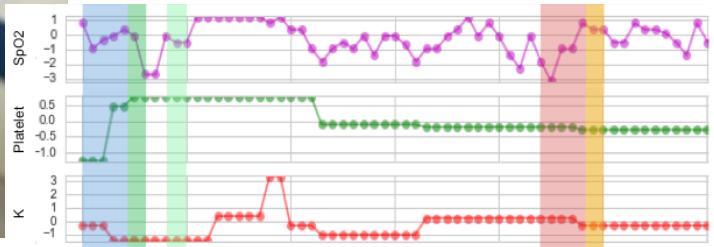
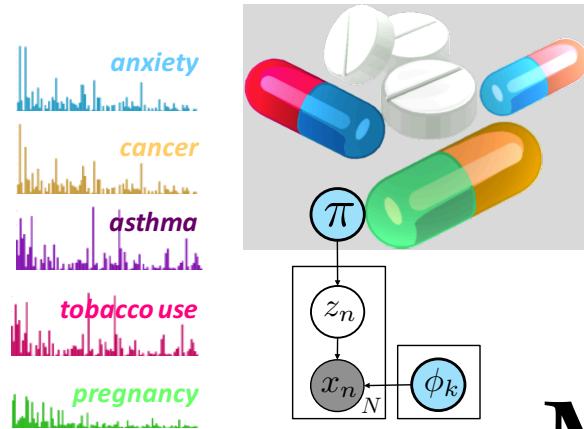


# Overcoming Misspecification with Prediction Constrained Probabilistic Models



## Mike Hughes

Assistant Professor of Computer Science, Tufts University

joint work with

Finale Doshi-Velez & Joe Futoma (Harvard)

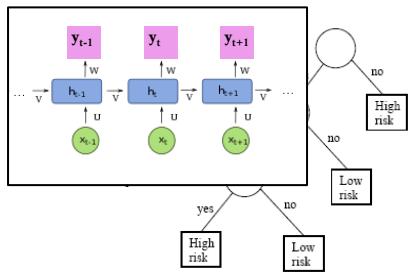
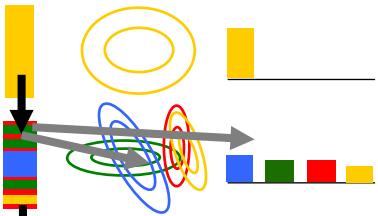
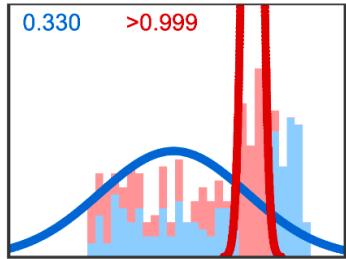
Erik Sudderth & Gabe Hope (UC Irvine)

slides / papers / code  
[www.michaelchughes.com](http://www.michaelchughes.com)

# Hughes Lab @ Tufts CS

Area: **statistical machine learning; clinical informatics**

Lab goal: *Reliable training of interpretable models for real-world decisions*



## New Training Goal: “Prediction-Constrained”

Avoids Model Misspecification for Decision Task via (Rare) Labels

- Semi-supervised topic models
- End-to-end training of POMDPs for reinforcement learning

*Hughes et al. AISTATS 2018*

*Futoma, Hughes, et al. AISTATS 2020*

## New Variational Algorithm: Scalable yet Reliable

Adapt Model Size to Data (Bayesian Nonparametrics)

- Add clusters during training
- Topic models for news articles
- HMMs for mocap and genomics
- Image composition models
- Speed-up model comparison

*Hughes & Sudderth NeurIPS 2013*

*Hughes, Kim & Sudderth AISTATS 2015*

*Hughes et al., NeurIPS 2015*

*Ji, Hughes, & Sudderth ICML 2017*

*Zhang & Hughes, AABI 2019*

## New Training Objectives for Deep Neural Nets

Optimize for interpretability, don't just interpret afterwards

- Find diverse explanations
- Find tree-like neural nets

*Ross, Hughes, Doshi-Velez IJCAI 2017*

*Wu, Hughes, Parbhoo, et al. AAAI 2018*

*Wu, Parbhoo, Hughes et al. AAAI 2020*

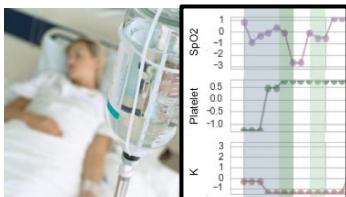
BNP Statistical Models : [github . com/bnpy/bnpy](https://github.com/bnpy/bnpy)

Time-series Prediction: [github . com/tufts-ml/time\\_series\\_predict](https://github.com/tufts-ml/time_series_predict)

# Hughes Lab @ Tufts CS

Area: statistical machine learning; **clinical informatics**

Lab goal: *Reliable training of interpretable models for real-world decisions*



## Personalize treatments for major depression

- Discover subtypes and best treatments with topic models

*Hughes et al. AISTATS 2018*

*Hughes et al. in submission to JAMA Open*  
Drs. McCoy and R. Perlis (MGH/Harvard)

## Personalize treatments in the Intensive Care Unit

- Suggest interventions
- Address non-stationarity features

*Ghassemi et al. AMIA CRI 2017*

*Nestor et al. MLHC 2019*

Drs. R. Kindle and L. Celi (Beth Israel)

## Predict mortality from chemotherapy for leukemia

- Balance costs to decide who gets high-risk treatment

*Siddiqui et al. Amer. Soc. Hematology 2019*

Drs. N. Siddiqui, A. Klein, et al. (Tufts Med.)

## Detect heart disease from few labeled images

*In progress, Dr. Ben Wessler (Tufts Med.)*

## Predict individual treatment effects from drug trials

*In progress, Dr. David Kent (Tufts Med.)*

# Roadmap

- Motivation: Improve interventions in ICU
- Models for Clustering Structured Data
- Method: Prediction-Constrained Training  
*Hughes et al. AISTATS 2018*
- Prediction-Constrained HMMs  
*Hope, Hughes, Sudderth, et al. In Progress*
- Prediction-Constrained POMDPs  
*Futoma, Hughes, Doshi-Velez AISTATS 2020*

# Problem: When will ICU patient need intervention?

*Ghassemi, Wu, Hughes, et al. AMIA CRI 2017*

Interventions:

- Ventilators to assist breathing
- Drugs to manage blood pressure

Early prediction helps:  
prepare patient  
plan staffing  
try less aggressive options early



# Data: ~30,000 ICU patients

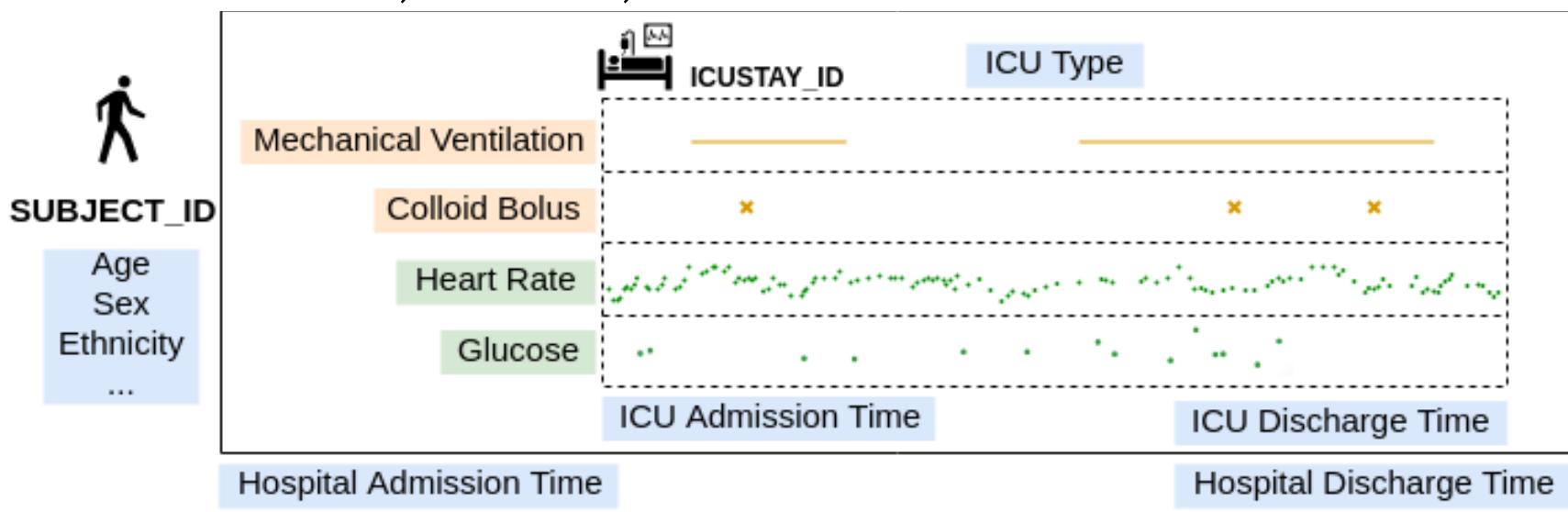
[mimic.physionet.org](https://mimic.physionet.org)  
*(Johnson et al. Sci. Data 2016)*

Nurse-validated vital signs (irregular, hourly)

heart rate, blood pressure, temp., SpO<sub>2</sub>, ...

Lab measurements (irregular, every few hours)

hematocrit, lactate, ...



# Key Goals for our Model

- How should we deal with missing data values?
  - We cannot draw blood every hour
- How to deal with missing labels?
  - Most patients never get some treatments of interest
- Punchline:  
Model is always wrong... **Is it sometimes useful?**

Can we **adjust fitting procedure** to make more useful?

# Approach: Model data and labels with a joint probabilistic graphical model

$$p(x, y) = p(y|x)p(x)$$

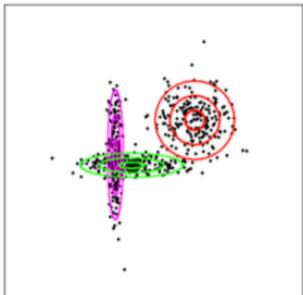
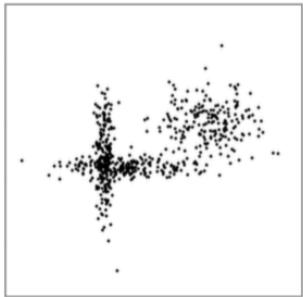
data    labels

Why a joint model?

- $p(x)$  can help us reason about missing data
- $p(y | x)$  can help us predict labels from data
  - even if some labels are missing from training set
- Tying these together makes
  - All parts work in unison
  - Simplifies training: solve one problem, not several disconnected pieces

# Structured Clustering Models

## Mixture Models



## Topic Models

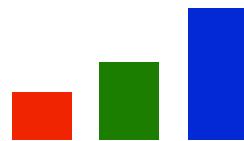


## Hidden Markov Models

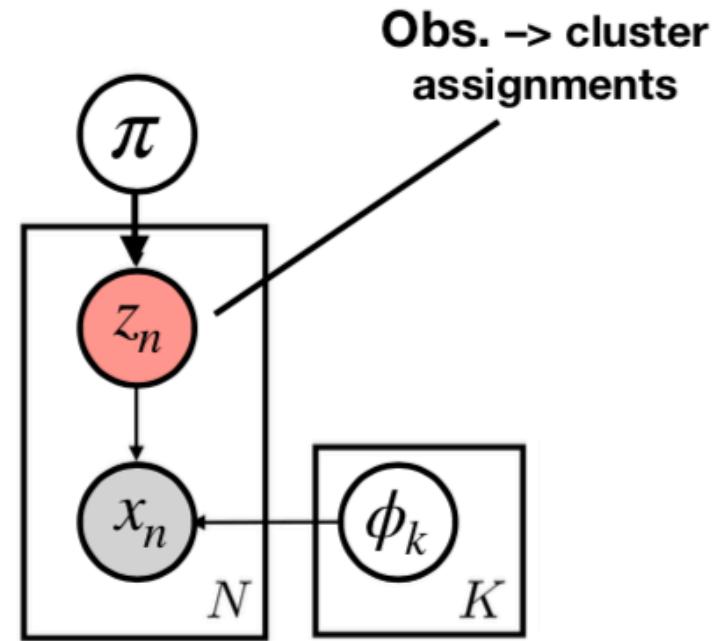
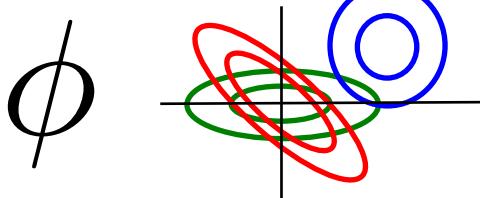


