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EFFICIENT ESTIMATES OF UNCERTAINTIES IN TSUNAMI INUNDATION FORECASTS

H. T.-S. Ko[1], W. D. Mayfield[1], M. Dumelle[1], H. Yeh[2], C. Fuentes[3], and J. M. Restrepo[4]

ABSTRACT

We present a methodology for efficient variance estimation of nonlinear tsunami inundation forecasts. The tsunami inundation dataset is produced using a numerical nonlinear shallow water equation solver with an initial deformation of the sea surface. We introduce a set of perturbation equations to the nonlinear shallow water equations, which use the dataset as the reference case. Perturbations to the initial condition of the inundation event are imposed to simulate an ensemble of outcomes. The linearized version of the shallow water perturbation equations is used to pre-calculate numerical Green's functions for a sparse grid of points within the domain. The Green's function approach allows for efficient calculations of ensembles of perturbations. Variances are computed from the ensembles, and the resulting statistical summaries are interpolated to the entire domain using the universal kriging. It is demonstrated that variance estimates, for the entire domain or a select grid points of interest, can be made with a much lower computational complexity than other traditional approaches, as well as exposing the task to simple parallelization. This methodology is applicable to other similar dynamic problems.

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Efficient Estimates of Uncertainties in Tsunami Inundation Forecasts

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ABSTRACT

We present a methodology for efficient variance estimation of nonlinear tsunami inundation forecasts. The tsunami inundation dataset is produced using a numerical nonlinear shallow water equation solver with an initial deformation of the sea surface. We introduce a set of perturbation equations to the nonlinear shallow water equations, which use the dataset as the reference case. Perturbations to the initial condition of the inundation event are imposed to simulate an ensemble of outcomes. The linearized version of the shallow water perturbation equations is used to pre-calculate numerical Green's functions for a sparse grid of points within the domain. The Green's function approach allows for efficient calculations of ensembles of perturbations. Variances are computed from the ensembles, and the resulting statistical summaries are interpolated to the entire domain using the universal kriging. It is demonstrated that variance estimates, for the entire domain or a select grid points of interest, can be made with a much lower computational complexity than other traditional approaches, as well as exposing the task to simple parallelization. This methodology is applicable to other similar dynamic problems.

Introduction

Efforts to improve tsunami mitigation and awareness in recent years have been accompanied by a drive to further develop tsunami-flood forecasts [1]. Flood forecasting has been moving in the direction of ensemble forecasting due its success for atmospheric processes [2]; ensemble forecasting is a prediction practice that utilizes repeated model runs with slightly different initial conditions, instead of running a single forecast. Ensemble forecasting has been shown to account for uncertainties that arise from model sensitivity to complex bathymetries, to resolution of simulated wave dynamics [3], and to initial conditions and other parameters [4]. Ensemble forecasting is a form of Monte Carlo analysis. Consequently, repeated runs of a numerical forecast model must be conducted, which puts a strain on computing resources. This presents a significant challenge for running operational forecasting systems [1].

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Here, we present an alternative and novel method for ensemble forecasting to assimilate initial condition uncertainty using the principles of numerical Green's functions. A linearized perturbation model to the nonlinear shallow-water equations is used to pre-compute, store, and recombine Green's Functions to efficiently calculate ensembles of outcomes under perturbation. These ensembles can be quickly computed for a given event using the Green's functions. Computed ensembles are supplemented with universal kriging [5] to interpolate statistical results to any arbitrary location within the domain. We show that this methodology is not only effective, but also much more efficient than the existing methods.

Methods

Demonstration Scenario

The variance calculations are performed for a single tsunami forecast. The output from the tsunami forecast is stored as the reference dataset used for the Green's function calculations and ensemble forecast. In the present study, the flood forecast is performed by NHWAVE [6] under hydrostatic conditions to generate a reference dataset equivalent to those obtained from nonlinear shallow-water wave theory. A hypothetical idealized tsunami scenario is used to demonstrate the methodology. The scenario is setup as a simple initial value problem. This scenario is chosen to facilitate convenient perturbations of the initial state for Monte Carlo simulation. The initial condition for the NHWAVE simulation is an idealized hump of water (ellipsoid Gaussian). The bathymetry used in this study is based on the nearshore region near Port Hueneme, California. This location has been thoroughly studied to establish a testbed for scenario-based tsunami engineering practice: dense and high-quality numerical data for a variety of tsunami scenarios (30 of them) are prepared and stored in the portal "PBTE: Data Explorer" [7].

Ensemble Computation

The developed methodology uses numerical Green's functions to efficiently compute "ensembles of perturbations" for tsunami flooding forecasts. Green's functions are defined as the response of a linear model to the Dirac delta function [8]. Numerically, they are implemented as responses of the system to unit impulses at specific locations. The Green's function approach requires a linear model, here it is developed for small perturbations to the reference scenario simulated by the foregoing numerical code NHWAVE.

The nonlinear shallow-water equations (SWE) are modified by adding a perturbation term to each of the dependent variables. The first order perturbation terms are shown in Eq. 1, which we denote as the linear shallow water perturbation equations (LSWPE):

$$\tilde{\eta}_t + \nabla \cdot (H\tilde{\boldsymbol{u}} + E_o\tilde{\boldsymbol{u}} + \tilde{\eta}\boldsymbol{U_o}) = 0; \qquad \tilde{\boldsymbol{u}}_t + (\boldsymbol{U_o} \cdot \nabla)\tilde{\boldsymbol{u}} + (\tilde{\boldsymbol{u}} \cdot \nabla)\boldsymbol{U_o} = -g\nabla\tilde{\eta}\,, \tag{1}$$

where $\tilde{\eta}$ and $\tilde{\boldsymbol{u}}$ represent small perturbations to the free surface elevation and horizontal velocity vector, respectively. The variables given by E_o and $\boldsymbol{U}_o$ represent the known solutions for the free surface and horizontal velocity vector, respectively, to the SWE obtained from the NHWAVE generated reference data. The variable g represents the gravitational acceleration, and H represents still water depth in the domain. The LSWPE characterize the interactions between small perturbations and the reference (unperturbed) case. Eq. 1 is solved numerically using pseudo-spectral Fourier methods with 4th order Runge-Kutta time stepping.

