Exam Feb 2024

Michael Koller

February 7, 2024

Contents

NOTE: For avoidance of doubt. This is a exam in person in the room allocated in your individual schedule.

1 NEW

Please find below the email addresses to which the presentations have to be sent (to all of them):

- michael.koller@bluemail.ch (If you have already sent no need to resend)
- marco.rodrigues@math.ethz.ch, lorian.krach@math.ethz.ch, jakob.heiss@math.ethz.ch

Please consider taking your laptop with you for presentation of you workings.

2 Aim

The aim of this document is to provide guidance for the upcoming exams and the respective preparations needs. It is expected that each student prepares the respective tasks themselves and does in particular not copy from other students. The usual plagiarism requirements are to be strictly adhered to and violations have corresponding consequences as outlined in the various guidelines of ETHZ.
The following paragraph answers the table in the student guide (section A):
We note that all material can be found on the git-hub used in the lecture, ie https://github.com/michaeldfkoller/lv12

Date, time, and duration; As per students portal on 8.2 and 9.2.24; amended schedule as agreed at the end of the document.

Allowed aids (books, notes, calculators, data tables, etc.; No restrictions - open book exam. Note time limit as per exam schedule applies

An email address or uploading functionality to transfer an electronic copy of your written notes, if so desired: **The primary email address is mikoller@ethz.ch. The presentation for the task needs to be sent at least 24h prior to the exam. As soon as the assistant for the exam is known I will send you the respective email also, and the task is to be sent to both email addresses**

Whether or not you will also be required to use postal mail to send your written notes to ETH (and the address to do so): Electronic in pdf format to me and the teaching assistant is sufficient.

Two independent ways of contacting the examiners in case of problems, including phone/SMS: **The two emails of me and the teaching assistant plus my phone +447826943343. Note it will be difficult to phone me during an exam before the scheduled time, hence I suggest an sms, what's app etc.**

And any other exam guidelines that require preparation on your part: See below

3 Process

1. I will send you these instructions at least one week before your exam. If you have high level questions please send them to me until one week before the exam date. I will try to answer them if deemed sensible and update this paper.

2. You prepare the below allocated task and send a presentation to me and the teaching assistant at least 24h before the actual exam - ie deadline for exams hold on 8.2.24: 7.2.24 no later than 23h, For exams hold on 9.2.24 the cut off date is 8.2.24 no later that 23h. Late submission might lead to penalties. The expected file format is pdf. You can share your workings as *.py or as a jupyter notebook.

3. On the day of exam you will present you findings and will be questioned correspondingly. Hence the exam is a mix between an expert discussion and theoretical questions according to the course. It is expected that the student shares his screen for the parts he is presenting. You do need to bring a printed copy of the exam, but if you want to present you need to bring your lap-top etc.

3.1 What is expected?

The aim of the expert discussion is to show to me you ability to apply the theory (and to prove certain parts of it) based on concrete questions. What is of utmost importance is the clarity of thought. You will be asked to define and use certain models to answer the questions below. I would follow the following grid:

- Why did I use the model I am presenting. What are the implicit and explicit assumptions I have made and which modelling risks are induced from it
- What is the basis of the model and why is the model adequate for the question
- Which parameters have I chosen and why. (Note I will give you some parameters, but you might need to tweak and amend them, If a certain parameter can not be determined exactly please make a sensible assumption. You can use all mortality tables etc from the course.
- What are the main finding of your analysis and what consequences do you draw.

Please note that your presentations should be adequately concise since we need to cover as much ground as possible - if you are not sure put a respective explanatory slide in the appendix which can be discussed if needed. Your mark will be determined based on a) the quality of the presentation b) The solution of the problem and c) The oral discussions and the theoretical questions. I expect that all the tasks are addressed in the presentation but I might focus during the oral part on one or two of them only.

4 Tasks for Preparation

4.1 Task 1: Markov Model (Theoretical Part):

Note: If you are doing the exam for Selected Chapters I obviously expect from you also the time-continuous versions addressed.

- Definition of a Markov chain and Chapman-Kolmogorov
- Model per se, induced cash flows
- Calculation of Cash flows, including the respective calculations and proofs
- Thiele Equation
- How would you do a decomposition of the premium into risk and savings part? How would you do a technical analysis? Note this was not explicitely covered in the lecture but was covered in the classical case. You might want to consult also chapter 10 of ram_sp.pdf

4.2 Task 2: Stopping to pay premium:

The aim of this task is to analyse the effect of policyholder paying premium. We consider the following policy and want to see what happens in the policyholder stops premium payment. For students taking Life Insurance Mathematics the question will focus what this means re equivalence principle and how benefits accumulate over time and for students taking selected chapters the question centre both around ALM; risk minimising portfolios and bonus strategies. The details will be provided as per below.
Product

