A Comprehensive Engineering Analysis of the 3-Point Shot in Basketball

Player-Specific Optimization Kinematics Model and Methods for Improved Training and Performance Outcomes

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Abstract. In the modern era of basketball, the 3PT shot (3PT) has become increasingly relevant to a player's fundamental skill set. At all levels of competition, successful implementation of 3PT shooting has transformed offensive schemes and defensive game plans. In recent years, 3PT shooting has correlated to on court success and winning teams, cementing long distance shooting as a desirable and necessary skill. Analysis of early shooting studies focused on the physics of shot release and ball flight, while recent studies examine kinematics of the shooting arm to develop insightful models. These studies debunk the *perfect* shooting form concept. Furthermore, research into standard training regimens to optimize shooting pinpoints the need for a player-specific method supported by quantitative evaluation. This study presents a player-specific release optimization kinematics model novel in its nature. The model inputs individual player measurements and outputs acceptable joint angles of the shooting arm that correlate to desired release parameters. In emphasizing the model's player-specific nature, which provides the computed outputs unique to a player's measurements, this study serves as a baseline for future basketball technologies, training, and performance outcomes. An engineering analysis outlines the background mechanics and kinematics of the shooting arm and ball during flight. A detailed overview provides comparison to current commercial products and market opportunity for the player-specific model, which can transform accepted understanding and implementation of shooting regimens, paving the way for new technologies at every level of the sport.

Keywords: Basketball Shooting· Three Point Shot · Kinematics · Optimization Model.

1 Introduction

Optimal shooting of a basketball is an important concept in basketball. Numerous studies analyzed shooting in order to provide an improved understanding for increased shot-making ability. Early studies (1951-1996) primarily focused on a comprehensive understanding of the ball's trajectory following release [13]. Specifically, these studies focused on release angle from the hand, peak height, entry angle into the hoop, and margins of error that provided a background for future studies. Recent studies (2015-2018) advanced towards kinematic models in two dimensions, providing unique insight into angular displacements and velocities of joints of the shooting arm during release [18,19,20]. These studies debunk the concept of a *perfect* shooting form. Rather, shooting is player-specific and should be viewed as pinpointing, developing, and improving the shot that is both optimal and comfortable to each individual. Traditionally, qualitative input has translated as the accepted universal methods of improved shooting mechanics: tucked in elbow, high arc, straight follow through, etc. However, in viewing shooting as a scheme of physics, it's apparent that calculated release mechanics correspondent to a player's specific measurements exist for optimal shooting release conditions. For example, two players of identical height, 6'3", do not necessarily require the same shooting mechanics. Instead, as later detailed by kinematic analysis, proper mechanics are dependent on each player's specific upper arm, forearm, and hand length measurements, among other variables. Despite this understanding, there is a surprising lack of shooting regimens or basketball technologies incorporating player-specific quantitative analysis.

To direct basketball shooting in this player-specific direction, this study presents a player-specific release optimization kinematics model. Combining this shifting overview in accepted understanding of basketball shooting aid with the growing relevance of the 3PT shot in the sport, this model provides an easy-use software that will pave the way for player-specific emphasis in future basketball training regimens, technologies, and commercial products.

2 Background

2.1 What is the 3PT (3PT) Shot?

The 3PT field goal (3-pointer, three, triple, trey) is a field goal made in a basketball game from beyond the 3PT line, a designated arc surrounding the basket (Fig. 1) [1]. Rewarding players for long range accuracy, the 3PT shot is the highest scoring field goal. In awarding three points for successful makes—in contrast to two points for field goals made within the 3PT line and one point for free throws—the 3PT field goal is important for offensive play calling and defensive scheming.

The 3PT line distance varies by competition level: in the National Basketball Association (NBA) the arc is 23'9" (7.24m) from the center of the basket; in the WNBA, FIBA, and men's play in NCAA Division I, this arc is 22'1.75" (6.75m); in men's play in NCAA Divisions II and III and women's play in NCAA Divisions I, II, and III, this arc is 20"9' (6.32m); and in High School and lower levels, this arc is 19'9" (6.02m) [1]. In the NBA, WNBA, FIBA, and NCAA men's Division I, the 3PT straightens at the points where the arc is 3" (0.91m) from each sideline. As a result, the distance from the basket at the corners decreases to 22' (6.71m)

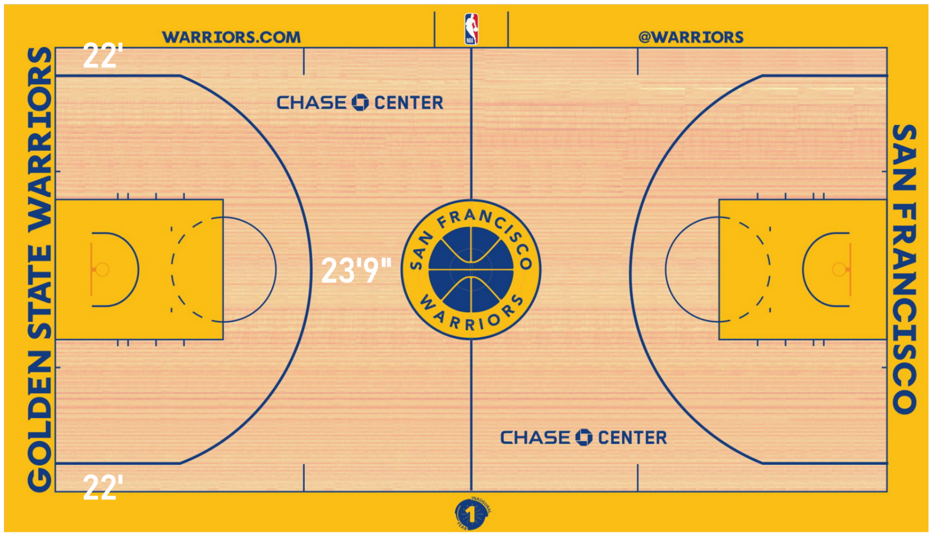

Fig. 1: NBA 3PT Line: Golden State Warriors Primary Court. From the direct center of the of hoop, the 3PT line is 23'9". As the arc reaches the corner, it decreases to 22' in distance. *NBCS Bay Area [2].*

in the NBA and 21'7.8" (6.60m) in WNBA, FIBA, and NCAA men's Division I play (Fig. 1) [1]. In NCAA Divisions II and III and High School competition and lower, the arc is continuous for its entirety around the basket [1].

Popularized in the inaugural 1967-68 American Basketball Association (ABA) season, the 3PT line was introduced to the NBA during the 1979-80 season [1]. A shift in the game's scoring mechanics, the 3-pointer required a fundamental change in teams' playing strategies, as well as players' mindsets, practice, and training [3]. Over the years, as the 3-pointer became a staple in the sport, natural progression foresaw the importance—and reliance—on the 3PT shot as players and coaches became skilled in its application.

2.2 Evolution of the 3PT Shot

Since the turn of the century, basketball has undergone a fundamental evolution in the way the game is played: the advent of the 3PT revolution has transformed modern offensive and defensive schemes at every level of the game.

In the 1998-99 NBA regular season, the Sacramento Kings led the league with 943 3PT attempts (3PA), or 18.9 long distance attempts per game [4]. The 2018-19 NBA regular season saw each team record over 2,000 3PA [4]. The Houston Rockets heaved a historic 3,721 3PA, or an average of 45.4 3PT shots a game—over 52% of their field goal attempts (FGA) [4]. Over the past five NBA seasons, the 3PT rate (3PAr) of teams has increased significantly (Fig. 2) [5].

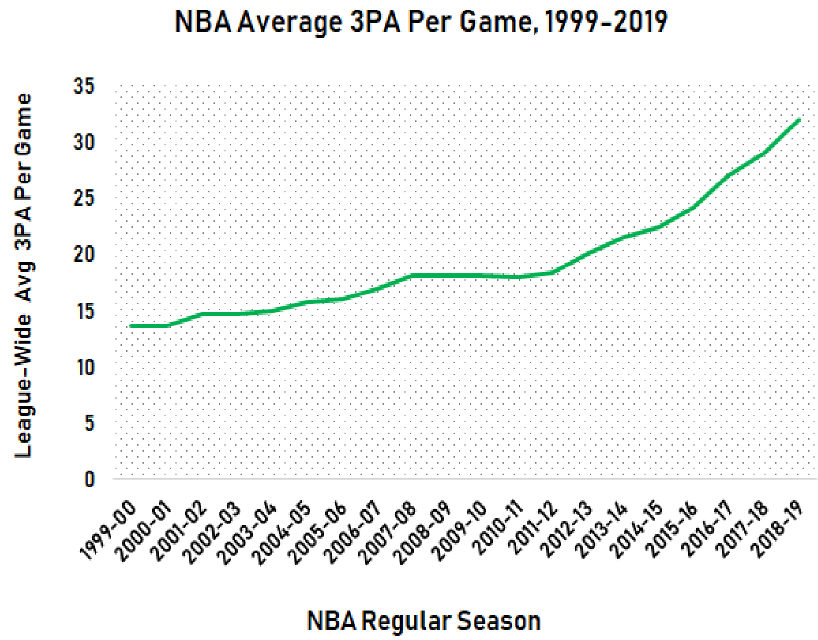

Fig. 2: League-Wide Average 3PA over the last 20 NBA Regular Seasons. Over the last five seasons, NBA teams have drastically increased 3PA, with the Houston Rockets leading the way. *Basketball Reference* [5].