# Simple case: Gaussian Mixture

cluster frequency  $\pi$



cluster shape  $\phi$



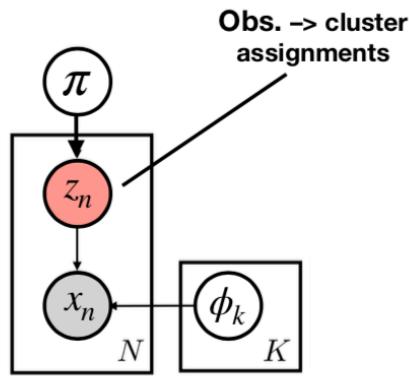
$$\phi_k = \{\mu_k, \sigma_k^2\}$$

obs.-to-cluster assignment  $z_n \sim \text{Discrete}(\pi_1, \dots, \pi_K)$

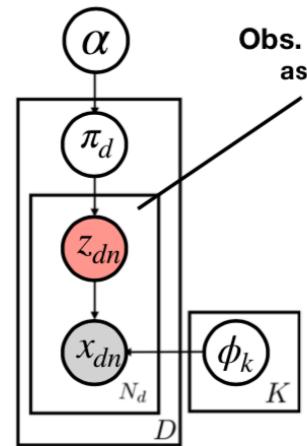
observed data  $x_n | z_n = k \sim \text{Normal}(\mu_k, \sigma_k^2)$

# Z: Per-Example Membership

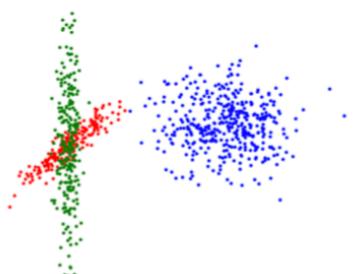
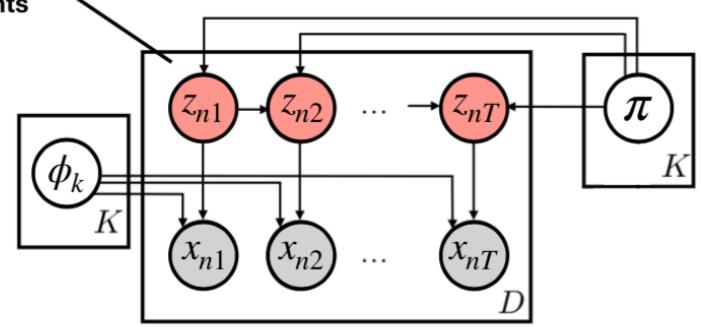
## Mixture Models



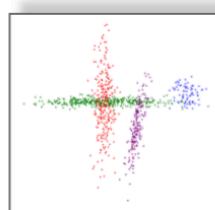
## Topic Models



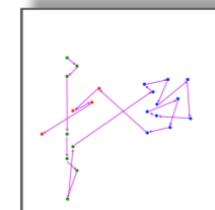
## Hidden Markov Models



(Generic data)



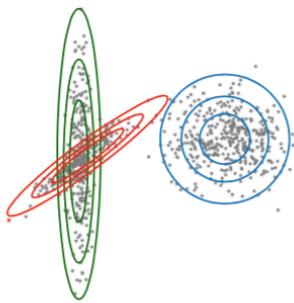
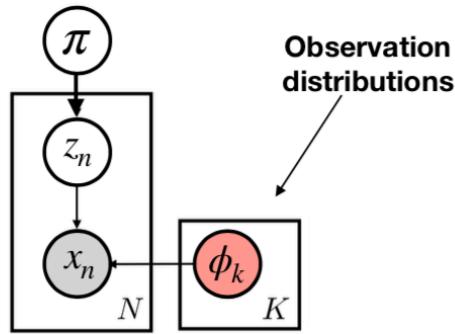
(Grouped data)



(Sequence data)

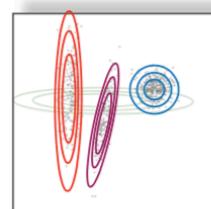
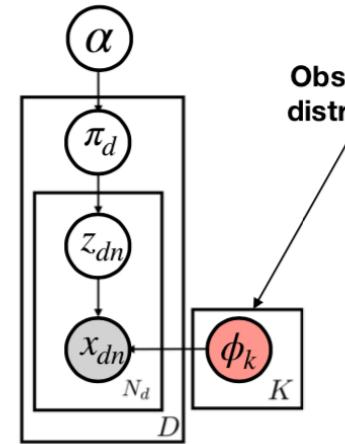
# Phi: Cluster Emission Parameters

## Mixture Models



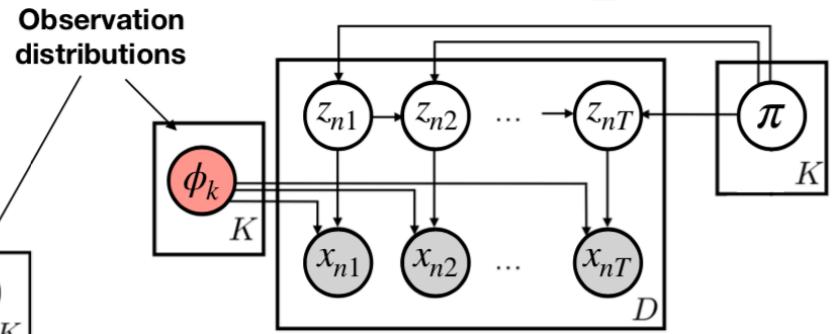
(Generic data)

## Topic Models



(Grouped data)

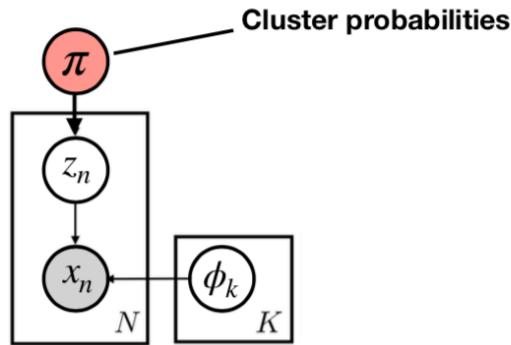
## Hidden Markov Models



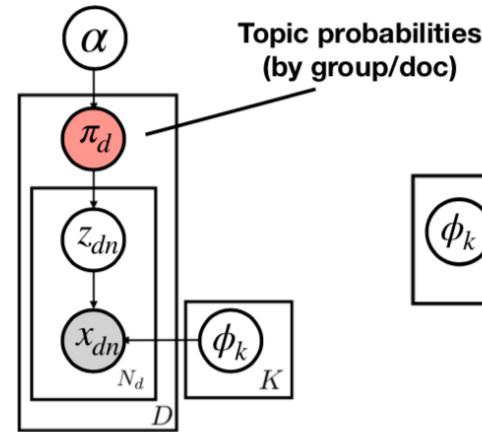
(Sequence data)

# Pi: Cluster Appearance Frequency

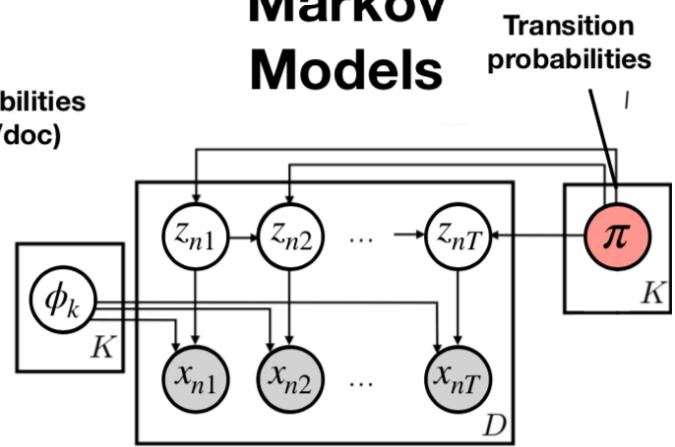
## Mixture Models



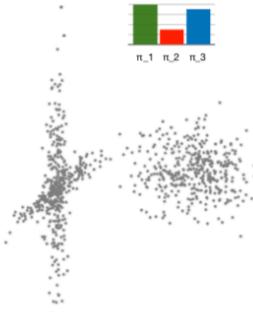
## Topic Models



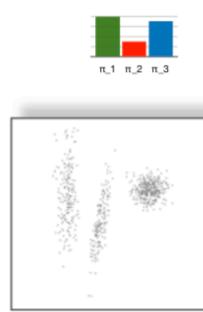
## Hidden Markov Models



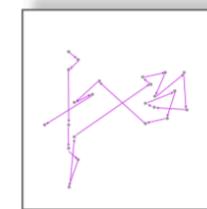
0.5	0.1	0.4
0.2	0.6	0.2
0.15	0.25	0.6



(Generic data)



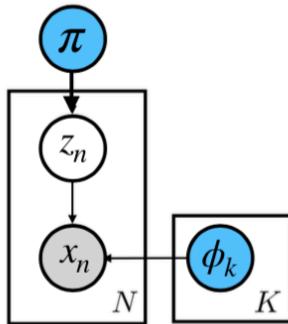
(Grouped data)



(Sequence data)

# How should we add supervision?

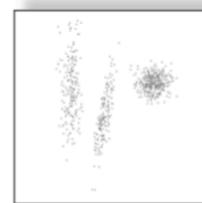
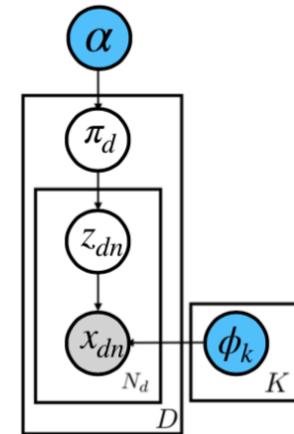
## Mixture Models



(Generic data)

+ Label per example

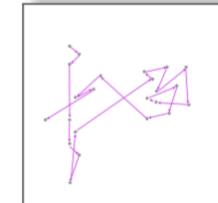
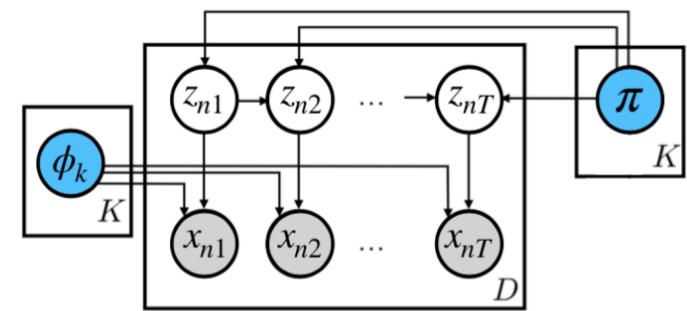
## Topic Models



(Grouped data)

+ Label per group

## Hidden Markov Models



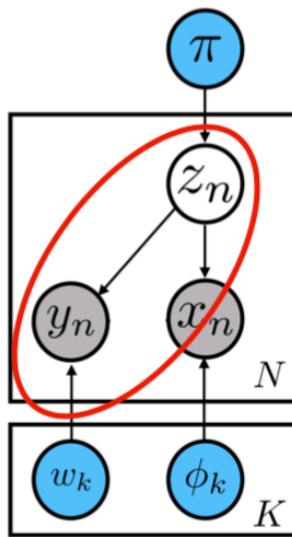
(Sequence data)

