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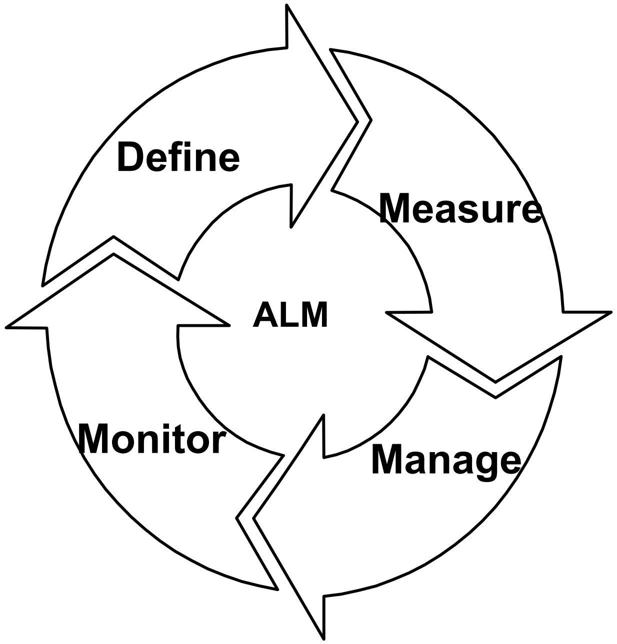
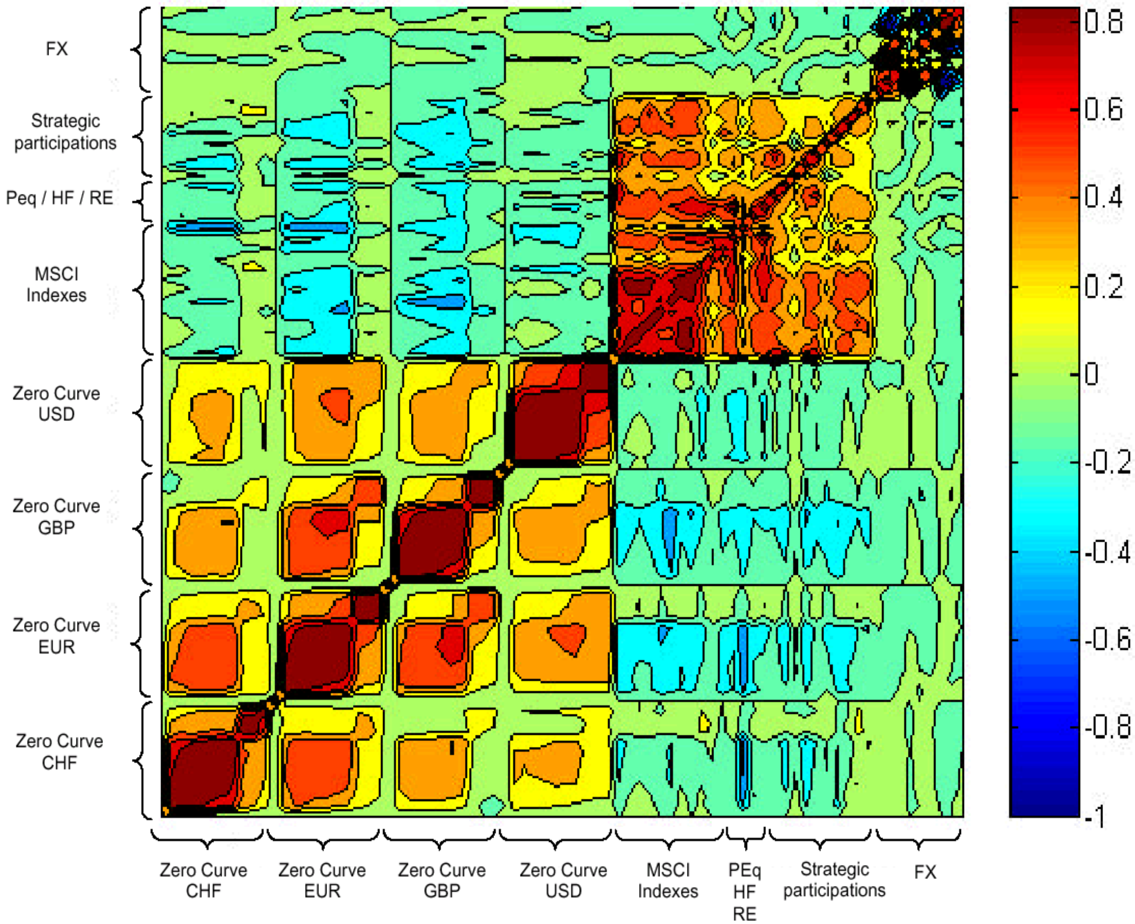
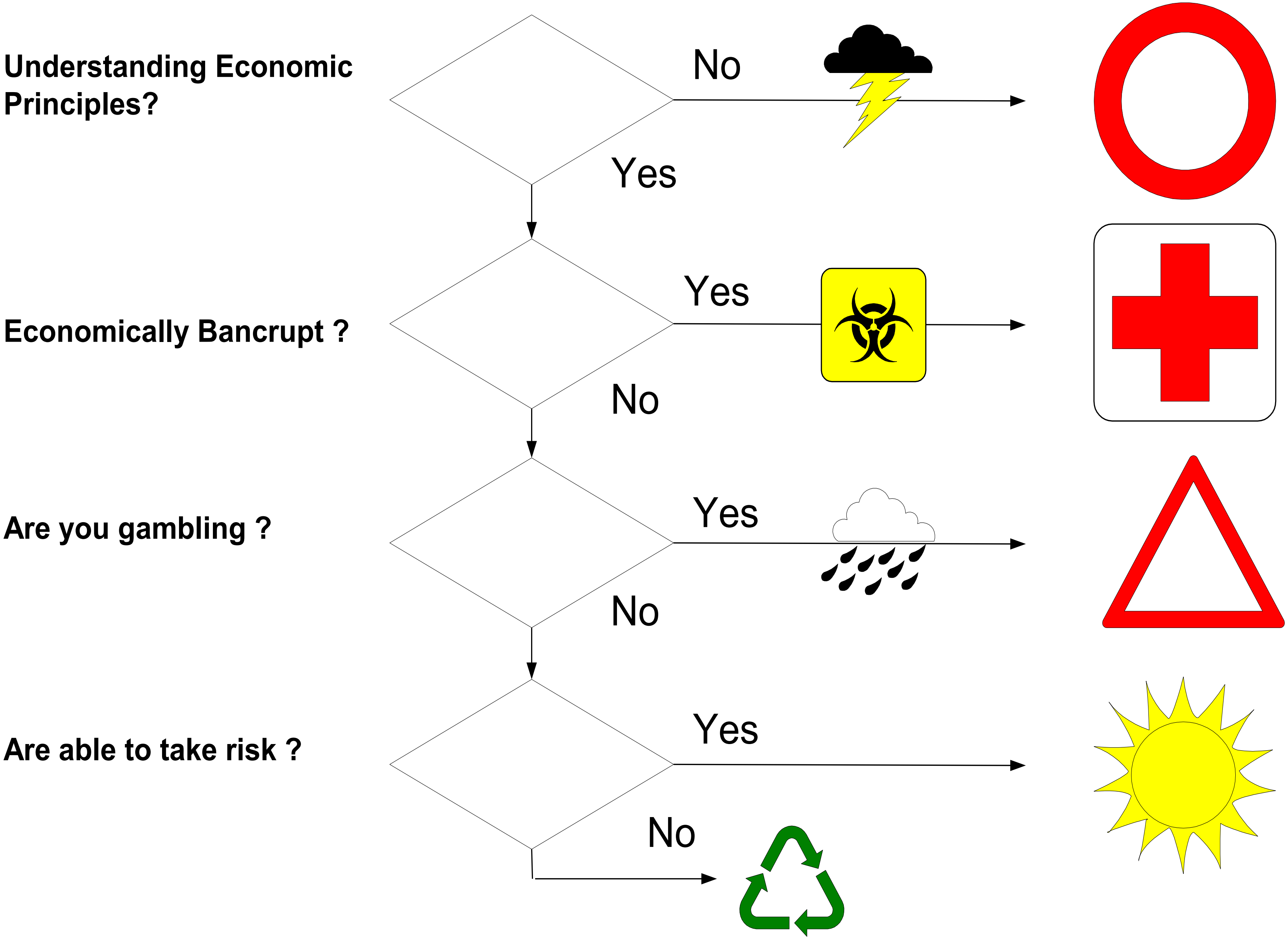




ALM

Michael Koller

Management Summary



Contents

Difference between a model and the reality

Reality	Model

<u>Observable Parts of the world:</u>

1) Share Prices
2) Discount Rates
3) Forward rates
4) etc.

<u>Observable Parts **in the model**:</u>

1) Share prices
2) Discount rates
3) Forward rates
4) etc.

<u>Real Questions:</u>

1) Value of an entity
2) Cost of a product
3) Required risk capital
4) etc.

?

<u>Deductions and solved questions **in the model:**</u>

1) Value of an entity
2) Cost of a product
3) Required risk capital
4) etc.

Why Fair Values (FV)

Traditional valuation methods focus on value creation and do not take into account the corresponding risks

This effects become apparent in particular for

- interest guarantees
- surrender values
- different strategic asset allocations

FV serve as a benchmark for additional reserves for liabilities

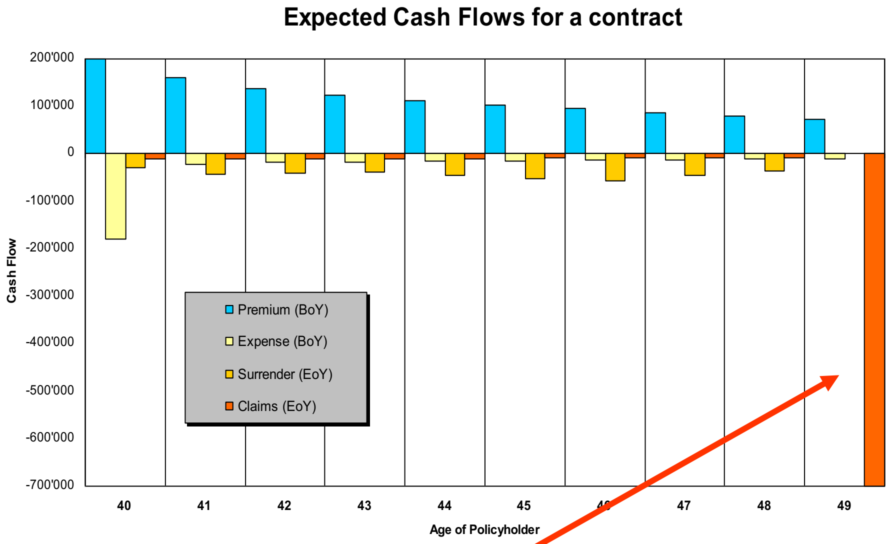
FV is in line with AFP and therefore consistent with capital markets

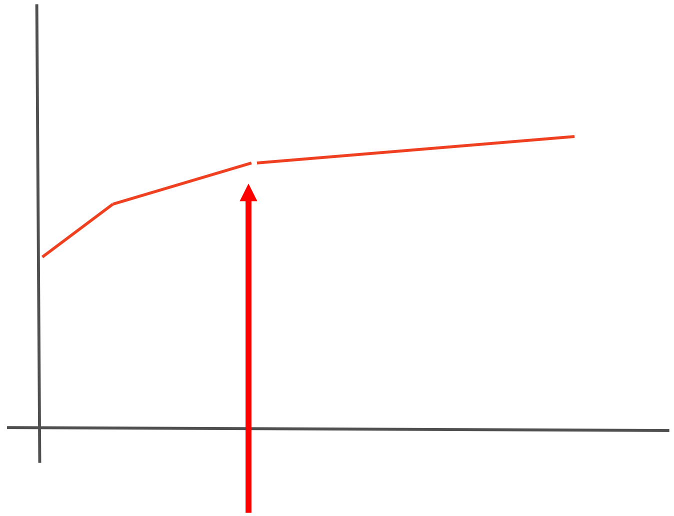
The application of FV in insurance leads to a competitive advantage

Mark to market B/S: Available capital (Step 1)

CF- Pattern

Yield curve (ZCB)





CF_t

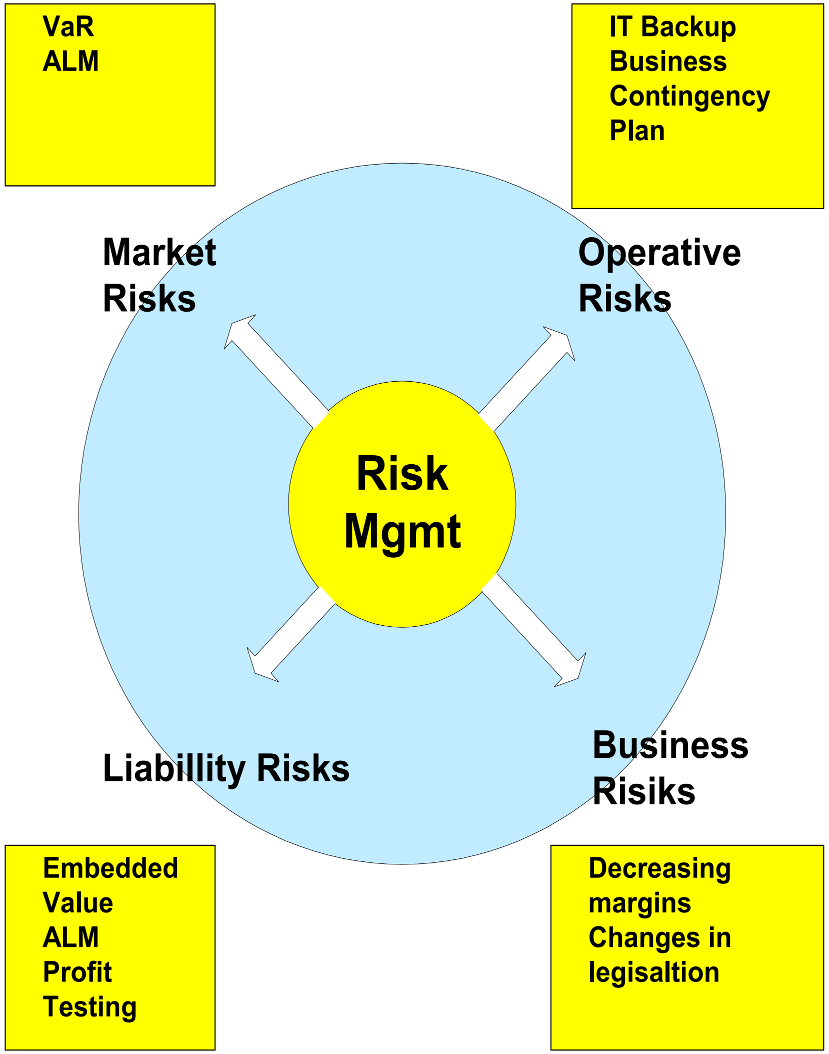
i_t

MV_Lia = MV(ins) - MV(outs)

Free_Capital = MV_Assets - MV_Lia

(Available Capital)

Risk-Landscape (Example)



Strategic Risks
Changes in business economies

+

Market Risks
Changes in market prices and/or liquidity

+

Life Risks
Changes in mortality, lapses

+

Non-life Risks
Changes in claims

+

Credit Risks
Changes in credit quality

+

Operational Risks
Internal / external one-off events

Method oriented view

Integrated RoRAC Model

Example ALM with capital efficient valuation

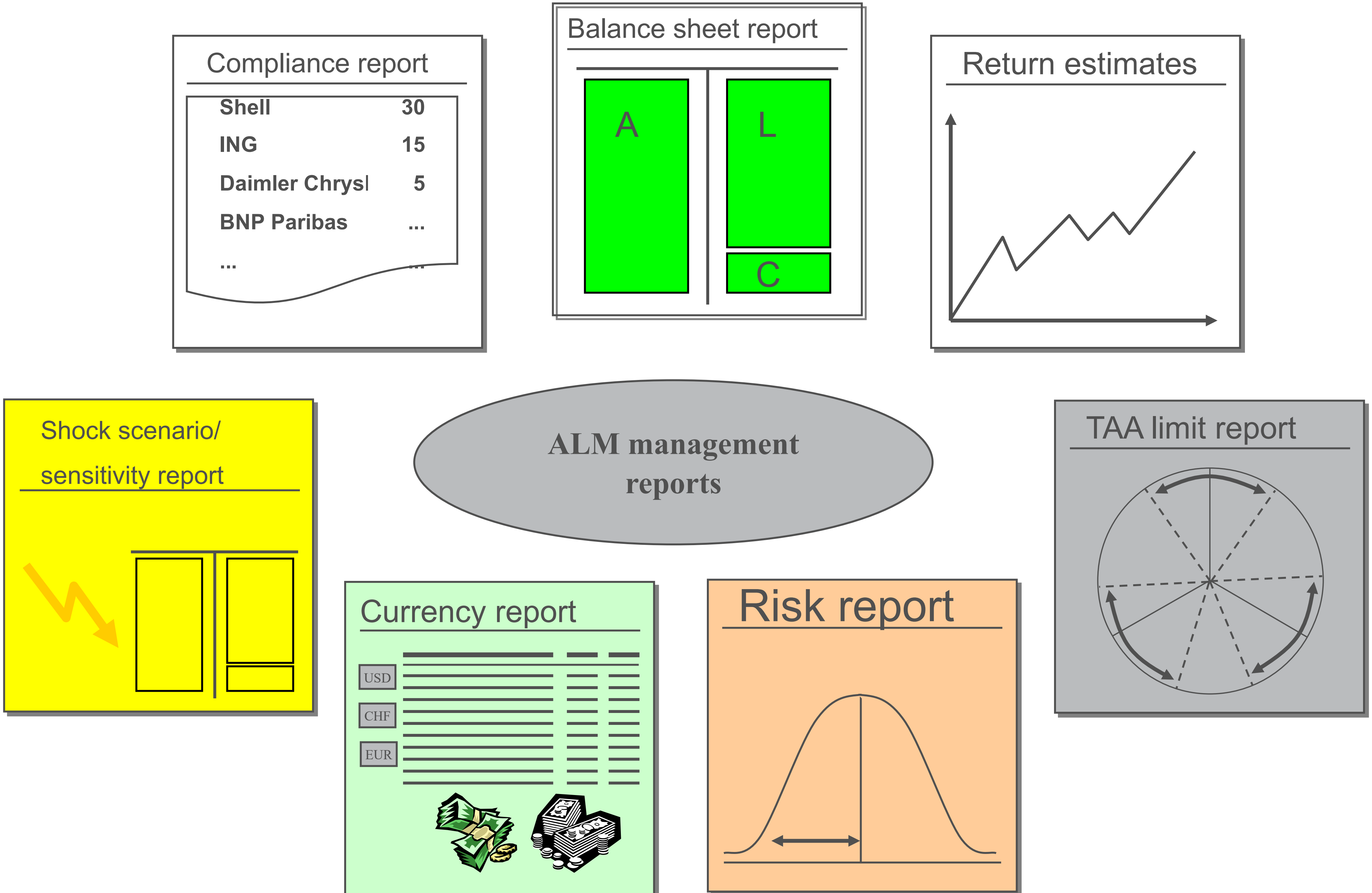
Company X wants to apply capital efficient pricing and valuation in its ALM process.