Perturbations are introduced in the numerical model as initial conditions, the behavior of which is monitored by the numerical model. The response of the model to the unit impulses is pre-computed and stored as the numerical Green's functions. Each Green's function is computed and stored as a timeseries corresponding to a specific "source" location (denoted $\boldsymbol{x}_j$) and point of interest (denoted $\boldsymbol{x}^*$). We denote such a Green's function as $G(\boldsymbol{x}_j; \boldsymbol{x}^*, t)$. Once computed, the Green's functions can be recombined to form any initial perturbation using Eq. 2:

$$\sum_j f(x_j) \cdot G(x_j; x^*, t), \tag{2}$$

where $f(x_j)$ is the initial water surface displacement at x_j. This summation yields the result of the perturbed simulation for the location x^*.

Spatial Interpolation

Universal kriging is the method employed to interpolate statistical results to the entire domain. Kriging allows the methodology to offer sensitivity analysis at a fine resolution. The technique finds unbiased, minimum variance estimates that take spatial and temporal correlation into account. Additionally, it provides confidence intervals at each point in the domain. The combination of the Green's function approach and kriging produces high resolution variance estimates for the entire numerical domain.

Results

Fig. 1 shows an example of results from combining Green's functions to generate an ensemble of perturbations to a simulated tsunami event.

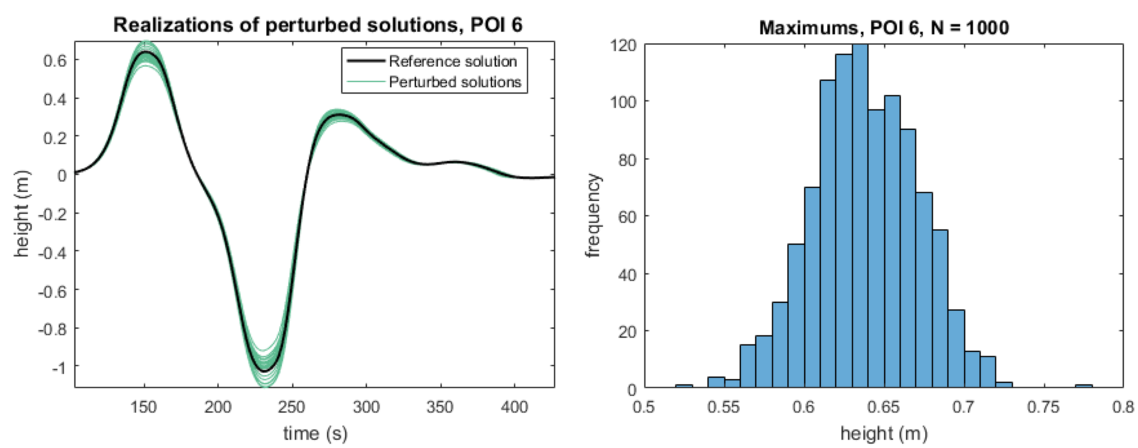

Figure 1. (Left) Green's Function realizations of perturbed solutions as seen at a chosen point of interest. (Right) histogram depicting an ensemble of maximum heights.

A specific point of interest (x^*) is chosen, and perturbations are made to the amplitude of the initial condition. Other types of perturbations, such as location, size, and shape may be analyzed as well. Due to the efficiency of Green's function calculations, large ensembles may be generated, allowing for statistical summaries of outcomes. For example, we take the maximum water height for a credible tsunami event as the quantity of interest, and generate statistical summaries at a grid of points of interest. After computing ensembles for a grid within the domain, kriging allows us to generate statistical summaries, such as the histogram from Fig. 2, for any point in the domain, down to a very fine spatial resolution.

Computational Complexity

Let N be the number grid points, M be the number of ensemble members, T represent the total number of time steps required for a model run, and $G = G_s G_r$ represents the total number of Green's functions (G_s is the number of Green's function source points and G_r is the number of receiver or grid points). If M ensemble members are required for an adequate estimation of variance, the nonlinear complexity is $\mathcal{O}(M \cdot N \cdot T)$. The LSWPE model has a cost of $\mathcal{O}(N \cdot T)$, but is only used for pre-calculation of Green's functions which has a total cost of $\mathcal{O}(G \cdot N \cdot T)$. For an ensemble of forecasts at a given location and time, an entire ensemble may be calculated for the cost of $\mathcal{O}(M \cdot G_s)$; this is an algebraic operation and is extremely efficient. For a time series at a receiver grid point, we calculate the resulting ensemble in $\mathcal{O}(M \cdot G_s \cdot T)$, a snapshot over space is of cost $\mathcal{O}(M \cdot G_s \cdot G_r)$ with the additional cost of Kriging which is of order $\mathcal{O}(N)$ but may be reduced through the use of local neighborhoods. We also note that precomputation of Green's and the sum in Eq. 2 are massively parallel problems, readily exposing the problem to further gains in computational efficiency.

Conclusions

Numerical Green's functions combined with kriging and a linear perturbation model can produce efficient ensemble forecasts for nearshore water levels and horizontal velocities in an idealized tsunami inundation simulation. Computational complexity calculations suggest that this methodology is more efficient than more traditional methods, enabling ensemble forecasts where they were otherwise intractable.

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