+ Label for sequence  
+ Label per timestep

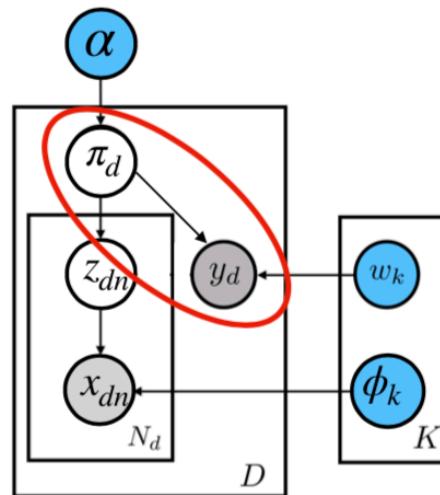
# Supervised clustering models

Predict labels as a function of the cluster membership:

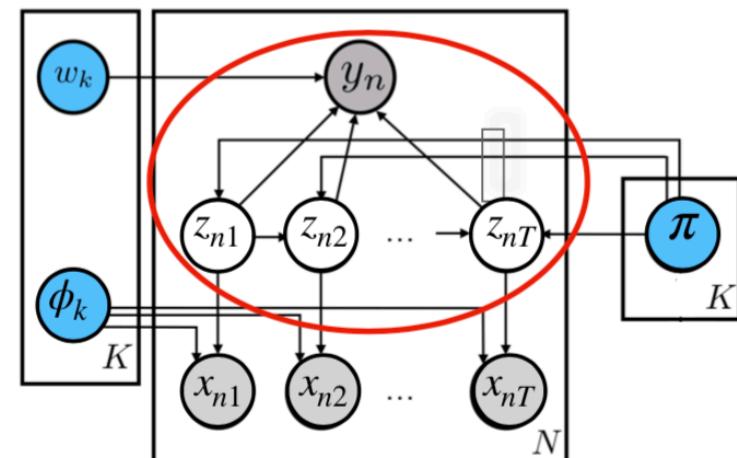
**Mixture Models**



**Topic Models**

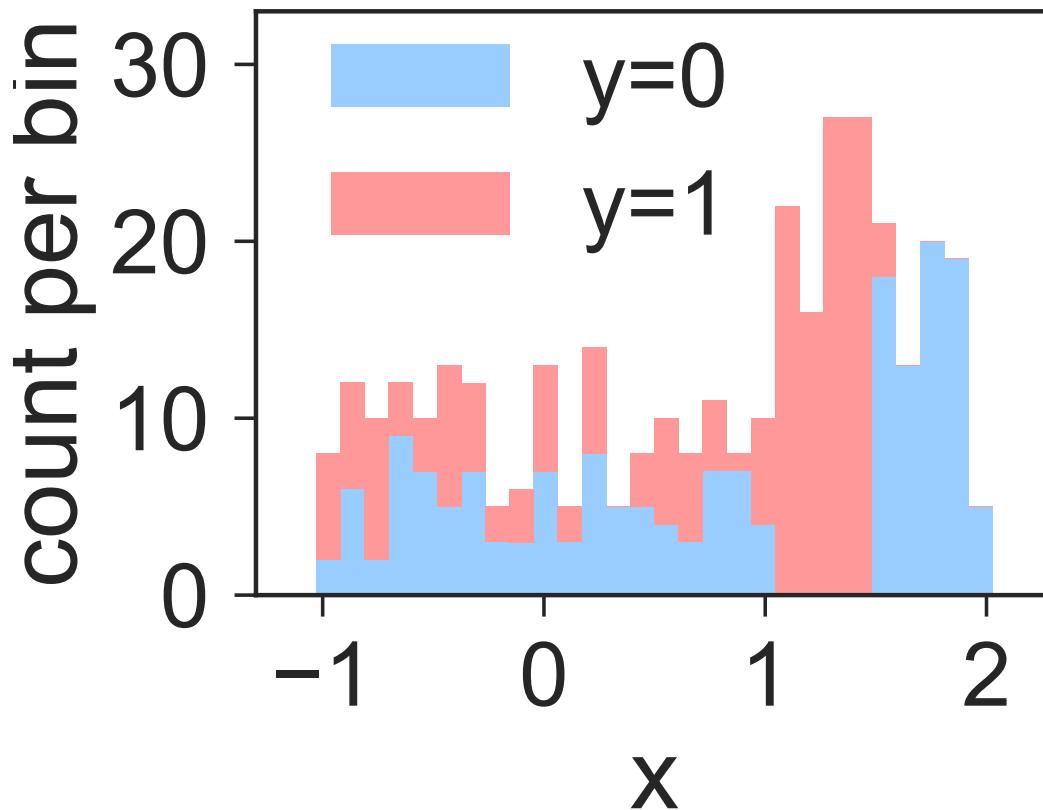


**Hidden Markov Models**



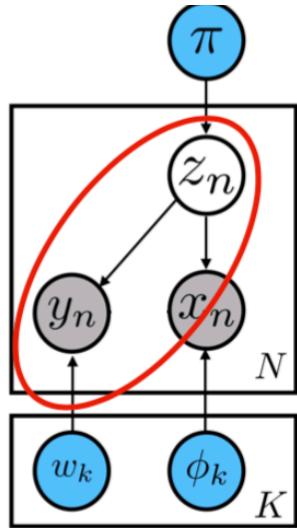
Directly modeling a label likelihood  $p(y | z)$  makes it easy when  $y$  is missing  
Class conditional models like  $p(z | y)$  would require expensive marginalization

# Simple Challenge: Model this Data!



# Supervised Gaussian Mixture Model

Assume K possible clusters

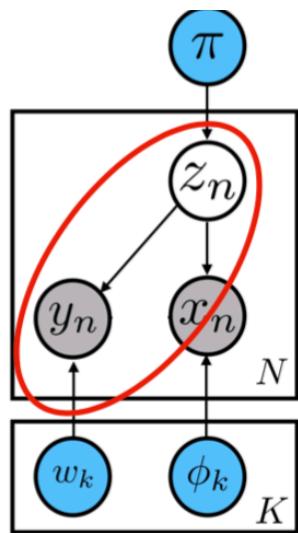


$$z_n \sim \text{Discrete}(\pi_1, \dots, \pi_K)$$

$$x_n | z_n = k \sim \text{Normal}(\mu_k, \sigma_k^2)$$

$$y_n | z_n = k \sim \text{Bern}(w_k)$$

# Supervised Gaussian Mixture Model



Assume  $K$  possible clusters

$$z_n \sim \text{Discrete}(\pi_1, \dots, \pi_K)$$

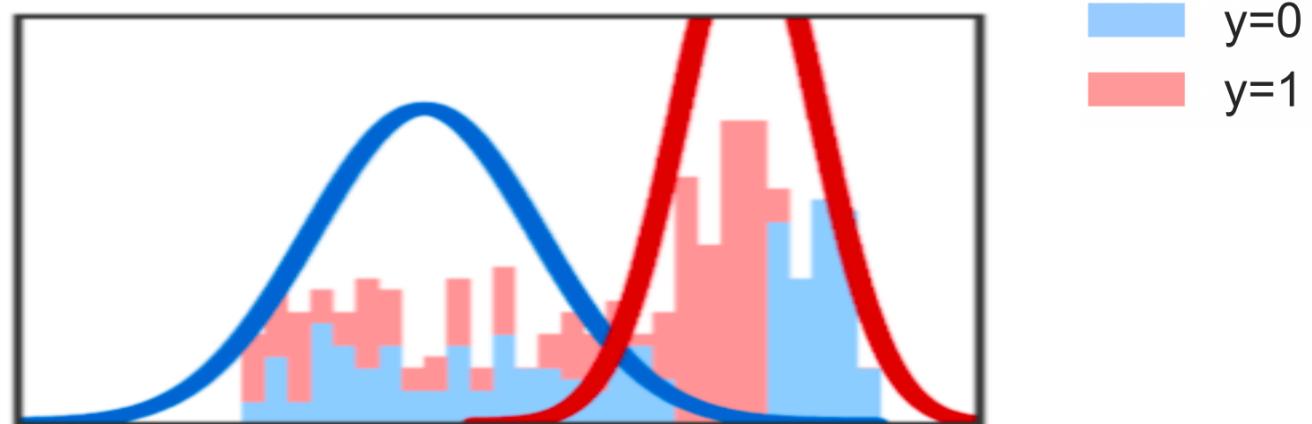
$$x_n | z_n = k \sim \text{Normal}(\mu_k, \sigma_k^2)$$

$$y_n | z_n = k \sim \text{Bern}(w_k)$$

$$K = 2$$

$$w_b = 0.4$$

$$w_r = 0.5$$



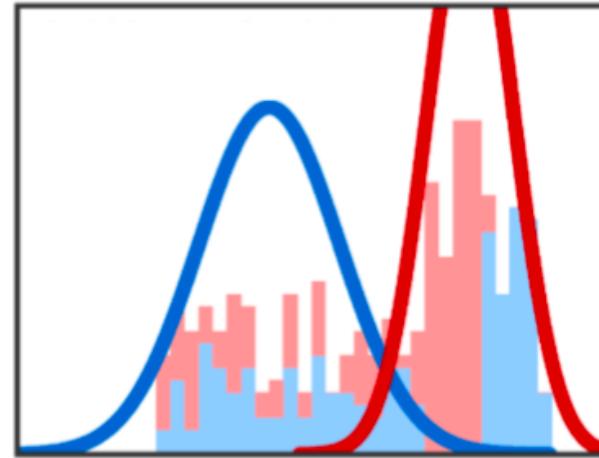
# Haven't we known how to fit these models for >30 years?

$$\max_{\pi, \phi, w} \sum_{n=1}^N \log p(x_n, y_n | \pi, \phi, w)$$

Result:

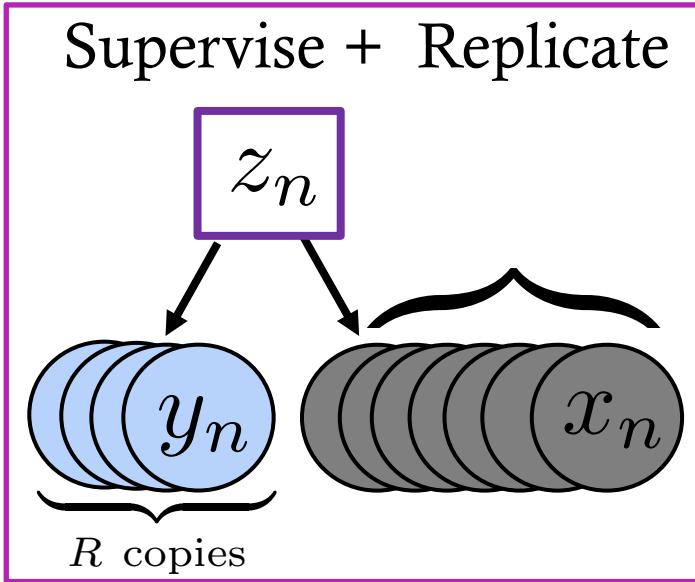
Terrible label prediction!

