Adjusting Standard ANOVA Methods to Account for Heterogeneous Variances With an Application to Turfgrass Management

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Background

- Statistics Consulting Practicum at Oregon State University
- Several turfgrass projects, Alec Kowalewski and Clint Mattox
- Collaboration with Horticulture Department
- One common problem: unequal variances (standard deviations) with treatment groups
- Variance is the standard deviation squared, important distinction!

Overview

1. Motivation
2. Why ANOVA?
3. Generalized Variance ANOVA (GV ANOVA)
4. Data Application
5. Conclusions

Motivation

- Analysis of Variance (ANOVA) - identify *significantly different* treatments
  - Focus on one-way ANOVA
- Several assumptions
- All observations have the same variance (variance homogeneity)
- Often violated in practice

Why ANOVA?

- Why do we use ANOVA?
- ANOVA estimates are

1. Correct on average (unbiased)
2. As precise as possible (minimum variance)

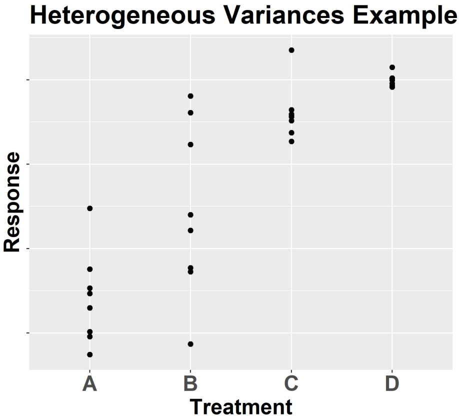
Shortcomings of ANOVA

- What changes with unequal variance (variance heterogeneity)
- Equal variance ANOVA estimates

1. Are still correct on average
2. Can give **incorrect p-values**
    - Misleading inference, poor policy decisions

How Do I Know?

- Graphics! If it looks off, it probably is



How Do I Know?

- Ratio of largest variance to smallest variance
    - Seen 1.5 to 9 as suggestions of cutoff points for variance
    - 1.22 to 3 for standard deviation
- Hypothesis tests for equality of variance
    - Levene's, Brown-Forsythe, etc.
    - Come with their own assumptions
    - Small p-value indicates evidence of unequal variance

How Do I Proceed?

- Transformations of the response
  - Difficult to find, often most effective with clear mean/variance relationship
  - Difficult to interpret, might not make statements on scale of interest
- Directly model unequal variance within groups
  - Generalized Variance ANOVA (GV ANOVA)

Generalized Variance ANOVA (GV ANOVA)

- GV ANOVA incorporates separate variances for each treatment level
    - No back transformation required
- Variance ratios known
    - `lm()` or `gls()` in **R**, `proc glm` or `proc mixed` in **SAS**
- Variance ratios unknown (usual case)
    - `gls()`, `lme()`, `lmer()` in **R**, `proc mixed` in **SAS**

Percent Green Cover

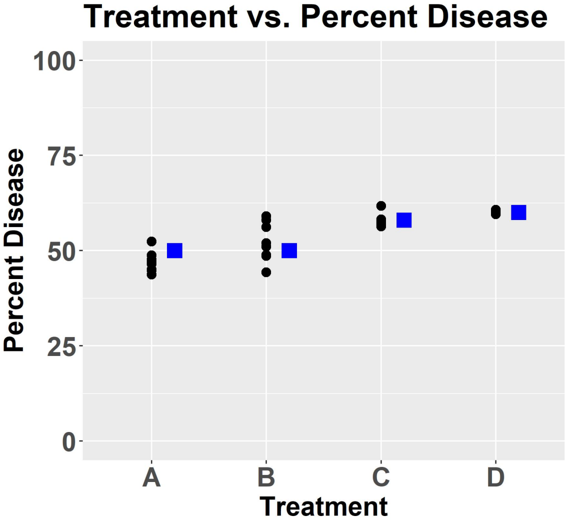
Data Setup

- Simulate turfgrass trial for percent green cover
- 4 treatments, 8 replicates, normally distributed, independent

