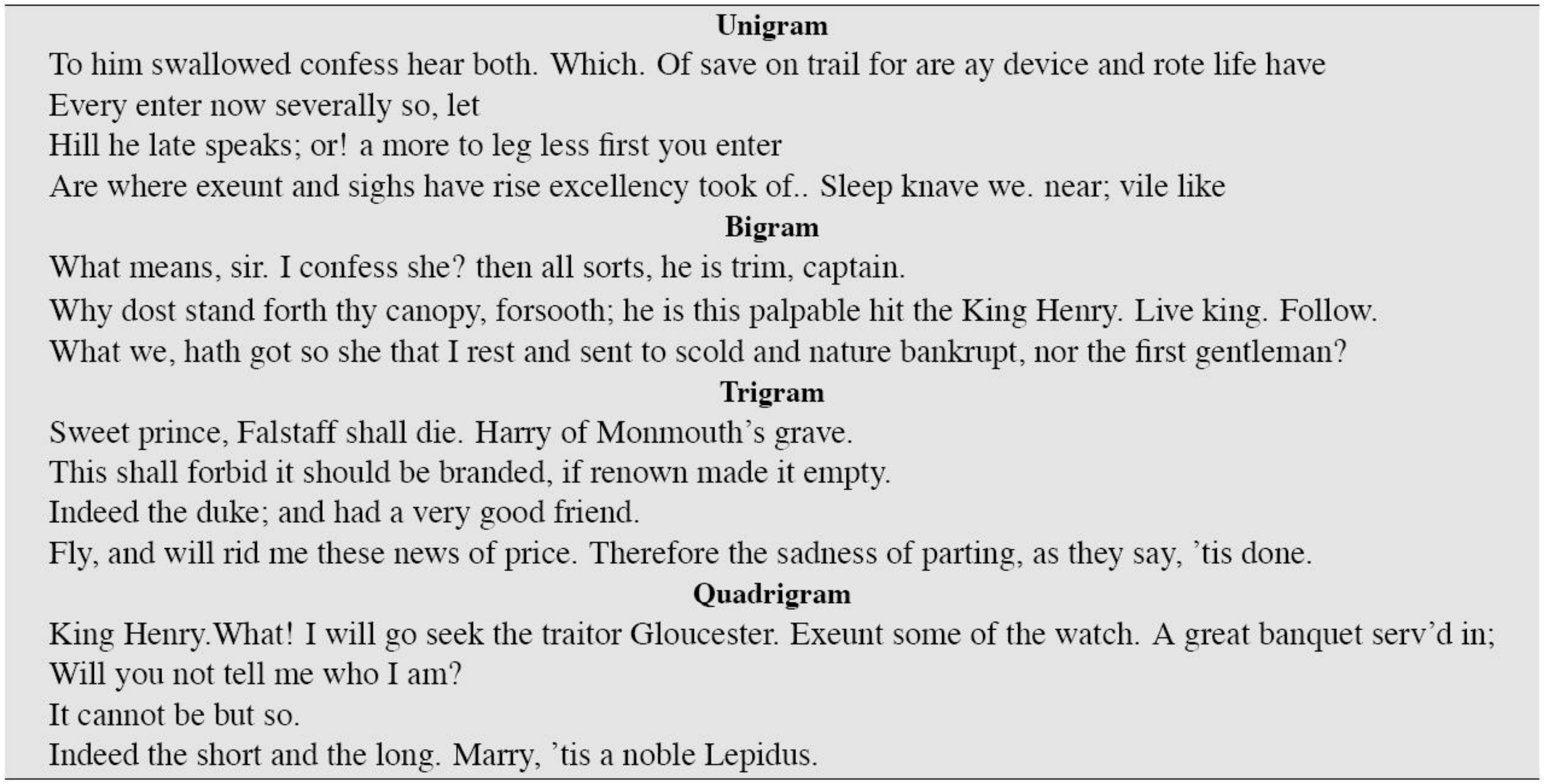


Language Modeling

The Shannon Visualization Method

```
<s> I
    I want
      want to
           to eat
              eat Chinese
                  Chinese food
                          food  </s>
I want to eat Chinese food
```

Approximating Shakespeare



Unigram

To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have
Every enter now severally so, let
Hill he late speaks; or! a more to leg less first you enter
Are where exeunt and sighs have rise excellency took of.. Sleep knave we. near; vile like

Bigram

What means, sir. I confess she? then all sorts, he is trim, captain.
Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow.
What we, hath got so she that I rest and sent to scold and nature bankrupt, nor the first gentleman?

Trigram

Sweet prince, Falstaff shall die. Harry of Monmouth's grave.
This shall forbid it should be branded, if renown made it empty.
Indeed the duke; and had a very good friend.
Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, 'tis done.