The following slides show some aspects of this project and in particular some of the reports

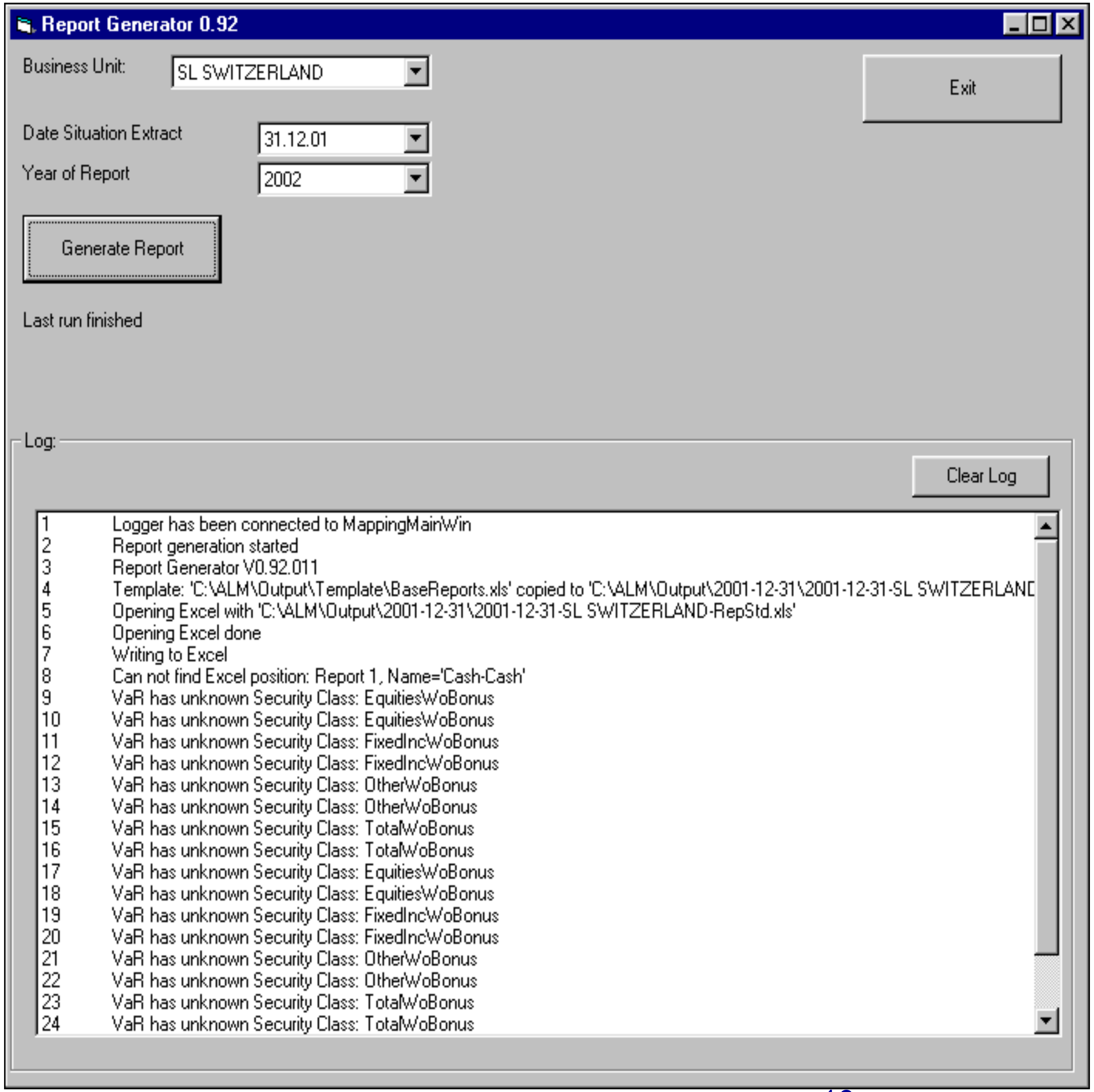
Some of the main challenges were:

- Data availability
- Modelling of liability cash flows and liability risk capital
- Defining a consistent ALM process

OVERVIEW OF ALM REPORTS



Tools



Input:

1) Asset position data

2) Risk Data

3) Lia. cash flows

Output:

1) MtM B/S report

2) Lia. report

3) TAA Limits

4) Scenario Report

5) Risk Capital Report

Liabilities

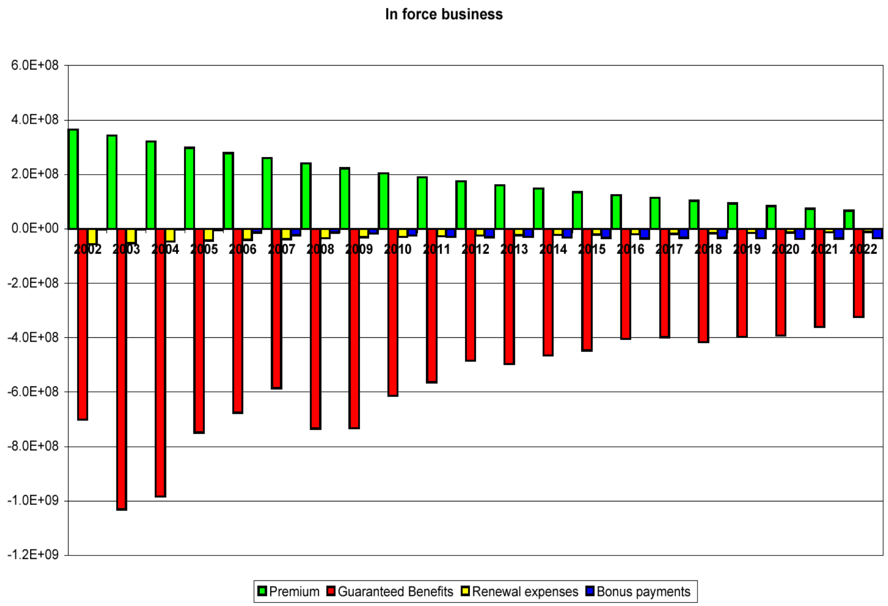
Liabillity Report

Example Company

December 2000 | Currency | **CHF**

SL_- Virtual Example	**In Force Portfolio**	**New Business**	**% Age**
Mathematical Reserve (statutory)	8'044'629'565		
Premium current year	365'137'994	351'985'212	96.4%
Number of Polices	126'480	6'951	18.2%
Expected P/L	6'324'000	-8'432'000	
Fair Value Reserves (absolute)			
Guaranteed Benefits	-10'044'629'565	-417'508'551	4.2%
< Premium >	3'250'317'943	496'985'499	15.3%
Administration Expenses	-492'945'712	-49'894'723	10.1%
Initial Commissions	0	-20'992'800	N/A
Bonus Payment (at Risk free)	-466'346'804	-8'271'933	1.8%
FV Guarnteed Benefits (total)	**-7'287'257'334**	**8'589'424**	
FV All Benefits (Bonus at AA + 150 bp)	**-7'656'132'845**	**1'995'467**	
FV All Benefits (max total)	**-7'753'604'138**	**317'491**	
FV Guaranteed in MR	**90.5%**		
FV All Benefits in MR	**96.4%**		
Distribution of Benefits			
Guaranteed Benefits	91.3%	84.1%	
Administration Expenses (Cost Ratio)	4.5%	10.0%	
Initial Commission	0.0%	4.2%	
Bonus Payments	4.2%	1.7%	
Premium in total guaranteed Benefits	29.5%	100.1%	

Client View	**In Force Portfolio**	**New Business**	**% Age**
Costratio	4.5%	10.0%	
Duration	10.1	12.4	
IRR (Guarantee Only)	2.70%	2.33%	
IRR (Guarantee + Admin Expenses)	3.35%	3.44%	
IRR (Total)	4.00%	4.11%	



Assets

- Position data for all instruments need to be available
- Including all derivatives
- The single assets are mapped to benchmarks
- Example: Bonds:

SecurityType	Benchmark	NominalAmount	Currency	Coupon	Frequency	MarketValue	DateMaturity
BO1	LB EURODOLL	8'000'000	AUD	7	S	6'953'928	01.04.04
BO1	LB EURODOLL	10'000'000	AUD	7	A	8'663'300	23.07.04
BO1	5 YEARS USD	5'000'000	AUD	6.5	S	4'363'929	14.06.05
BO1	5 YEARS USD	8'000'000	AUD	6.5	S	7'004'676	01.05.06
BO1	LB EURODOLL	7'700'000	AUD	6	S	6'549'575	14.07.09
BO1	SBI DOMESTIC	10'000'000	CHF	7.25	A	10'000'000	04.11.01
BO1	SBI DOMESTIC	1'000'000	CHF	6.5	A	1'000'500	10.11.01
BO1	SBI DOMESTIC	3'800'000	CHF	6.75	A	3'801'900	10.11.01
BO1	SBI DOMESTIC	5'500'000	CHF	6.75	A	5'502'750	12.11.01
BO1	SBI DOMESTIC	41'000'000	CHF	7	A	41'041'000	15.11.01
BO1	SBI DOMESTIC	35'000'000	CHF	4.5	A	34'965'000	19.11.01
BO1	SBI DOMESTIC	11'000'000	CHF	4.5	A	10'994'500	23.11.01
BO1	SBI DOMESTIC	7'000'000	CHF	4.375	A	7'010'500	30.11.01

Reports M-t-M Balance Sheet

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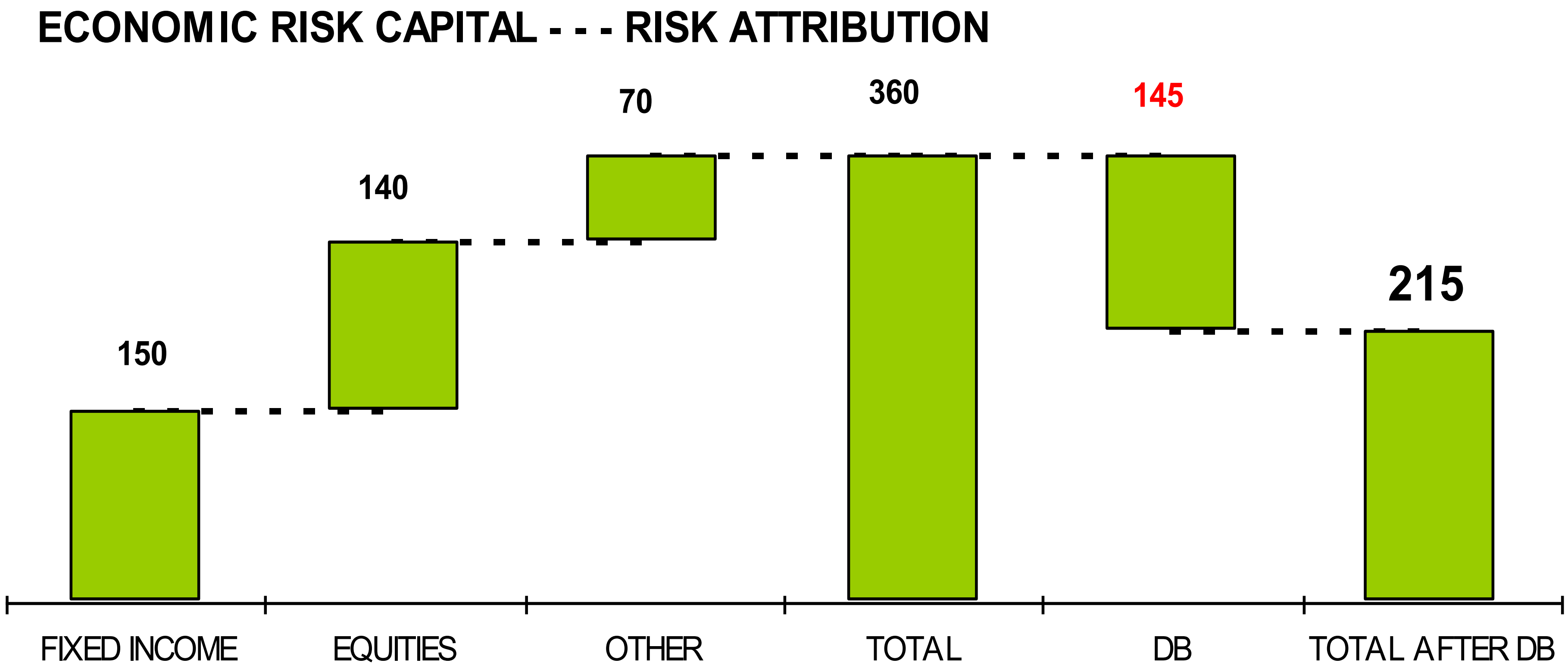
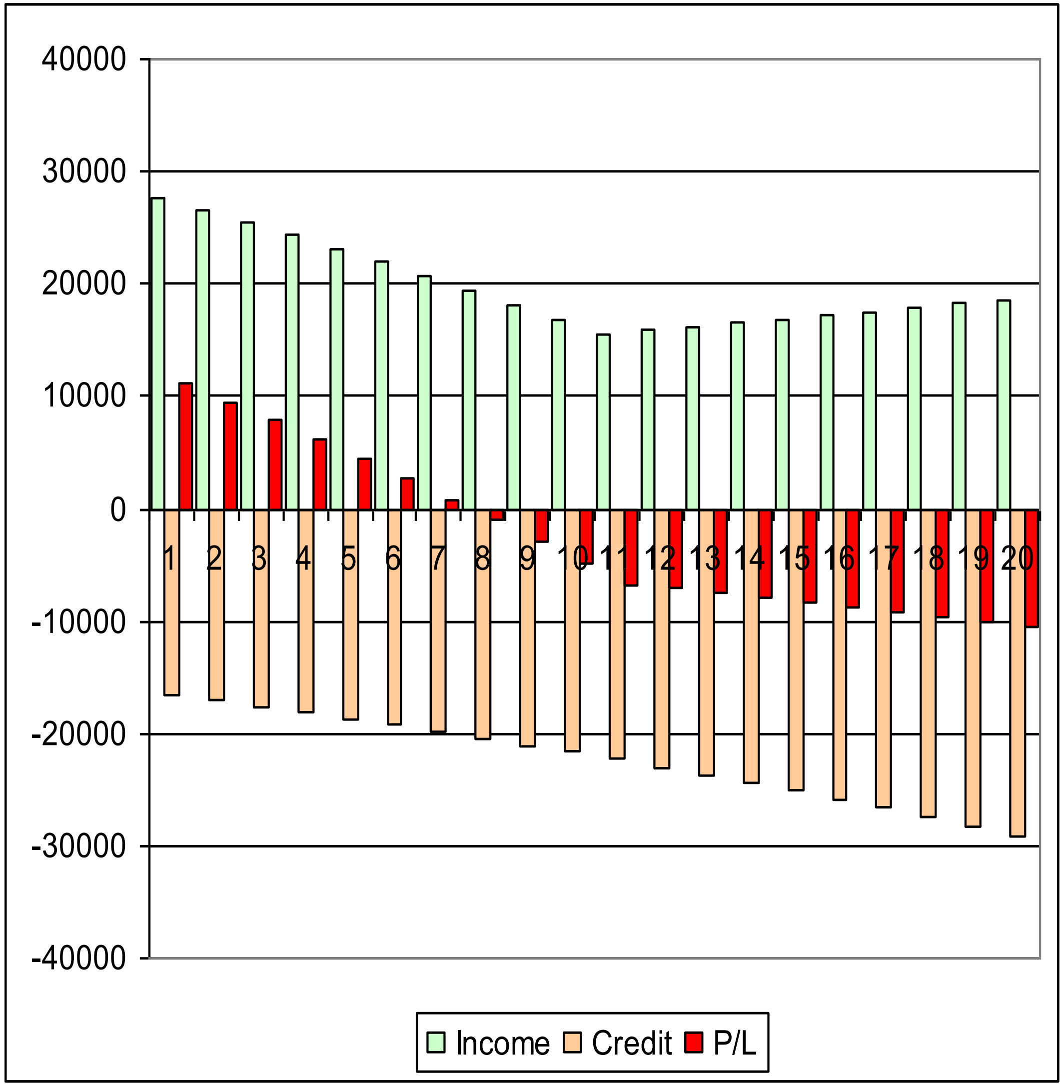
	Market value of relevant positions			Net position		Economic risk capital needs
ASSET TYPE	ASSETS	LIABILITIES				
FIXED INCOME*	2'000	-2'200		-200		150
EQUITIES	600			600		140
OTHER**	200			200		70
	2'800	-2'200		600		360
DIVERSIFICATION BENEFIT (DB)	-	-		-		145
TOTAL AFTER DIVERSIFICATION	2'800	-2'200		600		215

Reports Risk Capital



Duration mismatch properly understood

- Most life insurers have a duration mismatch
- Example Duration Assets 6 yrs, liabilities 10 yrs.
- Falling interest rates are a real danger (Japanese situation)
- Effects will be seen in the future if interest rates do not move upward
- **Losses will be seen in the future even if today the insurer is not economically bankrupt!**
- ALM and Duration management is needed in order to survive such conditions



Contents

- **Introduction**
- **Methodology**
  - Valuation of Assets and Liabilities
  - Market Value of Liabilities and replicating portfolios
  - Risk Capital
  - Optimization
- **Process**
  - Data requirements Assets
  - Data requirements Liabilities
- **Putting ALM in practice**

Principles for the valuation of liabilities

Mark-to-market principle: Replicating cash flows

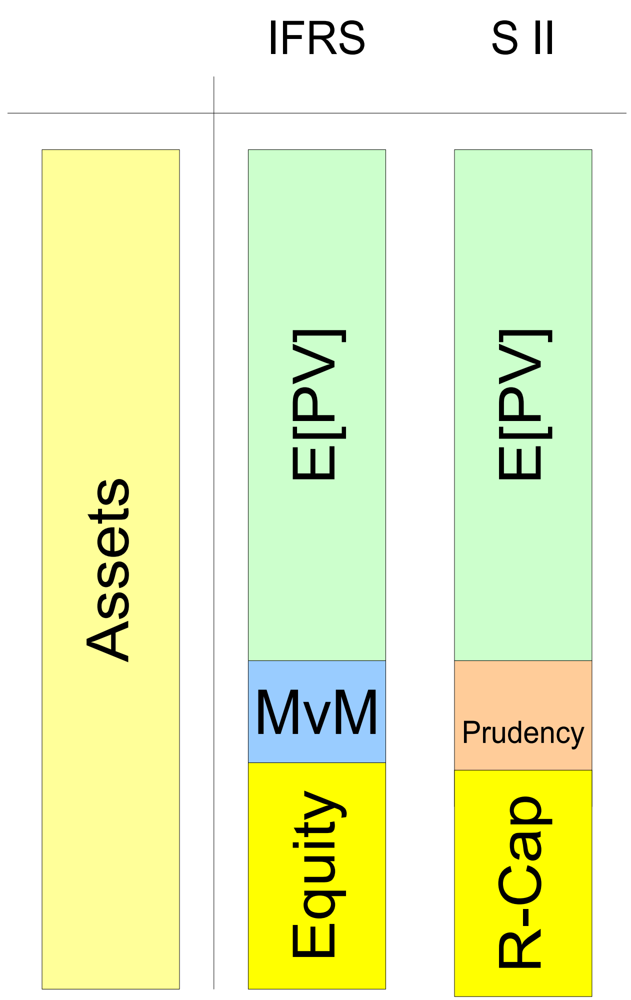
Two step approach

- Expected PV
- Market Value Margin

Options etc. to be considered

Two approaches for MVM

- Cost of Capital
- Quantile Approach



Annuity Portfolio

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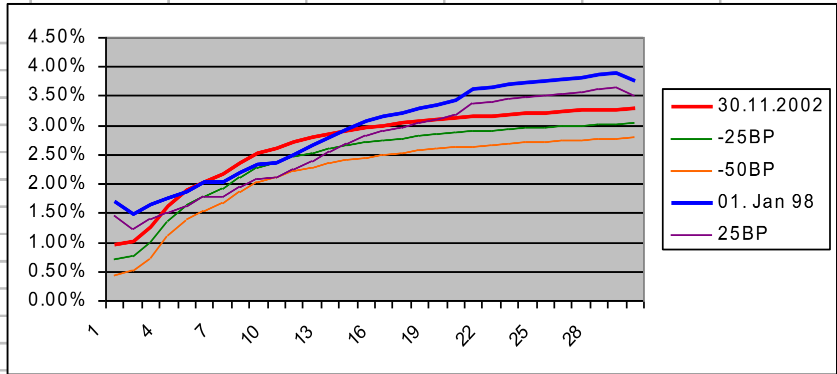
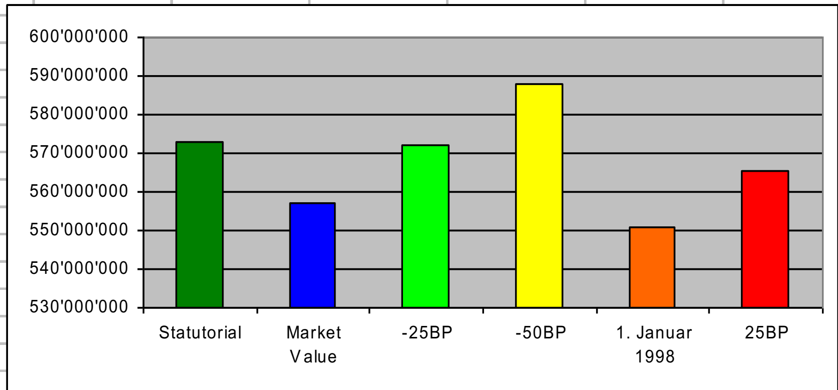
Yield Curve											
Base Year	2003										
Stat Interest	3.50%										
Scenario	1	2	3	4	5	6	7	8	9	10	11
30.11.2002	0.95%	1.02%	1.25%	1.62%	1.89%	2.03%	2.18%	2.35%	2.53%	2.62%	2.71%
-25BP	0.70%	0.77%	1.00%	1.37%	1.64%	1.78%	1.93%	2.10%	2.28%	2.37%	2.46%
-50BP	0.45%	0.52%	0.75%	1.12%	1.39%	1.53%	1.68%	1.85%	2.03%	2.12%	2.21%
01. Jan 98	1.71%	1.48%	1.66%	1.76%	1.87%	2.04%	2.04%	2.20%	2.34%	2.36%	2.50%
25BP	1.46%	1.23%	1.41%	1.51%	1.62%	1.79%	1.79%	1.95%	2.09%	2.11%	2.25%