Why are teams attempting more threes every season? The three provides value in two impactful ways: shot efficiency and transformation of offensive schemes. For background, the correlation of expected value between a 2PT and 3PT shot is straightforward mathematics: on the same number of attempts, sinking one-third of shots from beyond the arc is equivalent to making half of shots worth two points. These baseline percentages (33% for 3-pointers, 50% for 2-point shots) have become the focus of modern analytics, as shot tracking capabilities and computational models have transformed understanding of the game.

Over the last 20 years, NBA players have averaged 1.05 points per above-the-break (non-corner) 3PA and 1.16 points per corner 3PA [4]. In contrast, players have averaged just 0.79 points per 2PA outside of the restricted area and 1.20 points per 2PA inside the restricted zone [4]. For context, on 10 shots, non-restricted key and mid-range attempts would result in an expected 8 points, while purely 3PT attempts would result in 11 points. As expected, NBA teams have focused on layups, dunks, and high-percentage 3-pointers as the most efficient shots in the game. With this understanding, offenses have trended away from heavy reliance on the mid-range jump shot; the long 2-pointer is widely regarded as the worst shot in basketball. During the 2017-18 NBA season, shots between

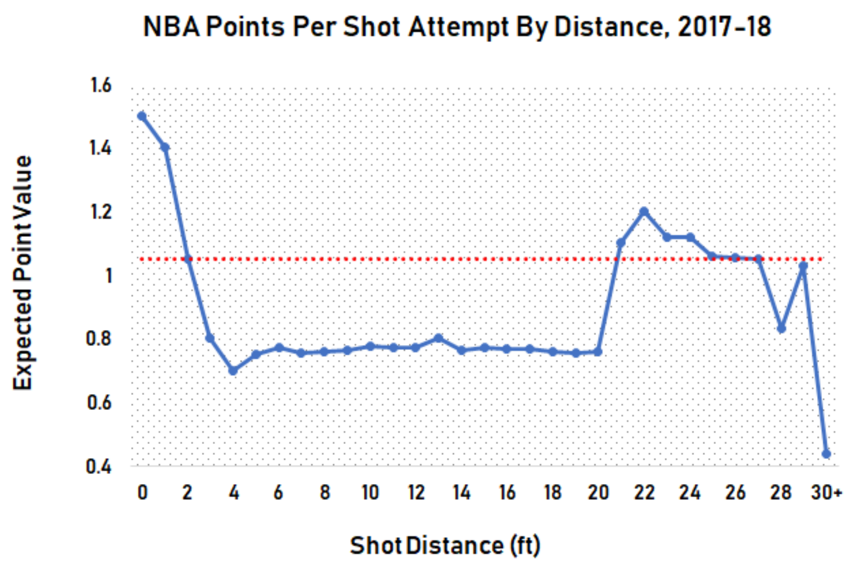

Fig. 3: NBA Points Per Shot Attempt During the 2017-18 Season. The red line represents the expected point value across every shot attempt. The value of any given shot dropped significantly beyond 3 feet but jumped back up at the 3PT line. *The Ringer* [6].

3 ft and the 3PT line fell below the average expected point value, while the most valuable shots proved to be at the rim or from downtown (Fig. 3) [6]. In the NBA today, over one-third of total shots are attempted from beyond the 3PT line [4].

The transformation of basketball offenses has paralleled the progression of the 3PT shot. By spacing the floor with 3PT shooting threats, teams can open up the court by providing space for players to drive, cut, screen, roll, and post-up. This emphasis on spacing produces open, high-quality shots from beyond the arc or directly at the rim. Recently, this evolution has resulted in remarkable offensive output, but most importantly, a strong correlation to winning teams at every level.

In the 2017-18 NCAA Division I men's basketball season, teams attempted an astonishing 37% of shot attempts from three [4]. With over 30 3PA per game, Belmont led a successful 24-9 season that recorded over 54% of their FGA from downtown, second in the nation [7]. This emphasis on spacing resulted in Belmont shooting a remarkable 61.7% 2PT field goal percentage (FG%), first in the nation by over 2% [8]. That season, Villanova, NCAA Division I Men's Basketball Champions, attempted 47.5% of their FGA from 3, making over 40% of these shots [4]. Purdue, a Sweet 16 contestant, nailed 42% of their 3s which accounted for 40% of their FGA [7]. Kansas, an Elite 8 finalist, drained 40% of their 3s, which accounted for 41.2% of their FGA [7].

Over the past decade, the three WNBA teams with the highest single-season 3PArs are the 2018 Phoenix Mercury, Seattle Storm, and Washington Mystics [6]. The Storm and Mystics met in the 2018 WNBA Finals after the Storm knocked out the Mercury in the semifinals [6].

Fig. 4: James Harden Shooting a 3. For the past three NBA post-seasons, the Houston Rockets have led the league in 3PA [9]. *Fansided* [10].

During the 2017-18 NBA season, the league's top two teams by regular season record, the Houston Rockets and Toronto Raptors, finished top three in 3PA [4]. The season's title favorites and Western Conference 2nd seed, the Golden State Warriors, led the league in 3PT FG% at 39.1% [4]. The Warriors and Rockets, arguably the league's two best 3PT shooting teams, were also the top teams in 2PT FG% [4].

During the 1980 NBA Finals, the Los Angeles Lakers and Philadelphia 76ers combined to make one 3-pointer during the entire series [6]. A testament to the game's evolution, since the 2014-15 NBA season, each champion (Warriors, Cavaliers, Warriors, Warriors, Raptors) has ranked in the top five of postseason 3PAr [9].

The 3PT revolution is particularly important for established "big man" Power Forward and Center positions, players who have traditionally played close to the basket. In the NBA, the evolution of the 3PT shot has paralleled the expansion of the "stretch 4" and "stretch 5" (Fig. 5) [9]. In the 2012-13 NBA season, players 6-foot-10 or taller attempted nearly 4,000 3-pointers [6]. During the 2017-18 NBA season, these players attempted more than 10,000 3-pointers [6].

Fig. 5: NBA Stretch Fours (L to R): Tom Chambers, Kevin Love, Ryan Anderson. In the 1986-87 NBA Season, Chambers became the first 6'10" or taller player to average one 3PA per game. Today, the "stretch four" is common among traditional Power Forwards. *NBA* [9]

Following this evolution, 3PT shooting has never been more instrumental to a player's skill set and ability to impact the game. The need to successfully implement optimal 3PT shooting mechanics for current and future generations of players has become critical for teams to develop a competitive advantage. This study will introduce a kinematic three-segment model in two dimensions adapted from Okubo and Hubbard and Schwark et al, detailed fully in **5.5** [18,19,20]. This software model aims to address the growing need for excellent 3PT shooting ability at every competition level by providing the foundation for a unique, player-specific output that provides optimal shooting mechanics for long range jump shots.

3 Basketball Shooting Mechanics

Shooting mechanics are often developed instinctively on an individual basis. Renowned free throw expert and former NBA shooting coach Tom Aberry believes players develop mechanics from a variety of sources: tips from other players and coaches, hand-me down information, or copying the styles of professional players [11]. He argues that many players never received proper instruction on how to shoot free throws; this study compels the same argument for the 3PT jump shot.

3.1 Need for Player-Specific Application among Current Training Regimens for Basketball Shooting

During early stages of learning the game, many players develop shooting mechanics that retain for the course of their careers. Without access to top coaches, many players rarely acquire proper education of effective shooting mechanics. For amateurs, form typically develops through comfort and casual play. Comfort is essential to developing consistency in a shot, and an important component to efficient mechanics.

Often, players who seek guidance on shooting obtain advice from coaches, books, videos, or online resources (unique shooting aid technologies exist, and are discussed in **4.1**). Examination of these traditional and generalized resources to improve shooting provide a baseline for the current landscape in training regimens. Nearly all of these methods present expertise qualitatively, in the form of words, inputs, and general suggestions. These resources provide insight on proper foot stance, knee bend, elbow tuck, hand placement, follow through, and more [11]. Enough research finds that many of these resources are generalized for all players—and in some cases present contradicting information. Notably, they lack quantitative analysis for individual application for each individual; extensive kinematic analysis studies, as discussed later, clarify that optimal shooting is dependent on individual player measurements. Furthermore, without understanding the physics behind the jump shot, many players are unfamiliar of the key differences between various shot types—a free throw, mid-range, or 3PT shot—and the required mechanical differences for each.

As expected, amateur and even experienced players are unaware of quantitative mechanics responsible for ineffective shooting forms. This study addresses the need to view shooting as an implementable science of kinematics and physics. In this way, players can understand effective methods for pinpointing and perfecting unique shooting mechanics. Indeed, this study asserts that optimal shooting mechanics exist unique to each individual that could drastically improve 3PT shooting. In implementing this optimization into an easy-use software model, this conception presents a novel technology in the realm of basketball shooting aid. Therefore, this model will provide a foundation for evolving the traditional sense in which shooting mechanics are accepted. By addressing the growing need for effective 3PT shooting across all traditional positions and following suit with the competitive advantage leveraged by modern analytics, this player-specific model provides a personalized depiction of optimal release mechanics for the 3PT shot that serves as a foundation for future training regimens.

3.2 Early Studies of Basketball Shooting Mechanics

Analysis of the free throw provides a baseline for understanding the mechanics of the shooting motion. Directly facing the backboard, the free throw is always taken 15 ft from the hoop and is typically characterized as a set shot—one in which the player's feet do not leave the ground upon release [12]. For consistency, early shooting studies centered on the free throw.