Quadrigram

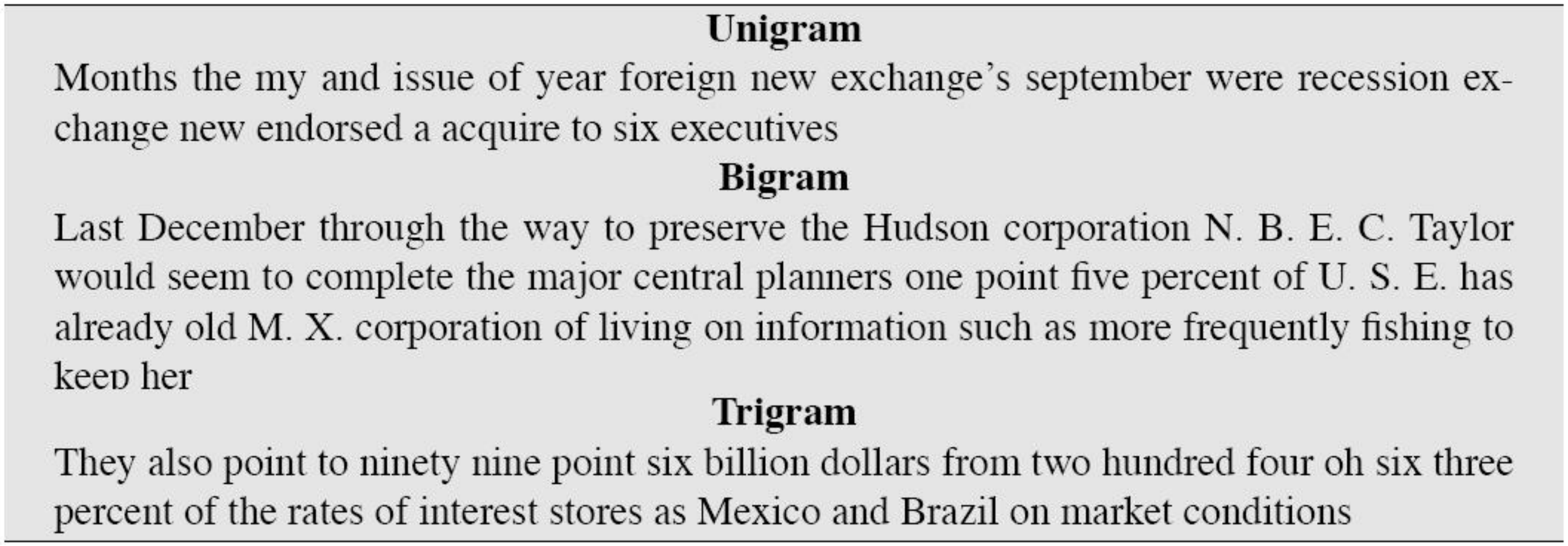
King Henry.What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv'd in;
Will you not tell me who I am?
It cannot be but so.
Indeed the short and the long. Marry, 'tis a noble Lepidus.

Shakespeare as corpus

-
-
    -
-

is

The wall street journal is not shakespeare (no offense)


Unigram

Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

Bigram

Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

Trigram

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

The perils of overfitting

Zeros

Zero probability bigrams

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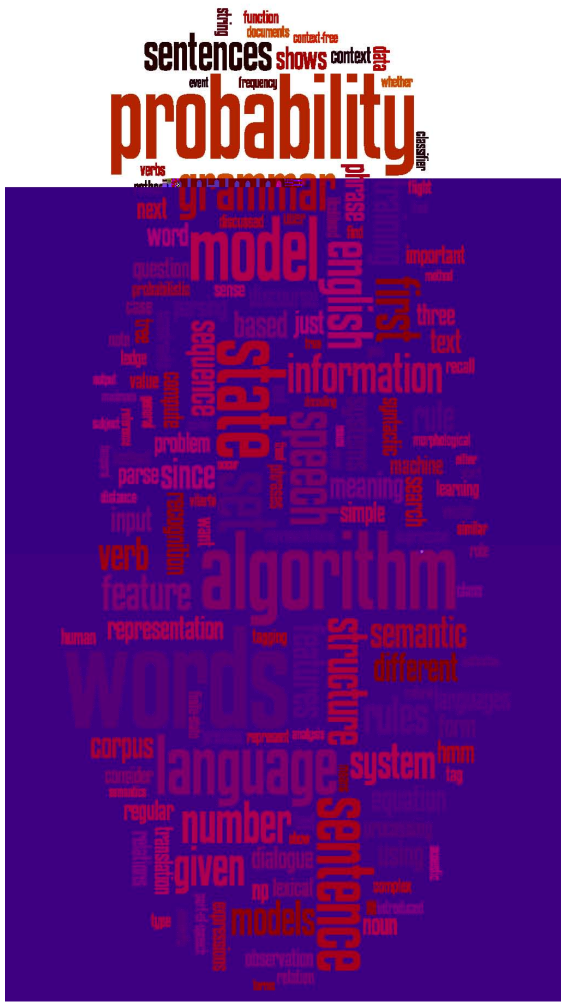
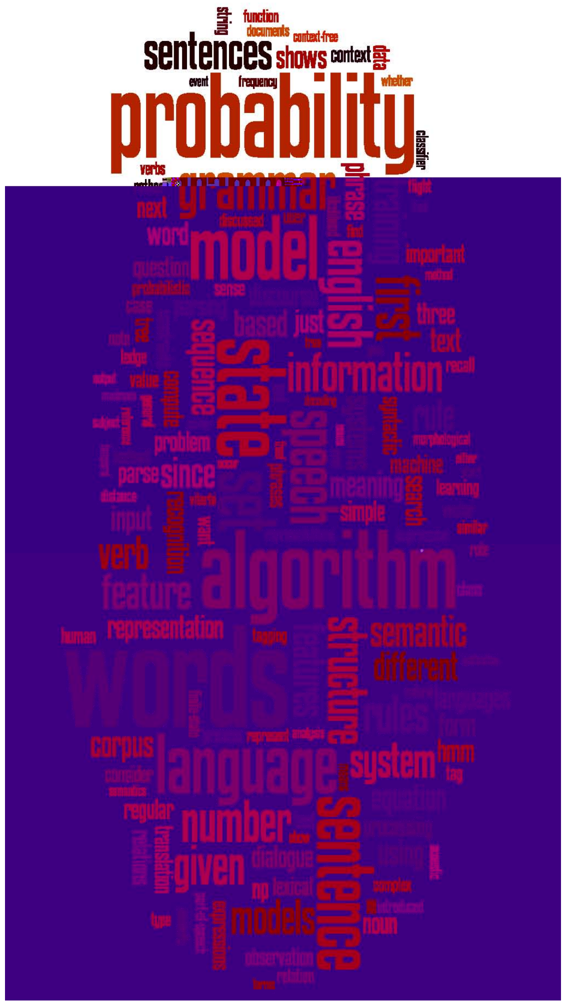