Calc Forward Rates

Results

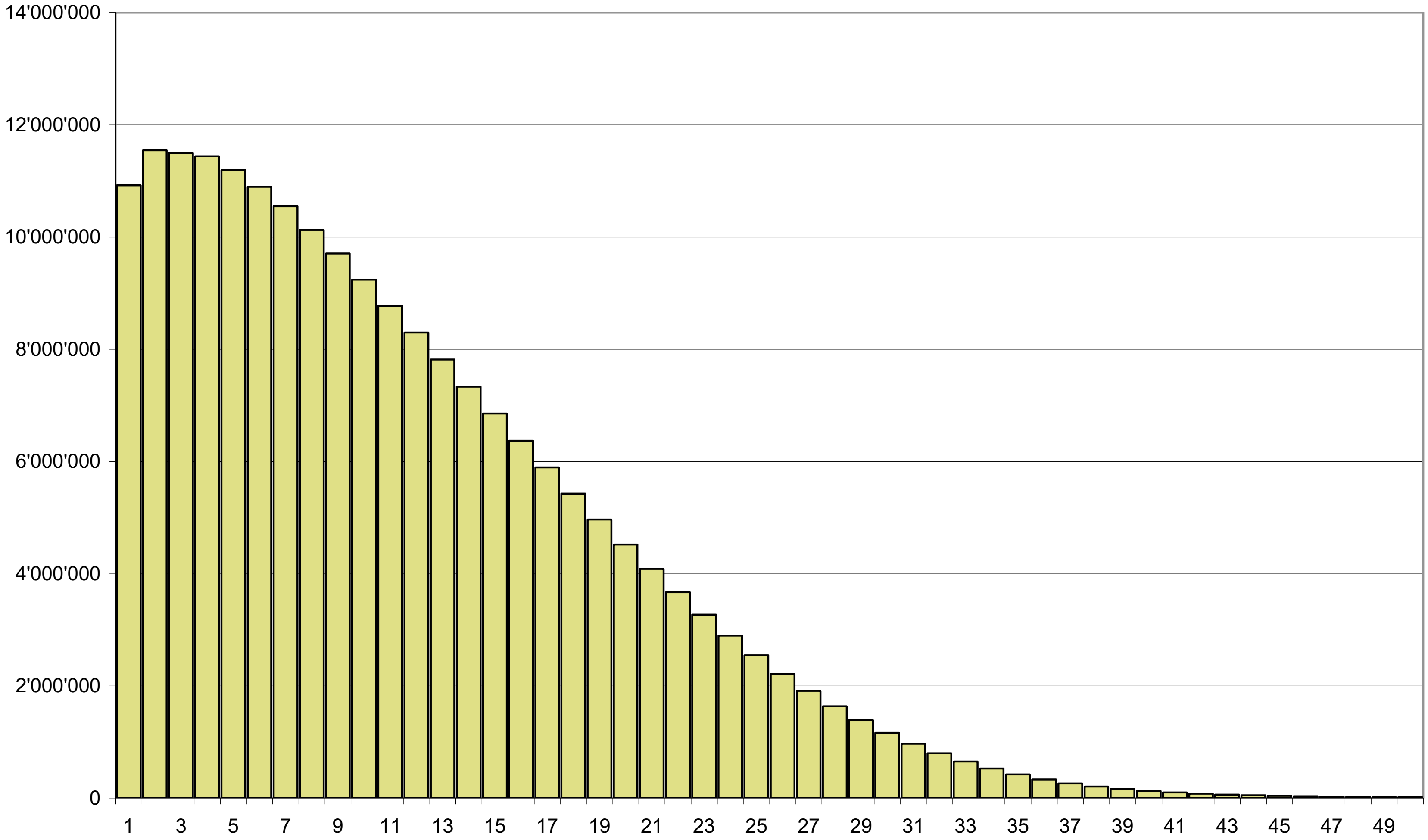
Sum of Annuities		38'964'389	
[E 1996/2000]			Difference
Mathematica	Statutorial	573'117'616	
Reserve	Market Value	556'879'280	-16'238'336
	-25BP	571'956'500	-1'161'116
	-50BP	587'755'665	14'638'049
	1. Januar 1998	550'684'111	-22'433'505
	25BP	565'553'755	-7'563'861





Effective **Group Life: Annuities in Payment**

Replicating Portfolio – Expected Values



Liquid **Not Liquid**

Model Risk

Cost of Capital

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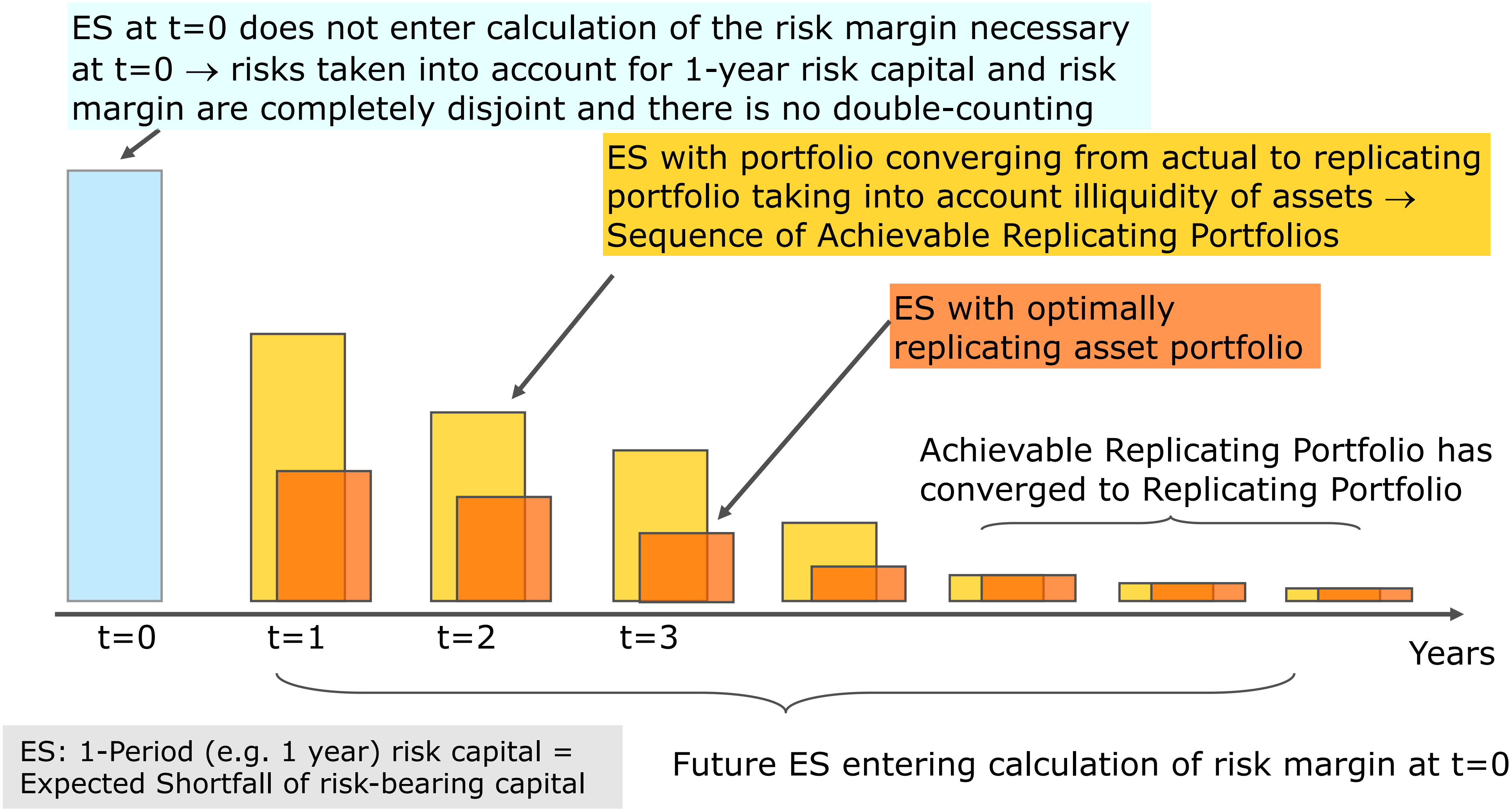


Key Ideas for the VaPo-Method

In the past the value of a insurance liability was always represented as a single number

The idea of the VaPo is to introduce an intermediate step within the valuation of a contract

In a first step **a life insurance liability is represented as a portfolio of different financial instruments**

In a second step **the VaPo is evaluated.**

Remarks:

- Several valuation principles can be applied at the same time (eg. Book values and market values)
- Different types of analyses can be performed on the level of VaPo and not as usually done yet on the level of values

The VaPo - Method

reality | model

$$
\begin{tikzcd}
\text{insurance policy} \arrow[r, "\text{translate}"] \arrow[d, "?"'] & \mathcal{G} \arrow[d, "\varphi"] \\
\mathbb{R} & \mathcal{F} \arrow[l, "\psi"']
\end{tikzcd}
$$

Representation

Value of the policy

Valuation-Principle

VaPo Representation

Required Definitions and Notation

The set of all representable insurance Policies (G)

The representation map (phi)

The Valuation portfolio (F)

The Valuation map (psi)

The set of all insurance policies (1)

The set of all insurance polices (G) serves to model the corresponding Valuation Portefolio (VaPo)

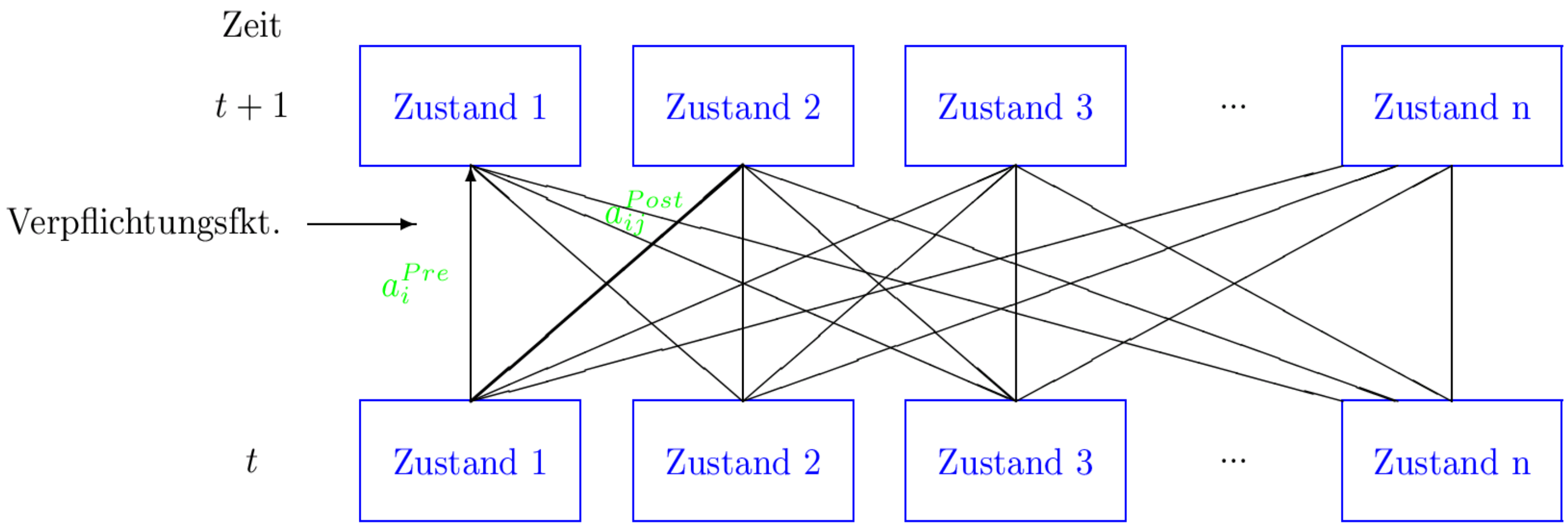
There are several possibilities to define this linear vector space

In the following we use a rather general markov respresentation in discrete time leading to a finite dimensional vector space

However other representations are also possible such as

G = { c(x) :   0 <= x }, c(x) representing the death benefits if a person dies at age x.

The set of all insurance policies (2)



$a_i^{Pre}(t)$: Annuity payment at time t, if the person stands in state i at time t;
$a_{ij}^{Post}(t)$: Capital payment at time $t+1$, if the person switches from state i to state j in the time interval $[t, t+1)$;
$p_{ij}(t)$: Probability of switching from state i to state j in the time interval $[t, t+1)$.

We consider a life insurance contract of a man aged x and write for this single contract g_x

$$g_x = \{a_i^{Pre}, a_{ij}^{Post}, p_{ij};\ i, j \in S\}$$

and for the set of all possible insurance contracts

$$\mathcal{G} = \{g_x; x = 0, \ldots, \omega\}$$

The Valuation Portefeuille

We describe each insurance contract in a model with financial instruments that span the vector space F.

The vector space F is spanned by the basis elements $\{e_1, \ldots e_m\}$

$F = \langle e_1, \ldots e_m \rangle$

Remarks:

- Different possibilities for F
- For traditional insurance it will normally consist out of the different zero coupon bonds with different maturities. We denote with $Z^{(t+k)}$ a zero coupon bond with maturity x+k at time k.
- With $P(Z^{(t+k)})$ we denote the price of this zero coupon bond. The interest of such a bond can be calculated by the formula $P(Z^{(t+k)})^{-1/k} - 1$.

Some mathematical remarks

Both vector spaces F and G are assumed to be finite dimentional. Therefore

- All topologies are equivalent and we use the Euclidean one
- The two maps phi and psi are assumed to be linear and therefore continuous
- An element x in G can always be represented as

  $x = \sum < x \mid e_i > e_i$

In financial mathematics the map psi is called pricing system for which the following axioms are required:

  - linearity
  - positivity

Procedure for the valuation

- First we define a linear map φ from the set of insurance contracts $\mathcal{G}$ into the vector space $\mathcal{F}$ spanned of the units $e_i, i = 1, \ldots, m$:

$$\begin{aligned} \varphi: \mathcal{G} &\rightarrow \mathcal{F} \\ g &\mapsto \sum_{i=1}^{m} \lambda_i(g) e_i, \end{aligned} \tag{1}$$

with $\lambda_i(g) = \langle \varphi(g), e_i \rangle$, whereby $\langle \cdot, \cdot \rangle$ denotes the inner product.

- Then we transform the valuation in units into money with the map ψ:

$$\begin{aligned} \psi: \mathcal{F} &\rightarrow \mathbb{R} \\ \sum_{i=1}^{m} \lambda_i(g) e_i &\mapsto \psi\Big(\sum_{i=1}^{m} \lambda_i(g) e_i\Big), \end{aligned} \tag{2}$$

defined as a linear map, i.e.

$$\psi\Big(\sum_{i=1}^{m} \lambda_i(g) e_i\Big) = \sum_{i=1}^{m} \lambda_i(g) \psi(e_i).$$

Example

We consider an endowment policy with premium payments of a 50 years old man:

death and maturity benefit	$C = 50'000$ CHF
age at entry	$x = 50$ years
cover period	$n = 5$ years

Remarks:

1. The time scale is in years. For ease of notation, all years are indicated as the age of the insured male.
2. Benefits are paid *at the end* of year when death occurs or at expiration.
3. Premiums Π are due *at the beginning* of the year.
4. All administration charges are ignored.

Convention: Payments *receivable* by the policy holder have *negative* sign. Payments to be *made* by the policy holder have *positive* sign.

Representation of the policy in G

DESCRIPTION OF THE INSURANCE CONTRACT:
State space $S = \{*, \dagger\}$, where $*$ symbolizes the *alive state* and $\dagger$ the *death*.
Contractual functions:

$$a_*^{Pre}(t) = \begin{cases} -\Pi, & t = 50, \dots, 54 \\ 0, & \text{else} \end{cases}$$

$$a_{*\dagger}^{Post}(t) = \begin{cases} C, & t = 50, \dots, 54 \\ 0, & \text{else} \end{cases} \qquad a_{**}^{Post}(t) = \begin{cases} C, & t = 54 \\ 0, & \text{else} \end{cases}$$

Transition probabilities:

$$\begin{aligned} p_{**}(t) &= p_t \\ p_{*\dagger}(t) &= q_t \\ p_{\dagger *}(t) &= 0 \\ p_{\dagger\dagger}(t) &= 1 \end{aligned}$$

Insurance contract: $g_{50} = \{a_*^{Pre}, a_{*\dagger}^{Post}, a_{**}^{Post}, p_{**}, p_{*\dagger}, p_{\dagger *}, p_{\dagger\dagger}\}$.

Task: Valuation of the contract

Construction of the VaPo

Step 1: Definition of the units

Premium:	The premiums Π are paid at age t, $t = 50, \ldots, 54$. The unit e_t is a zero coupon bond $Z^{(t)}$ with duration $t - 50$ paying CHF 1 at the beginning of age t. The premium Π is made of such zero coupon bonds.
Death benefits:	The death benefits payable at t are also regarded as ZCB $Z^{(t+1)}$, $t = 50, \ldots, 54$. REMARK: Because the death benefits are paid at the end of the year t they are represented by $Z^{(t+1)}$ paying CHF 1 at the beginning of age $t + 1$ and not by $Z^{(t)}$.
Maturity benefit:	The maturity benefit at age $t = 54$ is represented by $Z^{(55)}$.

We have defined the following six units:

$$Z^{(t)}, \quad t = 50, 51, 52, 53, 54, 55.$$

They forme a basis $\mathcal{B} = \{Z^{50}, Z^{51}, Z^{52}, Z^{53}, Z^{54}, Z^{55}\}$ of our valuation. We only need this basis to describe the $VaPo$.