The free throw provides insight into shooting mechanics for the 3PT shot—a jump shot—as similar kinematics translate despite differences in horizontal shot distance and the addition of the jumping motion. Okubo and Hubbard, as detailed in **3.4**, illustrate the main kinematic differences between set and jump shots as the vertical motion of the shooting-side shoulder joint and ball release height [19]. While set shots have a non-zero upward velocity of the shoulder, jump shots experience zero or minimal vertical velocity of the shoulder joint. The player-specific optimization model will focus on shooting mechanics of the 3PT jump shot.

Earliest shooting studies focused on the biomechanics of the free throw. The underhand free throw, popularized by Naismith Hall of Famer Rick Barry, was recommended by proponents for its stability and smaller release angle error [13]. As the game progressed, the single-handed overhand shots evolved as the standard shooting motion. Studies in favor of this technique highlight its advantages in comparison to underhand shooting: decreased distance to the hoop, minimized angle and velocity of release, and increased margin for error for successful entry into the hoop [13].

Hay analyzed the ball's entry angle and its influence on the effective shape of the hoop (Fig. 6) [13]. A standard basketball rim is 18 inches (0.254m) in diameter; the standard NBA basketball is 9.43-9.51 inches (0.24-0.241m) in diameter [14,15]. As the ball approaches the hoop at varying angles, the corresponding effective viewing shape of the rim changes (Fig. 6). For a high entry angle (90 degrees), the shape of the hoop is circular (Fig. 6a). As the angle decreases, the shape becomes increasingly elliptical. At small angles, it becomes impossible for the ball to enter the hoop, as the hoop's effective diameter is smaller than the ball (Fig. 6d). Although a perpendicular entry would provide the largest error margin, it requires a release speed greater than 40mph and a peak trajectory height above 70 ft—beyond the capability of any player using an orthodox technique [13]. At such velocities and distances, the corresponding error in shot length is enormous for any release angle error—more than offsetting the basket's large effective shape. Hay determined the optimal entry angles to be between 38-45 degrees, corresponding to release angles between 49-55 degrees [13]. These calculations were performed on free throw shots; necessary velocity and ball height for 3PT shots would be greater in magnitude.

As detailed by Hay, obtaining effective entry angle on a free throw is important for successful and consistent shotmaking. For longer distances, as in the 3PT shot, successful shotmaking becomes even more dependent on high-arching shots and corresponding entry angles. With this understanding, this study will

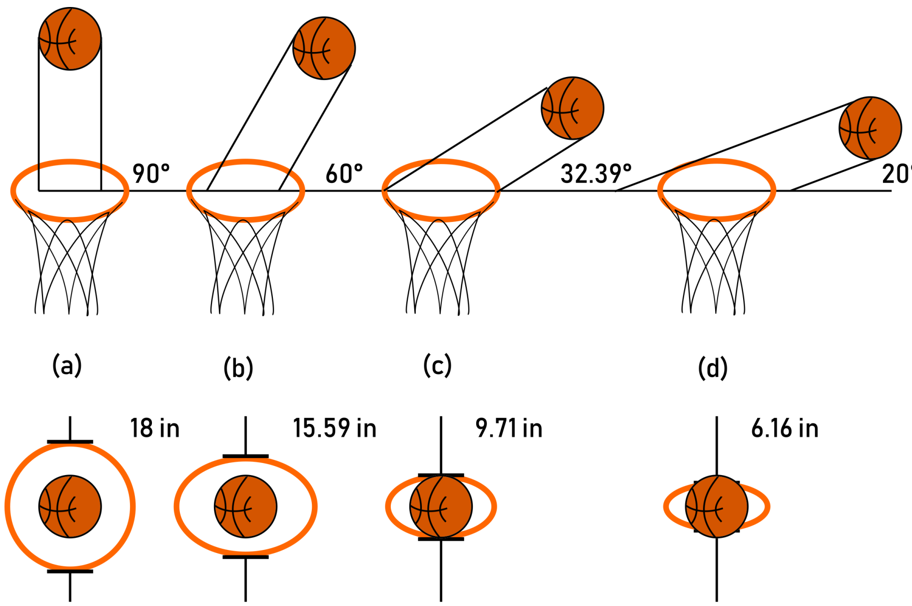

Fig. 6: Top Half: Geometry of Effective Viewing Shape for Various Entry Angles. Bottom Half: Corresponding Views Directly Behind the Basketball as it Approaches the Hoop. The effective entry diameter of the hoop changes from perfectly circular (a) to highly elliptical (d) as the entry angle decreases. *Adapted from Hay (1978), Fig. 112, pg. 216. [13]*

trend towards the upper limits of release angle ranges in favor of the larger corresponding viewing shapes.

3.3 Biomechanics of the Free Throw

Proper free throw technique involves a calculated pattern of unique biomechanics: bending the knees, aligning the elbow straight and beneath the ball in an "L", using fingertips rather than the palm, focusing eyes on the hoop, and upon release, using the elbow as a hinge, springing the forearm forward, snapping the wrist, arcing the ball high, and holding a proper follow-through [13]. The same general mechanics apply to 3PT shots with the addition of the jumping motion, in which the ball is released during the zenith of the jump.

For studies discussed and the optimization model detailed, release position of the shooting arm is the center of focus. While proper technique in the early catch and rise portion of the shooting motion are important to angular joint velocities, the kinematics of the release position are ultimately responsible for the ball release conditions and trajectory [13].

Hartle and Fulton provided insight into the relationship between trajectory, accuracy, and required strength for free throw shots [13]. As expected, a low trajectory arc required the most accuracy for successful shots, but necessitates the least strength. Alternatively, a high trajectory arc required the least accuracy but more strength. These results support the findings of Hay detailed in **3.2**. They concluded that a release angle of 55-60 degrees provides the optimal trade off between required power and accuracy.

Hudson analyzed biomechanics of collegiate players shooting free throws with a frame-by-frame video study [13]. Players were classified by skill: U.S. World University players, varsity players, and instructional class players. As expected, accuracy followed the skill hierarchy. An important outcome was the increased release height for high skill players: nearly 11 inches above the low skill group. Notably, the release angle ($\sim$52.5 degrees) and release velocity ($\sim$7.16 m/s) were not statistically different among the groups.

Tsarouchas and team investigated ball trajectory and movements of players' body segments [13]. Players were divided into efficient (80%$\geq$) and subpar ($\leq$65%) free throw shooters. Results highlight the impact of individual body segments. Lower extremities contributed mostly towards vertical displacement of the ball. The upper arm contributed in the vertical direction; the forearm contributed in the horizontal direction. While the hand contributed evenly to both components, the trunk had negligible contribution to either. They found that a high release point minimized travel distance of the ball. Similar to Hudson, there was no correlation between ball release height, release angle, ball displacement prior to release, and initial and final angles of the body's segments (with respect to the horizontal) and successful shotmaking.

Miller and Bartlett compared various joint positions and measurements between high level players [13]. Divided into positional groups (guards, forwards, centers), players were analyzed during 15 ft jump shots. Similar to Hudson and Tsarouchas, release angle and speed were similar between the three groups, inciting noncorrelation to successful shotmaking. Guards and forwards released the ball at 52 degrees and 6.28 m/s, while centers released the ball at 54 degrees and 6.4 m/s.

In *The Physics of Basketball*, physics professor John Fontanella explains that launch speed is a function of applied force on the ball upon release [16]. Necessary force is determined by shot distance to the goal, and more force is necessary for longer shots, as expected. He determined relationships between shot distance, release height, and ideal release angle for successful shotmaking. He provides a series of recommended release angles dependent on a player's height, detaild in **5.3**. In agreement with Hay, Fontanella asserts that higher arc increases the size of the target. However, higher arc requires more force at release and therefore more difficulty in shot control. Furthermore, the longer the ball is in the air, the more speed it gains, increasing the magnitude of collision with the rim. Alternatively, a low arcing shot requires more initial velocity to reach the rim, and therefore more force. From these observations, Fontanella concludes that as shot distance increases, launch angle should decrease to preserve decreased ball speed and therefore retain a "shooter's touch," a slow-moving ball that bounces favorably at the rim.

Hung and team expanded on many previous studies by performing a novel, comprehensive analysis on ball aerodynamics and shooting biomechanics [13]. Utilizing model simulations incorporating spin and drag, they employed accelerometers to record movement of upper body joints. Students, both experienced intramural and novice level players, were analyzed during free throws. As with previous findings, release angles and initial velocities were similar between the players. Instead, the defining difference between the flight trajectories correlates to spin rate. The experienced player exhibited twice the simulated spin rate (2.01 m/s) than the novice (1 m/s).

Spin is an important component that influences the vertical and horizontal trajectory of a shot. Spin largely contributes towards the Magnus force, which directs lift in a shot's trajectory (Fig. 7). The experienced player's greater backspin resulted in increased lift, helping the ball rise higher such that the peak height is reached further downstream by over 2' more than that of the novice. This preferred trajectory also resulted in a larger entry angle, and therefore increased success. In contrast, the novice's decreased backspin resulted in a smaller lift force. The peak height was reached earlier and farther from the basket, such that the ball fell consistently short and exhibited a reduced entry angle.

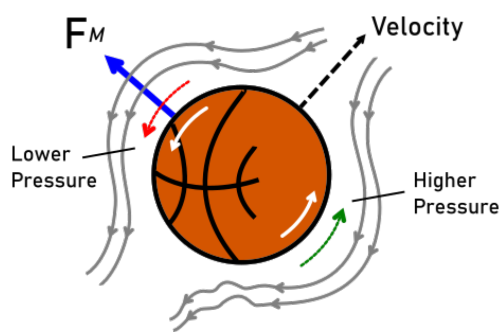

Fig. 7: Backspin of Ball Responsible for Magnus Force. The Magnus force is an observable phenomenon commonly associated with a spinning object moving through fluid, resulting in a lift force (F_M) perpendicular to the object's path [17].