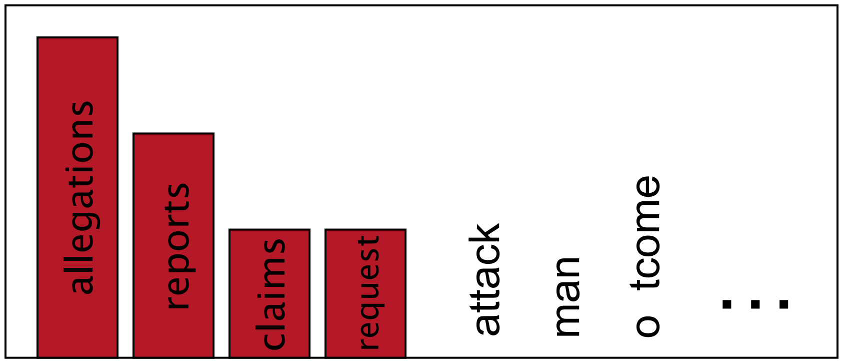
Language Modeling



Language Modeling

The intuition of smoothing (from Dan Klein)

-



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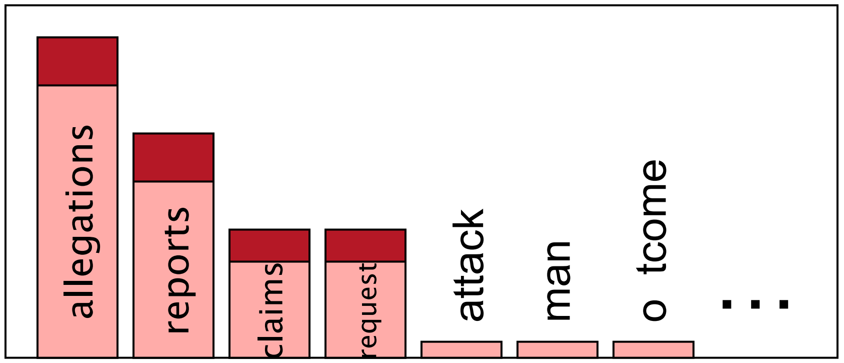

Add-one estimation

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

Maximum Likelihood Estimates

- **estimate** **most likely**

Berkeley Restaurant Corpus: Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n)+1}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Reconstituted counts

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n)+1] \times C(w_{n-1})}{C(w_{n-1})+V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
inese	1	0	0	0	0	82	1	0
ood	15	0	15	0	1	4	0	0
unch	2	0	0	0	0	1	0	0
pend	1	0	1	0	0	0	0	0

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-1 estimation is a blunt instrument