	A	B	C	D
Mean	50	50	58	60
SD	7	5	1	0.5

- Study pairwise differences between treatments, $\alpha = 0.05$

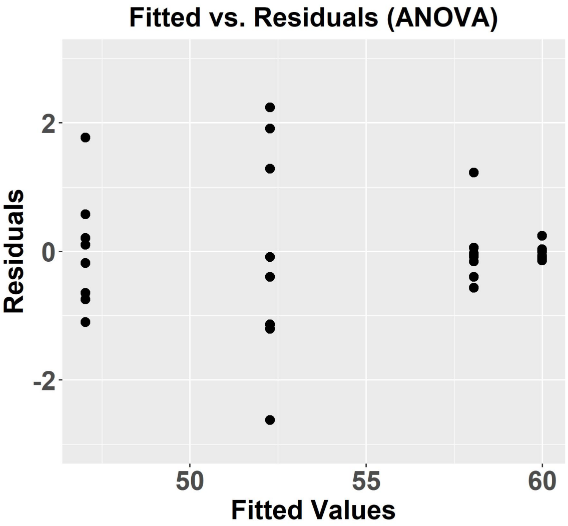
Data Layout


Treatment vs. Percent Disease
Percent Disease
100
75
50
25
0
A
B
C
D
Treatment

Pairwise Comparisons (ANOVA)

T1	T2	True.Diff	Est.	S.Err.	Adj.P.	Error
A	B	0	5.24	1.51	0.010	Type I
A	C	8	11.02	1.51	< 0.001	None
A	D	10	12.97	1.51	< 0.001	None
B	C	8	5.78	1.51	0.004	None
B	D	10	7.73	1.51	0.001	None
C	D	2	1.94	1.51	1.0	Type II

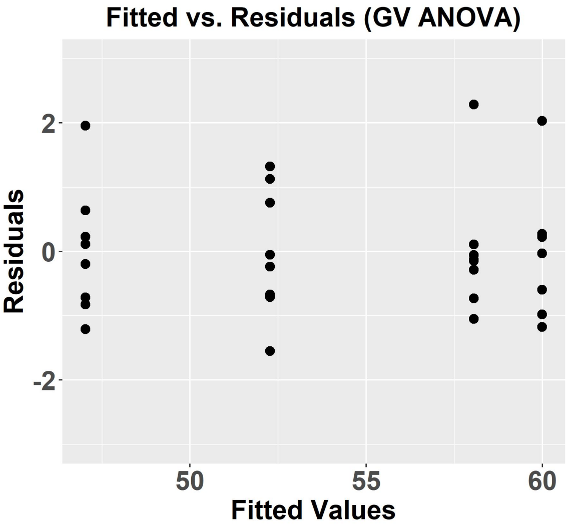
ANOVA Residuals (Studentized)



Pairwise Comparisons (GV ANOVA)

T1	T2	True.Diff	Est.	S.Err.	Adj.P.	Error
A	B	0	5.24	2.05	0.098	None
A	C	8	11.02	1.13	< 0.001	None
A	D	10	12.97	0.98	< 0.001	None
B	C	8	5.78	1.90	0.030	None
B	D	10	7.73	1.82	0.001	None
C	D	2	1.94	0.59	0.016	None

GV ANOVA Residuals (Studentized)



Conclusions

- ANOVA has great properties when assumptions satisfied
- Ignoring unequal variance and using ANOVA can give misleading inference
- How do I know if it is a problem in my data set? Explore graphics, variance ratios, hypothesis tests
- GV ANOVA accommodates unequal variance without transformations, interpret on original scale

Where to Start?

- Zuur, Alain, et al. *Mixed effects models and extensions in ecology with R*. Springer Science & Business Media, 2009.
  - Chapter 4
- Littell, Ramon C., et al. *SAS for mixed models*. SAS publishing, 2006.
  - Chapter 9

Acknowledgements

- Thank you to
  1. Everyone here
  2. Horticulture Department at Oregon State University
  3. Collaborators
  4. Special thanks to Alec Kowalewski and Clint Mattox