- We consider a mixed endowment ($A_{x:n}$).
- $x = 80$ and $n = 10$, hence maturing at the age of 90.
- Technical interest rate $i_T = 2\%$ with prevailing market rate of $i = 4\%$.
- Benefit level $L = 100000$, Premium to be determined by mean of *equivalence principle* at inception wrt i_T.
- Mortality given by

```
def Qx(gender,x,t,param =[]):
    # This is our default mortality
    if gender == 0:
        a =[2.34544649e+01,8.70547812e-02,7.50884047e-05,-1.67917935e-02]
    else:
        a =[2.66163571e+01,8.60317509e-02,2.56738012e-04,-1.91632675e-02]
    return(np.exp(a[0]+(a[1]+a[2]*x)*x+a[3]*t))
```

  Note we set $t = 2020$ for all ages to ensure comparable results!

- The policyholder is a man.
- Premium are paid 1/1 prenumerando.

Mechanism of stopping to pay premium A typical insurance policy allows the policyholder to top paying premium after having paid premium at every point in time. Hence in extremis the policyholder can only pay one premium over the whole policy term. In case a policyholder stops paying premium the policy is in the state "Premium Free" and the future benefits will be reduced by means of actuarial principles: The mathematical reserve at this point in time is taken as a single premium resulting in lower benefits going forwards. Example: Assume we have a benefit L and a premium P at inception. Assume the policyholder stops paying premium at $t = 2$, ie does not pay premium at time 2 (3rd premium). In such case:

- He is protected to the full extent between times $[0,2[$, and
- The benefit is reduced thereafter to $\tilde{L}$ with nil premium, and
- $\tilde{L} = \frac{{}_2V_x}{A_{x+2:n-2}}$.

Questions

1. Calculate the premium for product at inception (Students: **all**).
2. Calculate the Benefit $\tilde{L}$ if the policyholder stops after one premium (Students: **all**).
3. Calculate the Benefit Level as a function of the number of premium paid (Students: **LV**).
4. Which equivalence principle is fulfilled for the first premium assuming only one premium is paid. Proof your statement. (Students: **LV**).
5. Which equivalence principle is fulfilled for the second premium assuming only one premium is paid. Proof your statement. (Students: **LV**).
6. What is a risk minimising investment strategy for this type of risk. To do so consider the what would happen to the present value of benefits in case interest rates move at time $t = 1$ considering two cases a) We consider all cash flows disallowing for the optionality of premium free option; b) allowing for premium free option and the respective cash flows (Students: **Selected Chapter**):
   - Calculate the cash flows for both option a) and b) and see what happens if interest rates move $\pm 1\%$ starting from $i_t = 2\%$
   - Calculate the risk minimising portfolios for each premium paid. In the following form (example age 85; $\mathscr{Z}_x$ denotes the zero coupon bond at time x starting with $x = 0$ at age 80):

Premium paid at age 85

Age	Unit	Units for Mortality	Units for Premium	Total Units	Value $i = 2\%$	Value $i = 4\%$
85	$\mathscr{Z}_{5+}$	xxx	xxx	xxx	xxx	xxx
85	$\mathscr{Z}_{6-}$	xxx	xxx	xxx	xxx	xxx
...						
89	$\mathscr{Z}_{10-}$	xxx	xxx	xxx	xxx	xxx
90	$\mathscr{Z}_{10+}$	xxx	xxx	xxx	xxx	xxx
Total					**xxx**	**xxx**

   - Show your working as a graphic.
7. Consider the difference between the Present values (at interest rates 2% and 4%) of the table (**for the first premium only! – this number is somwhere between** $\in [-3000, 3000]$**; in case you can not calculate it, or it is outside this range, take** 1000**.)** above as amount of money to be invested for future policyholder participates, which invests (like in a variable annuity) in a equity and a put option to ensure that the fund never falls below its initial value (similar example is in the script). Calculate the respective replicating portfolios (assume market rate 4% and volatility $\sigma = 18\%$). The following working are expected: (Students: **LV**)
   - Calculation of the replicating portfolio - ie you need to enlarge the table above for additional instruments $\mathscr{S}$: shares, and $\mathscr{P}_k$: put options with term k. Which strike.
   - For the calculation of the replicating portfolio what is the condition of the value.
   - Calculate the value of the replicating portfolio (including guarantees $\mathscr{Z}_x$ for the following scenarios: a) $i = 4\%, \xi = 1$, where $\xi = 1$ is current equity level, b) a) $i = 3\%, \xi = 1$, c) $i = 4\%, \xi = 0.9$ (ie equities fall 10%), a) $i = 3\%, \xi = 0.9$
   - Can you calculate the Greeks for a) $i = 4\%, \xi = 1$?