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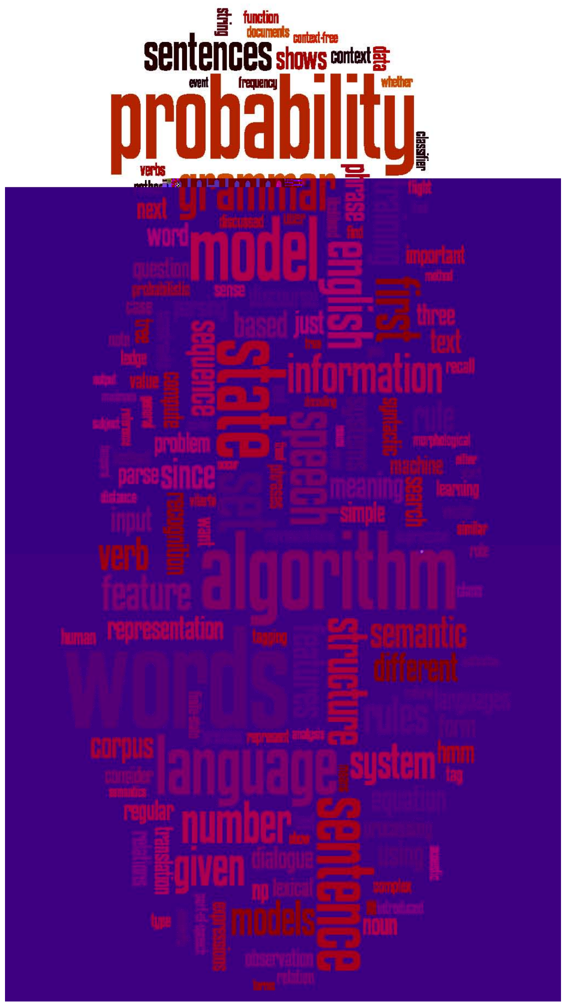
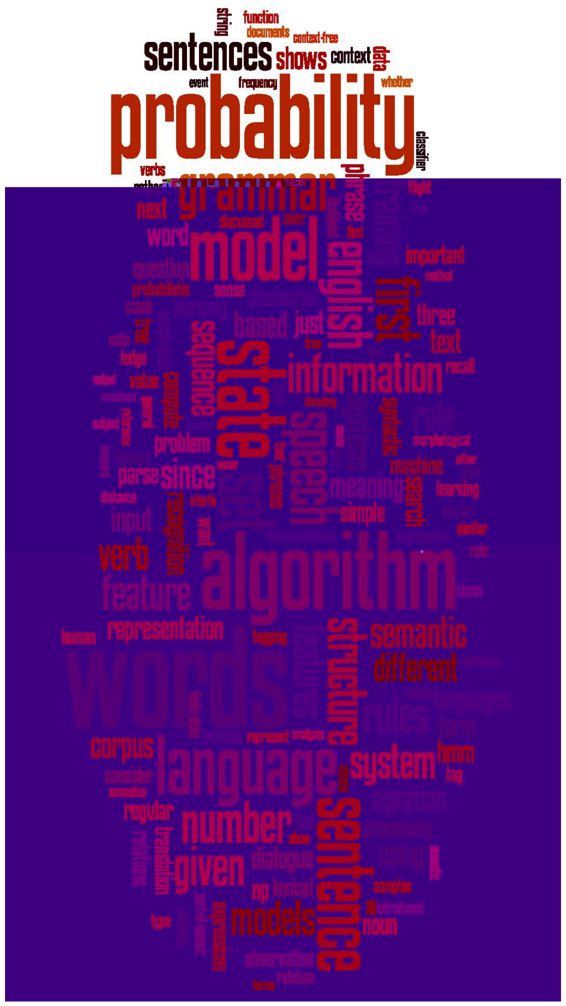


Language Modeling



Language Modeling

Backoff and Interpolation

- less
- **Backoff:**
- **Interpolation:**

Linear Interpolation

-

$$\begin{aligned}\hat{P}(w_n|w_{n-1}w_{n-2}) &= \lambda_1 P(w_n|w_{n-1}w_{n-2}) \\ &+\lambda_2 P(w_n|w_{n-1}) \\ &+\lambda_3 P(w_n)\end{aligned}$$

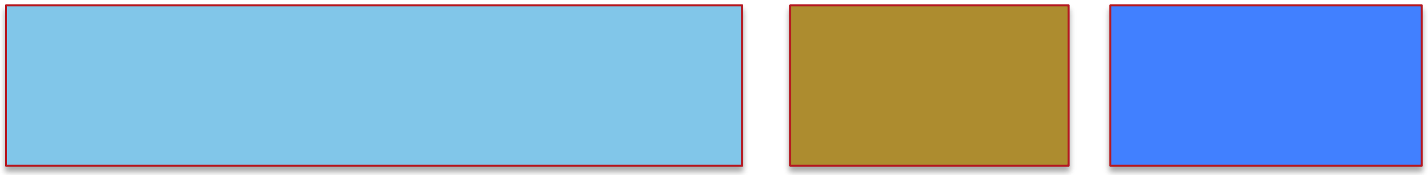
$$\sum_i \lambda_i = 1$$

-

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) &= \lambda_1(w_{n-2}^{n-1}) P(w_n|w_{n-2}w_{n-1}) \\ &+\lambda_2(w_{n-2}^{n-1}) P(w_n|w_{n-1}) \\ &+\lambda_3(w_{n-2}^{n-1}) P(w_n)\end{aligned}$$

How to set the lambdas?