Results in joint acceleration furthered these findings in spin differential. The novice recorded similar acceleration magnitudes for the fingertips and wrist, indicating a tendency to push rather than spin the ball. In contrast, the experienced player recorded large fingertip acceleration compared to the wrist. This *snap* of the wrist resulted in significantly more backspin and increased success. Snapping of the wrist is necessary for effective ball release velocity. Aerodynamic analysis concluded an optimal release angle between 51-56 degrees and velocity between 6.25-7.31 m/s.

3.4 Modern Studies of Basketball Shooting Mechanics

A series of studies by Okubo and Hubbard analyzed shooting arm motions in two dimensions utilizing a three-segment model derived from Schwark et al (Fig. 8) [18,19,20].

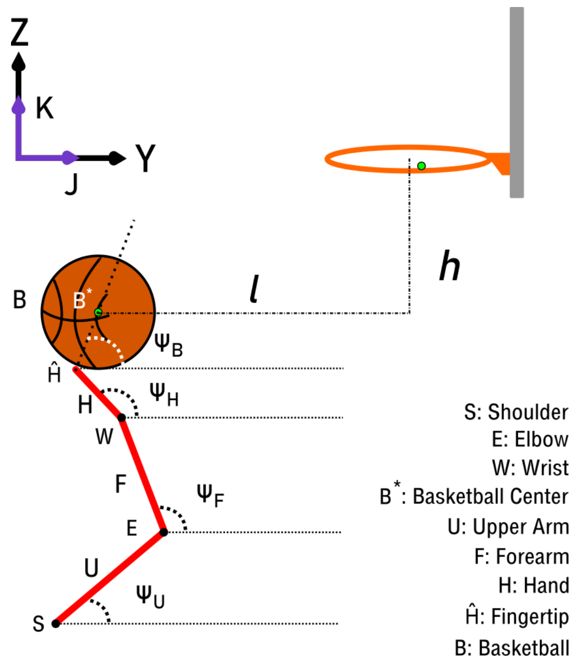

Fig. 8: A Three-Segment Shooting Arm Model in Two-Dimensions. The shooting arm is assumed to move in the vertical plane and have three rigid links of an upper arm, forearm, and hand with rotational joints at the shoulder, elbow and wrist joints. *Adapted from Okubo and Hubbard, Fig. 1, pg. 3. [19]*

A novel study analyzed the kinematics of shooting using a simulation model including the ball and shooting arm that estimated arm joint motions for a set of desired release speed, angle, and backspin [18]. For a given release position, there exists many angular displacement and velocity combinations of shoulder, elbow, and wrist joints that produce optimal release speed, angle, and backspin at release. This conclusion supports the basis for numerous optimal shooting forms rather than a single, perfect technique, as well as the need for player-specific application to determine these release forms. The developed model builds upon this study by determining the numerous angular displacement combinations for *any* given release position corresponding to a desired release angle.

They also evaluated the kinematic differences between set and jump shots, which are classified by the vertical velocity and acceleration of the shooting-side shoulder at release, previously touched upon in **3.2** [19]. During set shots, the player uses knee and hip extensions to accelerate the body vertically, and this upward motion contributes to ball release speed, angle, and backspin. During the jump shot, skilled shooters release the ball during the peak of the jump in which the body experiences no velocity. Therefore, the shooting arm motion is responsible for producing the ball release condition during the jump shot. As horizontal shot distance increases, as in the 3PT shot, upper arm angular speed also increases. In the jump shot, upper arm rotation produces effective arc in the shot; the motion of forearm and hand controls the horizontal shot distance and backspin.

Lastly, they provided insight into different shooting arm motions: pure hand, hand-forearm, and regular (hand, forearm, and upper arm) shots [20]. They determined that backspin angular velocity of the ball is a function of fingertip acceleration. In pure hand jump shots, there are limited release combinations in the release speed-angle-backspin space. In the hand-forearm jump shot, each configuration of the shooting arm is able to produce the desired ball release speed, angle, and backspin. The regular jump shot offers players unlimited release conditions for effective shotmaking given proper control and coordination of the upper arm, forearm, and hand segments.

3.5 Debunking the *Perfect* Shooting Form

Despite numerous studies, biomechanical analyses of shooting mechanics have not clearly identified the optimal coordination of arm components for the *perfect* shooting form. Indeed, there is no definitive, unanimous agreement of optimal shooting physics.

The studies previously explored support this observation, independently concluding that variable components of shooting mechanics do not directly correlate to shotmaking success. Hudson, Tsarouchas, and Miller and Bartlett agree that shotmaking is not dependent on a number of factors, most notably ball release height, angle, or velocity. Similarly, Okubo and Hubbard assert that numerous sets of angular displacements and velocities of shooting arm joints exist for a given release position. In other words, there are countless ways a player can shoot the ball to achieve a certain release.

Variables Independent to Shotmaking Success
Release Height
Release Angle
Release Velocity
Ball Displacement prior to Release
Initial Angle of the Body's Segments w. respect to the hor.
Final Angle of the Body's Segments w. respect to the hor.

As detailed, it is imprecise to depict a *perfect* shooting form because of the wide variations in a set of joint angles of the shooting arm that can result in consistent shotmaking ability. This is clear when analyzing the mechanics of historically great NBA 3PT shooters. These shooters utilize different shot mechanics in making jump shots at highly efficient rates.

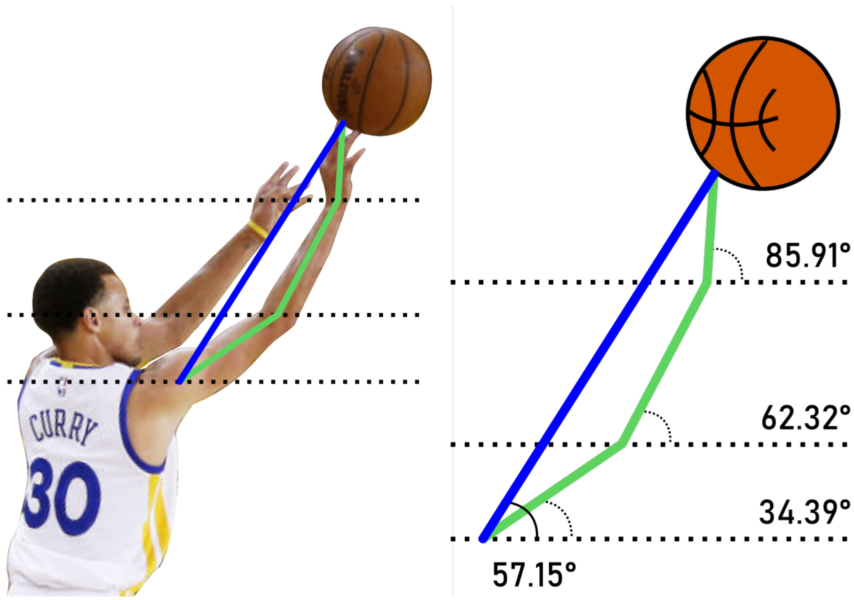

Fig. 9: Analysis of Stephen Curry's Joint Angles upon Release. During release of a 3PT shot, Curry exhibits a shoulder angle ∼35°, an elbow angle ∼ 60°, and a wrist angle ∼85° (w. the horizontal). Curry releases the ball ∼55° (w. the horizontal).

Stephen Curry, a career 43.6% 3PT shooter in the NBA, is considered one of the greatest 3 point shooters in the sport. On 3PT shots, Curry exhibits a consistent release angle near 55 degrees through ImageJ evaluation (Fig. 9). This is in agreement with ESPN Sport Science's analysis on Curry, depicting his consistent release angle between 51-55 degrees [21]. ImageJ evaluation on select release frames for other historic shooters provides similar insight. Klay Thompson releases the ball near 53 degrees, while Kyle Korver's release is slightly below 51 degrees. Due to limited availability of exact release frame images, determination of these angles was likely bounded in accuracy (exact release angles are slighty smaller than presented as the follow through pushes the fingertips further forward). Regardless, these depictions provide insight into the various release and joint angles of basketball's best 3PT shooters. For these players, the set of shoulder, elbow, and wrist angles (with respect to the horizontal) are unique upon release. These findings further suggest that optimal shooting is a matter of identifying a shot release that is favorable for the ball's trajectory and comfortable such that it can be performed consistently.

Following this comprehensive review, it's clear that the development of a player-specific release optimization model must incorporate two key components: (1) the sets of joint angles for desired release angles that has been extensively detailed and (2) the emphasis of wrist acceleration responsible for backspin and lift, as detailed by Hung et al in **3.3**. These key points of the model are detailed in **5.2**.

4 Market Opportunity

4.1 Current Shooting Aid Technologies

Beyond instructional guides, videos, and books, numerous current technologies exist as commercial products in the realm of basketball shooting aid [22].

Common accessories include shooting gloves that provide proper finger spacing on the shooting hand (Fig. 10a). The Pro Shot basketball shooting aid allows proper hand position around the ball during release [22]. The J-Glove Shooting Glove ensures an open-hand follow through by preventing finger flexion at the base of the fingers [22].

Specific equipment also develops proper off-hand placement and grip. The WetMitt Shooting Glove restricts non-shooting thumb interference when shooting (Fig. 10b) [22]. The Off-hand Perfect Jump Shot Strap stops rotation of the wrist that prevents off-hand hindrance during the shooting motion (Fig. 10c) [22].