Construction of the VaPo at age 50

Step 2: Valuation in units

We consider two valuation schemes:

- Scheme A describes
  - *when* the premiums are received and the benefits are paid.
  - the *amount* of this payments.
- Scheme B shows
  - the *sort of units* enclosed in the portfolio: $e_1, \ldots, e_m$.
  - the *number* of the different units: $\lambda_1(g_{50}), \ldots, \lambda_m(g_{50})$.

We get all information needed from the contractual functions and the transition probabilities.

Compiling these schemes for an endowment insurance, we will first carry out the valuation for l_{50} persons using the life table then an abstract valuation for one person. This valuation is more general and is valid for all insurance contracts.

Valuation Scheme A at age 50

Valuation at age 50 for l_{50} persons:

Valuation scheme A:

payments at age t	unit e_t	number of units for l_{50} persons: premium	death benefit	survival benefit
50	$Z^{(50)}$	$-l_{50} \cdot \Pi$		
	$Z^{(51)}$		$d_{50} \cdot C$	
51	$Z^{(51)}$	$-l_{51} \cdot \Pi$		
	$Z^{(52)}$		$d_{51} \cdot C$	
52	$Z^{(52)}$	$-l_{52} \cdot \Pi$		
	$Z^{(53)}$		$d_{52} \cdot C$	
53	$Z^{(53)}$	$-l_{53} \cdot \Pi$		
	$Z^{(54)}$		$d_{53} \cdot C$	
54	$Z^{(54)}$	$-l_{54} \cdot \Pi$		
	$Z^{(55)}$		$d_{54} \cdot C$	$l_{55} \cdot C$

Valuation Scheme B at age 50

Valuation scheme B:

unit e_t	number of units for l_{50} persons: premium	benefits		total
$Z^{(50)}$	$-l_{50} \cdot \Pi$			$-l_{50} \cdot \Pi$
$Z^{(51)}$	$-l_{51} \cdot \Pi$	$d_{50} \cdot C$		$-l_{51} \cdot \Pi + d_{50} \cdot C$
$Z^{(52)}$	$-l_{52} \cdot \Pi$	$d_{51} \cdot C$		$-l_{52} \cdot \Pi + d_{51} \cdot C$
$Z^{(53)}$	$-l_{53} \cdot \Pi$	$d_{52} \cdot C$		$-l_{53} \cdot \Pi + d_{52} \cdot C$
$Z^{(54)}$	$-l_{54} \cdot \Pi$	$d_{53} \cdot C$		$-l_{54} \cdot \Pi + d_{53} \cdot C$
$Z^{(55)}$		$d_{54} \cdot C$	$l_{55} \cdot C$	$l_{55} \cdot C + d_{54} \cdot C$

Abstract valuation of the policy at age 50 (1)

premium: $p_{**}(50,t) \cdot a_{*}^{Pre}(t) \cdot Z^{(t)}$

death benefit: $p_{**}(50,t) \cdot p_{*\dagger}(t) \cdot a_{*\dagger}^{Post}(t) \cdot Z^{(t+1)}$

maturity benefit: $p_{**}(50,t+1) \cdot a_{**}^{Post}(t) \cdot Z^{(t+1)}$

Valuation scheme A:

payments at age t	unit e_t	number of units for one person premium	death benefit	maturity benefit
t	$Z^{(t)}$	$p_{**}(50,t) \cdot a_{*}^{Pre}(t)$		
	$Z^{(t+1)}$		$p_{**}(50,t) \cdot p_{*\dagger}(t) \cdot a_{*\dagger}^{Post}(t)$	$p_{**}(50,t+1) \cdot a_{**}^{Post}(t)$

Valuation scheme B:

unit e_t	number of units for one person λ_t
$Z^{(t)}$	$p_{**}(50,t) \cdot a_{*}^{Pre}(t) + p_{**}(50,t-1) \cdot p_{*\dagger}(t-1) \cdot a_{*\dagger}^{Post}(t-1) + p_{**}(50,t) \cdot a_{**}^{Post}(t-1)$

Abstract Valuation of the policy at age 50 (2)

Advantage of this representation: The contractual functions are defined such that the valuation schemes are valid for all t.

With the mapping φ given in (1) we write in short:

$$\begin{aligned}
\varphi(g_{50}) &= \sum_{t=1}^{\omega} \Big[p_{**}(50,t) \cdot a_{*}^{Pre}(t) + p_{**}(50,t-1) \cdot p_{*\dagger}(t-1) \cdot a_{*\dagger}^{Post}(t-1) \\
&\quad + p_{**}(50,t) \cdot a_{**}^{Post}(t-1) \Big] Z^{(t)} \\
&= \sum_{t=1}^{\omega} p_{**}(50,t-1) \Big[p_{**}(t) \cdot a_{*}^{Pre}(t) + p_{*\dagger}(t-1) \cdot a_{*\dagger}^{Post}(t-1) \\
&\quad + p_{**}(t) \cdot a_{**}^{Post}(t-1) \Big] Z^{(t)}
\end{aligned}$$

Naturally the above described algorithms hold true for an arbitrary Markov model. The corresponding formulas have to be generalized slightly.

Valuation at age 51 (Scheme A)

Valuation at age 51 for l_{51} persons:

Our valuation scheme A at age 51 corresponds to the valuation scheme A at age 50 leaving out the premium payment and benefits at age 50.

VALUATION SCHEME A:

payments at age t	unit e_t	number of units for l_{50} persons		
		premium	death benefit	maturity benefit
51	$Z^{(51)}$	$-l_{51} \cdot \Pi$		
	$Z^{(52)}$		$d_{51} \cdot C$	
52	$Z^{(52)}$	$-l_{52} \cdot \Pi$		
	$Z^{(53)}$		$d_{52} \cdot C$	
53	$Z^{(53)}$	$-l_{53} \cdot \Pi$		
	$Z^{(54)}$		$d_{53} \cdot C$	
54	$Z^{(54)}$	$-l_{54} \cdot \Pi$		
	$Z^{(55)}$		$d_{54} \cdot C$	$l_{55} \cdot C$

Valuation Principles (1)

Step 3: Valuation in money (CHF)

In (2) we formally defined the map ψ which transforms the valuation into money. ψ can be regarded as an *accounting principle*:

$$\psi : \text{Portfolio in units} \rightarrow \text{Money amount}$$

QUESTION: What does ψ look like?

There are several possibilities. We consider two of them immediately an will show some extensions later.

$$\left.\begin{array}{l}\psi_1\\ \psi_2\end{array}\right\} \text{assigns the} \left\{\begin{array}{l}\text{book value}\\ \text{market value}\end{array}\right\} \text{to the units.}$$

First we look again at the endowment insurance of the 50 year old man and then at the annuity of the man aged 65.

Valuation Principles (2)

Book value (Traditional mathematical reserves)
The book value of the $VaPo$ gives us the mathematical reserves (MR), i.e. the value of the zero-coupon bonds is calculated with the technical interest rate.

EXAMPLE: The book value of the zero-coupon bond $Z^{(t)}$ with duration $t - 50$ has the value

$$\psi_1(Z^{(t)}) = v^{t-50},$$

where $v = \frac{1}{1+i}$, i the technical interest rate.

Market value
The market value of the $VaPo$ gives us the *fair value*. Each unit has its market value.

EXAMPLE: $\psi_2(Z^{(51)}) := P(50, 51)$ correspond to the value of a zero-coupon bond with duration one year at age 50 (see appendix **??**).

NOTATIONS: We denote the value of the $VaPo$ at age t under the map ψ_1 by MR_t and under ψ_2 by $V_{t|t}$.

The valuation scheme B gives us the value of the $VaPo$: We multiply the number of every unit $Z^{(t)}$ by the value of $Z^{(t)}$ and add them up.

Application to the endowment example (1)

By using the commutation functions of the life table EKM95 with the technical interest rate 2.5% we receive:

$$\text{premium} \quad \Pi = \frac{M_{50}-M_{55}+D_{55}}{N_{50}-N_{55}} \cdot C$$

$$= \frac{13'245-12'613+23'237.84}{562'903-435'605} \cdot 50'000 \text{ CHF} = 9'375.21 \text{ CHF}$$

Valuation scheme B:

unit	number of units for one person			
e_t	premium	benefits		total
$Z^{(50)}$	$-9'375$			$-9'375$
$Z^{(51)}$	$-9'336$	208		$-9'128$
$Z^{(52)}$	$-9'293$	229		$-9'064$
$Z^{(53)}$	$-9'246$	251		$-8'995$
$Z^{(54)}$	$-9'194$	275		$-8'919$
$Z^{(55)}$		302	$48'734$	$49'036$

Application to the endowment example (2)

payments at age	$V_{50\|50}$, term structure 2000	$V_{50\|50}$, term structure 2002	MR_{50}
50	$-9'375.21$	$-9'375.21$	$-9'375.21$
51	$-8'784.52$	$-9'043.42$	$-8'905.20$
52	$-8'365.82$	$-8'824.58$	$-8'627.53$
53	$-7'946.17$	$-8'552.66$	$-8'352.58$
54	$-7'543.31$	$-8'252.87$	$-8'080.14$
55	$39'689.57$	$44'037.07$	$43'340.67$
total	$-2'325.45$	-11.67	0

Valuation at age 51 for one person:

VALUATION SCHEME B:

unit	number of units for one person			
e_t	premium	benefits		total
$Z^{(51)}$	$-9'375$			$-9'375$
$Z^{(52)}$	$-9'332$	230		$-9'102$
$Z^{(53)}$	$-9'285$	252		$-9'033$
$Z^{(54)}$	$-9'233$	277		$-8'956$
$Z^{(55)}$		303	$48'938$	$49'241$

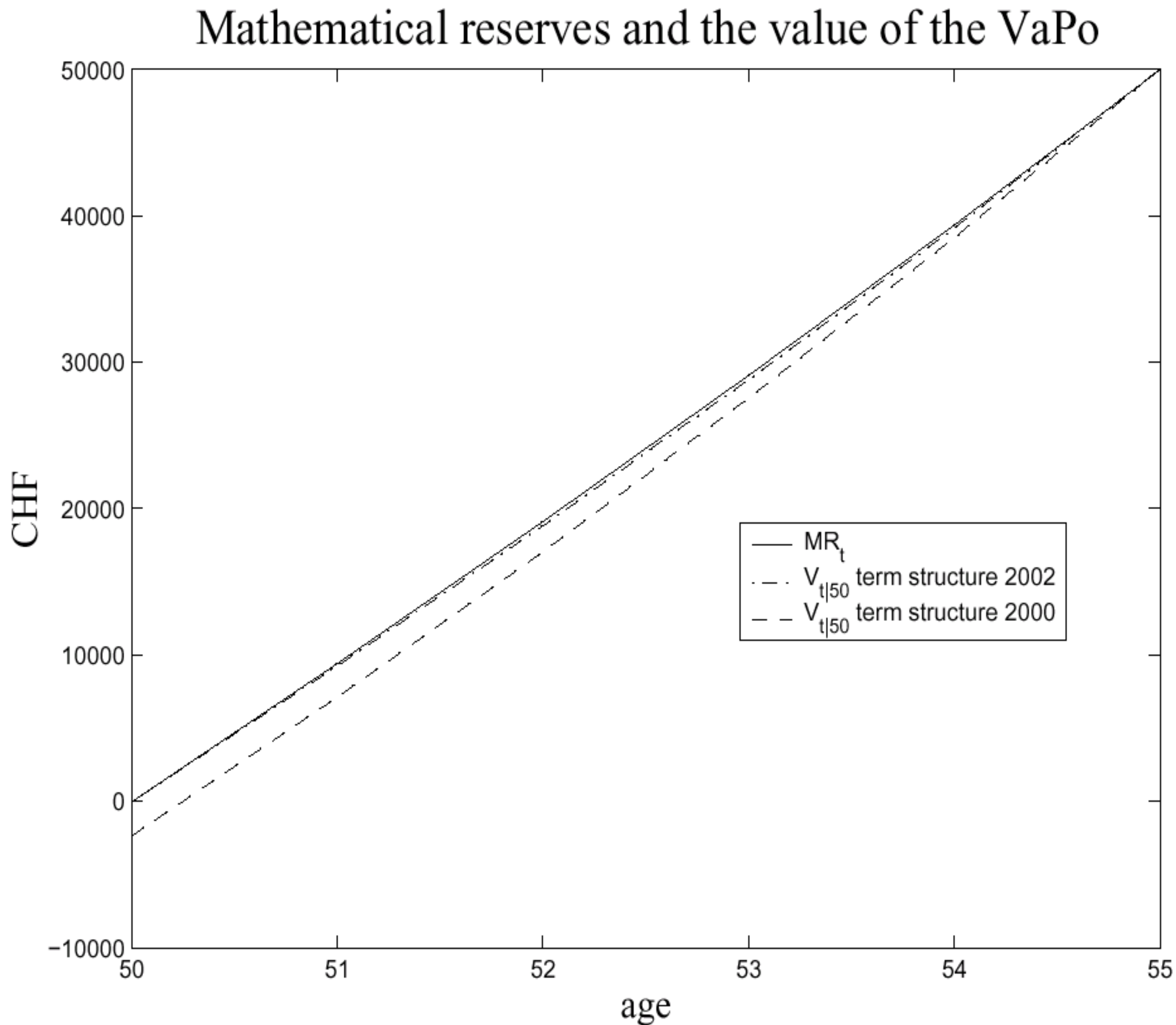
Application to the endowment example (3)

payments at age	$V_{50\|50}$, term structure 2000	$V_{50\|50}$, term structure 2002	MR_{50}
51	$-9'375.21$	$-9'375.21$	$-9'375.21$
52	$-8'729.14$	$-8'944.22$	$-8'880.22$
53	$-8'291.26$	$-8'668.61$	$-8'597.22$
54	$-7'870.91$	$-8'364.76$	$-8'316.80$
55	$41'413.25$	$44'634.08$	$44'610.04$
total	$7'146.74$	$9'281.30$	$9'440.61$

Overview of the different values of the $VaPo$:

payments at age t	$V_{t\|50}$, term structure 2000	$V_{t\|50}$, term structure 2002	MR_t
50	$-2'325.45$	-11.67	0
51	$7'146.74$	$9'281.30$	$9'440.61$
52	$17'076.73$	$18'842.83$	$19'144.35$
53	$27'521.46$	$28'784.46$	$29'126.71$
54	$38'475.03$	$39'151.72$	$39'405.28$
55	$50'000.00$	$50'000.00$	$50'000.00$

Application to the endowment example (4)



Thiele's Difference Equation for market values

The mathematical reserves can be calculated recursively with the Thiele's differential equation:

$$MR^i_{t+s} = a_i^{Pre}(t+s) + \sum_{j \in S} v_{t+s}\, p_{ij}(t+s) \left\{ a_{ij}^{Post}(t+s) + MR^j_{t+s+1} \right\}. \qquad (3)$$

We imagine to be at age t and identify this t to coincide with the year 2000 or 2002. v_{t+s} denotes the annual discount during the time interval [t+s,t+s+1) and MR^i_{t+s} the mathematical reserves at age $t+s$ if the policy holder is in state i.

The same method is also applicable to the valuation of the $VaPo$:

QUESTION: What does v_{t+s} look like with respect to the calculation of MR_{t+s} and $V_{r+s|t}$?

ANSWER:

- MR_{t+s}: We have the normal annual discount factor

$$v_{t+s} = v.$$

- $V_{r+s|t}$: The forward short rate $f(t, r+s)$ gives us the right answer. In the formula (3), replace t by r and v_{r+s} by

$$v_{r+s} = \frac{1}{1 + f(t, r+s)} = \frac{P(t, r+s+1)}{P(t, r+s)}.$$

Example considered

Mixed Endowment policy:

- ERM 1995
- No expense loading
- Single premium payment

Mixed Endowment

Traditional Product sold in individual life. We consider a life insurance policies, which all mature at age 65.

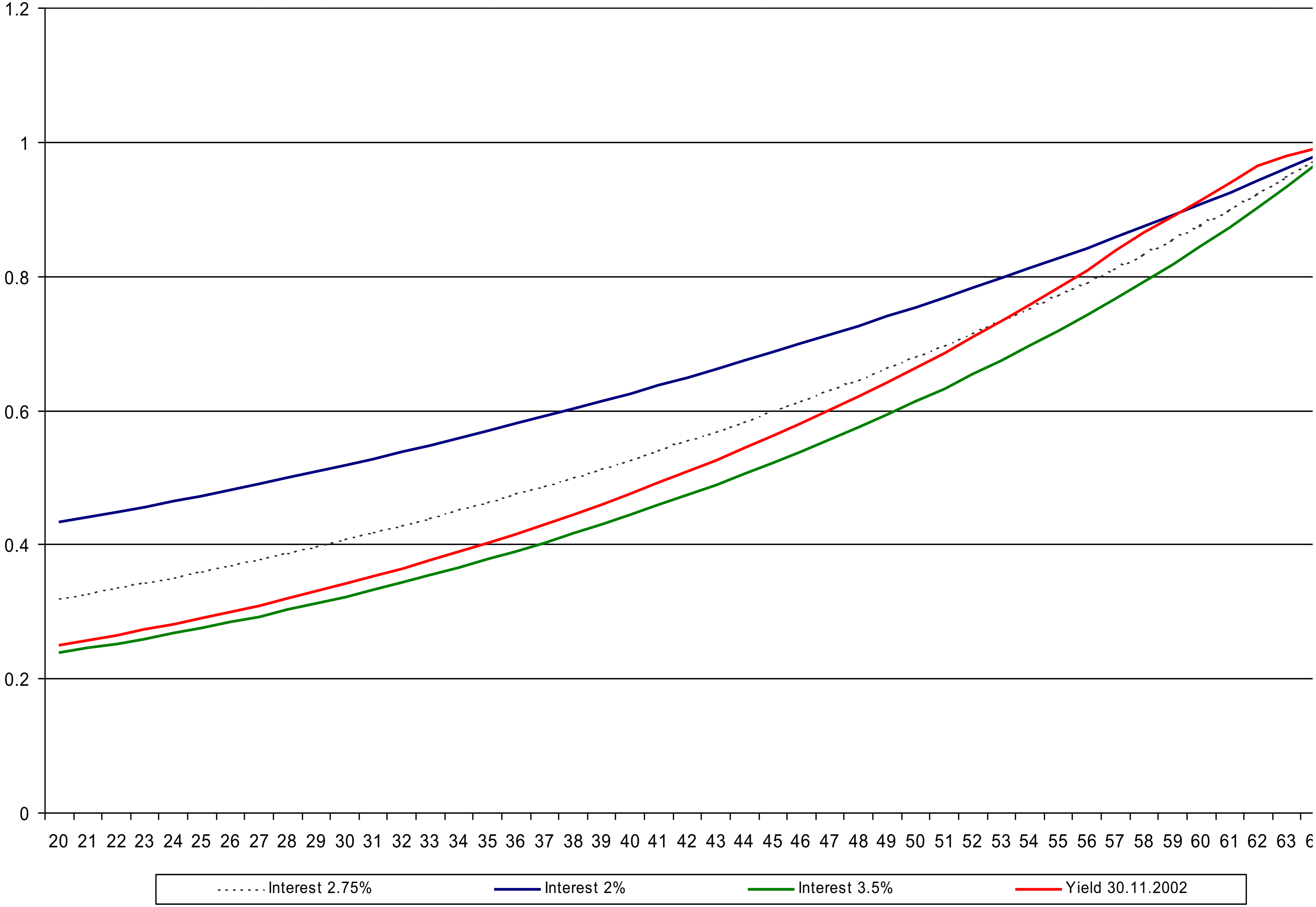
Typically the policies are written with a guaranteed minimal crediting rate (Technischer Zins), which was for a long time at 3.5%.