- held-out



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$$\log P(w_1 \ldots w_n \mid M(\lambda_1 \ldots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \ldots \lambda_k)}(w_i \mid w_{i-1})$$

Unknown words: Open versus closed vocabulary tasks

- Out Of Vocabulary

Huge web-scale n-grams

Smoothing for Web-scale N-grams

- *et al*
-

$$S(w_i \mid w_{i-k+1}^{i-1}) = \begin{cases} \dfrac{\text{count}(w_{i-k+1}^{i})}{\text{count}(w_{i-k+1}^{i-1})} & \text{if } \ \text{count}(w_{i-k+1}^{i}) > 0 \\ 0.4 S(w_i \mid w_{i-k+2}^{i-1}) & \text{otherwise} \end{cases}$$

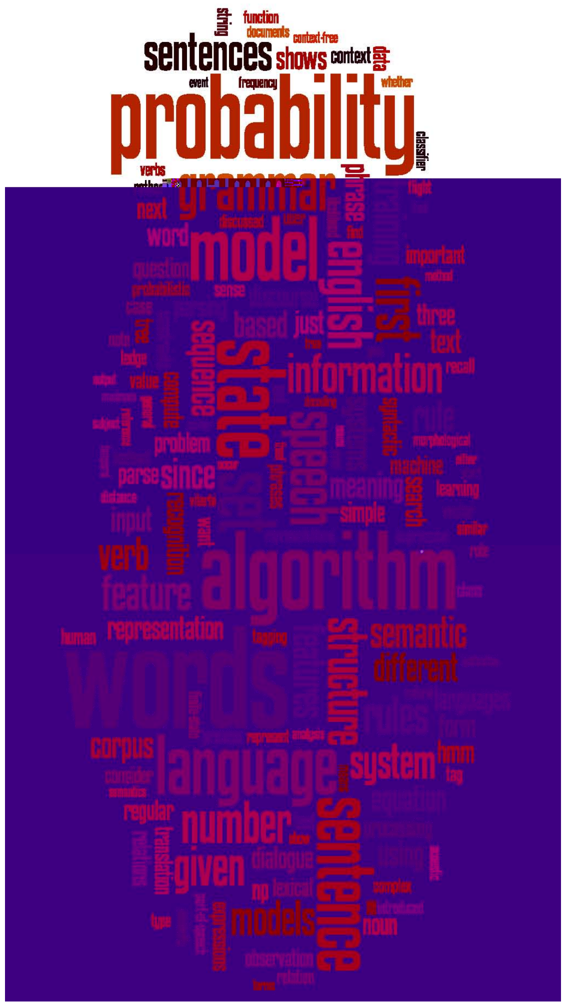
$$S(w_i) = \frac{\text{count}(w_i)}{N}$$

N-gram Smoothing Summary

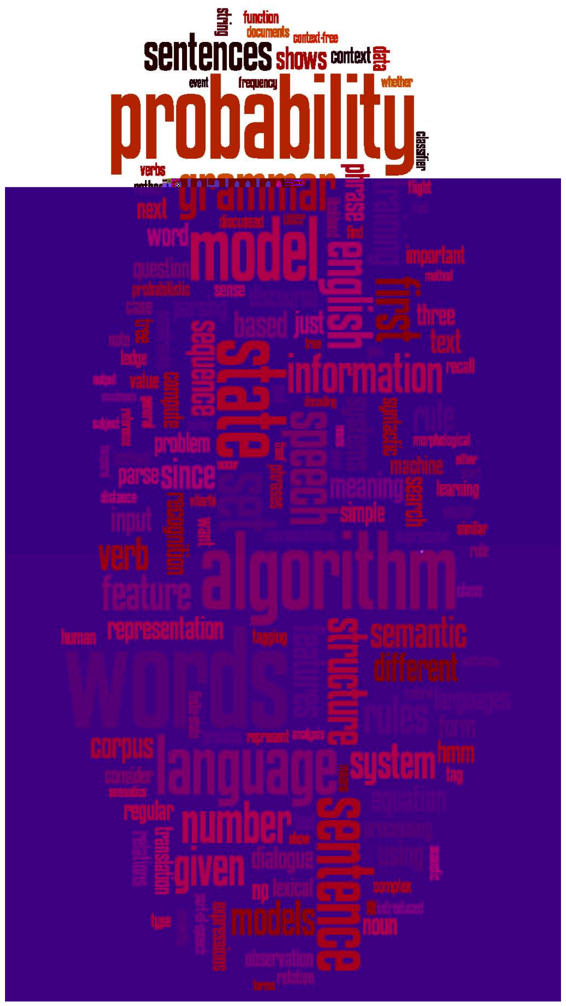
-
  -
-
  -
-
  -

Advanced Language Modeling

$$P_{CACHE}(w \mid history) = \lambda P(w_i \mid w_{i-2} w_{i-1}) + (1-\lambda) \frac{c(w \in history)}{\mid history \mid}$$



Language Modeling



Language Modeling

Advanced: Good Turing Smoothing

Reminder: Add-1 (Laplace) Smoothing

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

More general formulations: Add-k

$$P_{Add-k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + k}{c(w_{i-1}) + kV}$$

$$_{-}\,(\quad \mid \quad _{-1}) = \frac{(\quad _{-1}, \quad) + \quad (\frac{1}{\quad})}{(\quad _{-1}) + }$$