Other training aids target shooting straight. The Straight Shooter Basketball Training Aid straps the shooting bicep across the torso on the user's hip (Fig. 10d) [22]. This ensures that the elbow and follow through align straight with the basket. The MarksMan Shooting Aid is worn on the wrist and prevents lateral movement of the hand during the shot's follow through (Fig. 10e) [22].

Unique devices aim to develop muscle memory for increased shooting arc. The Get It Up Shot Trainer forces players to shoot above an elevated loop which can be adjusted for desirable arc (Fig. 10f) [22]. The Bulls Eye Basketball Shooting Aid restricts a perpendicular upper arm and forearm that minimizes use of the shoulder, requiring more shot arc during the shooting motion (Fig. 10g) [22].

An overview of these shooting accessories depicts the current landscape of commercial shooting aid products. While they provide excellent tools to fine tune common mistakes within shooting, they are generalized and fail to incorporate player-specific quantitative analysis. For example, despite the sometimes drastic differences in player heights, and upper arm, forearm, and hand length measurements, these products typically exist as use for one size fits all. Similarly, inconsistencies likely develop between two players utilizing these products independently, paving the way for poor habits ingrained in muscle memory. A lack of numerical data reasons that these products are limited in their effectiveness. In addressing issues without confirming proper correction, these products may ultimately result in negligible shooting improvement.

Lastly, and most advanced, is the development of the SOLIDshot smart sleeve by Vibrado Technologies, not available for commercial use (Fig. 10h) [23]. The

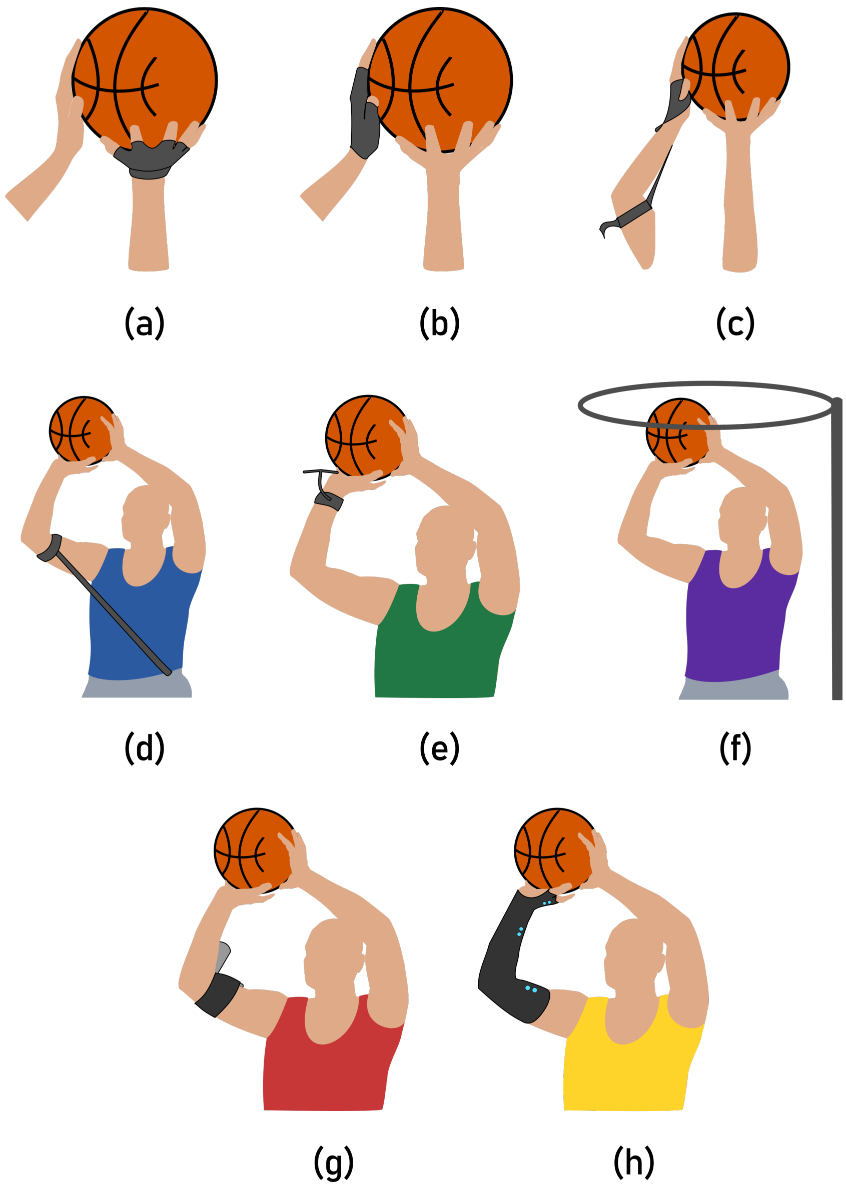

Fig. 10: Current Basketball Shooting Aid Products. Various commercial products exist to improve the shooting motion [22]. These products products are usually generalized as one size fits all use and do not take into account player-specific application.

sleeve incorporates three sensors, each containing an ARM processor, a gyroscope, and an accelerometer, positioned along the arm to detect whether shooting mechanics provide proper shot arc on a shot-by-shot basis. LEDs and speakers provide audible and visual cues when proper mechanics are performed, specifically targeting arm height, elbow tuck, and snapping of the wrist. In conjunction with a mobile application, the SOLIDshot sleeve provides real-time analysis and feedback. The SOLIDshot sleeve most resembles the goals of the player-specific release optimization model in providing a personable, easy-use technology that helps players develop optimal arc in their shot. However, the SOLIDshot sleeve, advertised as one size fits all, does not provide the unique sets of joint angles at release for optimal shooting, which allows players optionality in deciding comfortable mechanics. The combination of the player-specific model with a technology like the SOLIDshot smart sleeve would provide the foundation for transformative products in the scope of basketball shooting aid.

4.2 Market Need

The commercial landscape for shooting aid products depicts a clear need for player-specific methods. In this direction, the proper development of basketball shooting mechanics, specifically 3PT shooting, must be viewed as a science. If proper shooting mechanics is accepted as a series of calculated kinematics, knowledgeable players agree that calculated and specific release mechanics provide an attainable baseline to center training methods on. The developed player-specific release optimization model, while not complete by any means, provides a foundational technology that players of all competition levels can utilize to transform individual understanding of shooting mechanics, training regimens, and habits.

5 Development of a Player-Specific Optimization Release Model

5.1 Inspiration for the Model

In *Straight Shooter: A Game-Changing New Approach to Basketball Shooting*, renowned basketball shooting expert Bob Fisher provides an integrated approach to maximizing shooting success [24]. In covering topics of physics, biomechanics, and mindset, Fisher sets out to develop a new standard for basketball shooting instruction. He highlights an important fallacy within the sport: *why have all aspects of basketball evolved over the years except the approach to shooting?* This study draws inspiration from Fisher's approach by innovating a software-centric optimization model for the everyday player specific to long-range shooting.

5.2 Overview of the Model

From the extensive studies detailed, we transition to the main objective of this study: the development of a player-specific release optimization model. Similar to Okubo and Hubbard, a three-segment model in two dimensions is utilized to depict the segments of the shooting arm (Fig. 11).

We developed a player-specific release optimization model that will benchmark widely accessible and easy-to-use training programs for basketball shooting mechanics. By inputting player specific measurements (upper arm, forearm, and hand length), the model suggests optimal sets of joint angles for a given release angle. The model determines all valid ball release positions and corresponding sets of shoulder, elbow, and wrist joint angles for a specific release angle unique to a player.

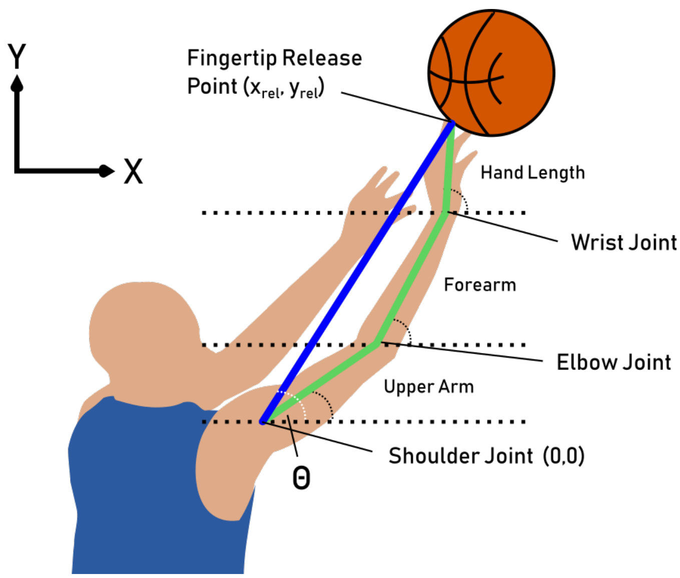

Fig. 11: A Three-Segment Shooting Arm Model in Two-Dimension derived from Okubo and Hubbard [19]. The model is adapted by including a coordinate grid, with the shoulder at the origin, and release coordinates at the fingertip position. All angles are measured counter clockwise from the horizontal.

Previous kinematic shooting models primarily focus on estimations for angles and angular velocities of a player's shoulder, elbow, and wrist for set values of ball release speed, angle, and backspin [18,19,20]. These models fall short in two important areas: 1) optimizing combinations of joint angular displacements for high percentage shots and 2) player-specific application for optimal shooting forms. This model aims to incorporate individual player measurements as well as kinematic analysis of the shooting motion to develop a player-specific application in 3PT shooting that will provide useful feedback for trainers, coaches, and players.