Forced to compromise  $p(y | x)$   
to make  $p(x)$  look good



If my application need prioritizes  $p(y | x)$ , maximizing joint likelihood may not yield useful results

# Past Work Attempted Fix: Label Replication



$$\max_{\phi, w} \log p(x, y, y, \dots y | \phi, w)$$

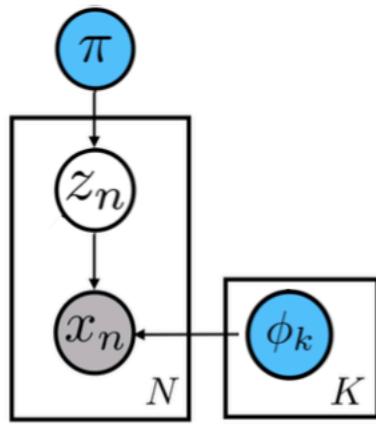
Proposed by

- Zhang & Kjellstrom (2014) as “Power sLDA”
- Zhu et al. (2012) as “Med-LDA”
- Ganchev et al. (2010) as ”Posterior Regularization”

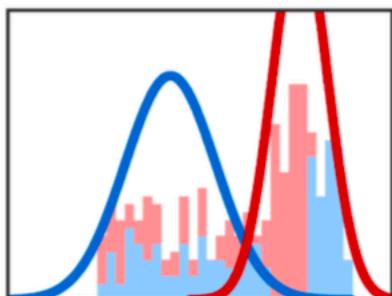
*Hughes et al. AISTATS 2018 contribution:  
Show many previous efforts equivalent to this basic idea.*

# No known alternatives work well

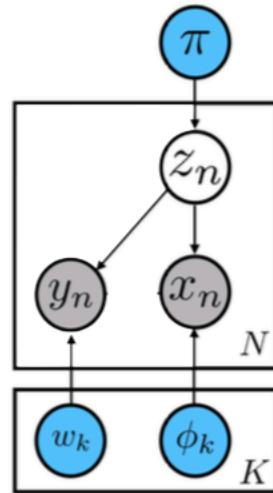
Unsupervised model



$$\max_{\pi, \phi} p(x | \pi, \phi)$$

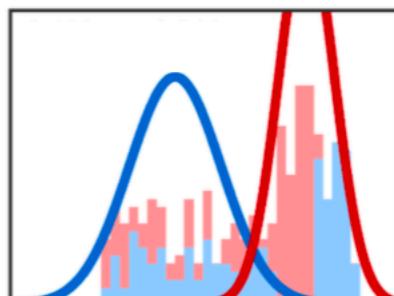


Joint model

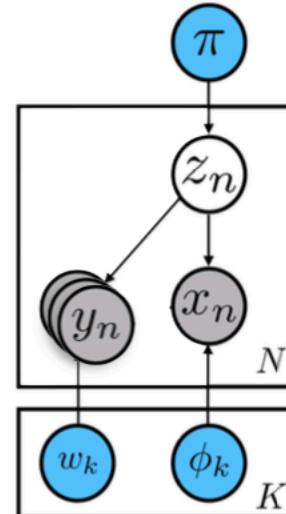


$$\max_{\pi, \phi, w} p(x, y | \pi, \phi, w)$$

$$|\log p(x_n | z_n)| >> |\log p(y_n | z_n)|$$



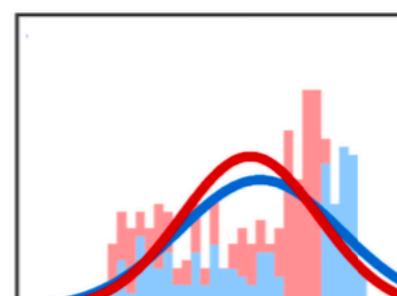
Label replication



$$\max_{\pi, \phi, w} p(x, \beta y | \pi, \phi, w)$$

Equiv. to

$$p(y_n | z_n, w) \rightarrow p(y_n | z_n, w)^\beta$$



# Prediction-Constrained Training

**Key idea:** Maximize likelihood of observations...

$$\max_{\pi, \phi} \log p(x | \pi, \phi)$$

**Subject to:**  $-\log p(y | x, \pi, \phi, w) < \epsilon$

Subject to a **constraint** that we can achieve a given performance threshold for predicting labels **given observations**

How to compute?

$$p(y_n | x_n, \pi, \phi, w) = \sum_{k=1}^K p(y_n, z_n = k | x_n, \pi, \phi, w)$$

# How to optimize?

$$\max_{\phi, \eta} \lambda \log p(y|x, \phi, \eta) + \log p(x|\phi)$$

Use Lagrange multiplier to form unconstrained objective

Optimize via stochastic gradient descent

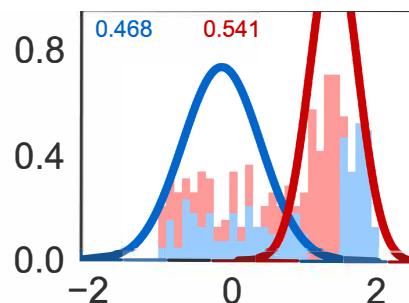
- Write objective as Python code
- Automatic gradients from Tensorflow/Pytorch



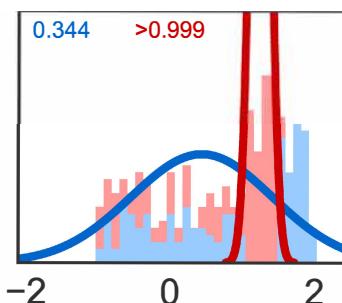
# PC can overcome misspecification

# Weak constraint

$$\lambda = 1.0$$



$$\lambda = 4.0$$

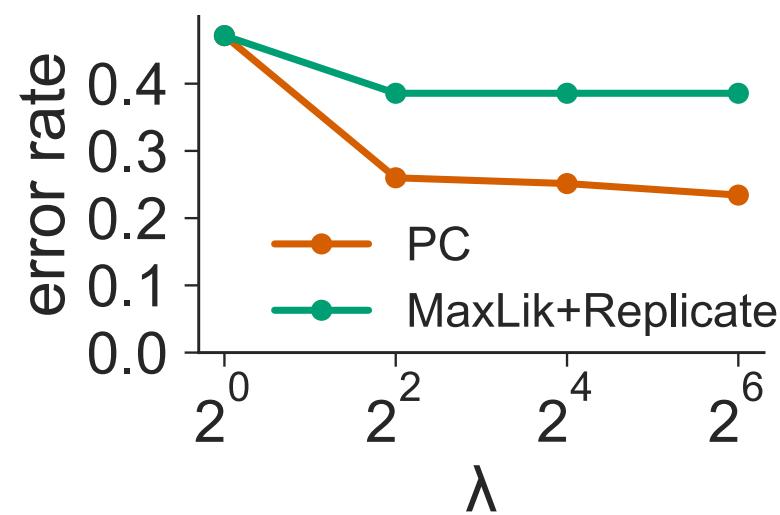


$$\lambda = 16.0$$



## Stronger constraint

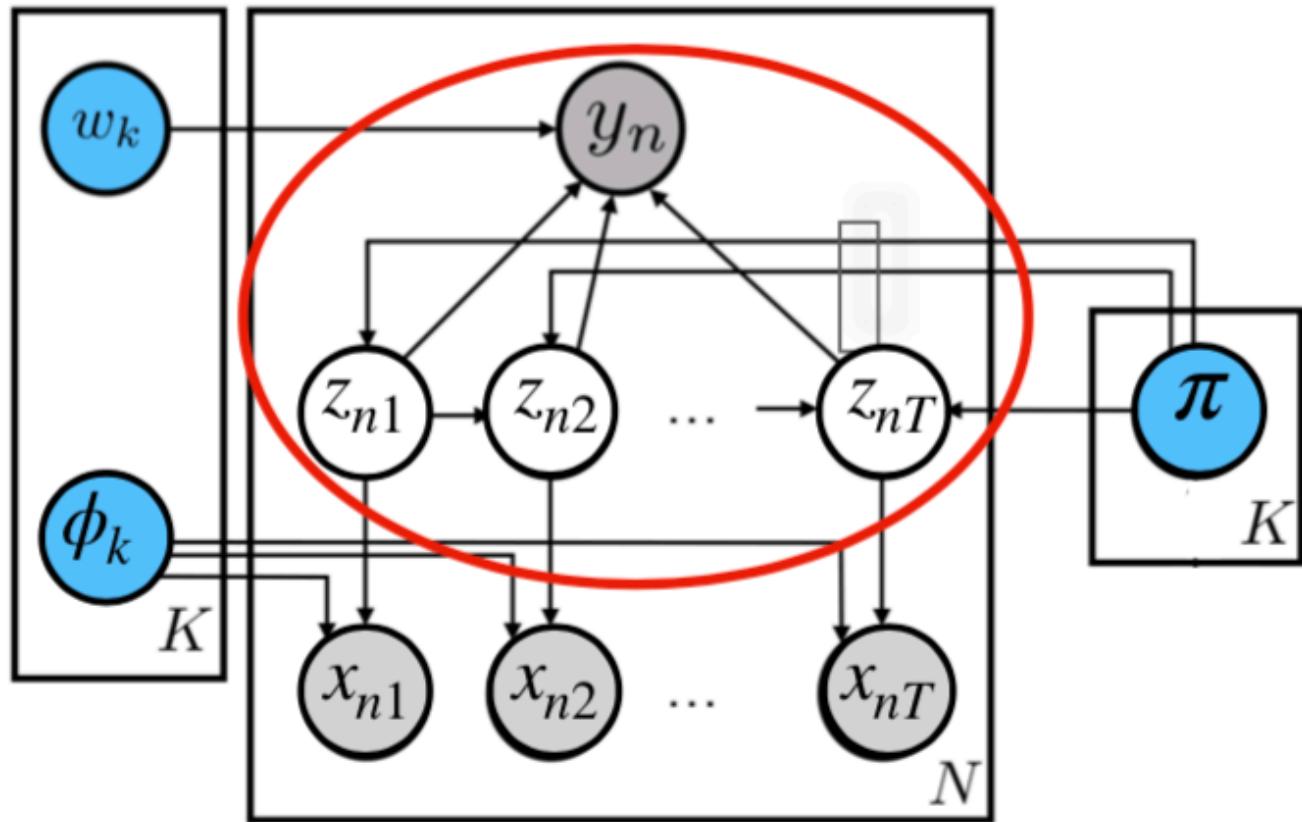
$$\lambda = 64.0$$



# Roadmap

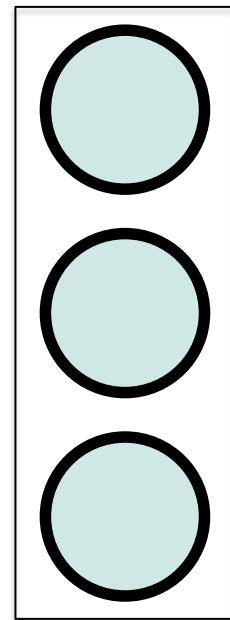
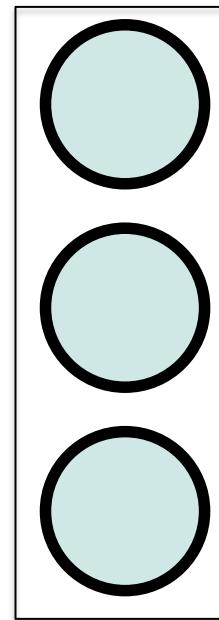
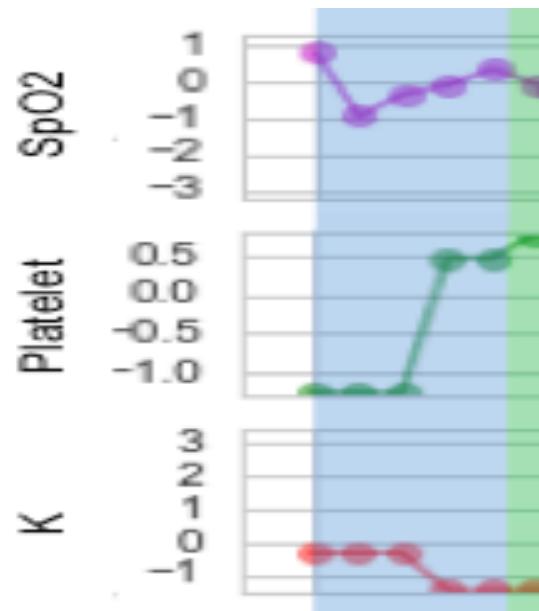
- Motivation: Improve interventions in ICU
- Model Family for Clustering Structured Data
- Method: Prediction-Constrained Training
- **Prediction-Constrained HMMs**  
*Hope, Hughes, Sudderth (in progress)*
- Prediction-Constrained POMDPs