Unigram prior smoothing

$$_{-}(\quad|\quad_{-1}) = \frac{(\quad_{-1},\quad)+\quad(\frac{1}{\quad})}{(\quad_{-1})+}$$

$$P_{\text{Unig amP i}}(w_i \quad w_{i-1}) = \frac{c(w_{i-1},w_i)+mP(w_i)}{c(w_{i-1})+m}$$

Advanced smoothing algorithms

- seen once
  - never seen

Notation: N_c = Frequency of frequency c

-
-

I	3
sam	2
am	2
do	1
not	1
eat	1

Good-Turing smoothing intuition

-
    -
-
    -
-
    -
    -
-
    -
    -

Good Turing calculations

$$P^*_{GT}(\text{things with zero frequency}) = \frac{N_1}{N} \qquad c^* = \frac{(c+1)N_{c+1}}{N_c}$$

- Unseen (bass or catfish)
  - c = 0:
  - MLE p = 0/18 = 0

  - P^*_{GT} (nseen) = N_1/N = 3/18

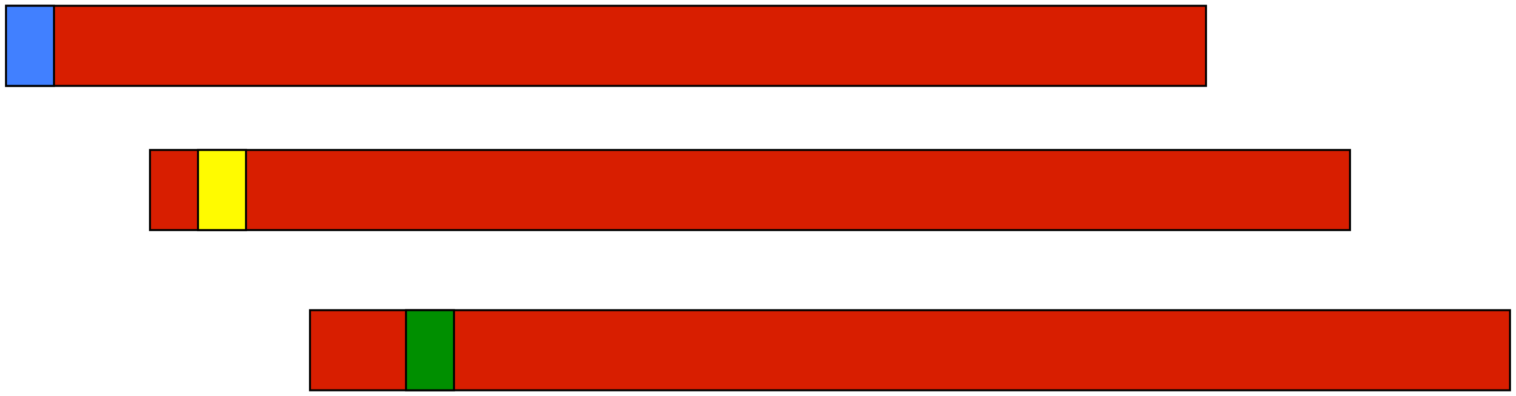
- Seen once (tro t)
  - c = 1
  - MLE p = 1/18

  - C*(tro t) = 2 * N2/N1

    = 2 * 1/3

    = 2/3
  - P^*_{GT}(tro t) = 2/3 / 18 = 1/27

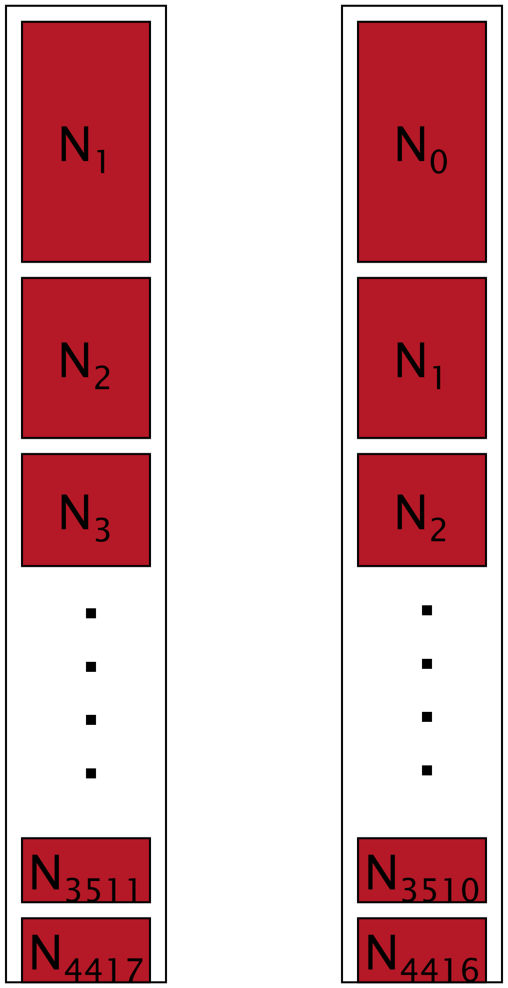
Ney et al.'s Good Turing Intuition



Held-out words:

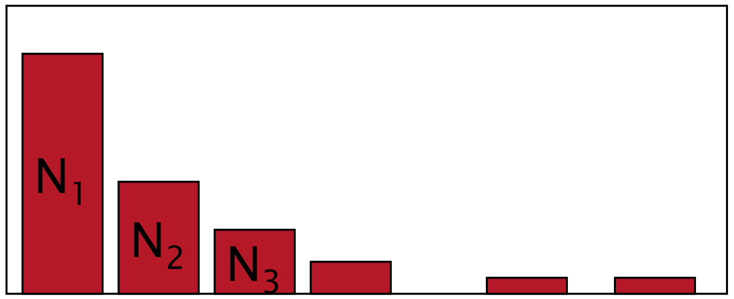
Ney *et al.* Good Turing Intuition (slide from Dan Klein)

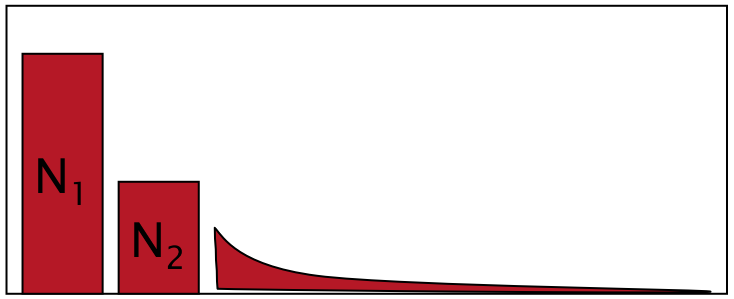
-
  - c
  - c c
  -
    - N c
  - k
    - k N_k c
  - k N_k c k
  - N_k k
  -
    - k N_k c N_k
  - $$k^* = \frac{(k+1)N_{k+1}}{N_k}$$



Good-Turing complications

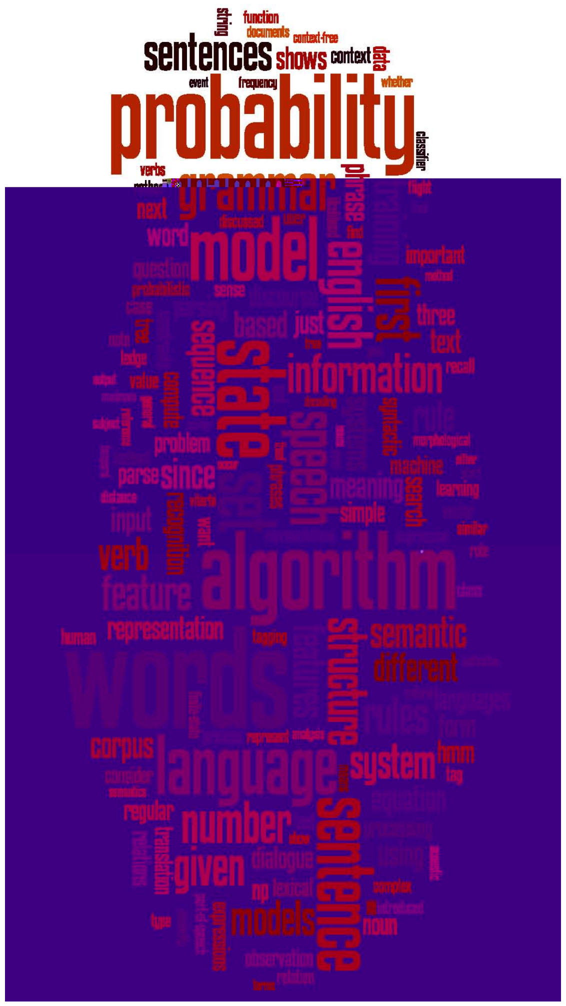
(slide from Dan Klein)