In utilizing the measurements specific to a player, the model provides optimal ranges for each joint angle that determine which set of arm angles provides the specific release targets from a 3PT shot. Naturally, during the motion of a jump shot, the set of joint angles in the shooting arm that a player finds comfortable varies among individuals. Furthermore, the model provides quantitative magnitude values for ball velocity and acceleration, as well as fingertip velocity and acceleration. These values are important because they directly correlate to the snapping of the wrist and the ball's flight trajectory, backspin, and entry angle. Specifically, fingertip acceleration provides valuable insight into necessary adjustments for snapping of the wrist and follow through mechanics. The innovation of this model capitalizes on this player-to-player variance to develop consistency towards a highly optimal, comfortable jump shot for any individual. Furthermore, the model can analyze players who struggle to successfully execute the jump shot and offer guidance towards improved mechanics.

The model is developed in Python 3, with data exported as .txt files and subsequently plotted in MATLAB.

5.3 Creating a Player-Specific Profile

User input sets the foundation for the player-specific model. The player's name, height (converted to meters), and handedness are first recorded to create a unique profile. Users input the size of the basketball used (Sizes 7, 6, 5, or 4), which determines the radius of the basketball used by the model's equations, later detailed in **5.5**. Similarly, competition level (Professional, Collegiate, or High School and lower) is clarified to distinguish the distance of the 3PT shot, d_{shot}. If the professional level is selected, the user is asked to specify the association (NBA, FIBA, or WNBA). If the collegiate level is selected, the user is asked to specify the program (Men's or Women's) and division (I, II, III). Users are also asked to select corner or above-the-break threes, which determines the exact distance of the 3PT line.

Based on the player's height, a table of recommended release angles is provided. This table is adapted from Fontanella and based on shots from the free throw line [27]. Because the major differences between the free throw and 3PT shot are vertical shoulder velocity and acceleration, release height, and ball release speed rather than release angle, the recommended release angles from the free throw line effectively translate for the 3PT shot. Angles are rounded up to the nearest half degree because of the favorable corresponding entry angles, detailed by Hay in **3.2**. The table is adjusted to include a larger scope of player heights as well as a range of recommended release angles. A one degree error margin is used for each range of heights.

Player Height	**Recommended Release Angle [°]**
5' - 5'4"	53 - 56
5'5" - 5'8"	52 - 55
5'9" - 6'	51 - 54
6'1" - 6'4"	50 - 53
6'5" - 6'8"	49 - 52
6'9" - 7'	48 - 51
7'1"+	≤47

From the input player height, an estimation of shoulder height determines the release point of the ball, as detailed in **5.6**. In inches, measurements of the player's upper arm, forearm, and hand length are input. After providing recommended release angles, a desired release angle in degrees, θ, is requested. The model will determine optimal series of joint angle sets (in degrees) that result in release position of the fingertips corresponding to the input release angle. An average set of joint angles is determined from the numerous series of joint angles (Fig. 15). The coordinates of the fingertips in this set corresponds to the release point, (x_{rel}, y_{rel}) (Fig. 11). This is in reference to the shoulder joint, which functions as the origin at (0,0) (Fig. 11).

5.4 Functionality

Minimums and maximums are set for the ranges of shoulder, elbow, and wrist angles (with respect to the horizontal) (Fig. 11). These ranges can be adjusted per user request but are set to default values found in literature [19]. At minimum, the shoulder angle is set to 15 degrees and reaches 50 degrees at maximum. The lower bound for the elbow angle is equal to that of the shoulder, as the forearm cannot extend past the upper arm during full extension. The elbow maximum is set to 60 degrees. The wrist minimum and maximum are 40 and 89 degrees, respectively. The release position of the wrist should be less than 90 degrees to ensure the ball is shot forward.

Following user input of arm measurements and desired release angle, the model utilizes three nested while loops to determine acceptable sets of joint angles given the conditions. All angles are determined counter clockwise from the horizontal in degrees. The outermost loop corresponds to the shoulder angle; the middle loop corresponds to the elbow angle; and the inside loop corresponds to the wrist angle. In the inside loop, the inverse tangent of the fingertip coordinates at release determines the angle with respect to the shoulder joint. If this angle is within the margin of the desired release angle, this set of joint angles is valid amongst a series of sets. The coordinates for the elbow, wrist, and fingertips are saved to be plotted later. By default, the error margin is set to 0.01 and the while loops end by incrementing the angles by 1 degree. These values are chosen to reduce the number of saved joint angle sets to a feasible size, but can be adjusted for user preference. Pseudocode of the entire release optimization of joint angle sets is provided below.

Algorithm 1 Release Optimization of Set of Joint Angles

Result: Determine the series of joint angle sets corresponding to a specific release angle and store the coordinates in lists

while *shoulder angle* $\leq$ *max shoulder angle* **do**
    store x elbow coordinate
    store y elbow coordinate

    set elbow angle to elbow angle minimum

    while *elbow angle* $\leq$ *max elbow angle* **do**
        store x wrist coordinate
        store y wrist coordinate

        set wrist angle to wrist angle minimum

        while *wrist angle* $\leq$ *max wrist angle* **do**
            store x fingertip coordinate
            store y fingertip coordinate

            determine the angle of this release point

            if *determined angle - error margin* $\leq$ *release angle* $\leq$ *determined angle + error margin* **then**
                save x elbow coordinate
                save y elbow coordinate
                save x wrist coordinate
                save y wrist coordinate
                save x fingertip coordinate
                save y fingertip coordinate
            end
            increment wrist angle
        end
        increment elbow angle
    end
    increment shoulder angle
end

5.5 Equations Used

The player-specific release optimization model utilizes numerous equations established in literature [19,25]. A visual depiction of the kinematic model incorporating the three-segment shooting arm and the ball upon release is depicted (Fig. 12). The Newtonian frame XYZ with unit vectors ***I, J, K*** is fixed relative to the hoop center. The XZ plane is horizontal and the Y axis is up. The XY plane includes the three-segment arm links.

Shoulder height in meters, $h_{shoulder}$, is derived from input player height, h_{player}, through an equation determined through experimentation (Eq. 1), as

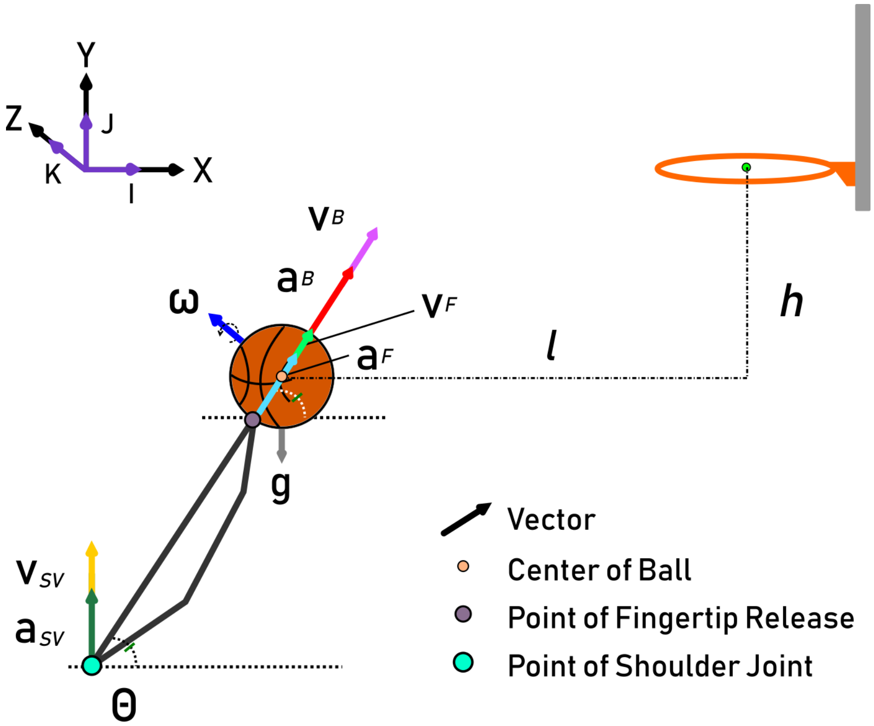

Fig. 12: Kinematics Model used by the Player-Specific Optimization Model. Adapted from Okubo and Hubbard and Schutzengel, Varin, and Quinlivan [19,25].

detailed in **5.6** (Fig. 13). Jump height, h_{jump}, is set to 0.1524m (6"), an estimation based on experimental research by taking the average jump heights of professional players during the jump shot (ImageJ evaluation). Jump height can be adjusted for more specific results. Shoulder height, $h_{shoulder}$, release angle in degrees, θ, release point, *(*x_{rel}*,* y_{rel}*)*, radius of the ball, R_b, and 3PT shot distance, d_{shot}, are used to calculate shot parameters as h meters below (Eq. 3) and l meters away (Eq. 4) from the hoop, which is 3.048m tall (10 ft); h and l are measured from the center of the ball upon release (Fig. 12). Three-point shot distance, d_{shot}, is specified by individual user input, dependent on competition level and location of the shot (corner or above-the-break 3-pointer), as various distances of the 3PT line exist detailed in **2.1**. The equation for distance between two points (Eq. 2) is used to calculate the hypotenuse distance, d_{hyp}, between the shoulder joint origin, *(0,0)*, and the fingertip release point *(*x_{rel}*,* y_{rel}*)*.