Note: the above results can be presented in graphics

4.3 Task 3 (Life Insurance)

We consider a deferred annuity with complete payback of remaining premium as a waiver against a regular premium. This means that a policyholder is entitled to get back the sum of the paid premium minus the sum of the paid out annuities in case of death, as long as this sum is positive. Typically this is considered as three distinct parts: a) the deferred annuity (LA), b) the death benefit before payout of annuitties (LD_1) and c) the death benefit after the withdrawal has started (LD_2).
In our concrete example we assume the following

1. Man, age 30, start of withdrawal at age 65, we sell this product in the year $t = 2024$
2. Annuity $R = 60000$, prenumerado payable
3. Technical interest rate 2%, $\omega = 110$
4. Mortality as per $qx(x,t) = e^{a_0+a_1x+a_2x^2+a_3t}$; note we model the product using a generational table.

The parmaters for a are given by a =[2.34544649e+01,8.70547812e-02,7.50884047e-05,-1.67917935e-02]
Tasks

1. Calculate the Premium for this insurance, and split the premium into the three benefits – this is used to calculate the correct risk premiums
2. Calculate the mathematical reserve for the ages 30,40,50,65,70,75,85
3. Calculate the Risk premium by type in tabular form for the above ages. To check correctness please also calculate the respective savings premium, and the sum of savings- and risk premium (per component and the sum of the three risk- ans savings premiums respectively)
4. What should the sum of risk and savings premium equate to? Can you proof?
5. Assuming that the above mortality is second order (ie best estimate), propose a mortality table which has a 10% margin
6. Discuss the construction of such a first order table in case of most policyholder taking out policies between 30 and 65

4.4 Task 3 (Selected Chapters)

Discuss the impact of policyholder behaviour on the value and the hedging of a variable annuity impact.

A Student Guide

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

v.8 20.08.2020

STUDENT GUIDELINES FOR ZOOM-PROCTORED ORAL EXAMS

Due to the Corona situation, selected exams will be conducted online in individual settings. To enable oral exams, they will be conducted online via the video-conference tool Zoom. Your examiners will be ETH lecturers and teaching staff. As in face-to-face exams, a witness will be present online.

Video data may be recorded subject to prior informed consent by you; if your examiners expect to record your exam, but you do not agree, it will be left to the judgment of the examiner on how to proceed – this may include cancellation or postponement of the exam. If recordings are made, this will be clearly visible to you by a window asking you to confirm your consent prior to the start of the recording, and by a blinking red "Recording" icon in Zoom during the session. At no point in time will the video stream or the recording be made available to anybody but ETH personnel on a strictly need-to-know base. However, network connection data on bandwidth, latency, and incidents will be collected for quality assurance; these data are available to ETH staff only.

Preparation: your instructor will announce

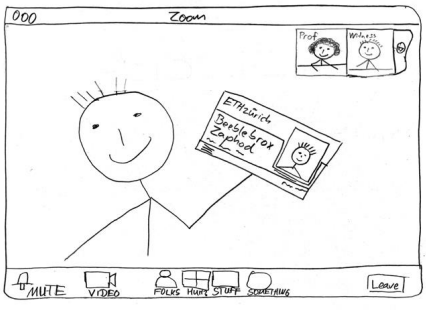
1) date, time, and duration;
2) allowed aids (books, notes, calculators, data tables, etc.);
3) an email address or uploading functionality to transfer an electronic copy of your written notes, if so desired;
4) whether or not you will also be required to use postal mail to send your written notes to ETH (and the address to do so);
5) two independent ways of contacting the examiners in case of problems, including phone/SMS;
6) if the exam will be recorded; if so, the examiner will ask for your consent via e-mail and inform you who has access to the recording for what purpose, and where and how long it is stored; and
7) any other exam guidelines that require preparation on your part.

In any case, **you will need**

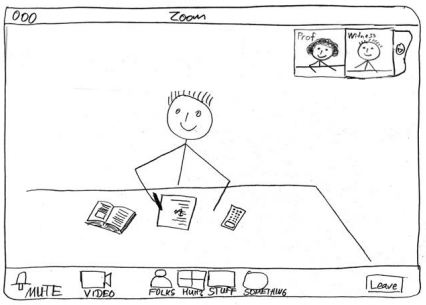
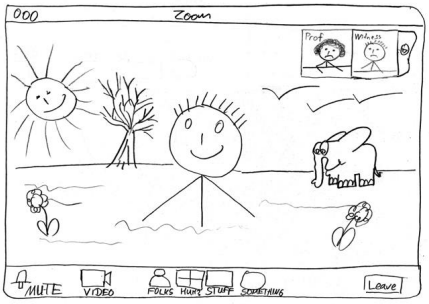
8) a computer with a camera, speaker, and microphone;
9) a stable broadband internet connection (ideally wired);
10) your ETH student ID;
11) the latest release of the Zoom client software; and
12) a camera, phone or scanner to reproduce your written notes electronically and transfer them to the examiners. Scanner apps on phones are perfect, since they also provide parallax-correction.