# Prediction Constrained Hidden Markov Models



# Probabilistic time-series model

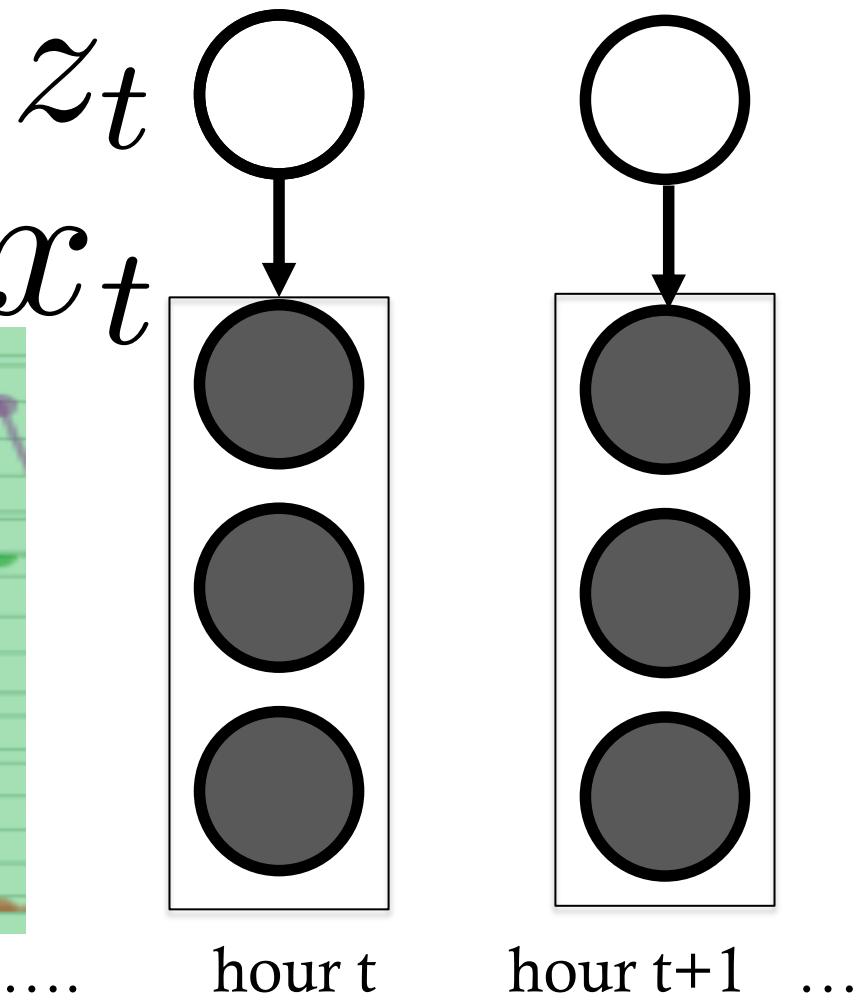
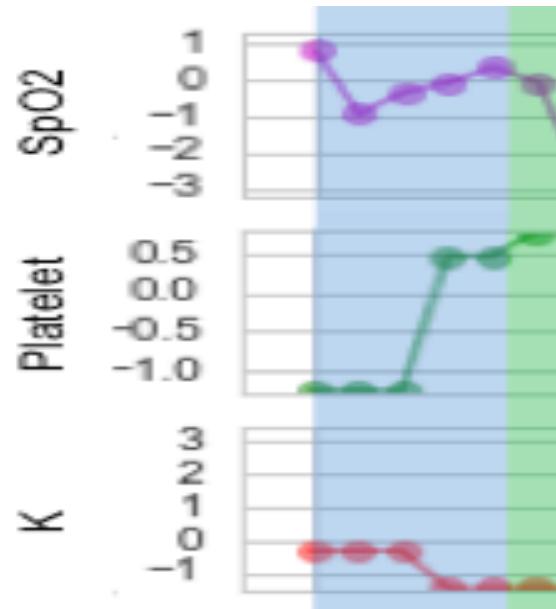
Observed Feature Vector  $\mathcal{X}_t$



# Probabilistic time-series model

Hidden Patient State  
*one of  $K$  possible values*

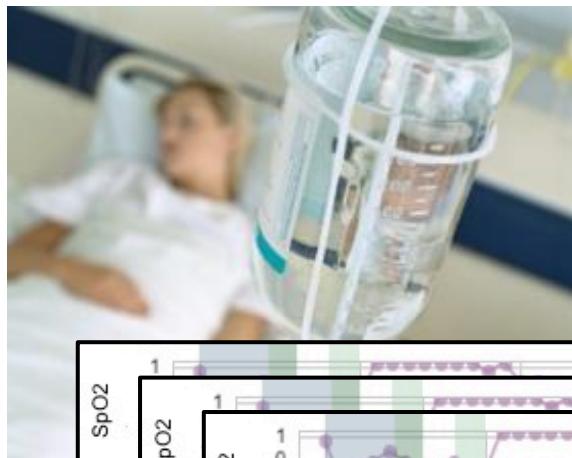
Observed Feature Vector  $\mathcal{X}_t$