$$h_{\text{shoulder}} = \frac{21}{25} h_{\text{player}} \quad [\mathbf{m}] \tag{1}$$

$$d_{\text{hyp}} = \sqrt{(x_{\text{rel}})^2 + (y_{\text{rel}})^2} \quad [\mathbf{m}] \tag{2}$$

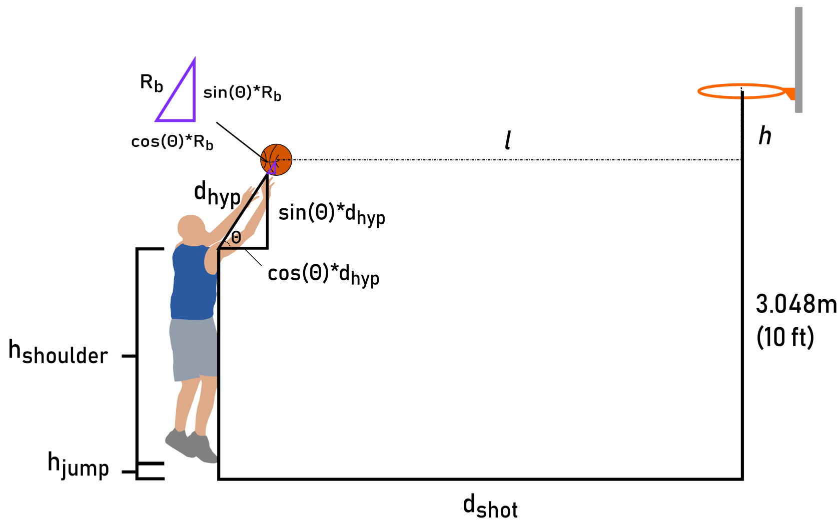

Fig. 13: Determination of Shot Release Height and Distance. The parameters h and l were determined by the shot distance, jump height, shoulder height, release point, and release angle.

$$h = 3.048 - (h_{\text{shoulder}} + h_{\text{jump}} + d_{\text{hyp}} sin\theta + R_{\text{b}} sin\theta) \quad \textbf{[m]} \tag{3}$$

$$l = d_{\text{shot}} - (d_{\text{hyp}} cos\theta + R_{\text{b}} cos\theta) \quad \textbf{[m]} \tag{4}$$

Okubo and Hubbard established a relationship for ball release velocity, v^B, as a function of gravity, g, release angle in degrees, θ, and shot parameters, h and l (Eq. 5) [19]. They also derived backspin of the ball, ω, as a function of shot distance upon release, l (Eq. 6). The backspin equation is modified for units in Hertz (s^{-1}).

$$v^B = \sqrt{gl/2cos^2\theta(tan\theta - h/l)} \quad \textbf{[m/s]} \tag{5}$$

$$\omega = ((l + 2)/3)\boldsymbol{K} \quad \textbf{[Hz]} \tag{6}$$

Further equations are derived for fingertip velocity, v^F, (Eq. 7-9) and fingertip acceleration, a^F, (Eq. 10-12) as functions of ball release velocity, v^B, release angle in degrees θ, backspin ω, gravity, g, and ball radius, R_b [19]. A significant modification of this model is the substitution of the release angle in place of Ψ_{B}, the angle including the fingertip and ball center and the horizontal, as detailed in **5.6** (Fig. 8). The equations also include the addition of the vertical shoulder velocity, v_{sv}, and acceleration, a_{sv}.

$$v^{F_{\text{Hor}}} = (v^B cos(\theta) + R_b \omega sin(\theta))\boldsymbol{I} \quad \textbf{[m/s]} \tag{7}$$

$$v^F{}_{\text{Ver}} = (v^B sin(\theta) - R_b cos(\theta) - v_{\text{sv}})\boldsymbol{J} \quad [\mathbf{m/s}] \tag{8}$$

$$v^F = \sqrt{(v^F{}_{\text{Hor}})^2 + (v^F{}_{\text{Ver}})^2} \quad [\mathbf{m/s}] \tag{9}$$

$$a^F{}_{\text{Hor}} = R_b \omega^2 cos(\theta)\boldsymbol{I} \quad [\mathbf{m/s^2}] \tag{10}$$

$$a^F{}_{\text{Ver}} = ((R_b \omega^2 - g) - a_{\text{sv}})\boldsymbol{J} \quad [\mathbf{m/s^2}] \tag{11}$$

$$a^F = \sqrt{(a^F{}_{\text{Hor}})^2 + (a^F{}_{\text{Ver}})^2} \quad [\mathbf{m/s^2}] \tag{12}$$

Schutzengel, Varin, and Quinlivan, winners from the 2011 University Physics Competition, published *The Physics of a Three Point Shot* [25]. They derive an equation (Eq. 13) for the acceleration of the ball upon release, a^B, as a function of gravity, g, ball radius, R_b, ball mass, m_b, cross-sectional area of the ball, A, density of the air, ρ, drag coefficient of the ball in the air, C_d, backspin ω, ball velocity, v^B, and forces acting on the ball [25]. During flight, motion of the ball is governed by the forces of gravity, buoyancy, drag, and lift (Magnus force) (Fig. 14) [26]. The first term of Eq. 13 accounts for gravity, the second term for the Magnus force, and the third term for drag. Buoyancy can be neglected in calculations due to its diminutive magnitude in comparison to gravity, which is over 67× larger. During flight, $\hat{v}^B$ distinguishes the unit vector in the direction of the ball. To align the different axes used, the velocity component vectors are scaled by cos(45°).

$$a^B = \overrightarrow{g} + \frac{1}{m_b}\Big(\frac{16}{3}\pi^2 R_b^3 \rho \overrightarrow{\omega} \times \overrightarrow{v}^B - .5 C_d \rho A \left|\overrightarrow{v}^B\right|^2 \hat{v}^B\Big) \quad [\mathbf{m/s^2}] \tag{13}$$

5.6 Model Assumptions

Following the parameters set by Okubo and Hubbard, vertical shoulder velocity, v_{sv}, and acceleration, a_{sv} are set equal to 0 and -g in Eq. 8 and 11, respectively. This assumption asserts that the shooter releases the ball during the zenith of the jump shot, creating a zero velocity condition at release in which gravity is the only force present. Furthermore, in adjustment to Fontanella's findings, we round release angles up by half degrees as mentioned in **3.3**. First, a larger release angle provides optimal entry angles and therefore effective viewing shape, as detailed by Hay in **3.2**, which provides a larger margin of error consistent with the required increase in release velocity and ball height for a 3PT shot. Additionally, a larger release angle corresponds to a higher effective release height, which minimizes distance that a 3PT shot must travel and thus less force required at release during the shooting motion.

The relationship between player height and shoulder height (Eq. 1) is an estimation based upon 30 separate experimental data collections both in-person

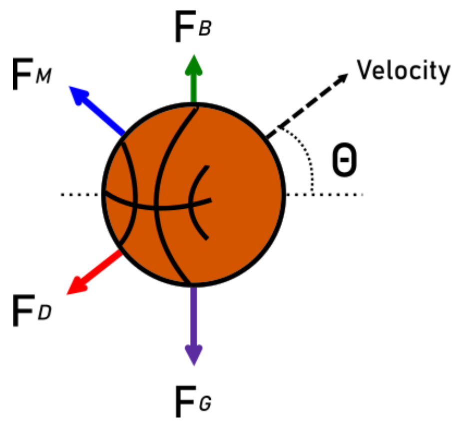

Fig. 14: Complete Force Diagram of Basketball in Motion. Gravity (F_G), the most dominant force, acts in the downward vertical direction. It is directly opposed by the buoyant force (F_B), caused by pressure differences above and below the ball. The Magnus force (F_M) acts perpendicular to the ball's path, while the drag force (F_D), caused by air resistance, directly opposes the ball's path [26].

and through analysis using ImageJ. This estimation is used for simplification, as shoulder height is not easily measured for many users. However, the model can be easily adapted for a specific shoulder height for more accurate calculations.

As previously mentioned, an adjustment to the equations for fingertip velocity and acceleration presented by Okubo and Hubbard is the assumption that upon release, the angle of ball release with the horizontal corresponds with that between the line connecting the fingertip and shoulder joint and the horizontal (Fig. 12). This assumption was constructed through image analysis using ImageJ on 30 separate experimental release frames of amateur, experienced, and professional players shooting 3PT shots.

Under the same assumptions by Schutzengel and team, the ball is assumed to be spinning with constant angular velocity and that spin was purely backspin. It is not possible for the ball to exhibit topsin using any conventional shooting technique. Similarly, the drag coefficient of the basketball is assumed to be 0.54, a value found in literature [25].

Despite variance in the size of a basketball, the model assumes ball radius values dependent on the user's input of the basketball size. A standard size 7 basketball (29.5" in circumference) is set to a radius value of 0.119m (4.7 inches). Size 6 (28.5"), size 5 (27.5"), and size 4 (26.5") basketballs have radius values of 0.1152m (4.54 inches), 0.111m (4.38 inches), and 0.107m (4.22 inches), respectively. The radius of the basketball can be specified for exact values for user preference.

5.7 Model Output

Following the completion of user input, the model provides the various series of joint angle sets corresponding to a specified release angle. In MATLAB, these series of joint angle sets are plotted as dashed lines. An average set of angles among the series is emphasized in red, providing the user a consistent set of target joint angles for successful shotmaking unique to the player's specific profile and measurements (Fig. 15). For example, a unique player profile is created for Michael, who is 5'5" with upper arm, forearm, and hand length measurements of 11", 9.5", and 7.25", respectively. Using a standard size 7 basketball, Michael is targeting guidance for corner threes at the NCAA men's Division I level. Michael's release optimization of joint angles for a 54 degree release is depicted (Fig. 15).