At the time of writing this presentation (May 2003) the government bond with a term of 10 yrs yield only about 2.5%.

For a portfolio written with 3.5% the different reserves (no bonuses considered) might look as follows:

Sum Insured				492'729'000		
PV of outfows	2.50%			374'528'000		120%
PV of outfows	*3.50%*			*311'455'000*		*100%*
PV of outfows	**Yield**			**336'566'000**		**108%**

Mixed Endowment (Pricing)



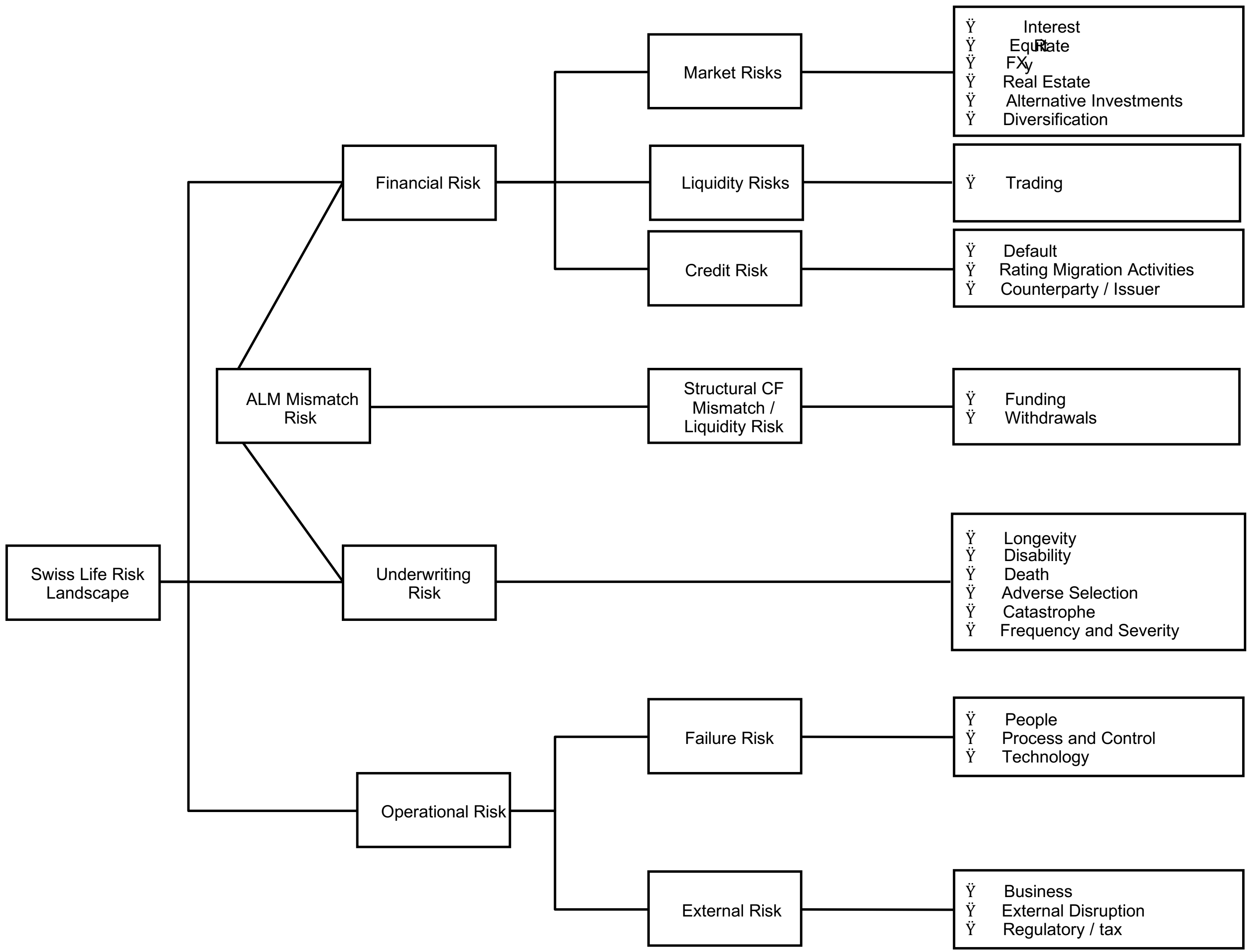
Contents

Required Capital: How to determine

- **The following steps are necessary for the definition of the required risk capital**
  - Definition of the risk map
  - Model for the different risks
  - Calculation of the probability distribution and the required risk capital

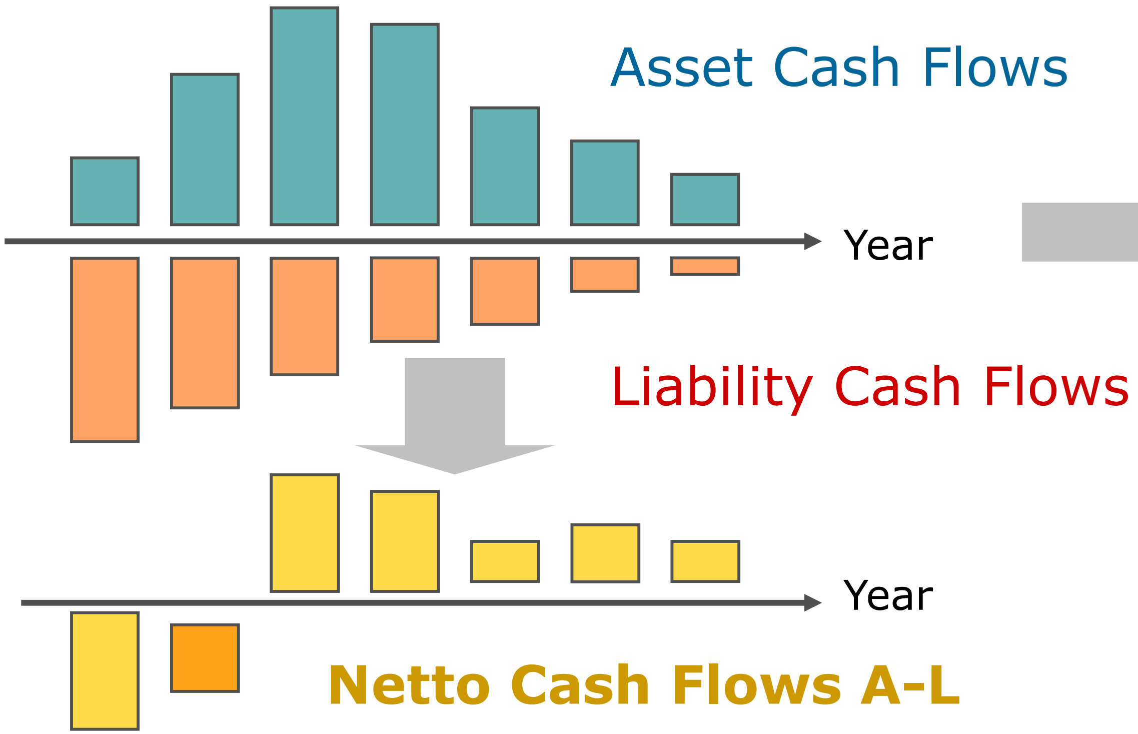
- **In order to measure market risk capital a standard methodology adjusted for insurance liabilities is needed such as**
  - Market Risk: Risk Metrics (TM)
  - For insurance liabilities some adjustments were required
  - Or the SST

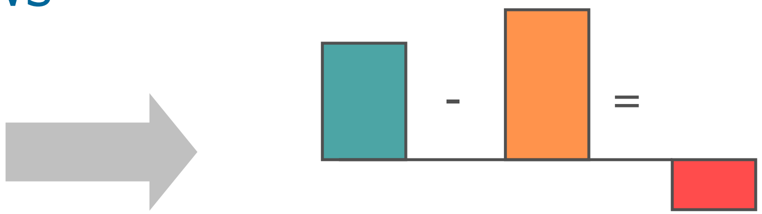
Risk management landscape



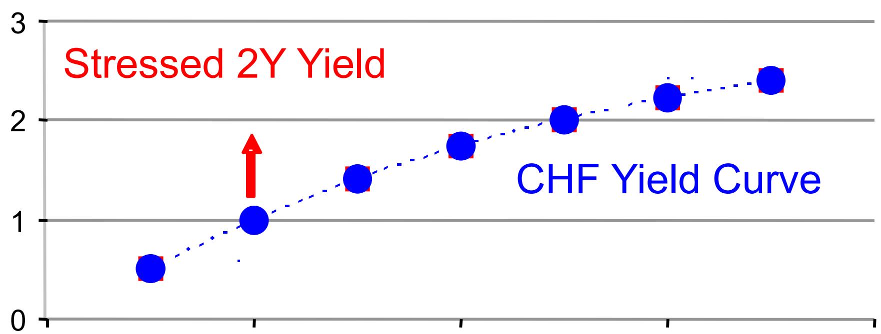
Cash Flow Concept

Example: Sensitivity to 2 Year CHF Yield





Present Value von Asset
- Liabilities



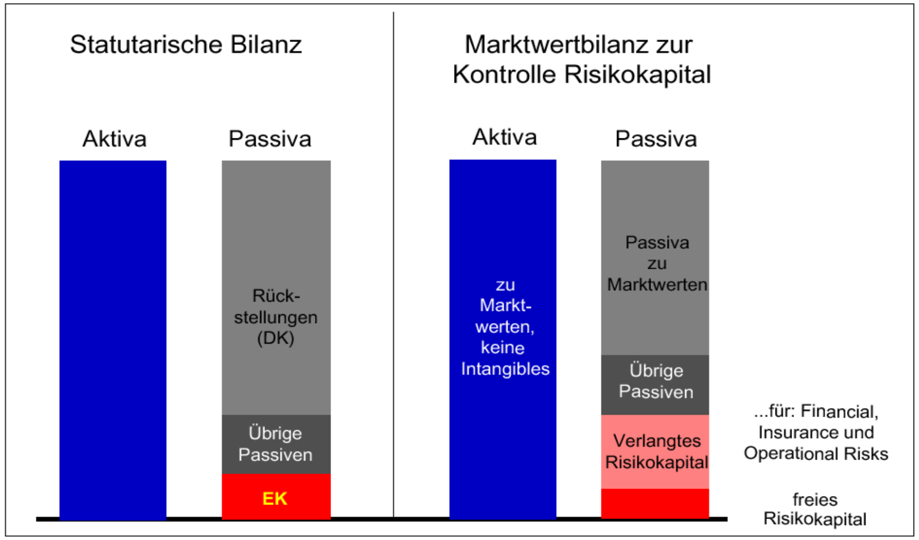
Change of present value of net cash flow (assets-liabilities) due to change in the 2 year CHF yield

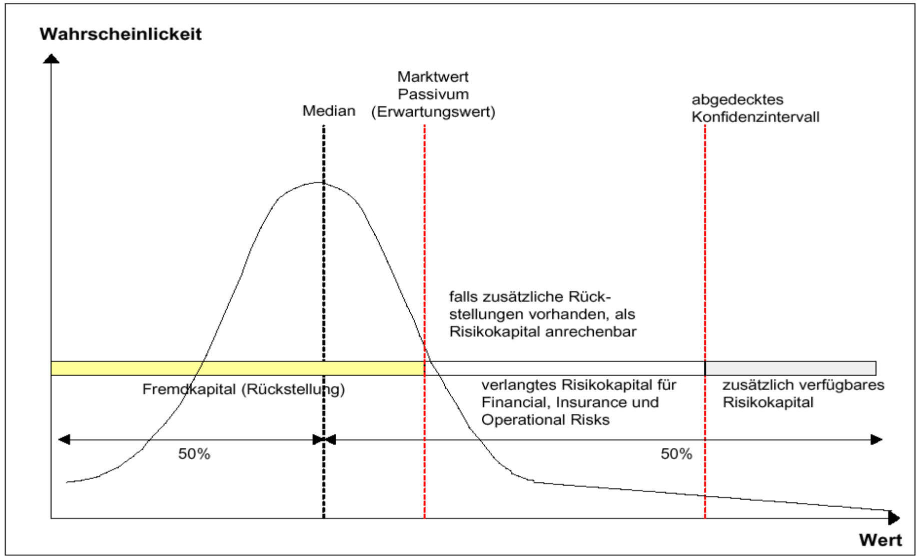
Aims of the “Swiss Solvency Test” (SST)

The SST should help the supervising authorities to answer the following questions:

- Which economic risk capital is available (Risk bearing capital = Market value of assets – market value of liabilities)?
- The possible change of the risk bearing capital within a year
- How much risk bearing capital („target capital“) in order that the company remains solvent with a certain probability during a year.

Principles





Comment

- Starting point is the statutory balance sheet
- A reconciliation leads to the m-t-m balance sheet
- The risk bearing capital is the difference between market value (mv) of the assets and the mv of the liabilities
- The determination of the required risk capital to absorb shocks is based on:
  - A time interval of 1 yr and
  - A given probability

How to determine the target capital

Starting point: market near b/s

- Bonds
- Property
- Shares
- Alternatives
- Other

- Individual Life
  - Cat A
  - ...
- Group Life
  - Annuities
  - Savings
  - other
- Debt
- Other Lia.

Economic „Equity“

Sensitivities and scenarios

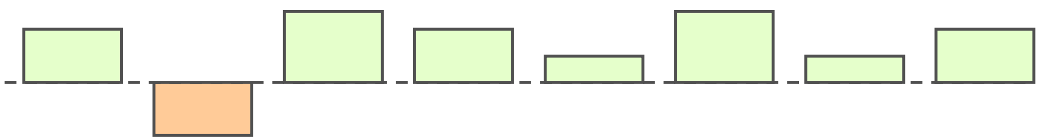
financial

- interest
- shares
- Fx
- Property
- Credit
- ...

Insurance

- mortality
- disability
- customer behaviour
- ...

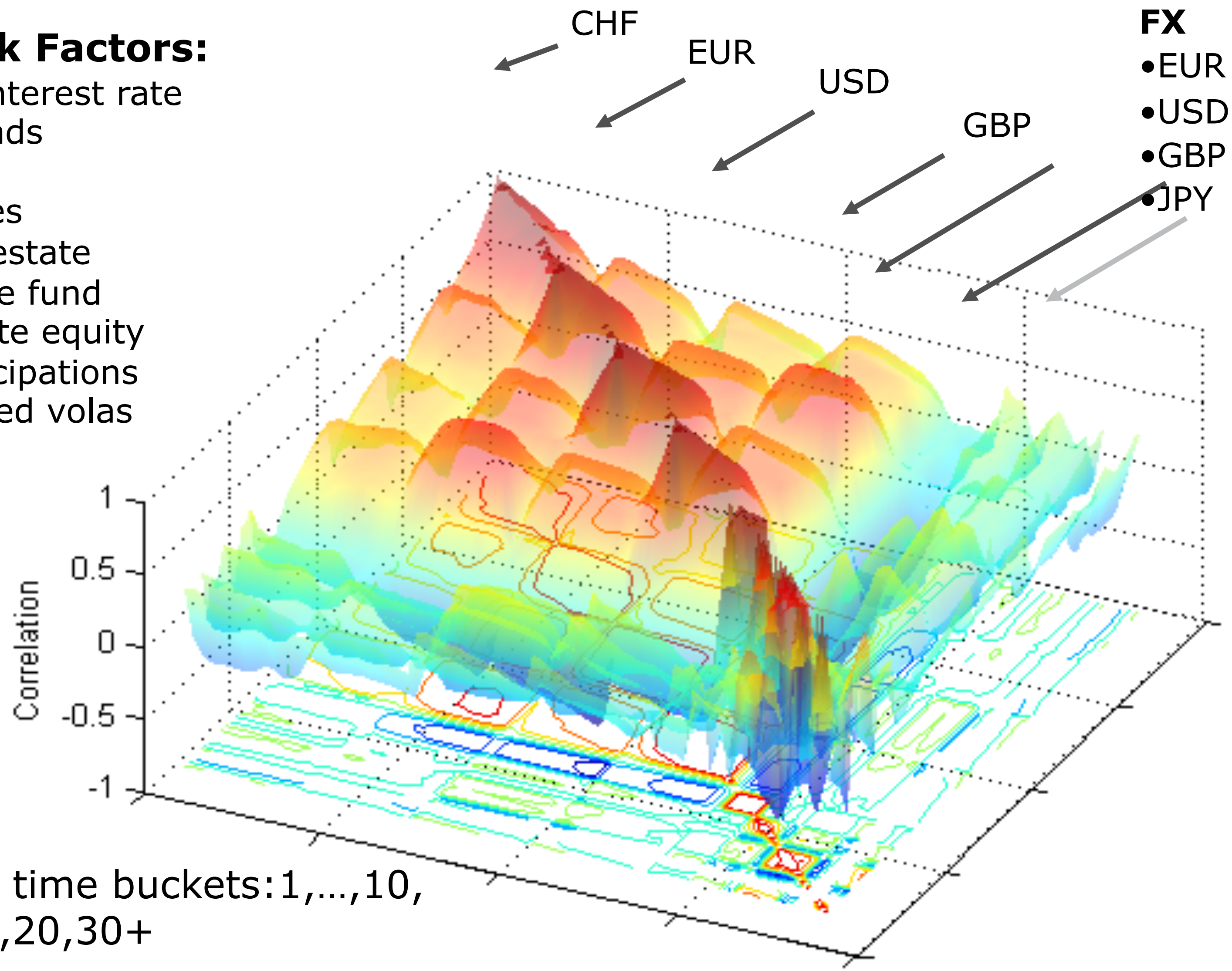
Changes in assumptions have an impact on the economic „equity“



Standard Model: Market Risk

75 Risk Factors:

- 4*13 interest rate
- 4 spreads
- 4 FX
- 5 shares
- 4 real estate
- 1 hedge fund
- 1 private equity
- 1 participations
- 3 implied volas



i.r. time buckets:1,...,10, 15,20,30+

FX

- EUR
- USD
- GBP
- JPY

Spreads

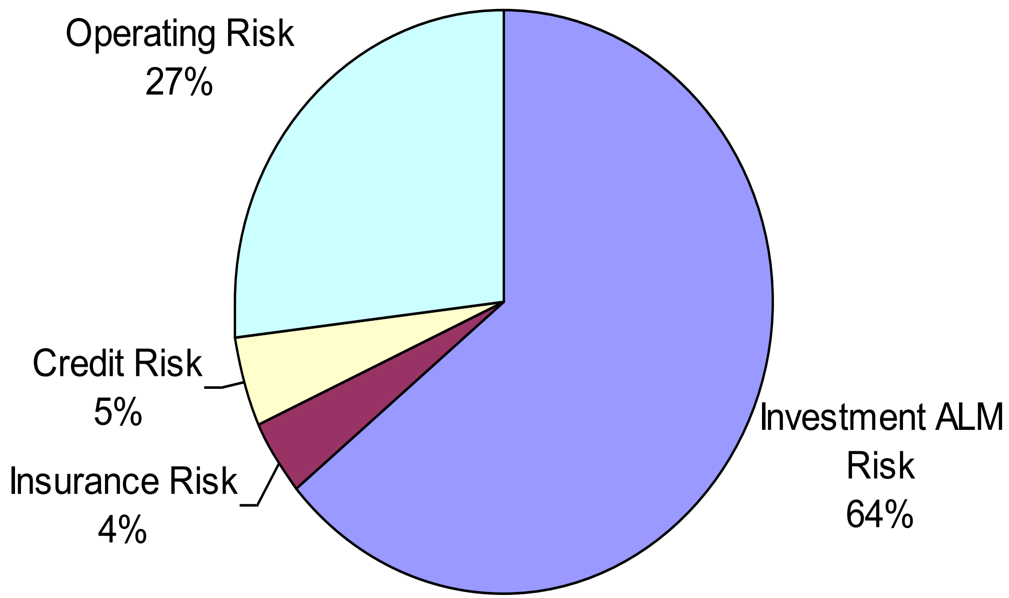
- AAA
- AA
- A
- BAA

Equity

- Shares
  - CHF
  - EUM
  - USD
  - GPB
  - JPY
- Real Estate
  - IAZI
  - Commercia l
  - Rüd Blass
  - WUPIX A
- Hedge Funds
- Private Equity