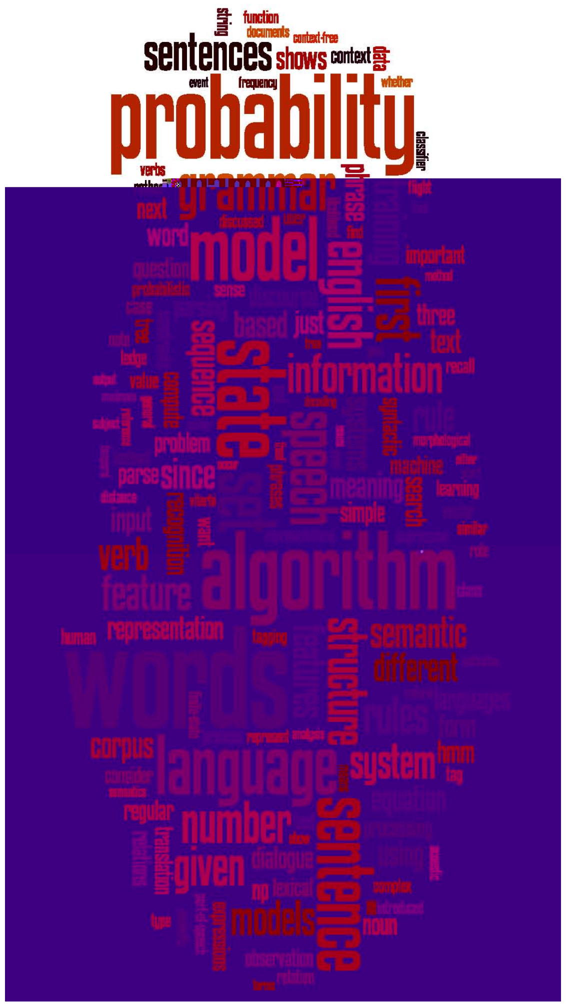
Resulting Good-Turing numbers

$$c^* = \frac{(c+1)N_{c+1}}{N_c}$$



Language Modeling

Advanced: Good Turing Smoothing



Language Modeling

Advanced: Kneser-Ney Smoothing

Resulting Good-Turing numbers

$$c^* = \frac{(c+1)N_{c+1}}{N_c}$$

Absolute Discounting Interpolation

•

$$P_{\text{AbsoluteDiscounting}}(w_i \mid w_{i-1}) = \overbrace{\frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})}}^{\text{discounted bigram}} + \overset{\text{Interpolation weight}}{\lambda(w_{i-1})} \underset{\text{unigram}}{P(w)}$$

•

•

Kneser-Ney Smoothing I

- *I can't see without my reading*____________ *Francisco* *glasses*

$$P_{CONTINUATION}(w) \propto \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

Kneser-Ney Smoothing II

- $$P_{CONTINUATION}(w) \propto \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$
- $$\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|$$

$$P_{CONTINUATION}(w) = \frac{\left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|}{\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|}$$

Kneser-Ney Smoothing III

$$|\{w_{i-1} : c(w_{i-1}, w) \quad 0\}|$$

$$P_{CONTINUATION}(w) = \frac{\left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|}{\sum_{w'} \left|\{w'_{i-1} : c(w'_{i-1}, w') > 0\}\right|}$$

Kneser-Ney Smoothing IV

$$P_{KN}(w_i \quad w_{i-1}) = \frac{\mathrm{ma}\ \ (c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1}) P_{CONTINUATION}(w_i)$$

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} \left| \{ w : c(w_{i-1}, w) > 0 \} \right|$$

the normalized discount

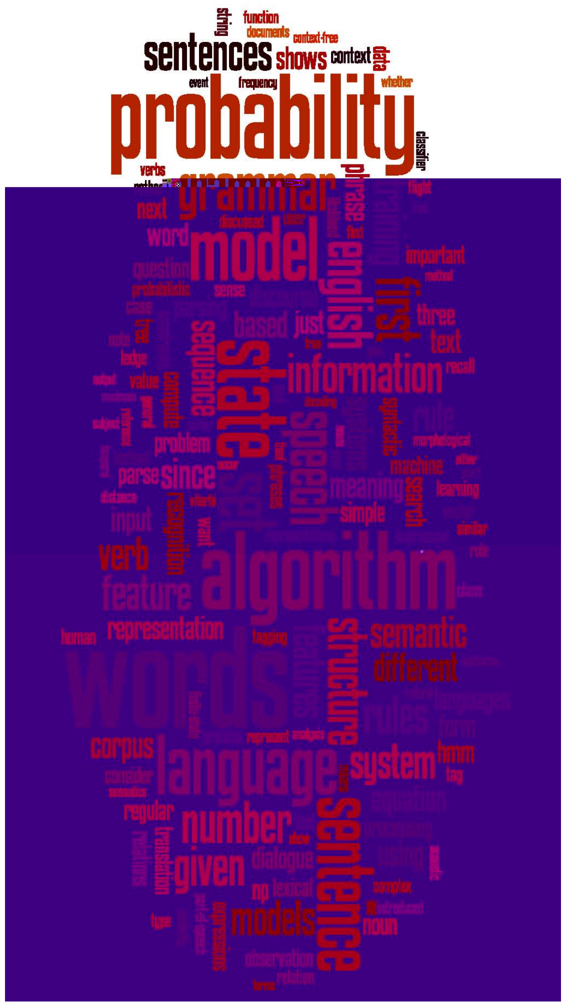
The number of word types that can follow w_{i-1}
= # of word types we discounted
= # of times we applied normalized discount

Kneser-Ney Smoothing: Recursive formulation

$$P_{KN}(w_i \mid w_{i-n+1}^{i-1}) = \frac{\max(c_{KN}(w_{i-n+1}^{i}) - d, 0)}{c_{KN}(w_{i-n+1}^{i-1})} + \lambda(w_{i-n+1}^{i-1}) P_{KN}(w_i \mid w_{i-n+2}^{i-1})$$

$$c_{KN}(\bullet) = \begin{cases} count(\bullet) & \text{for the highest order} \\ continuationcount(\bullet) & \text{for lower order} \end{cases}$$

Continuation count = Number of unique single word contexts for •



Language Modeling

Advanced: Kneser-Ney Smoothing