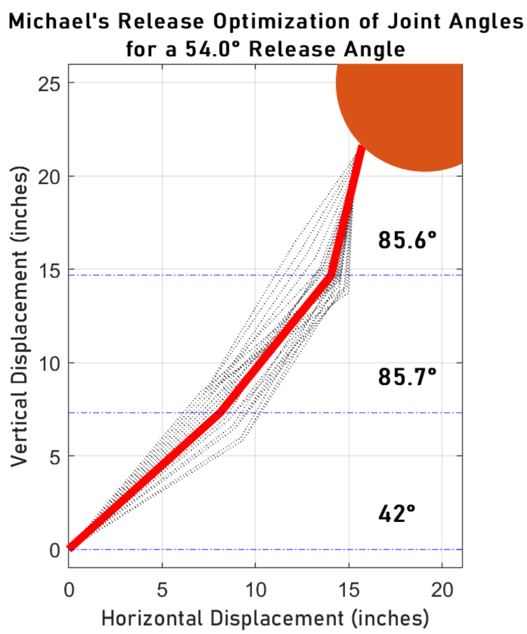

Fig. 15: Release Optimization of Joint Angles for Michael. For specific player measurements, a set of joint angles is provided correspondent to a 54° release. The average of this set is depicted in red and provides a set of target values for Michael to base training on.

The model utilizes Eq. 1-13 to determine the release properties of the ball, fingertips, and shot parameters, providing quantitative values of h, l, ball release velocity, ball acceleration, fingertip velocity, and fingertip acceleration usable by players. These values serve as benchmark targets that users can implement to guide basketball shooting practice and training regimens. For Michael, these values are depicted in the table below.

Release Property	**Magnitude**
h	0.87m
l	6.14m
Ball Velocity	8.40 m/s
Hor. Fingertip Velocity	5.19 m/s
Ver. Fingertip Velocity	6.73 m/s
Fingertip Velocity	8.50 m/s
Hor. Fingertip Acceleration	0.52 m/s^2
Ver. Fingertip Acceleration	0.88 m/s^2
Fingertip Acceleration	1.02 m/s^2

5.8 Potential Applications

Providing an optimal set of joint angles for a desired release angle or ball release properties is not particularly applicable or intuitive in standard application. Indeed, the average player will not seamlessly apply the results of the model to everyday training. Rather, this study is presented as a novel technology towards the foundation of player-specific application in basketball shooting aid. In general, a player should utilize this model to understand target values for the placement and motion of the shooting arm upon release unique to the player. For example, a player who has maintained a consistent, ineffective shooting form for most of his or her playing career can be made aware of key adjustments that guide corrective training. Perhaps a player releases the ball with a high shoulder angle resulting in long shots; the model can immediately provide corrective guidance by advising the player to aim for an optimal shoulder angle near 39 degrees to decrease the distance, for example. Similarly, if a player consistently shoots the ball short, the model may instruct for a wrist angle of 82 degrees to propel the ball farther forward, for example. Furthermore, for a player who consistently shoots the ball short on 3PT shots, the model can suggest this player increase the force of the ball upon release. A calculated output may advocate increasing fingertip velocity nearly 3 m/s faster, achieved by accelerating the fingertips around 1 m/s^2 more, for example. Though these specific values—39 degrees, 82 degrees, 3 m/s, and 1 m/s^2—cannot be easily measured for most players, they provide general guidance in broad ranges for understanding where a player should make necessary adjustments. As emphasized, this study is far from complete, and the intention of the model is to be used in conjunction with other technology for more concrete application.

The use of visual recording, common among professional and collegiate basketball teams, provides a useful means to review shooting angles as well as angles of the joints upon release. Specifically, motion tracking technology provides an advantageous mechanism to record, store, and analyze these data targets. For example, collegiate coaches currently incorporating motion tracking may utilize the model to provide guidance in 3PT shooting drills of his or her players. Based on the results of motion tracking data, coaches can suggest that players raise the elbow higher or follow through further by a calculated angle in direct comparison to the results of the model. Through motion tracking software, the application of these specific angle values will be easier to apply for players utilizing such guidance.

Similarly, the use of sensors like that of the SOLIDshot sleeve will provide feedback on ball release properties in comparison to the model. For example, collegiate coaches applying wearable sensors may utilize the model to provide guidance on follow through mechanics during the shooting motion. Coaches can suggest that players increase the snap of the wrist by a calculated magnitude for increased fingertip acceleration and therefore ball velocity if the player consistently shoots the ball short. In contrast, the player should snap the wrist less by a certain degree if his or her shots are consistently long. Through wearable sensors and similar equipment, the application of these specific velocity and acceleration magnitudes can be integrated effectively by instant feedback suggesting magnitudes too high or too low.

The conjunction of the model with motion tracking and sensors paves the way for optimized training regimens in the basketball shooting realm, providing continuous and instant feedback that will allow players to drastically improve 3PT shooting in reduced time.

The model can also be incorporated into a mobile application that provides instant feedback and ease of use. Through these means, players of all levels can access a release optimization model unique to the player's measurements that can provide advantageous training during shooting practice. Eventually, the model will pave the way for future shooting aid technologies, whether commercial products like shooting gloves, sleeves, equipment or software like mobile applications and programs.

5.9 Source Code

Source code for the entire project can be found on Github at the following link:

6 Future of Study

6.1 Improvements

A next step for the model is the incorporation of three dimensions, which would provide input of the ball's lateral movement during flight. Utilizing the same XYZ Newtonian frame, hand placement of the ball during release can provide insight into the ball's movement in the Z axis (Fig. 16). The angle between the vertical and the center of the hand, ϕ, is equivalent to the vertical angle across the intersecting vertical line. Lateral offset of the ball can be determined as a scaling factor of $\cos(\phi)$. This angle can provide insight into lateral offset for shooters consistently shooting the ball left or right of the basket. A useful way to incorporate this feature in a mobile application would be a sliding scale from 0 to 60 degrees that corresponds to the user's preferred shooting hand placement.

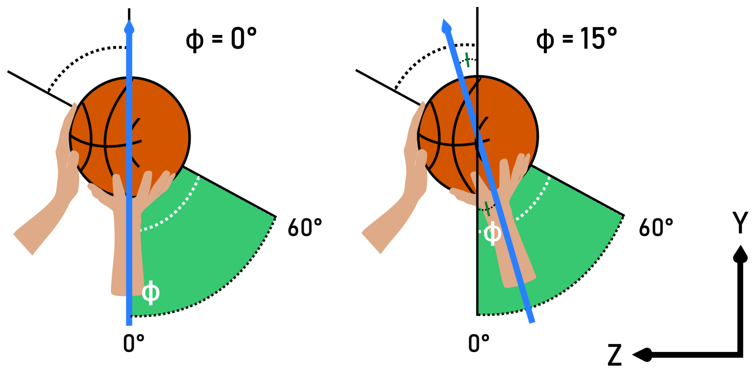

Fig. 16: Incorporation of Three Dimensions in the Z Axis. A significant addition to the model would account for lateral offset of the ball at the release position, providing insight into the ball's final location in reference to the hoop.

Furthermore, a useful addition to the model would be the incorporation of a trajectory plot of the ball during flight. A depiction of trajectory will provide additional insight into the ball's properties following release. Specifically, the player can attain information regarding optimal entry angle, peak of ball trajectory, and time of flight. This additional information provides valuable guidance for players who lack effective arc on 3PT shots. Players who shoot consistently short can reference trajectory plots to understand when the ball should reach peak height and the necessary adjustments for release angle and ball velocity; alternatively, a player who shoots long shots can inquire for the opposite. These additional insights can be used in conjunction with visual recording, motion tracking, and other technologies that will further cement the player-specific model as an informative tool during shooting training regimens.

Lastly, a significant addition to the model would be the incorporation of the body's lower extremities, including kinematics analysis of the trunk and legs. Adapting the model as full body and multi-segmented will provide a comprehensive understanding of the player-specific application that distinguishes optimal shooting between players. Incorporation of the joints of the ankles, knees, and hips will provide a more complete set of joint angles that can introduce the capability of player-specific adjustments to the model. For example, if a player prefers a specific foot stance during the shooting motion, a full body model will be able to adapt to these adjustments and output a still optimal set of joint angles for a corresponding release motion. Similarly, it would be insightful to understand the adjustments necessary to the segments of the shooting arm if a player prefers using more knee bend and vertical hip velocity. This incorporation of player specific habits and player preference would pave way for a truly comprehensive and novel foundation of player-specific application in the realm of basketball shooting aid.

7 Glossary

Kinematics: the branch of mechanics concerned with the motion of objects without reference to the forces which cause the motion.

Optimization: the selection of a best element from some set of available alternatives

Basketball Terminology

2PA: 2-point attempt; a field goal attempt inside the 3PT line

3PA: 3-point attempt; a field goal attempt beyond the 3PT line

Above-the-Break 3: any 3-pointer that is not taken from the corners

Corner 3: any 3-pointer that is taken from the corners

Free Throw (Foul Shot): an unopposed attempt to score a point by shooting from behind the free throw line, situated at the end of the restricted area

Key (Paint): the area located under each basket that begins at the endline and ends at the top of the key, with the free throw lane as the side boundaries; known as the paint because this area of the court is usually painted in a different color than the rest of the court

Mid-range: any 2-point field goal attempt outside of the key, but inside the 3PT line

Restricted Area: the portion of the key denoted by an arc that is positioned four feet from the basket; denotes area where a defending player cannot force a charging foul

8 Acknowledgments

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