# Goal: Health States Trajectories

Ghassemi, Wu, Hughes, et al AMIA CRI '17

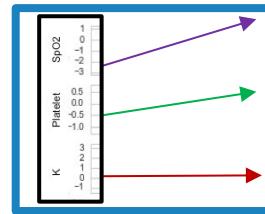
*ICU signals from many patients*



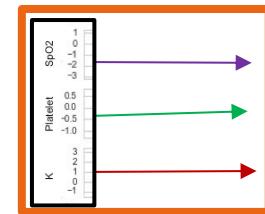
*Fit the model*

*Health state trajectories*

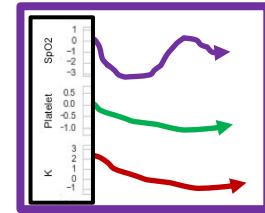
Improving kidney function



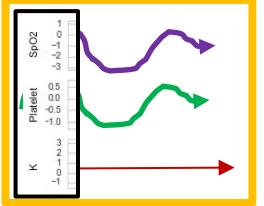
Steady-state kidney function



Dropping lung function



Steady-state lung function



Subj. 1



Subj. 2



Subj. 3



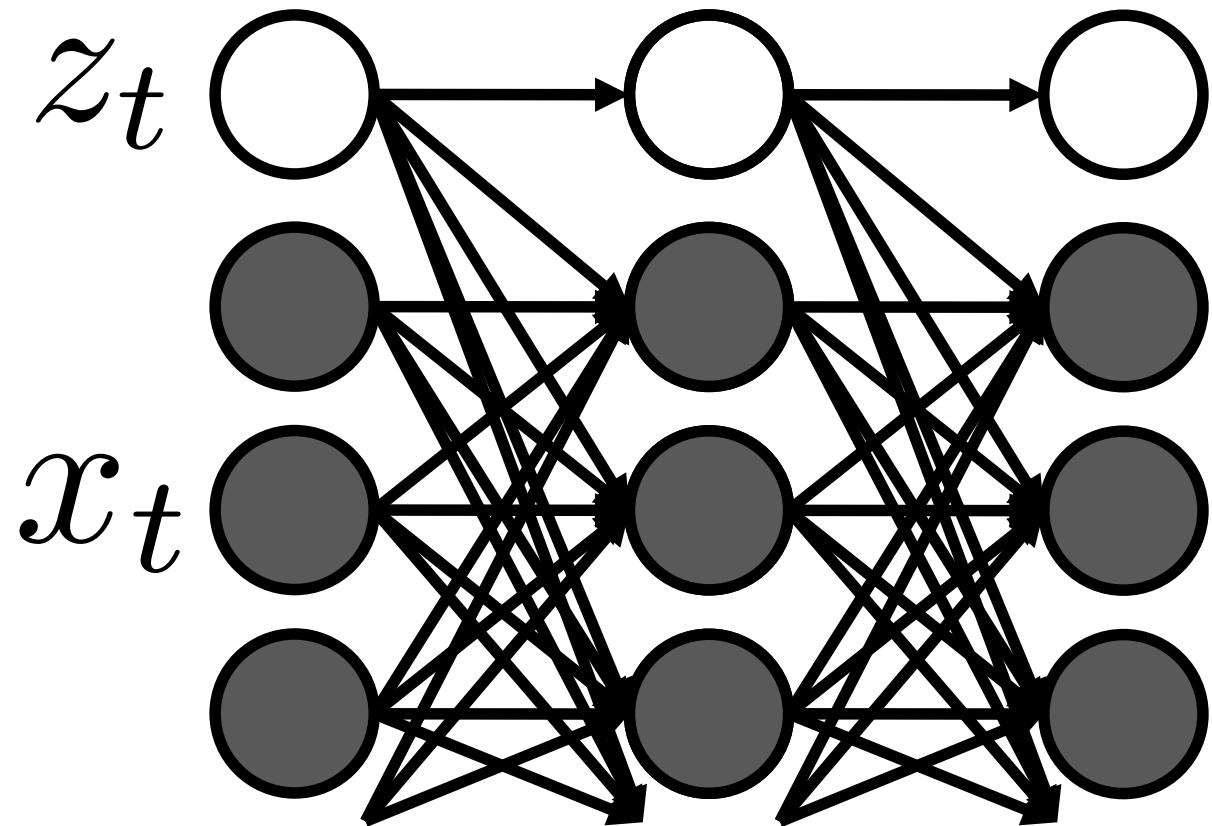
Subj. 4



# $p(x,z)$ : Autoregressive HMM

Hidden  
Patient State

Observed Vitals

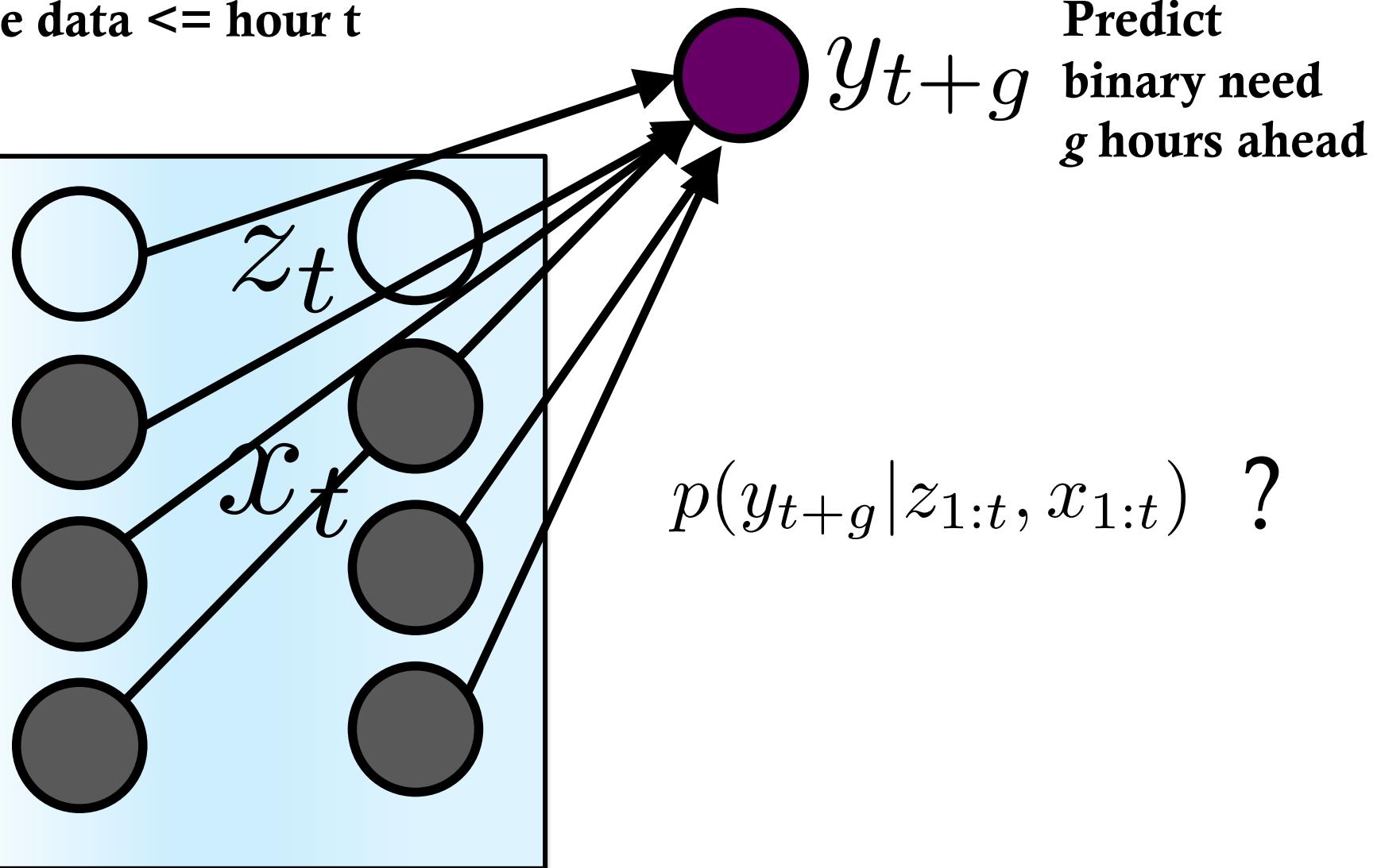


$$z_t | z_{t-1} = j \sim \text{Discrete}(\pi_{j1}, \dots, \pi_{jK})$$

$$x_t | z_t = k \sim \mathcal{N}(A_k x_{t-1} + \mu_k, \Sigma_k)$$

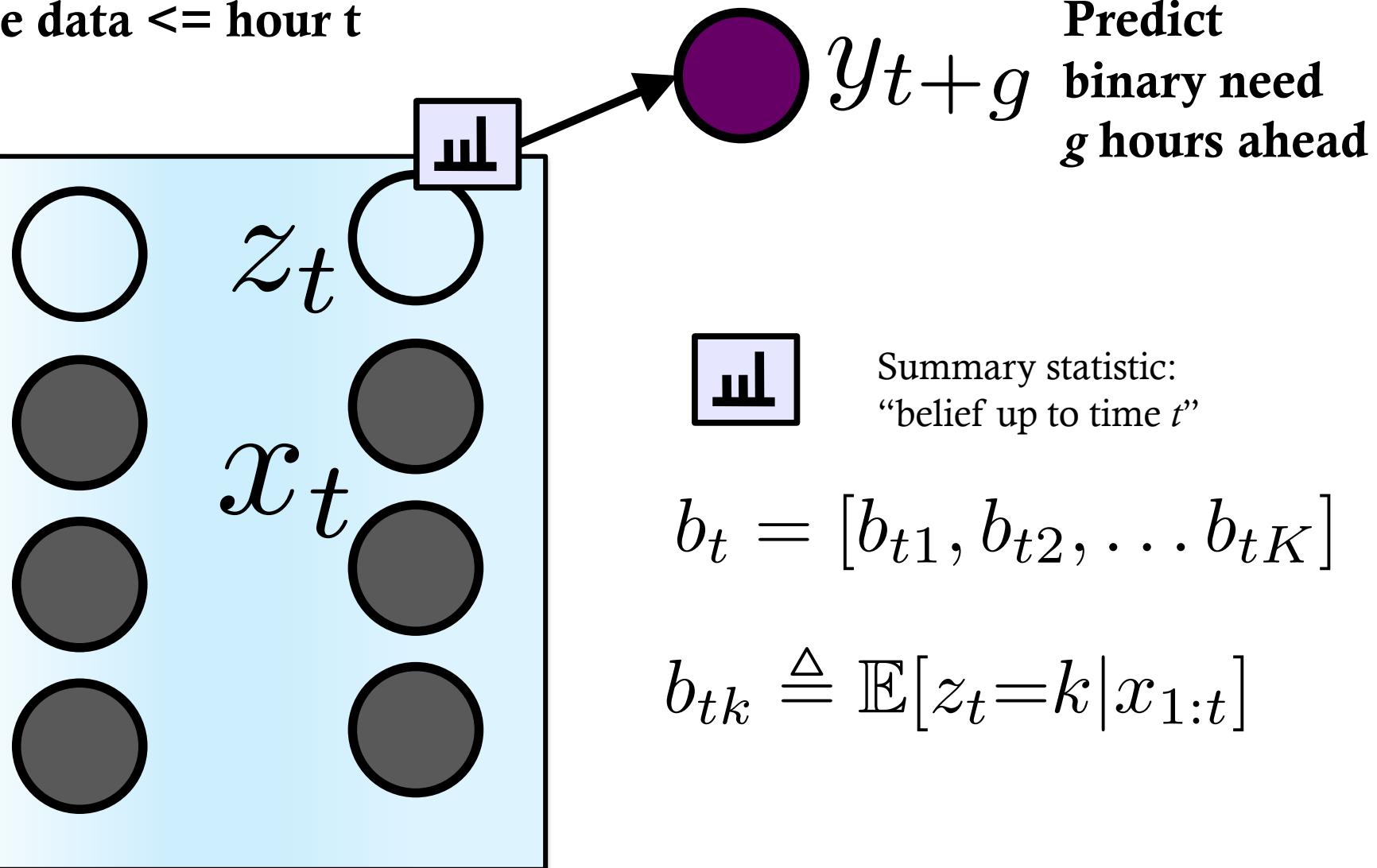
# $p(y | z, x)$ : Binary Label Prediction

Use data  $\leq$  hour t

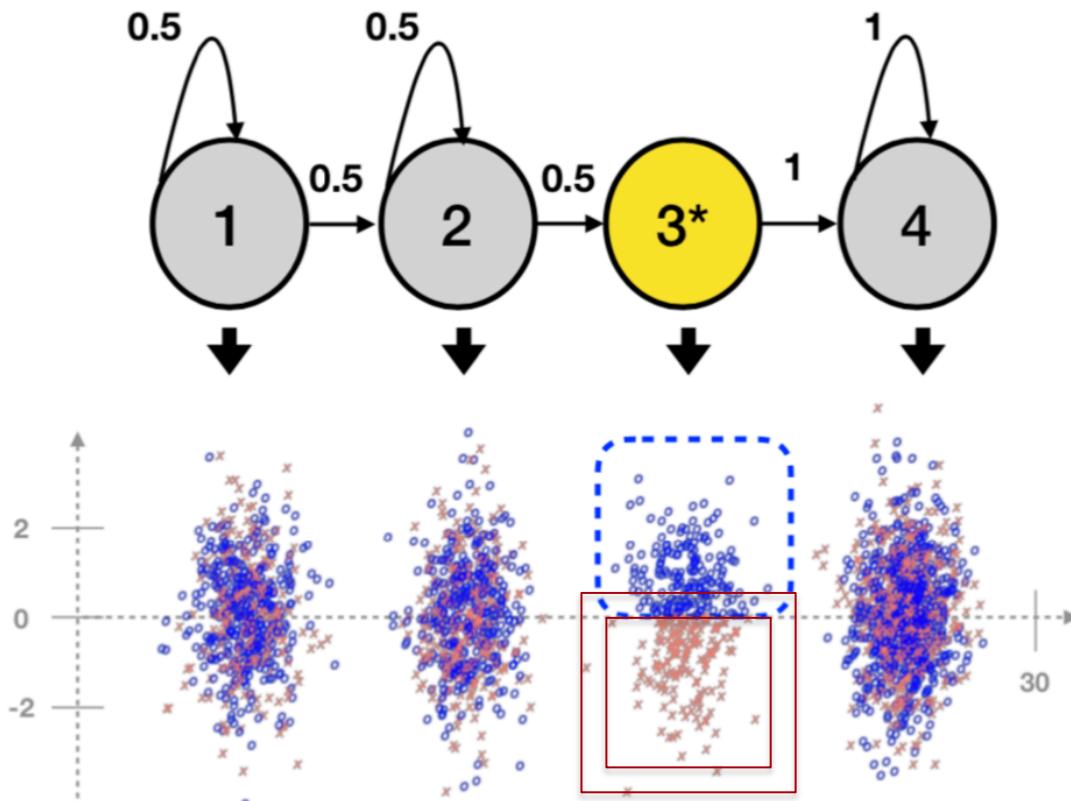


# $p(y | z, x)$ : Binary Label Prediction

Use data  $\leq$  hour t



# Example HMM



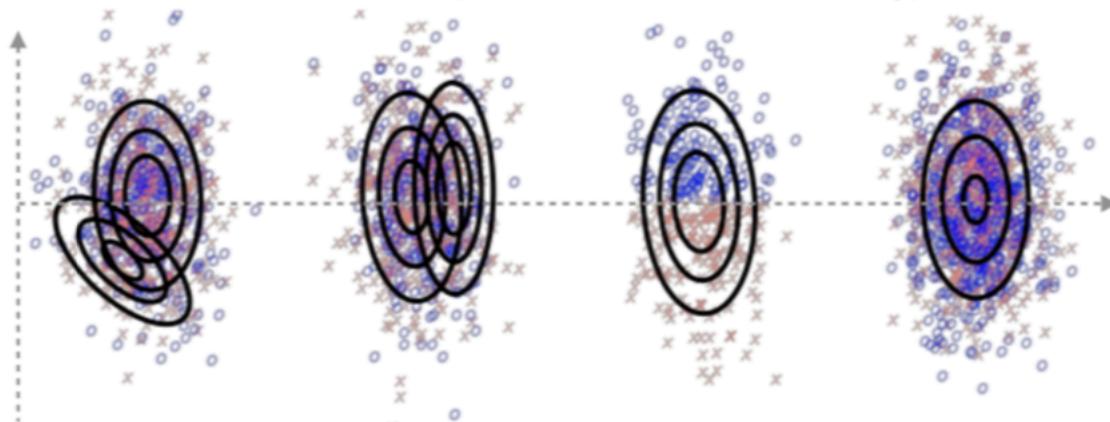
Each sequence gets  
binary label

1 if above x-axis  
0 if below x-axis

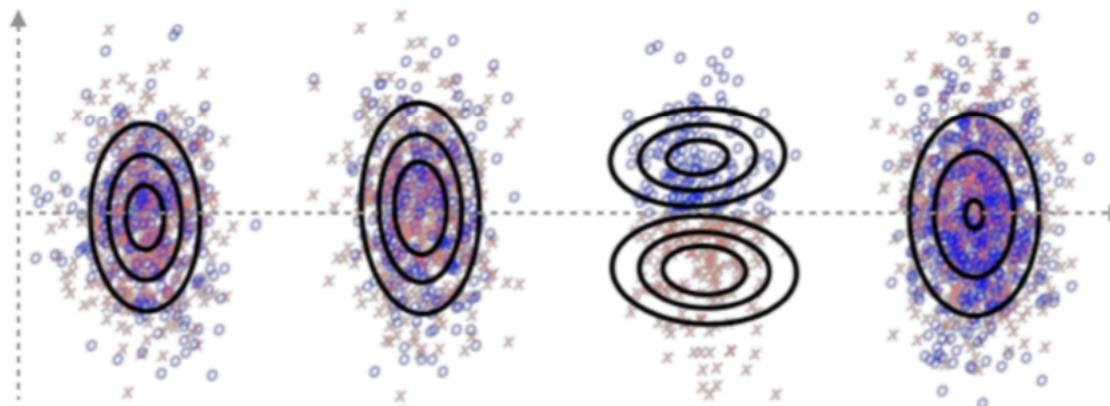
**Data generation of  
500 sequences**

# Fit with 100% sequences labeled

EM Result (48.6% held-out accuracy):

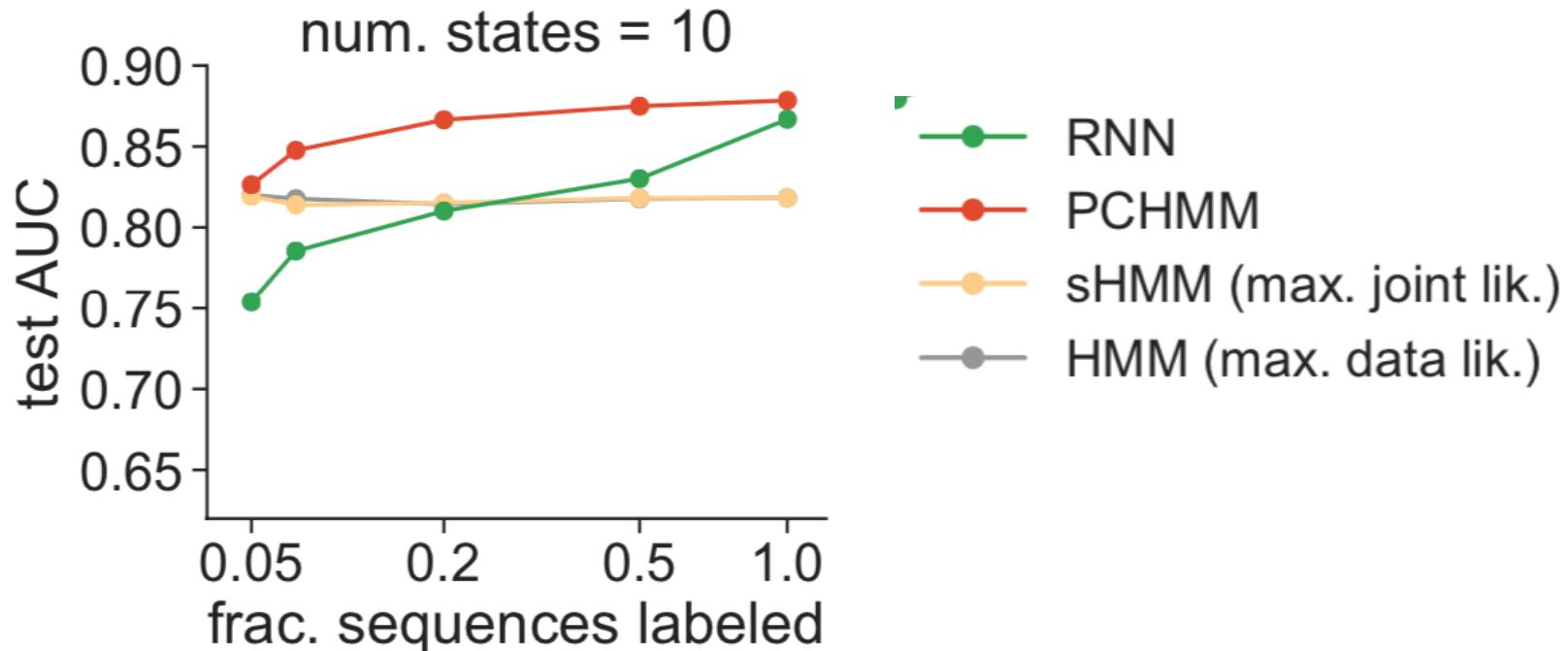


PC-HMM Result (97.3% held-out accuracy):