Drivers for required rsik capital

MOW finds in a study the following distribution of required risk capital as per end 2002:



Quelle: Mercer Oliver Wyman „Life at the End of the Tunnel“ (February 2004)

The relatively high amount of required capital for operating risk has the following reasons:

- Regulatory compliance risks
- Expense risks

Achieving Stakeholder Balance

Policyholders View
- Security
- Competitive bonus
- Regular bonus payments

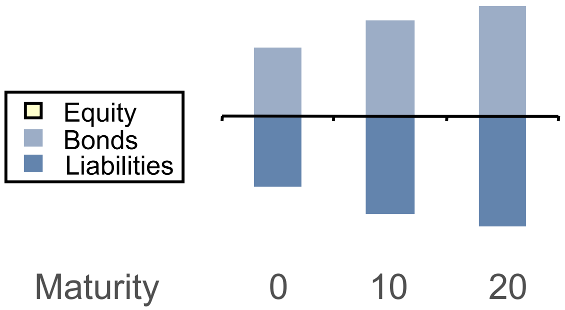
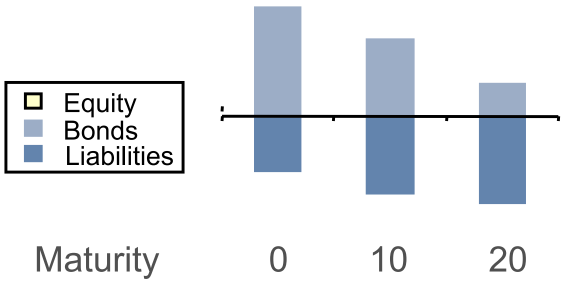
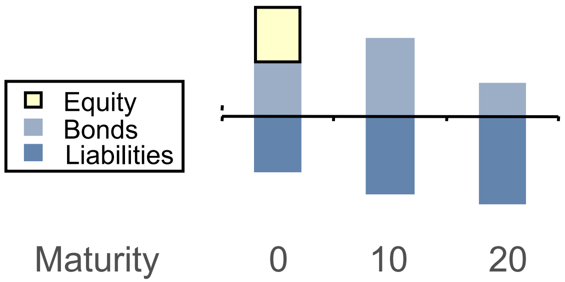
Shareholders View
- Risk adjusted return optimisation
- High dividends
- Regular dividend payments

Company View
- Sustainability
- Achieve corporate strategy
- Ensure sufficient reserves

Structuring the Balance sheet & Business model

Assets	Liabilities	Functions	Benefits to Policyholder	Benefits to Shareholder
Assets (for guaranteed benefits)	**Mathematical Reserves** (guaranteed benefits)	→ Cash Flow Matching and immunisation of the guaranteed benefits based on Mark-to-Market	→ Guaranteed benefits → Risk-Coverage	→ "Fees" → Risk result
Assets (for non-guaranteed benefits and return on equity)	**Mathematical Reserves** (Reserves - Non guaranteed benefits)	→ „Alpha“-Management and limited risk taking (e.g. „Floor“)	→ Bonus	
	Shareholder Equity	→ Absorbs unexpected volatility		→ Capital gains

Illustrative example Asset Allocation

<table>
<tr><th>DESCRIPTION</th><th>CASH FLOWS</th><th>EXPECTED RETURN</th><th>ECONOMIC RISK CAPITAL[1)]</th></tr>
<tr><td>PERFECT MATCHING
▪ Liability cash flows perfectly matched
▪ 0% equity</td><td>
</td><td>3%</td><td>0.5%</td></tr>
<tr><td>SHORT DURATION
▪ Short duration on bonds
▪ 0% equity</td><td>
</td><td>2%</td><td>7%</td></tr>
<tr><td>HIGH EQUITY
▪ Short duration on bonds
▪ 25% equity</td><td>
</td><td>4%</td><td>18%</td></tr>
</table>

1) In % of assets

Contents

- **Introduction**
- **Methodology**
  - Valuation of Assets and Liabilities
  - Market Value of Liabilities and replicating portfolios
  - Risk Capital
  - Optimization
- **Process**
  - Data requirements Assets
  - Data requirements Liabilities
- **Putting ALM in practice**

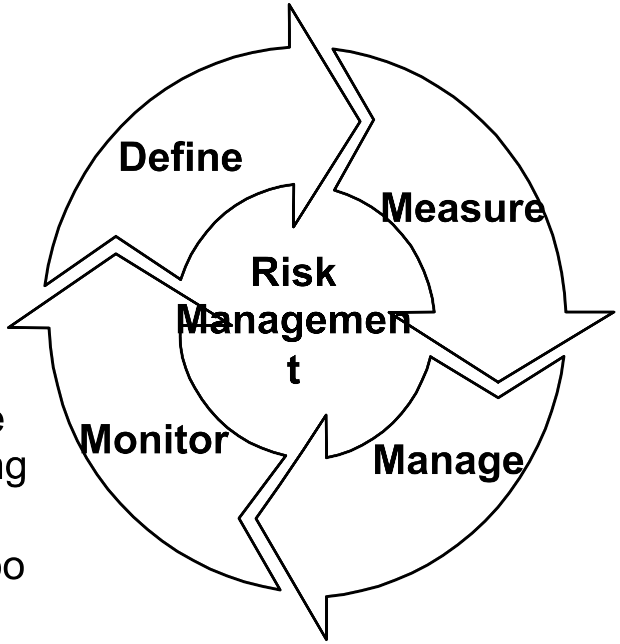
Functional Risk Management Process

Risk Assessment

- Ildentify and define all risks that need to be managed systematically (risk map)

Risk Management

- Monitor/measure risks on an ongoing basis
- Take actions if too much risk is taken



- Define risk metric and measurement methodology
- Assess/measure the risks

- Based on risk appetite implement desired risk/return profile
- Translate risk/return-profile into risk limits

ALM

Investment Management
- Historical focus: total return
- Responsible: CIO

ALM Management
- Focus: Economic Contribution
- Responsible: all

CFO

CIO EC/FV(1) CMO

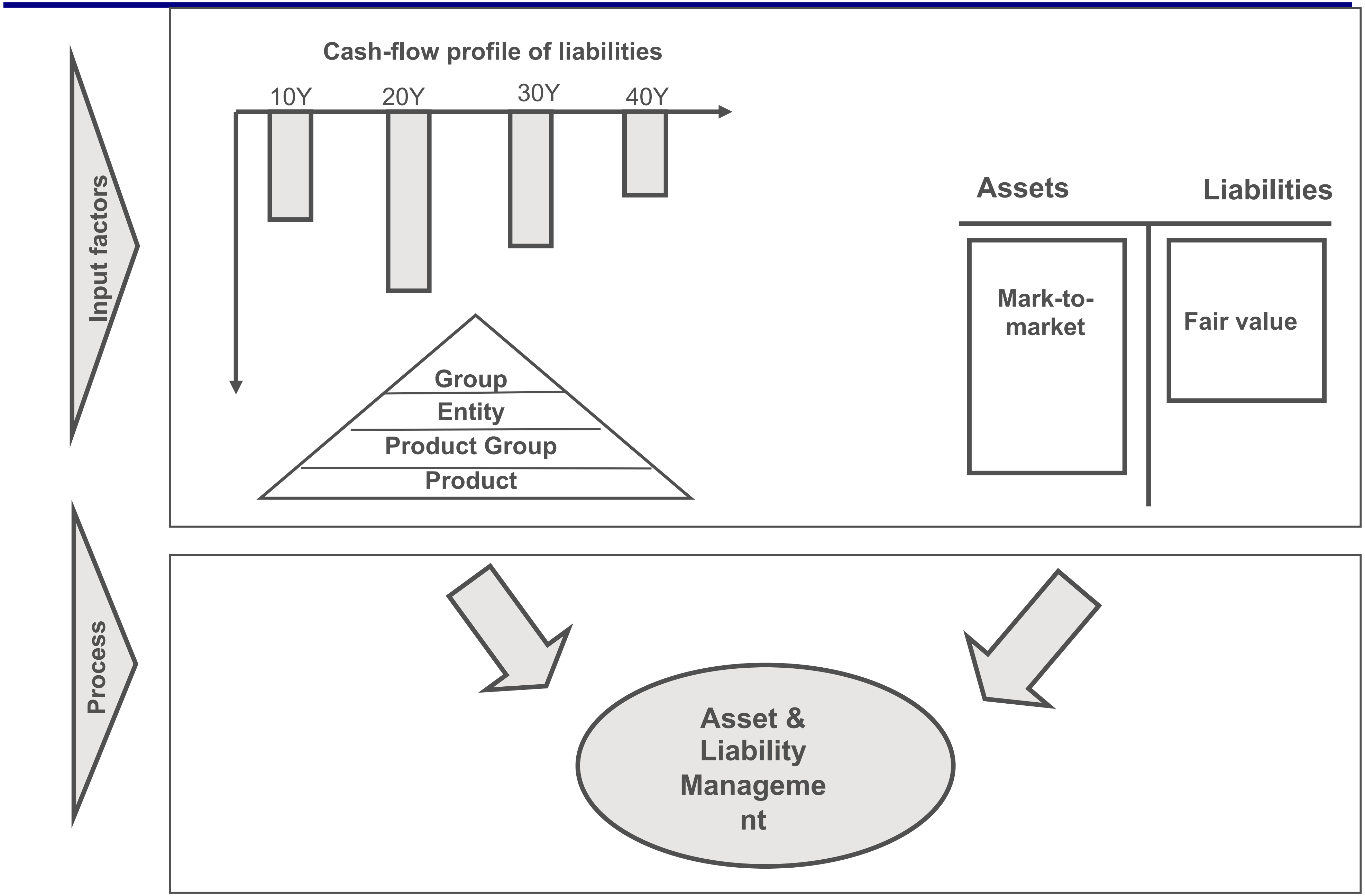
CRO

Product Economics
- Historical focus: Economic Margin
- Responsible: CRO

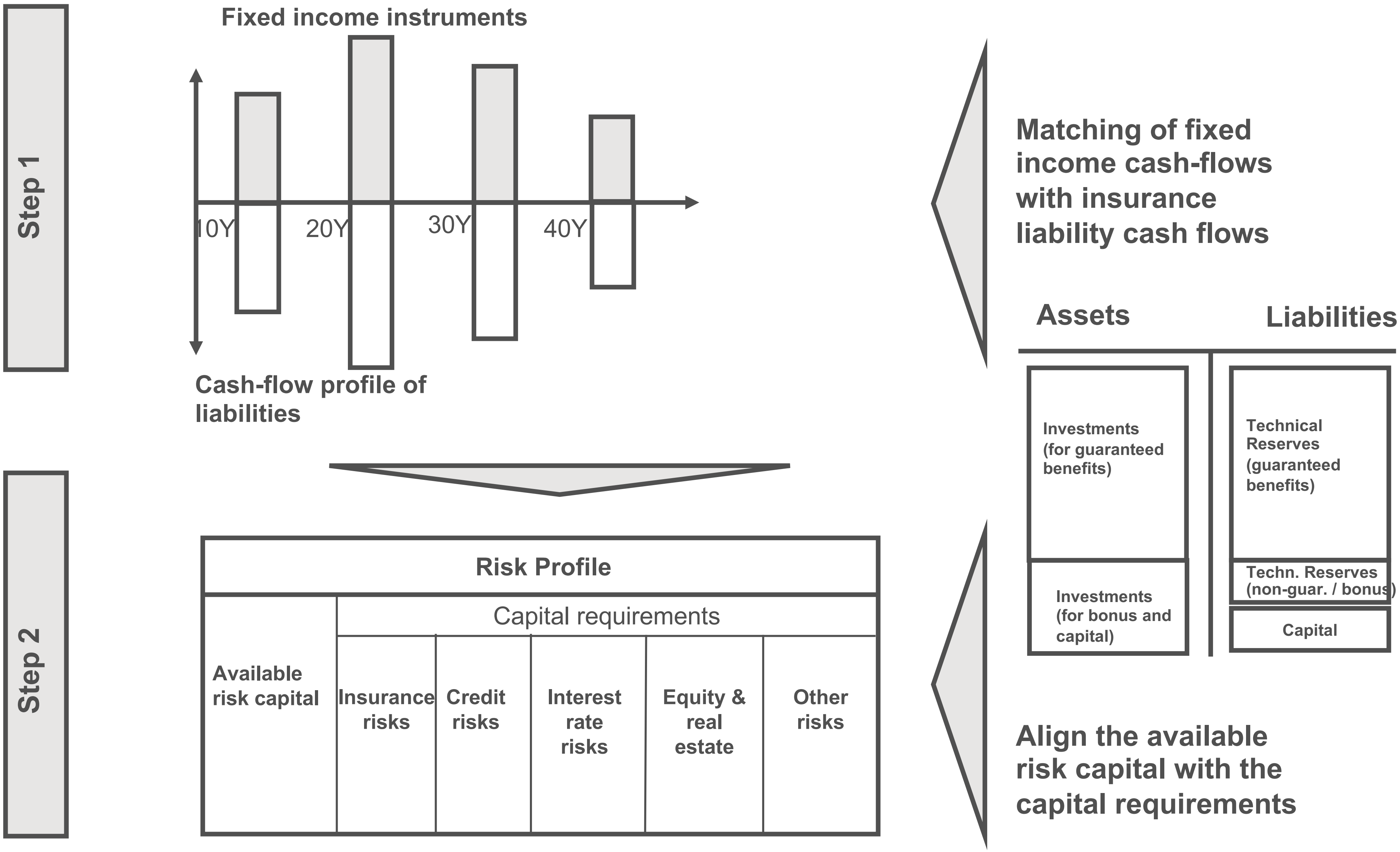
Commercial Aspects
- Historical focus: Volume
- Responsible: CMO

FV: Fair value (1) EC: Economic Risk Capital

Reduce risk profile - “Input”



Reduce risk profile - “Output”



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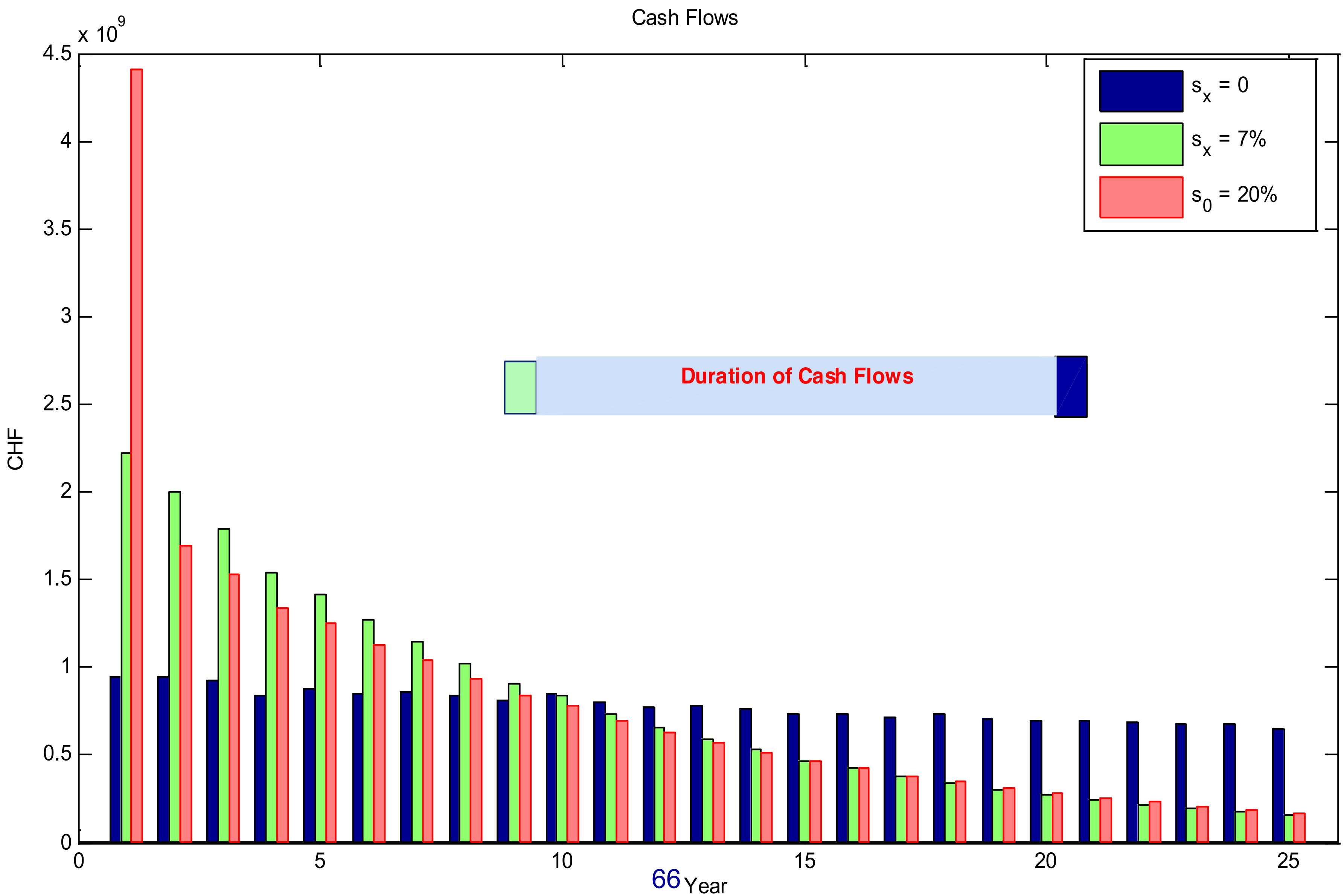
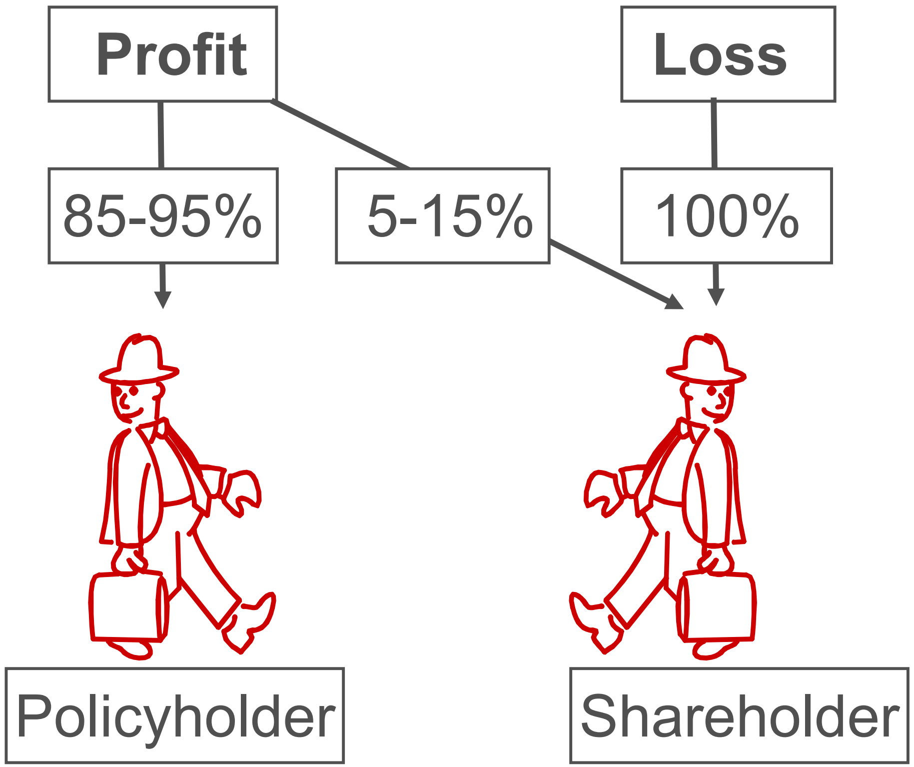
  - Data requirements Liabilities
- **Putting ALM in practice /Challenges (-;))**