Failure to comply with these preparation guidelines by either party will render the exam void or result in a failing grade. In case of technical difficulties during the exam, these will be documented, and it will be left to the judgment of your examiners if, how and when the exam will be continued or repeated.

Starting the exam:
Make sure your place is well lit (no light from the back, no window) and quiet, preferably lockable. Test your setup prior to the exam using https://ethz.zoom.us/test
Your examiners will announce the Zoom Meeting ID for entering the exam at least 30 minutes prior to the beginning of the exam. This ID will need to be kept confidential. Please open the Zoom Client and type the meeting ID and your name as it appears on your student ID.
You will enter your exam session at least 5 minutes prior to the announced starting time. Depending on settings, you will probably find yourself in a virtual waiting area for admission.
Your examiners may be using your stored student photo to identify you or require you to show your ETH student ID in clear view of the camera. Your examiners may take a static screenshot at this point in time for the exam record.



1/2

Taking the exam:	
Your workplace should ideally be arranged such that the computer is generally out of reach, and that your face, hands, notes, aids, and background are clearly visible. The desk and the room should be clear of clutter. Your examiners may ask you to move the camera around the room. Camera, microphone, and speaker need to be turned on or under the control of the examiners (Zoom settings). Do not use earphones, headsets, or any earbuds (Bluetooth or otherwise, including noise-canceling devices).	
Virtual backgrounds need to be turned off; your surroundings need to be visible for the examiners. Once again, your examiners may ask you to move the camera around the room.	 **No**
If the examiners would like to see your work, please hold it up to the camera. Please make sure there is sufficient lighting. Clear and easy communication will contribute to your success on the exam!	

After the exam:
While still in the video conference, photograph or scan your notes, if so required.
While still in the video conference, email or upload your notes according to the guidelines set out in 3) above, if so required.
Your examiners might delegate you to the waiting room to assess your exam; they will then bring you back to the meeting room.
Do not leave the video conference until your examiners tell you that it is fine to hang up.
If you are also required to mail your notes according to 4) above, please do so as soon as possible. Mail service tends to be somewhat unpredictable during Corona times, so the sooner the better.

Very best of success!

Start	End	Do 8. Feb	Fr 9. Feb
07:30	07:50		~~Protopapas, Kimon Dimitri~~
07:50	08:10	~~Tian, Yafu~~	
08:10	08:30		
08:30	08:50	Tian, Yafu	Herrmann, Tim
08:50	09:10	Koster, Floris Jan	Protopapas, Kimon Dimitri
09:10	09:30	Wiesinger, Moritz Maximilian	Marty, Alexander
09:30	09:50	Edera, Patrick	Brunner, Joël Luca
09:50	10:10	Brems, Amalie Riff	Brüderlin, Jason
10:10	10:30	Caillat, Maxime Jean Zacharie	Erdin, Matthias
10:30	10:50	Faller, Micha Maurice	Arnold, Jonas
10:50	11:10	Marty, Alexander	Bieri, Selim
11:10	11:30	Turin, Riccardo	Brosche, Tim
11:30	11:50	Schraff, Marius	Kreuzer, Fabian
11:50	12:10	Han, Luyang	Russ, Tom
12:10	12:30	Kuhn, Felix Arjun	Mogasale, Anarghya
13:00	13:20	Abegglen, Johannes Christian	Chen, Yurong
13:20	13:40	Rakutt, Rebekka Luise	Pouget, Angéline Nathalie
13:40	14:00	Thomann, Joël Pascal	
14:00	14:20	Himmelsbach, Dominik	Pouget, Angéline Nathalie
14:20	14:40	Wittwer, Melena	Chen, Yurong
14:40	15:00	Schönenberger, Vera Pascale	
15:00	15:20	Moghimikheirabadi, Ahmad	
15:20	15:40	Schaer, Thomas Patrik	Russ, Tom
15:40	16:00		~~Brosche, Tim~~
16:00	16:20	~~Rakutt, Rebekka Luise~~	
16:20	16:40	~~Han, Luyang~~	Marty, Alexander
16:40	17:00	~~Turin, Riccardo~~	~~~~
17:00	17:20	~~Himmelsbach, Dominik~~	
17:20	17:40		
17:40	18:00	~~Faller, Micha Maurice~~	
18:00	18:20	~~Brems, Amalie Riff~~	
18:20	18:40	~~Wiesinger, Moritz Maximilian~~	