# ICU Need for Ventilator Prediction

Using autoregressive HMM with 10 states



- PC is strictly better than maximum likelihood training
- When labels are rare, PC > deep learning on labels only

# Roadmap

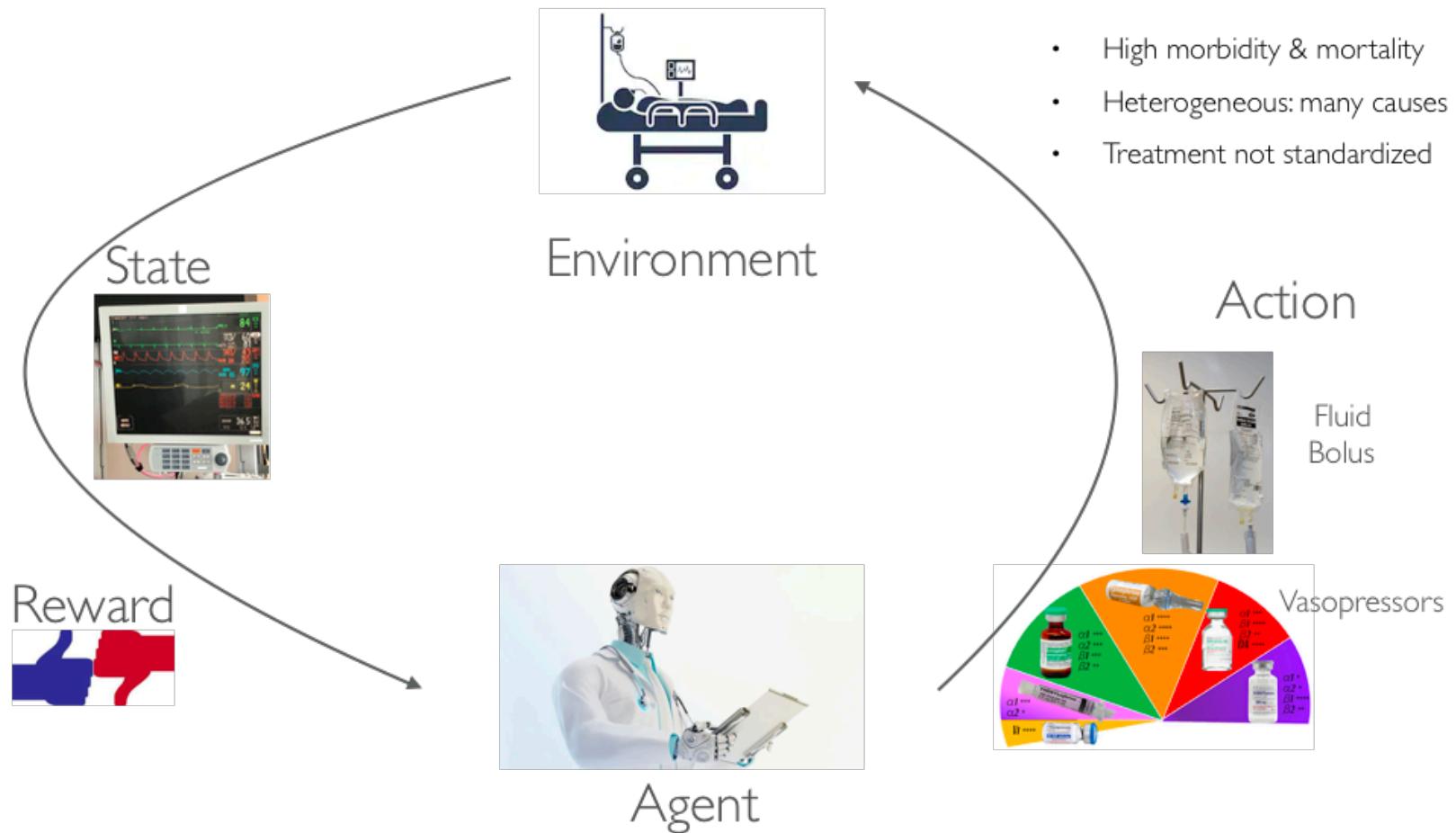
- Motivation: Improve interventions in ICU
- Model Family for Clustering Structured Data
- Method: Prediction-Constrained Training
- Prediction-Constrained HMMs
- **Prediction-Constrained POMDPs**