Parameters: real Life-portfolio

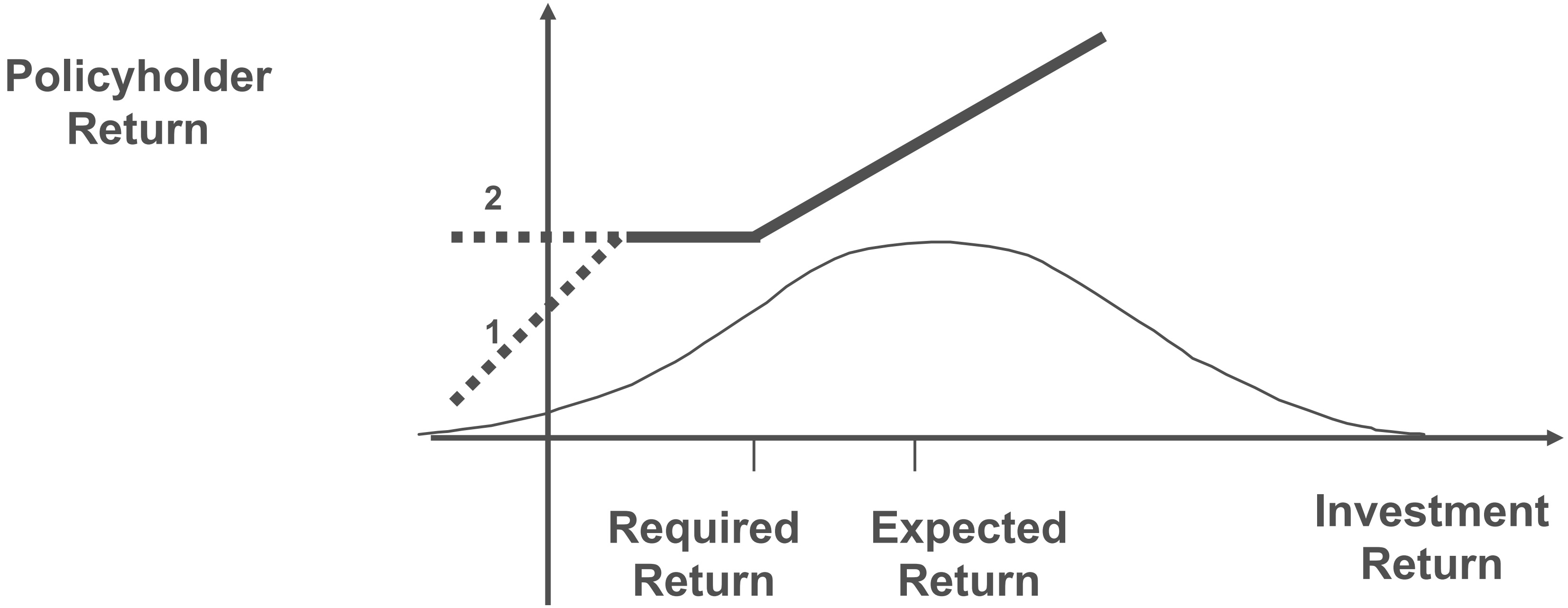


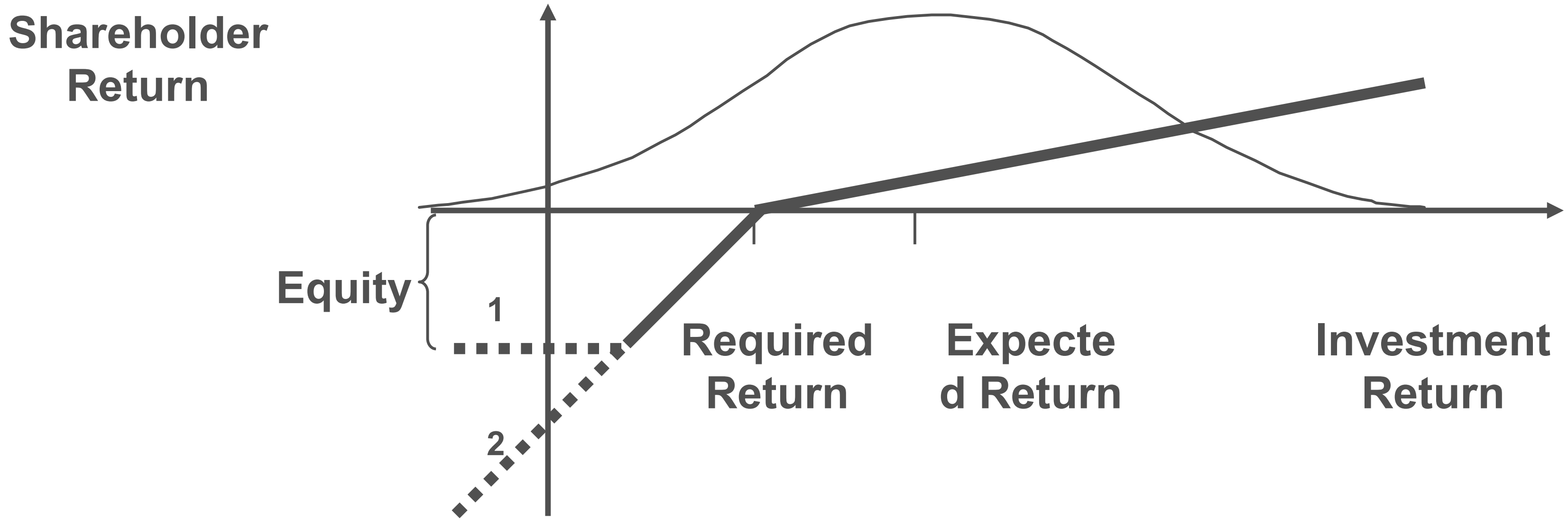
Profit sharing between policyholder and shareholder (1)



Establish fair and transparent profit sharing rules	
- internal /contractual rules	- external / regulatory rules (legal quote)

Profit sharing between policyholder and shareholder (2)

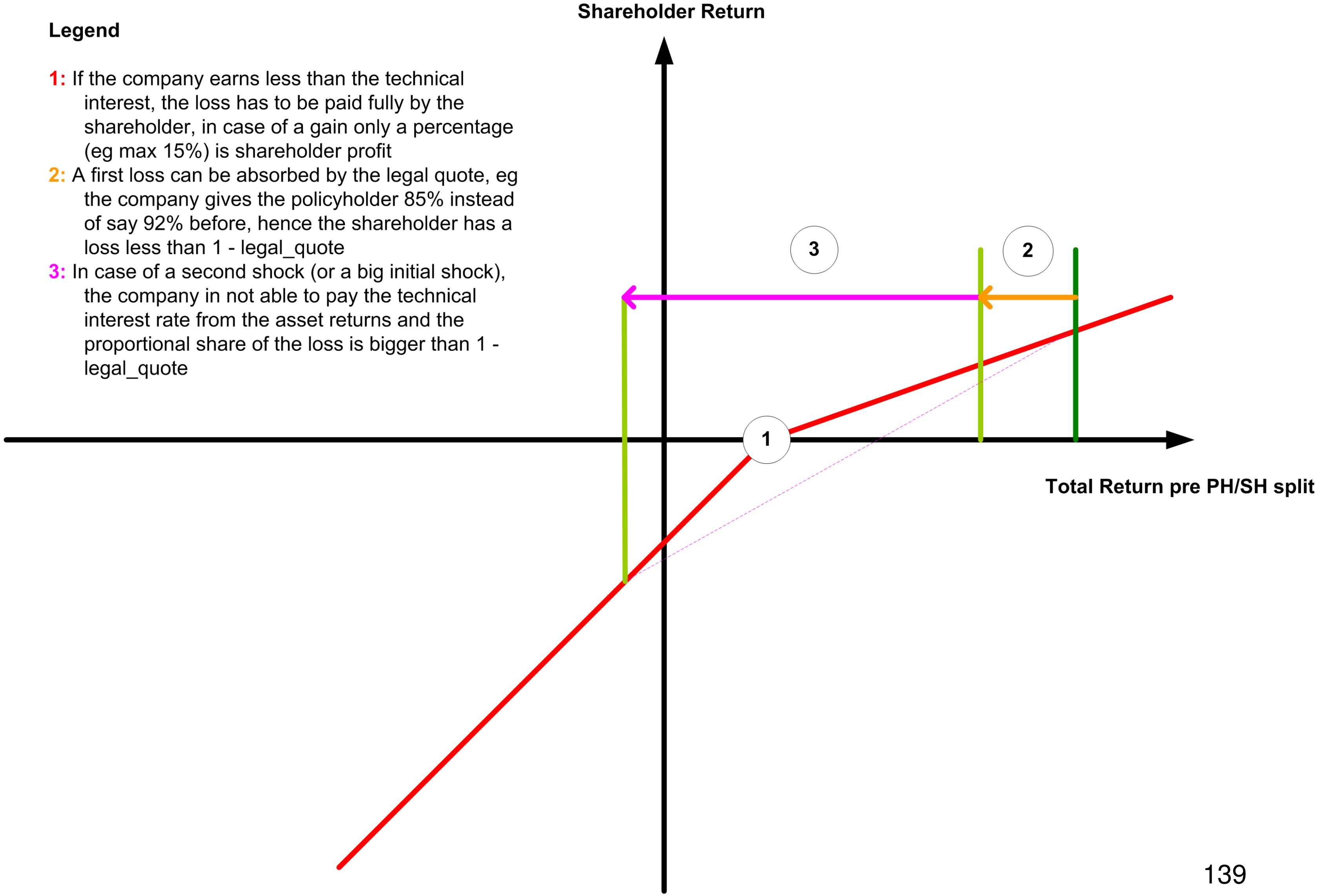




Investment Guarantees and Bonus Rates (1)

Legend

1: If the company earns less than the technical interest, the loss has to be paid fully by the shareholder, in case of a gain only a percentage (eg max 15%) is shareholder profit
2: A first loss can be absorbed by the legal quote, eg the company gives the policyholder 85% instead of say 92% before, hence the shareholder has a loss less than 1 - legal_quote
3: In case of a second shock (or a big initial shock), the company in not able to pay the technical interest rate from the asset returns and the proportional share of the loss is bigger than 1 - legal_quote



Investment Guarantees and Bonus Rates (2)

One reason for big issues with insurance products is that sometimes the existence and the level of interest guarantees the absence of a reasonable ALM. In order to understand the corresponding issues better we look at Swiss pension schemes.

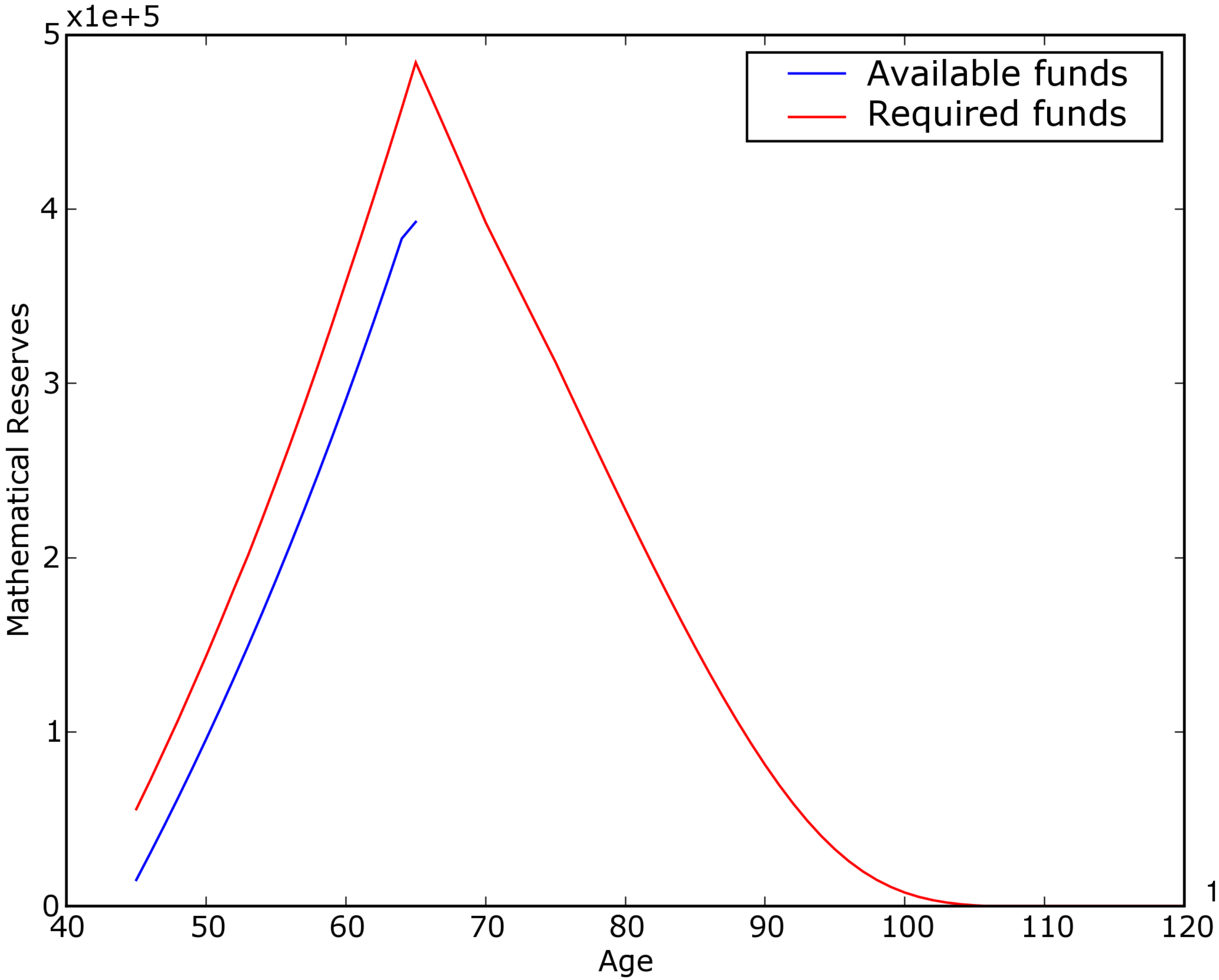
Interest during deferral period	$i = 2.0\%$
Current age of the insured	45
Current age of the partner	$\Delta_{XY} = -1$
Conversion rate at 65	7.2 % including widow pension
Contribution rate	15 %
Pensionable salary	100000
Single Premium at age 45	300000
Valuation date	29.12.2006
Profit share mechanism	Reserves for longevity provision can be deducted. Shareholder can claim 10 % of the remaining profit.

Investment Guarantees and Bonus Rates (3)

Age	Saving amount	Cash Flow	Discount	MR
45	15000	-15000	1.00000	55609
50	95816	-15000	0.88600	143506
60	290703	-15000	0.68753	358147
64	383169	-15000	0.62467	457903
65	**392749**	28277	0.61004	**484246**
66	0	27990	0.59656	466274
67	0	27682	0.58350	448090
68	0	27349	0.57086	429716
69	0	26995	0.55862	411183
70	0	26626	0.54677	392517
80	0	21605	0.41803	227625
90	0	13183	0.32582	81052

From the above table we see that the savings amount at age 65 equals 392749 CHF and that we need 484246 CHF for paying the liabilities assuming a risk free investment return. Hence c. 23 % of funds are missing at age 65 and the present value of the loss at age 45 equals 55609 CHF which equals 3.7 times the yearly contribution. For women the situation is worse because they live longer.

Investment Guarantees and Bonus Rates (4)



Investment Guarantees and Bonus Rates (5)

But what does this loss now mean for the business. There are three ways how one can look at this:

- Do not do such business,
- Take the loss up front,
- Invest in a asset allocation where one can in average achieve the goal.

Hence we have the following situation, assuming that shares yield 400 bps more than risk free:

Required yield	3.5 %
Risk free yield	2.5 %
Required uplift	1.0 %
Required equity backing ratio	25.0 %

Investment Guarantees and Bonus Rates (6)

If you now go back to the balance sheets of the Swiss life insurers at the beginning of this century, you will find that they were investing heavily in equities with equity backing ratios of 25 % and more.

	SAA	in CHF	Yield		Return
Shares	20%	7%	$4000000 \cdot 7\%$	=	280000
Bonds	60%	4%	$12000000 \cdot 4\%$	=	480000
Properties	10%	5%	$2000000 \cdot 5\%$	=	100000
Mortgages	10%	4%	$2000000 \cdot 4\%$	=	80000
			Total		940000
Math. Res.		3.5%	$-18000000 \cdot 3.5\%$	=	-630000
			Total		310000

Hence the insurance has an average return to both shareholders and policyholders of 310 M CHF.

Investment Guarantees and Bonus Rates (7)

In the year 2001, the equity index fell by 21 %. What has happend to the insurers income statement?

	SAA	in CHF	Yield		Return
Shares	20%	-21%	$4000000 \cdot$ (-21%)	=	-840000
Bonds	60%	4%	$12000000 \cdot$ 4%	=	480000
Properties	10%	5%	$2000000 \cdot$ 5%	=	100000
Mortgages	10%	4%	$2000000 \cdot$ 4%	=	80000
			Total		-180000
Math. Res.		3.5%	-$18000000 \cdot$ 3.5%	=	-630000
			Total		-810000

PH vs SH risk (8)

The following table shows a comparison between two different investment strategies, assuming a legal quote of 85 % and a tax-rate of 0 %. We assume the following:

Mathematical Reserve	1000000000 EUR
Technical interest	3.0 %
Yield of a bond investment	4.0 %
Expected yield shares	7.0 %
Volatility of shares	18.0 %
Strategy 1	100 % invested in bonds
Strategy 2	25 % invested in shares , 75 % in bonds.

For strategy 1 we know that we have a gross profit of 10 M EUR and hence the shareholder (SH) gets 1.5 M EUR and the policyholder (PH) 8.5 M EUR. For strategy 2, the situation is more complex and we need to look at the corresponding probability distribution:

PH vs SH risk (9)

Return Shares	Probability Return	Portfolio Gross	P/L $\sum$	P/L SH	P/L PH
-40 %	0.00086	-7.00 %	-100000000	-100000000	0
-35 %	0.00169	-5.75 %	-87500000	-87500000	0
-30 %	0.00426	-4.50 %	-75000000	-75000000	0
-25 %	0.00962	-3.25 %	-62500000	-62500000	0
-20 %	0.01948	-2.00 %	-50000000	-50000000	0
-15 %	0.03530	-0.75 %	-37500000	-37500000	0
-10 %	0.05730	0.50 %	-25000000	-25000000	0
-5 %	0.08331	1.75 %	-12500000	-12500000	0
0 %	0.10851	3.00 %	0	0	0
5 %	0.12659	4.25 %	12500000	1875000	10625000
10 %	0.13229	5.50 %	25000000	3750000	21250000
15 %	0.12383	6.75 %	37500000	5625000	31875000
20 %	0.10383	8.00 %	50000000	7500000	42500000
25 %	0.07799	9.25 %	62500000	9375000	53125000
30 %	0.05247	10.50 %	75000000	11250000	63750000
35 %	0.03162	11.75 %	87500000	13125000	74375000
40 %	0.03097	13.00 %	100000000	15000000	85000000
Expected Value	1.00000	5.34 %	23472093	-1518018	24990112

Longevity (1)

$\ddot{a}_{65}(i = 3.5)\%$	men	Δ men	women	Δ women
ERM/F 70	12.491	3.958	13.923	3.820
ERM/F 80	13.199	3.250	14.789	2.954
ERM/F 90	14.387	2.062	16.221	1.522
ERM/F 00 @ 2005	16.450		17.744	

Longevity (2)

Tariff generation	MR reserve Original base M CHF	MR reserve ERM/F 2000 M CHF	Difference M CHF
ERM/F 70	400	526	126
ERM/F 80	800	997	197
ERM/F 90	2000	2286	286
ERM/F 00 @ 2005	400	400	0
Total	3600	4210	610

Imperfect Cash Flows Matching (1)

We have seen that there are no prices for long dates bonds in some currencies. Moreover even if there are prices for these bonds, there might be only a limited market for long dated bonds, as a consequence of states not wanting to issue long term bonds. A typical example is the CHF, where the market is liquid only up to durations of about 15 years. As a consequence insurance companies and pension funds are not able to match their guaranteed cash flows with corresponding bonds. In this section we want to have a closer look at this question and the corresponding risks. The best way to understand this risk is to look at concrete examples:

- A portfolio of annuities in payment,
- A portfolio of deferred annuities,
- A portfolio of endowment policies.