*Futoma, Hughes, Doshi-Velez AISTATS 2020*

# RL IN GENERAL...

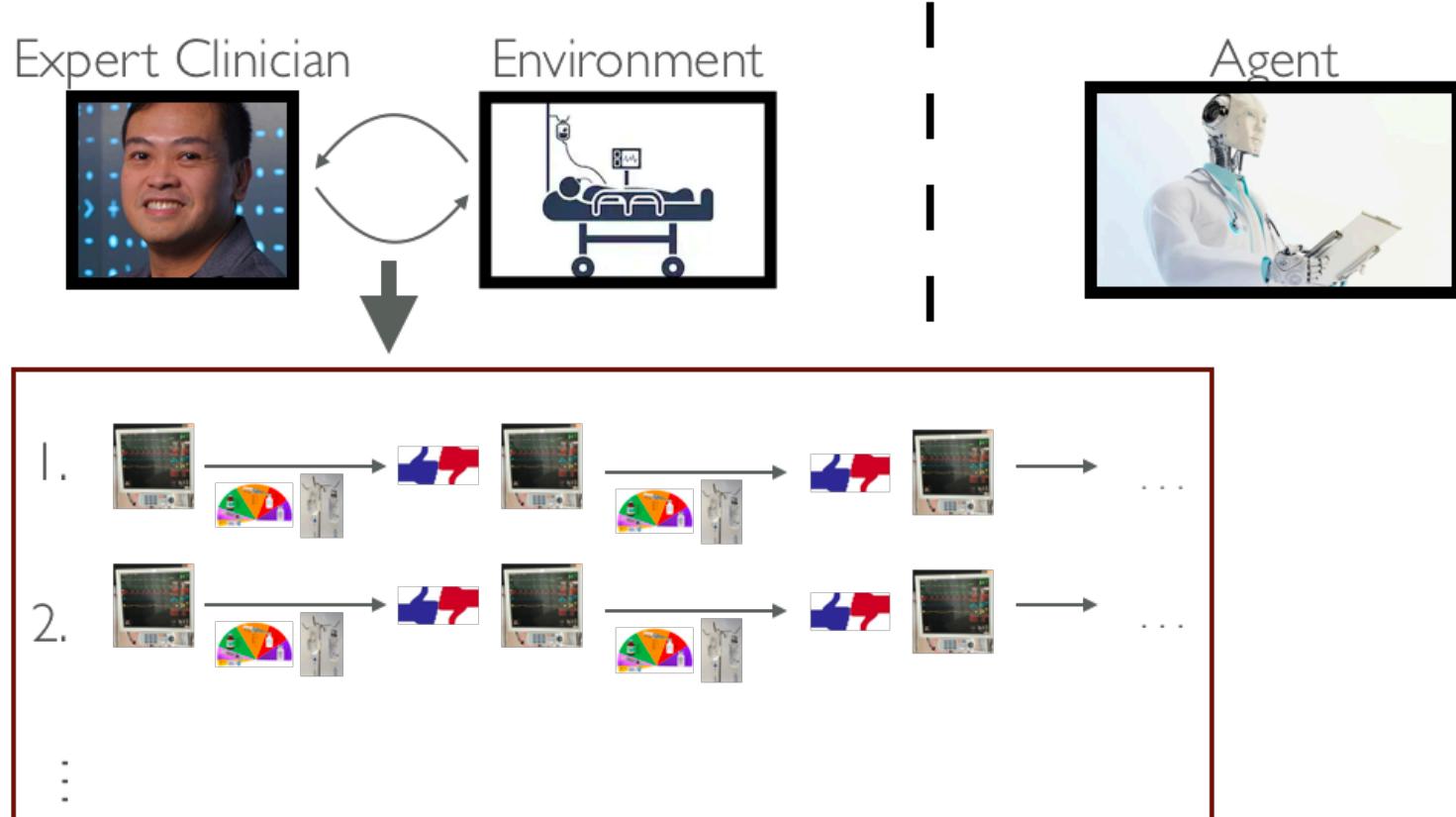


# RL FOR ACUTE HYPOTENSION



# RL FOR ACUTE HYPOTENSION

**Retrospective data ONLY!**

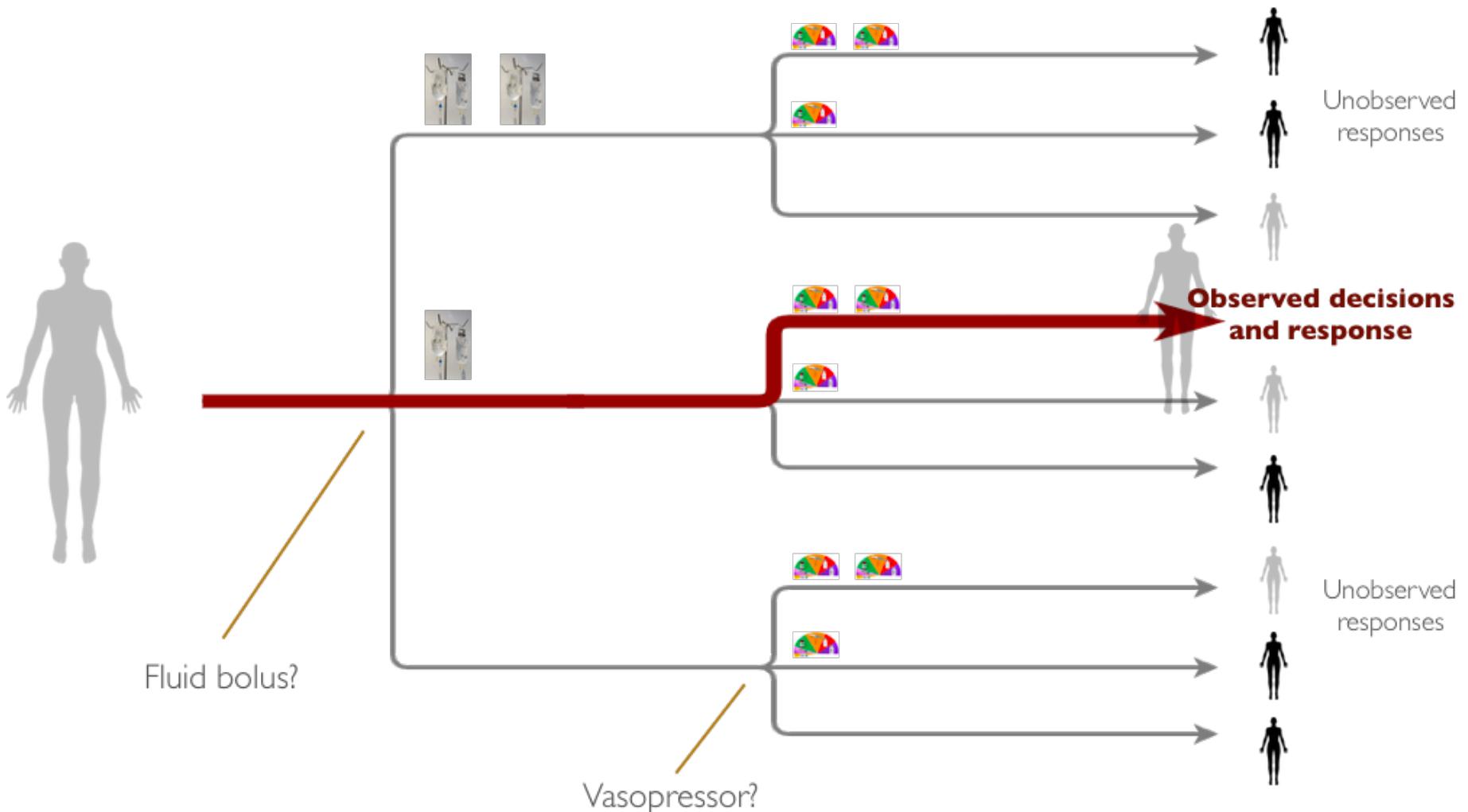


# What is needed for Clinical RL

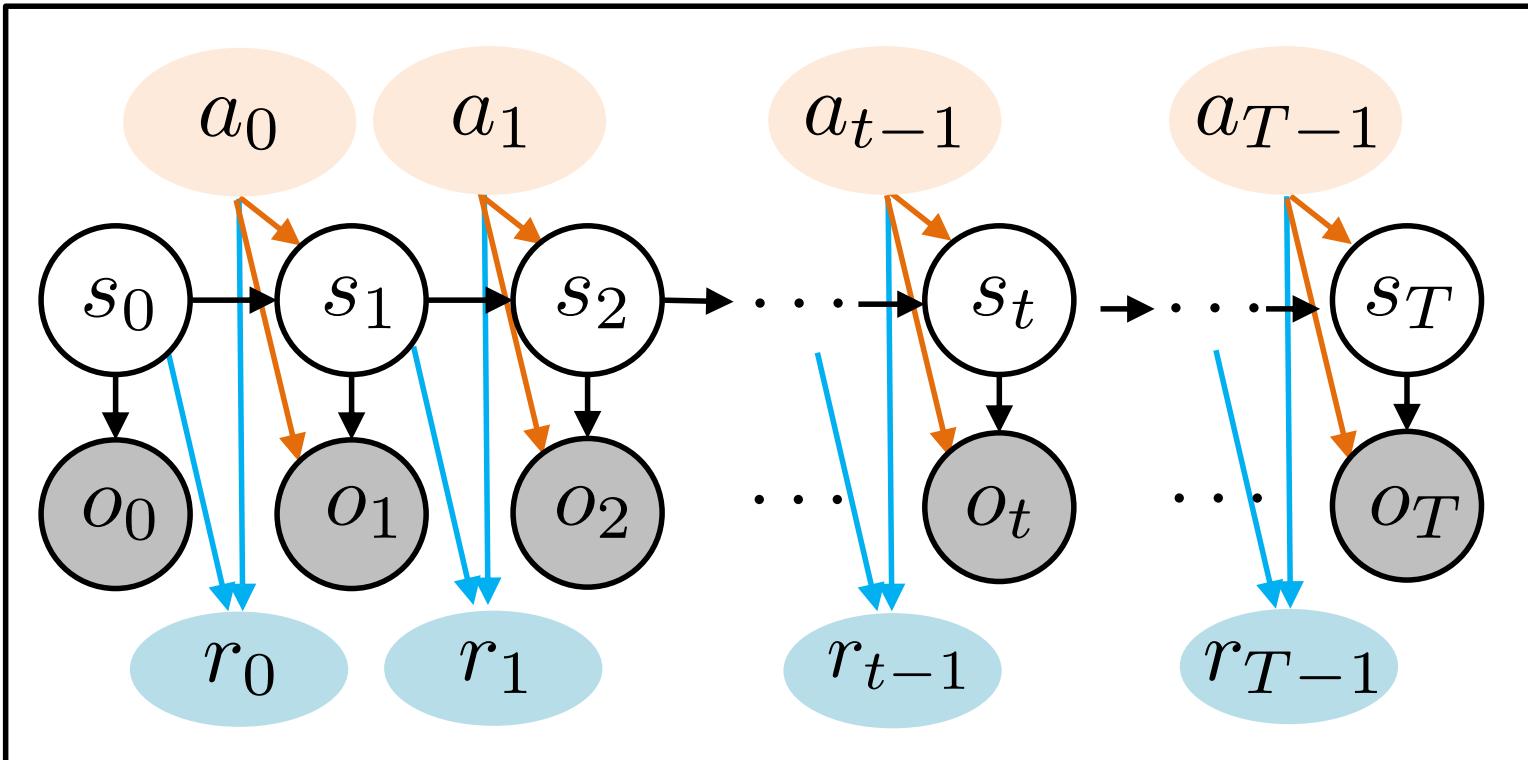
- Learn from retrospective histories only
  - Called “batch” setting of RL
- Use model-based RL
  - Can deal with **little data** and **missing data**
  - Can do **forecasting** and **simulation**
- Handle unknown state space
  - “POMDP”: partially observed Markov decision process

Need: to **avoid misspecification** and get high reward

# Actions: fluid & vaso at each hour



# POMDP as structured clustering model: Input/Output-HMM



# Estimating Value from Off Policy Data

$$V^{\text{CWPDIS}}(\pi_\theta) \triangleq \sum_{t=1}^T \gamma^t \frac{\sum_{n \in \mathcal{D}} r_{nt} \rho_{nt}(\pi_\theta)}{\sum_{n \in \mathcal{D}} \rho_{nt}(\pi_\theta)},$$
$$\rho_{nt}(\pi_\theta) \triangleq \prod_{s=0}^t \frac{\pi_\theta(a_{ns} | o_{n,0:s}, a_{n,0:s-1})}{\pi_{\text{beh}}(a_{ns} | o_{n,0:s}, a_{n,0:s-1})}.$$

Consistency Weighted Per-decision Importance Sampling  
(CWPDIS, Thomas 2015)  
Lower bias but high variance

# Prediction Constrained POMDPs

$$\max_{\theta} \quad \mathcal{L}_{gen}(\theta) + \lambda V(\pi_{\theta})$$

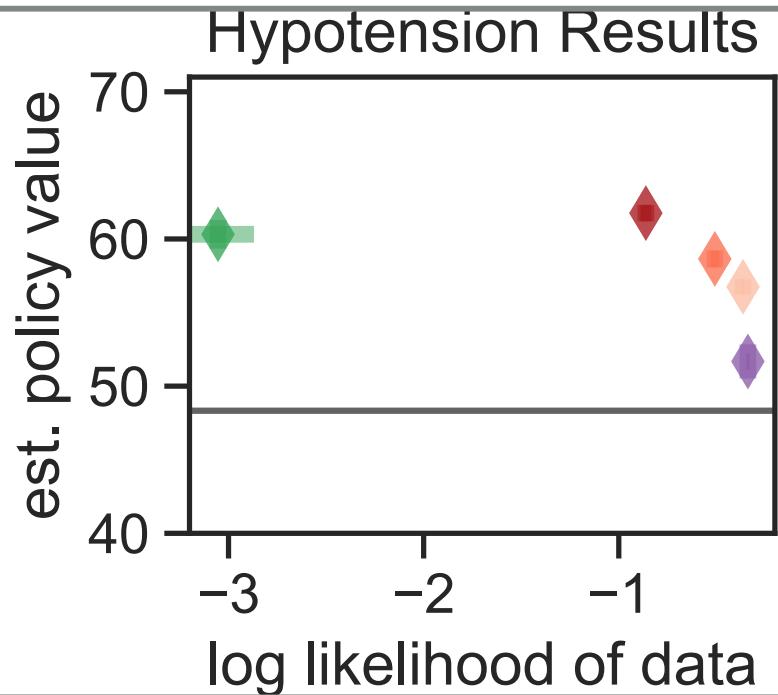
Generative likelihood of the  
observations given the model

Value of policy  
Given the model

We call our method “POPCORN”:  
Partially Observed Prediction Constrained  
Reinforcement Learning

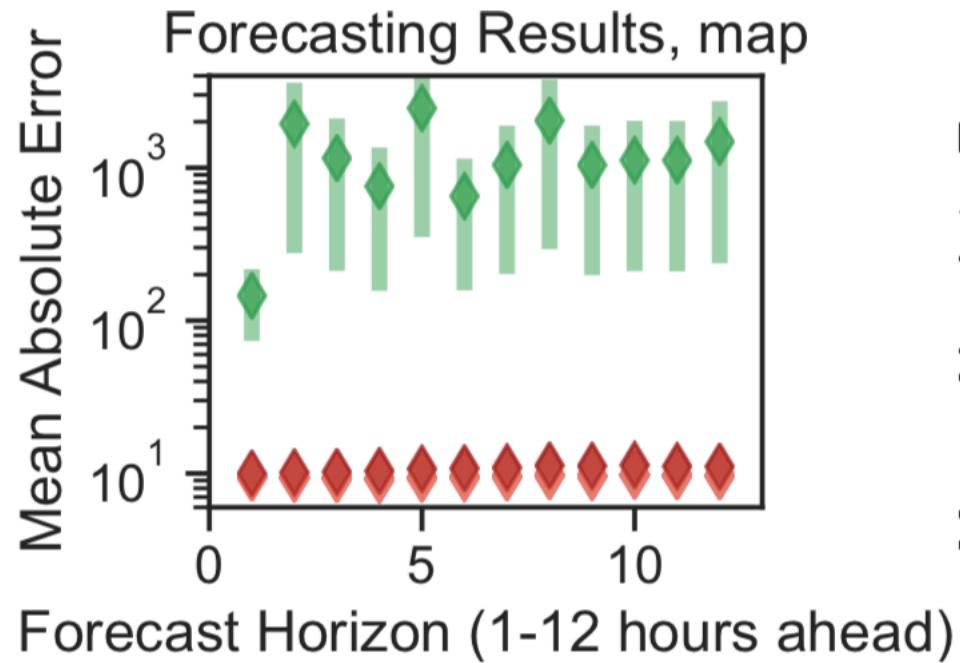
# Results: PC-POMDP best for reaching sweet spot of value and likelihood

- ◆ Value term only (ESS:  $79 \pm 5$ )
- ◆ POPCORN  $\lambda = .316$  (ESS:  $87 \pm 4$ )
- ◆ POPCORN  $\lambda = .031$  (ESS:  $78 \pm 3$ )
- ◆ POPCORN  $\lambda = .003$  (ESS:  $77 \pm 3$ )
- ◆ 2-stage (EM then PBVI) (ESS:  $52 \pm 2$ )
- Behavior policy value

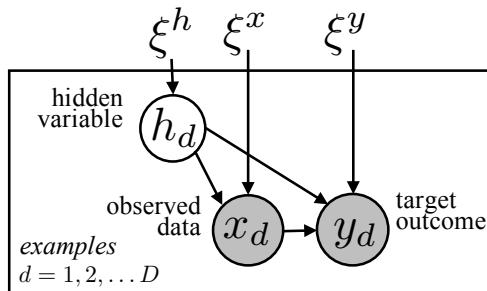


# Can use model for forecasting!

- ◊ null model (predict mean)
- ◊ 2-stage (EM then PBVI)
- ◊ POPCORN  $\lambda=.003$
- ◊ POPCORN  $\lambda=.031$
- ◊ POPCORN  $\lambda=.316$
- ◊ Value term only



# Future: PC training for rich family of deep generative models



- Mixture models
- Topic models
- Hidden Markov models
- Network models (MMSB)
- PCA or factor analysis
- Non-negative matrix factorization
- Probabilistic encoder/decoder (VAE)

- Disease progression over time
- Models of many data sources