Imperfect Cash Flows Matching (2)

In all this three cases we assume that the benefits are denominated in CHF and we furthermore assume that there is only a liquid market for CHF bonds until year 15 and that hence the best thing to do is to use investments according to this. In order to valuate what could happen we look at three different scenarios:

1. Yield curve and investment opportunities as seen today,
2. At time 15 there is a flat yield of 0%, 1%, 2 % and 3 % respectively.

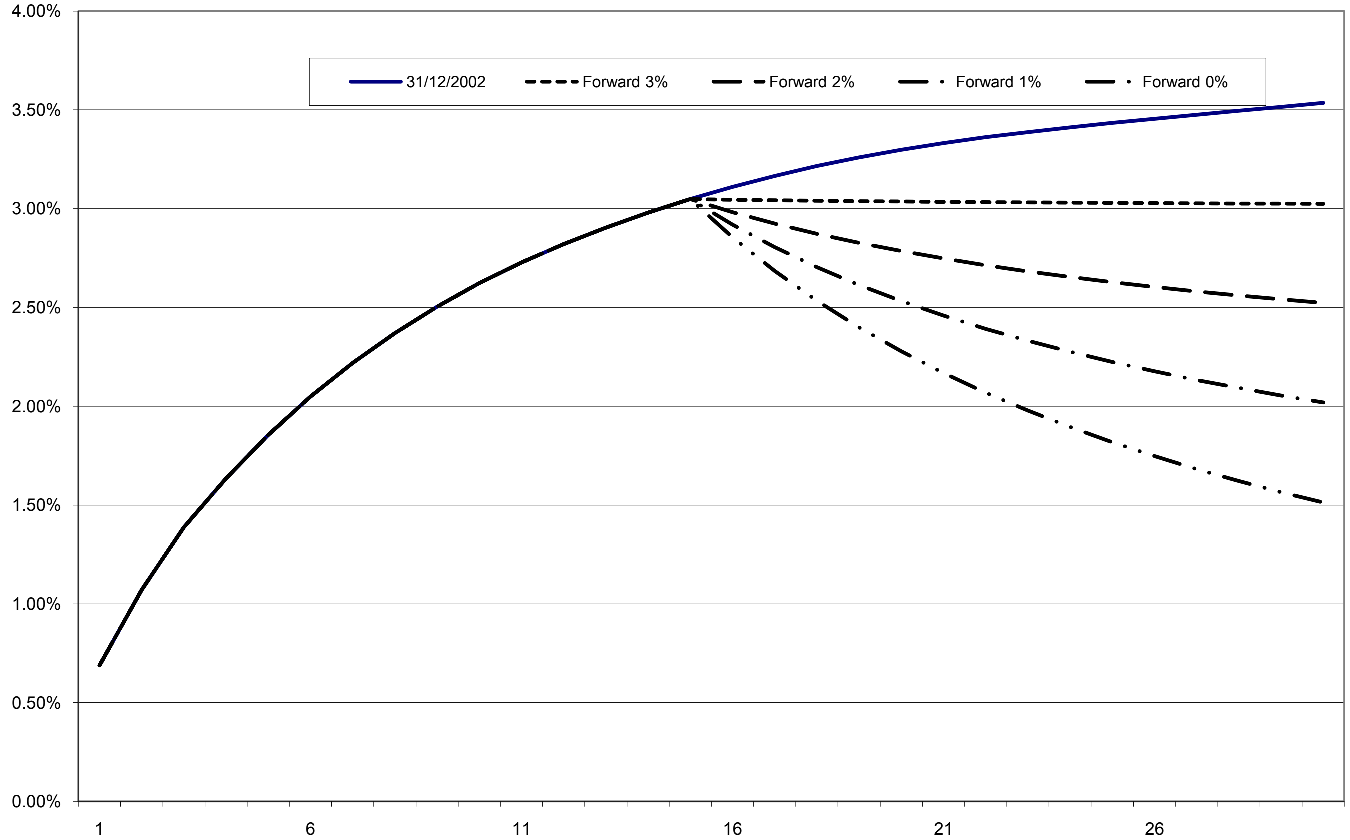
In order to be able to better describe this problem, we denote with $(CF_k)_{k \in \mathbb{N}}$ the vector of expected cash flows and we do for the moment neglect the fact that this cash flows are actually random and can depend on the market environment.

Imperfect Cash Flows Matching (3)

For the analysis we assume that the company invests as follows in $\sum_{k \in \mathbb{B}} \alpha_k \mathscr{Z}_{(k)} \in \mathscr{X}$:

$$\alpha_k = \begin{cases} CF_k & \text{if } k < 14, \\ \sum_{k \geq 15} CF_k & \text{else.} \end{cases}$$

Imperfect Cash Flows Matching – Modified Yield Curves (4)



Imperfect Cash Flows Matching – Modified Yield Curves (5)

$$
\begin{aligned}
\pi_t(\mathscr{B}) &= \sum_{k=0}^{\infty} CF_k \times \pi_t(\mathscr{Z}_{(k)}) \\
&= \sum_{k=0}^{\infty} CF_k \times (1 + y_t(k))^{-k}.
\end{aligned}
$$

Moreover the forward rates can in this case be calculated by

$$
f_t(n,m) = \left(\frac{\pi_t(\mathscr{Z}_{(n)})}{\pi_t(\mathscr{Z}_{(m)})} \right)^{\frac{1}{m-n}} - 1,
$$

and hence the following equation holds:

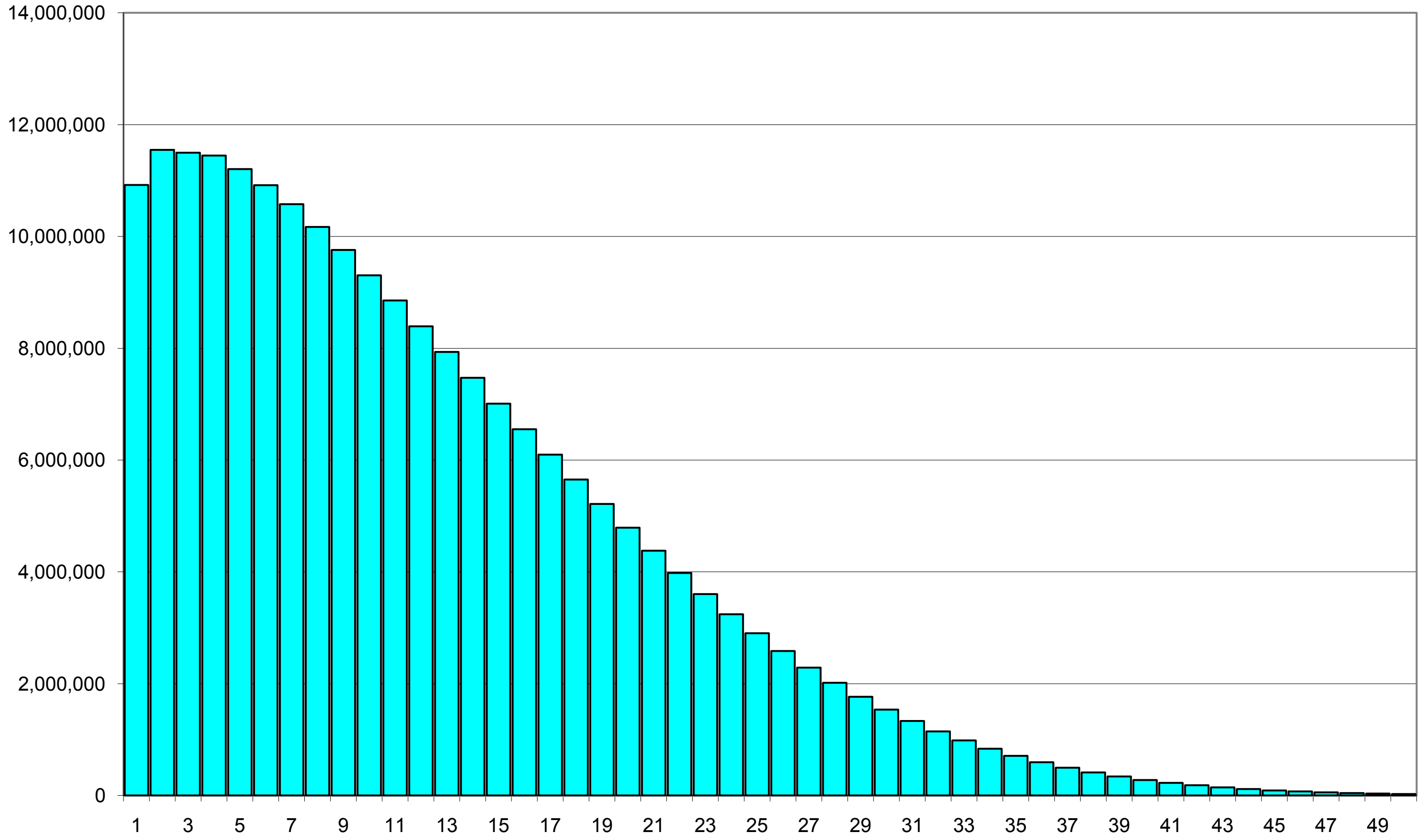
$$
(1 + y_t(n))^n = \prod_{k=0}^{n-1} (1 + f_t(k, k+1)).
$$

Imperfect Cash Flows Matching – Modified Yield Curves (6)

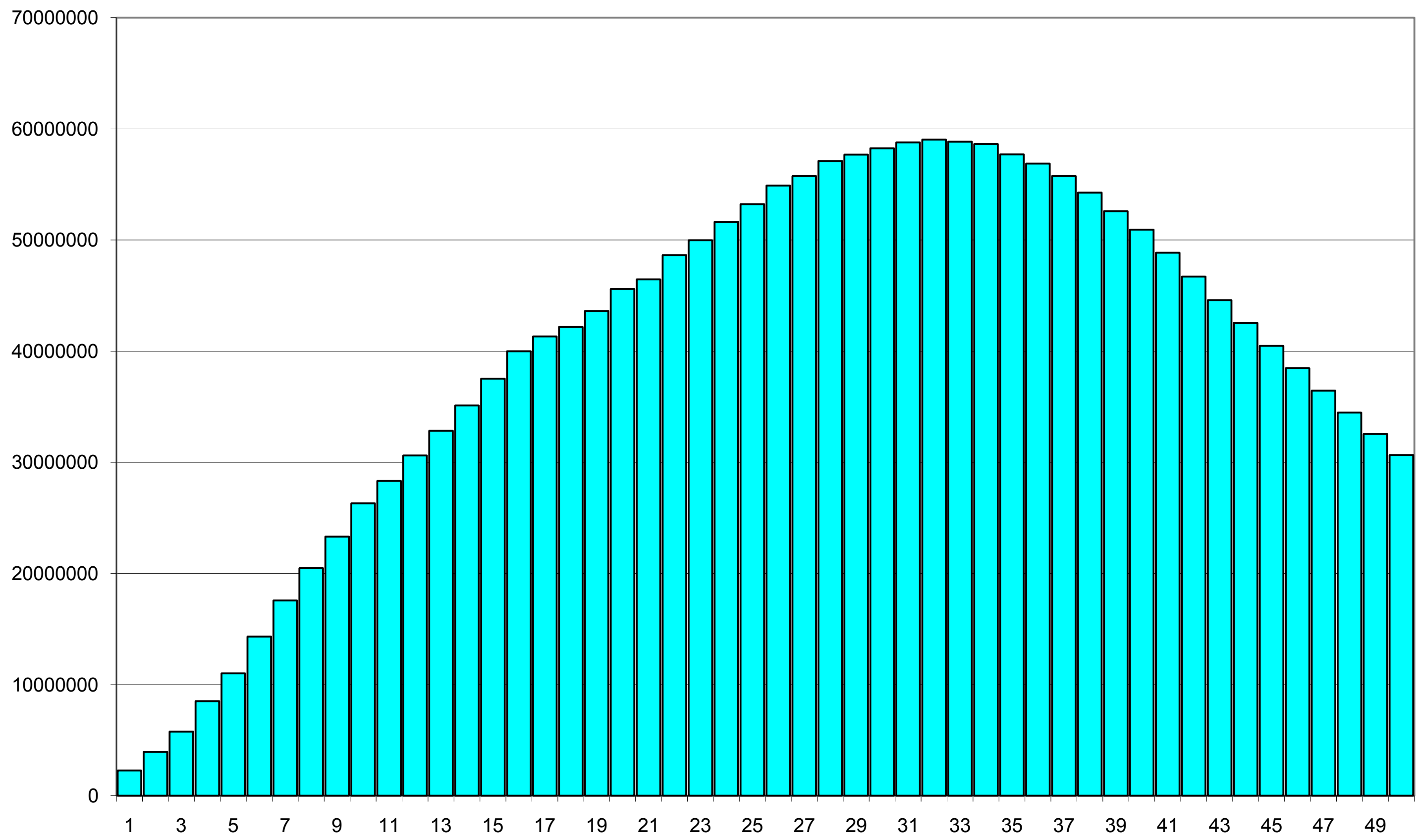
At this stage it is now easy to “construct” suitable yield curves representing the scenarios above by setting:

$$f_t(n, n+1) = \theta,$$

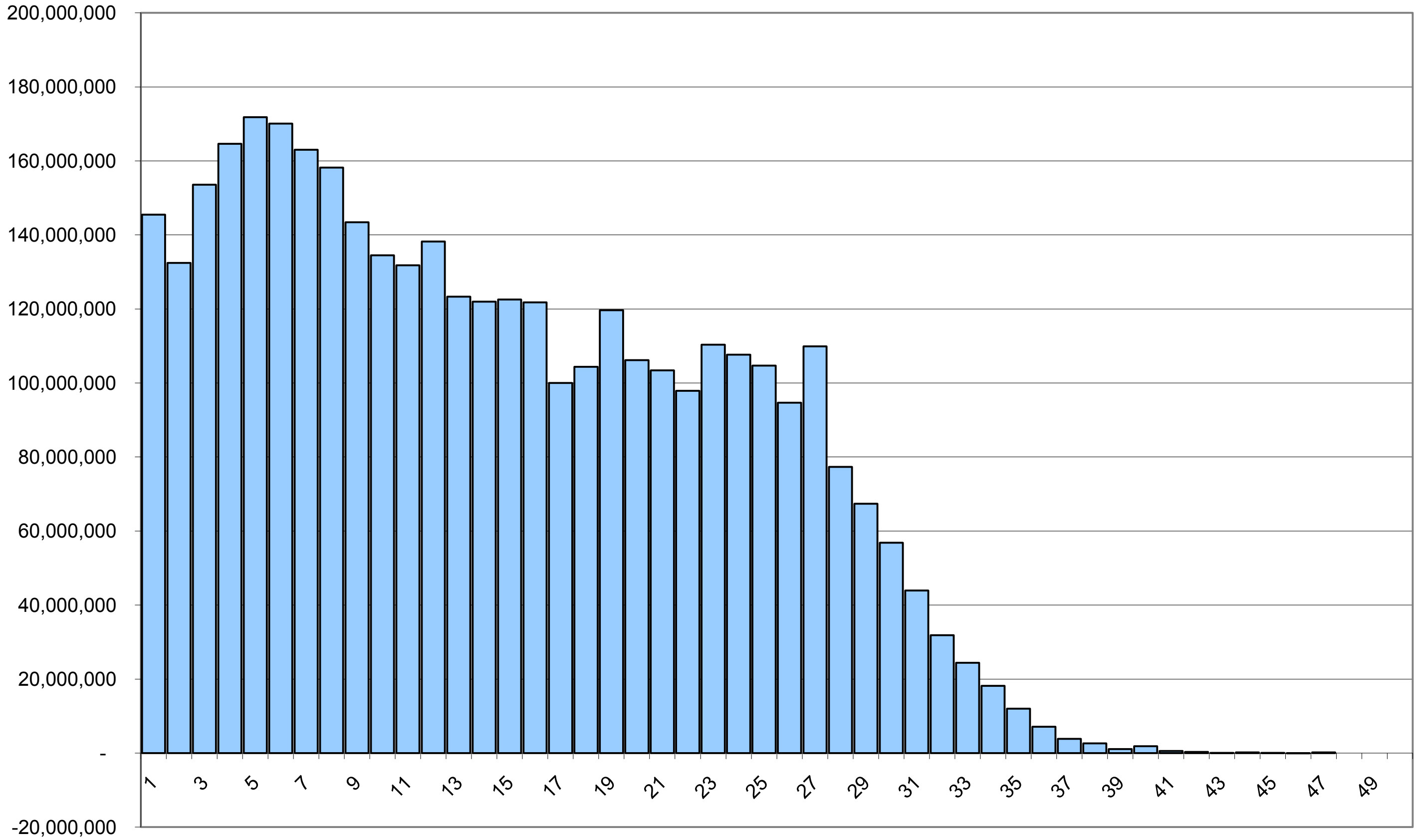
Imperfect Cash Flows Matching (7)



Imperfect Cash Flows Matching (8)



Imperfect Cash Flows Matching (9)



Imperfect Cash Flows Matching (10)

in M CHF	Portfolio A	Portfolio B	Portfolio C
Benefit	13.6 p.a.	108.1 p.a	4211.1
Statutory Reserves	162.7	841.5	2474.4
Premiums	–	–	20.4
Duration	8.9	25.3	11.0

Imperfect Cash Flows Matching (11)

in M CHF	Portfolio A	Portfolio B	Portfolio C	Total
Statutory Reserves 3.5 %	162.7	841.5	2474.3	3478.6
31/12/2002	159.4	880.0	2617.3	3656.9
Forward 2%	163.7	1135.6	2746.0	4045.4
Forward 1%	166.3	1322.3	2821.5	4310.2
Forward 0%	169.2	1575.5	2906.7	4651.5
Coverage in %				
31/12/2002	102.0 %	95.6 %	94.5 %	95.1 %
Forward 2%	99.3 %	74.1 %	90.1 %	85.9 %
Forward 1%	97.8 %	63.6 %	87.6 %	80.7 %
Forward 0%	96.1 %	53.4 %	85.1 %	74.7 %
Coverage absolute				
31/12/2002	3.2	-38.5	-143.0	-178.3
Forward 2%	-1.0	-294.0	-271.7	-566.8
Forward 1%	-3.6	-480.8	-347.2	-831.6
Forward 0%	-6.5	-734.0	-432.3	-1172.9

Gender Directive

in M CHF	Portfolio A	Portfolio B	Portfolio C	Total
Men as men	137.2	663.7	1749.0	2549.9
Women as women	25.4	177.8	725.3	928.6
Total	162.7	841.5	2474.3	3478.6
All as men	157.7	804.2	2483.5	3445.5
All as women	197.1	1037.5	2451.1	3685.8
Impact	4.9	37.3	23.2	207.2
Relative Impact	**3.0 %**	**4.4 %**	**0.9 %**	**5.9 %**